



GAUTENG PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

REMOTE LEARNING ACTIVITY BOOKLET

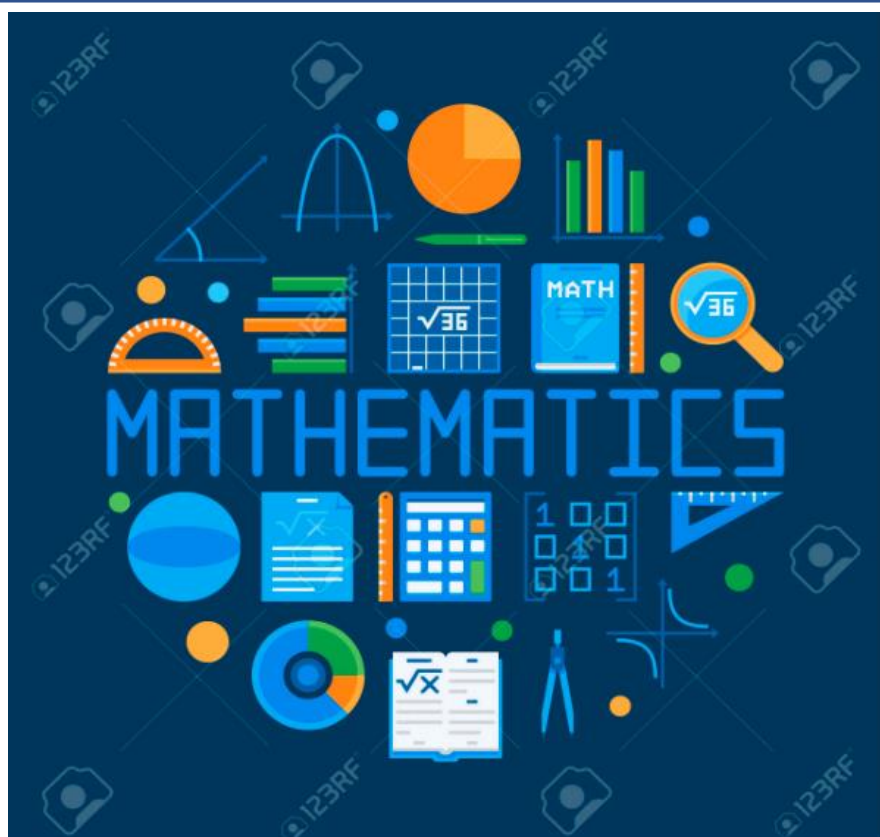
(RELAB)

SUBJECT: MATHEMATICS

GRADE: 11

TERM 1 – TERM 4

TEACHER



GGT2030
GROWING GAUTENG TOGETHER

A TABLE OF CONTENTS

		PAGE
1.	Introduction and purpose	3
2.	Term 1	4 - 63
3	Term 2	64 - 106
4	Term 3	107 - 147
5	Term 4	148 - 154

A. INTRODUCTION AND PURPOSE OF THE RELAB

The pandemic has forced schools to resort to the implementation of rotational timetables-where learners who are at home during normal schooling must continue learning. Hence RELAB as a strategy towards the deployment of remote learning.

The RELAB is underpinned by the following Legislative demands:

- a) Responding to GDE Strategic goal 2 promoting quality education across all classrooms and schools
- b) **DBE Circular S13 of 2020** the requires the GDE to support the implementation of the Recovery Annual Teaching Plan (RATP)
- c) **GDE Circular 11 of 2020** requiring districts to issue Learning Activity Packs to support schools for lockdown learning. Understanding learning constraints at home as majority of learners do not have access to devices or data to use for online learning. Many households are depending on schools to provide them with learning resources packs

RELAB is designed in a study guide format, where the content is briefly explained with related concepts as revision, in the form of e.g. notes, mind-maps, concept progression from the previous grade/s followed by exemplar exercises then practice exercises/problems . The exercises are pitched at different cognitive levels to expose learners at Grade 10 & 11 to these different cognitive levels of questioning. The NSC diagnostic reports in different subjects have revealed that learners fail to analyse questions and as a result fail to respond accordingly.

The RELAB is intended to ensure that learners work on exercises as per topics taught while at school. These exercises must be completed at home, fully and learners will receive feedback as groups or individually at school. It is therefore of paramount importance that teachers mark the work with learners in class, as a way of providing feedback. Educators must diagnose learner responses, remediate where necessary and plan further intervention.

Educators are encouraged to create WhatsApp groups to remind learners on what is expected of them in a particular week/ day(s). There shouldn't be a backlog on curriculum coverage as content will be covered simultaneously. Feedback from learners at home will confirm usage of the RELAB material.

RELAB further prepares learners for formal assessment.

B. TABLE OF CONTENTS

WEEK	TOPIC	SUBTOPIC	PAGE
1	Exponents and Surds	1. Simplify expressions and solve equations using the laws of exponents for rational exponents where $x^{\frac{p}{q}} = \sqrt[q]{x^p}; x > 0; q > 0$ 2. Add, subtract, multiply and divide simple surds. 3. Solve simple equations involving surds.	4 - 24
2	Equations and Inequalities	1. Complete the square 2. Solve Quadratic equations (by factorization and by using the quadratic formula) ; K- Method	
3		3. Solve Quadratic inequalities in one unknown (Interpret solutions graphically.) NB: It is recommended that the solving of equations in two unknowns is important to be used in other equations like hyperbola-straight line as this is normal in the case of graphs	
4		4. Equations in two unknowns, one of which is linear and the other quadratic 5. Nature of roots	
5	Euclidean Geometry	Teach past grades concepts required for smooth content progression into Gr 11 such as: Angles, lines, triangles, etc.	24 - 45
6		Accept results established in earlier grades as axioms and also that a tangent to a circle is perpendicular to the radius, drawn to the point of contact. Then investigate and prove the theorems of the geometry of circles: - The line drawn from the centre of a circle perpendicular to a chord bisects the chord. - The line drawn from the centre of the circle to the midpoint of the chord is perpendicular to the chord - The perpendicular bisector of a chord passes through the centre of the circle; - An angle subtend by the arc at the centre of the circle is two times an angle subtended by the same arc at the circumference of the circle	
7		- Angles subtended by a chord of the circle, on the same side of the chord, are equal; - The opposite angles of a cyclic quadrilateral are supplementary; - An exterior angle of a cyclic quad is equal to the interior opposite angle - Two tangents drawn to a circle from the same point outside the circle are equal in length;	
8		- The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment. - Use the above theorems and their converses, where they exist, to solve riders.	
9		1. Derive and use the identities: $\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\theta \neq k. 90^\circ, k$ an odd integer; and $\sin^2 \theta + \cos^2 \theta = 1$. 2. Derive and use reduction formulae to simplify the following expressions: 2.1. $\sin (90^\circ \pm \theta)$; $\cos (90^\circ \pm \theta)$; 2.2. $\sin (180^\circ \pm \theta)$; $\cos (180^\circ \pm \theta)$ and $\tan (180^\circ \pm \theta)$; 2.3. $\sin (360^\circ \pm \theta)$; $\cos (360^\circ \pm \theta)$ and $\tan (360^\circ \pm \theta)$;	46 - 63
10		2.4. $\sin (-\theta)$; $\cos (-\theta)$ and $\tan (-\theta)$; 3. Determine for which values of a variable an identity holds.	

WEEK 1 – 4: EXPONENTS, SURDS AND EQUATIONS

SUB TOPIC

1. Simplify expressions and solve equations using the laws of exponents for rational exponents

$$\text{where } x^{\frac{p}{q}} = \sqrt[q]{x^p}; x > 0; q > 0$$

2. Add, subtract, multiply and divide simple surds.

3. Solve simple equations involving surds.

QUADRATIC EQUATIONS, SIMULTANEOUS

1. Complete the square

2. Solve Quadratic equations (by factorization and by using the quadratic formula) ; K- Method

3. Solve Quadratic inequalities in one unknown (Interpret solutions graphically.)

NB: It is recommended that the solving of equations in two unknowns is important to be used in other equations like hyperbola-straight line as this is normal in the case of graphs

4. Equations in two unknowns, one of which is linear and the other quadratic

NATURE OF ROOTS

To deal with this topic teachers must ensure:

- ❖ Basic factorization involving fractions and surds.
- ❖ Basic algebra and factorization.
- ❖ Basic linear and quadratic equations.
- ❖ This topic is the fundamental of Mathematics because it can be integrated in all topics

- ❖ The topic is in Paper 1
- ❖ The topic accounts for 45 ± 3 at the end of the year Examination
- ❖ The topic is taught for 4 weeks
- ❖ The topic of Algebra is question 1 in grade 12 P1
- ❖ In grade 12 the topic accounts for 25 ± 3

ACTIVITY 1

Tick the correct columns in the table below:

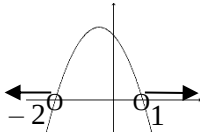
Number	Real ($\mathbb{R}$)	Non- Real ($\mathbb{R}'$)	Rational ($\mathbb{Q}$)	Irrational ($\mathbb{Q}'$)	Whole ($\mathbb{N}_0$)	Natural ($\mathbb{N}$)	Integer ($\mathbb{Z}$)
0	✓		✓		✓		✓
-9	✓		✓				✓
$\sqrt{3}$	✓			✓			
$\frac{13}{16}$	✓		✓				
-1,45	✓		✓				
$\sqrt{-27}$		✓					
$(\sqrt{-3})^2$	✓		✓				✓
2π	✓			✓			
$\sqrt[3]{-343}$	✓		✓				✓

ACTIVITY 2

QUESTION 1

1.1	$x = -5$	
1.2	$x = -11$ and $x = 0$	
1.3	$x = 15$	

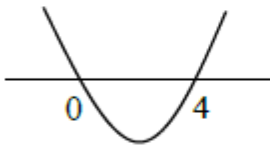
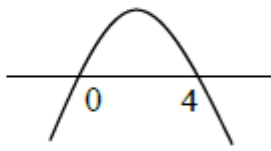
QUESTION 2

2.1.1	$x^2 - 6x = 0$ $x(x-6) = 0$ $x = 0 \text{ or } x = 6$	✓ common factor ✓ answers (2)
2.1.2	$x^2 + 10x + 8 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-10 \pm \sqrt{10^2 - 4(1)(8)}}{2(1)}$ $= \frac{-10 \pm \sqrt{68}}{2}$ $x = -0,88 \text{ or } x = -9,12$	✓ substitution into the correct formula ✓ - 0,88 ✓ -9,12 (3)
2.1.3	$(1-x)(x+2) < 0$ Critical values: $x = 1 \text{ or } x = -2$ $\leftarrow 0 \quad 0 \rightarrow$ $\quad - \quad + \quad -$ $\quad -2 \quad 1$ or  $x < -2 \text{ or } x > 1$	✓ critical values ✓✓ answer (3)
2.1.4	$\sqrt{x+18} = x-2$ $x+18 = x^2 - 4x + 4$ $0 = x^2 - 5x - 14$ $(x-7)(x+2) = 0$ $x = 7 \text{ or } x \neq -2$	✓ squaring both sides ✓ standard form ✓ factors ✓✓ answer with selection (5)
2.2	$x + y = 3$ $y = 3 - x \dots\dots\dots (1)$ $2x^2 + 4xy - y = 15 \dots\dots\dots (2)$ Substitute (1) into (2): $2x^2 + 4x(3-x) - (3-x) = 15$ $2x^2 + 12x - 4x^2 - 3 + x - 15 = 0$ $-2x^2 + 13x - 18 = 0$ $2x^2 - 13x + 18 = 0$ $(2x-9)(x-2) = 0$ $x = \frac{9}{2} \text{ or } x = 2$ $y = -\frac{3}{2} \text{ or } y = 1$	✓ y subject of the formula ✓ substitution ✓ standard form ✓ factors ✓ x-values ✓ y-values (6) OR

	OR $x + y = 3$ $x = 3 - y \dots\dots\dots (1)$ $2x^2 + 4xy - y = 15 \dots\dots\dots (2)$ Substitute (1) into (2): $2(3 - y)^2 + 4(3 - y)y - y = 15$ $2y^2 - 12y + 18 - 4y^2 + 12y - y - 15 = 0$ $-2y^2 - y + 3 = 0$ $2y^2 + y - 3 = 0$ $(2y + 3)(y - 1) = 0$ $y = -\frac{3}{2} \text{ or } y = 1$ $x = \frac{9}{2} \text{ or } x = 2$	✓ x subject of the formula ✓ substitution ✓ standard form ✓ factors ✓ y-values ✓ x-values (6)
2.3	$n^{200} < 5^{300}$ $(n^2)^{100} < (5^3)^{100}$ $(n^2)^{100} < (125)^{100}$ $n^2 < 125$ Maximum value of n is 11. OR $200 \log n < 300 \log 5$ $n < 10^{\frac{3}{2} \log 5}$ $n < 11,18$ $\therefore n = 11$ OR $n^{200} < 5^{300}$ $n < 5^{\frac{300}{200}}$ $n < \sqrt{125}$ $n = 11$	✓ $(n^2)^{100} < (5^3)^{100}$ ✓ 125 ✓ 11 (3) OR ✓ use of logs ✓ $n < 11,18$ ✓ 11 (3) OR ✓ $n < 5^{\frac{300}{200}}$ ✓ $n < \sqrt{125}$ ✓ $n = 11$ (3)
		[22]

QUESTION 3

3.1.1	$x^2 - 5x - 6 = 0$ $(x - 6)(x + 1) = 0$ $x = 6 \text{ or } x = -1$ OR/OF $x^2 - 5x - 6 = 0$ $x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(-6)}}{2(1)}$ $x = \frac{5 \pm \sqrt{49}}{2}$ $x = 6 \text{ or } x = -1$	
3.1.2	$(3x - 1)(x - 4) = 16$ $3x^2 - 13x - 12 = 0$ $x = \frac{13 \pm \sqrt{(-13)^2 - 4(3)(-12)}}{2(3)}$ $x = \frac{13 \pm \sqrt{313}}{6}$ $x = 5,12 \text{ or } x = -0,78$ OR/OF $3x^2 - 13x - 12 = 0$ $x^2 - \frac{13}{3}x = 4$ $x^2 - \frac{13}{3}x + \left(-\frac{13}{6}\right)^2 = 4 + \left(-\frac{13}{6}\right)^2$ $\left(x - \frac{13}{6}\right)^2 = \frac{313}{36}$ $x = \frac{13 \pm \sqrt{313}}{6}$ $x = 5,12 \text{ or } x = -0,78$	

3.1.3	$x(4-x) \geq 0$ $-x(x-4) \geq 0 \quad \text{or} \quad -x(x-4) \geq 0$ $x(x-4) \leq 0$   $0 \leq x \leq 4 \quad \text{or} \quad x \in [0 ; 4]$	
3.1.4	$\frac{5^{2x} - 1}{5^x + 1} = 4$ $\frac{(5^x + 1)(5^x - 1)}{5^x + 1} = 4$ $5^x - 1 = 4$ $5^x = 5$ $x = 1$ OR/OF $\frac{5^{2x} - 1}{5^x + 1} = 4$ $5^{2x} - 1 = 4 \cdot 5^x + 4$ $5^{2x} - 4 \cdot 5^x - 5 = 0$ $(5^x - 5)(5^x + 1) = 0$ $5^x = 5 \quad \text{or} \quad 5^x \neq -1$ $x = 1$	

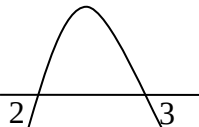
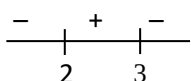
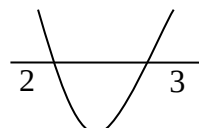
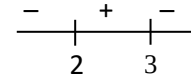
3.2	OR $x = 2 - 3y \dots\dots\dots(1)$ $x^2 + 4xy - 5 = 0 \dots\dots\dots(2)$ Substitute (1) in (2): $(2 - 3y)^2 + 4y(2 - 3y) - 5 = 0$ $4 - 12y + 9y^2 + 8y - 12y^2 - 5 = 0$ $-3y^2 - 4y - 1 = 0$ $3y^2 + 4y + 1 = 0$ $(3y + 1)(y + 1) = 0$ $y = -\frac{1}{3} \text{ or } y = -1$ $x = 3 \text{ or } x = 5$ OR $y = \frac{2}{3} - \frac{x}{3} \dots\dots\dots(1)$ $x^2 + 4xy - 5 = 0 \dots\dots\dots(2)$ Substitute (1) in (2): $x^2 + 4x\left(\frac{2}{3} - \frac{x}{3}\right) - 5 = 0$ $3x^2 + 8x - 4x^2 - 15 = 0$ $-x^2 + 8x - 15 = 0$ $x^2 - 8x + 15 = 0$ $(x - 5)(x - 3) = 0$ $x = 3 \text{ or } x = 5$ $y = -\frac{1}{3} \text{ or } y = -1$	
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3.3	$ab = 2\sqrt{10}$ $bc = 3\sqrt{2}$ $ac = 6\sqrt{5}$ $ab.bc.ac = 2\sqrt{10}.6\sqrt{5}.3\sqrt{2}$ $(abc)^2 = 36\sqrt{100}$ $abc = \sqrt{360} = 6\sqrt{10}$ OR/OF $ac = 6\sqrt{5} \quad \therefore a = \frac{6\sqrt{5}}{c}$ $bc = 3\sqrt{2} \quad \therefore b = \frac{3\sqrt{2}}{c}$ $ab = 2\sqrt{10}$ $\left(\frac{6\sqrt{5}}{c}\right)\left(\frac{3\sqrt{2}}{c}\right) = 2\sqrt{10}$ $18\sqrt{10} = 2\sqrt{10}.c^2$ $c^2 = 9$ $c = 3$ $\text{Volume} = abc = 2\sqrt{10}.3 = \sqrt{360} = 6\sqrt{10}$	

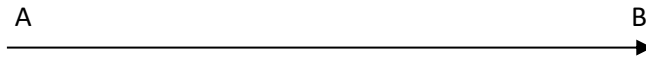
QUESTION 4

4.1.1	$x(x-1) = 0$ $x = 0 \quad \text{or} \quad x = 1$	$\checkmark x = 0$ $\checkmark x = 1$ (2)
4.1.2	$2x^2 - 4x - 5 = 0$ $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-5)}}{2(2)}$ $= \frac{4 \pm \sqrt{56}}{4}$ $x = -0,87 \quad \text{or} \quad x = 2,87$ OR/OF	$\checkmark$ correct substitution into correct formula $\checkmark\checkmark$ answers (3)

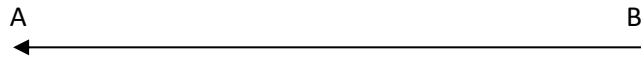
	$x^2 - 2x - \frac{5}{2} = 0$ $(x-1)^2 = \frac{5}{2} + 1$ $x-1 = \pm \sqrt{\frac{7}{2}}$ $\therefore x = 1 \pm \sqrt{\frac{7}{2}}$ $x = -0,87 \text{ or } x = 2,87$	✓ completing the square/ <i>voltooiing van die vierkant</i> ✓✓ answers (3)
4.1.3	$5^x = \frac{1}{125}$ $5^x = 5^{-3}$ $x = -3$ OR/OF $5^x = \frac{1}{125}$ $\left(\frac{1}{5}\right)^{-x} = \left(\frac{1}{5}\right)^3$ $-x = 3$ $x = -3$ OR / OF $5^x = \frac{1}{125}$ $x = \log_5\left(\frac{1}{125}\right)$ $= -3$ OR / OF $5^x = \frac{1}{125}$ $5^x \cdot 125 = 1$ $5^x \cdot 5^3 = 1$ $5^{x+3} = 5^0$ $x+3 = 0$ $x = -3$	✓ 5^{-3} ✓ answer (2) ✓ $\left(\frac{1}{5}\right)^{-x} = \left(\frac{1}{5}\right)^3$ ✓ answer (2) ✓ use of logs ✓ answer (2) ✓ 5^3 ✓ answer (2)

4.1.4	$(x-3)(2-x) > 0$  OR/OR  $2 < x < 3$ OR/OR $(x-3)(2-x) > 0$ $(x-3)(x-2) < 0$  OR/OR  $2 < x < 3$	✓ critical values ✓ solves an equality ✓ answer (3)
4.2.1	$x = 3$	✓ answer (1)
4.2.2	$x+1 = \frac{-4}{x-3}$ $(x+1)(x-3) = -4$ $(x+1)(x-3) + 4 = 0$ $x^2 - 2x + 1 = 0$ $(x-1)^2 = 0$ $x = 1$	✓ $(x+1)(x-3) = -4$ ✓ standard form ✓ factors ✓ answer (4)

4.3



$$\begin{aligned}\text{Speed/Spoed} &= y \text{ km/h} \\ \text{Distance/Afstand} &= x \text{ km} \\ \text{Time/Tyd} &= \frac{x}{y}\end{aligned}$$



$$\begin{aligned}\text{Speed/Spoed} &= \frac{3y}{2} \text{ km/h} \\ \text{Distance/Afstand} &= x \text{ km} \\ \text{Time/Tyd} &= \frac{x}{\frac{3y}{2}} = \frac{2x}{3y}\end{aligned}$$

OR/OF

	S	D	T
A to/na B	y	x	$\frac{x}{y}$
B to/na A	$\frac{3y}{2}$	x	$\frac{2x}{3y}$

Average speed travelled/Gemiddelde spoed afgelê:

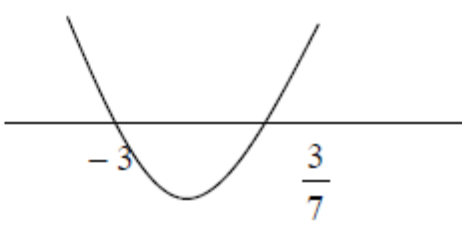
$$\begin{aligned}\frac{\text{Total distance travelled/Totale afstand gereis}}{\text{Total time taken/Totale tyd geneem}} &= \frac{2x}{\frac{x}{y} + \frac{2x}{3y}} \\ &= \frac{2x}{\frac{3x + 2x}{3y}} \\ &= \frac{6xy}{5x} \\ &= \frac{6y}{5} \text{ km/h}\end{aligned}$$

✓ time from A
to B is $\frac{x}{y}$ ✓ time from B
to A is $\frac{2x}{3y}$

$$\begin{aligned}&\checkmark \quad 2x \\ &\checkmark \quad \frac{x}{y} + \frac{2x}{3y} \\ &\checkmark \quad \frac{6xy}{5x} \\ &\checkmark \quad \frac{6y}{5} \text{ km/h}\end{aligned}$$

(6)
[23]

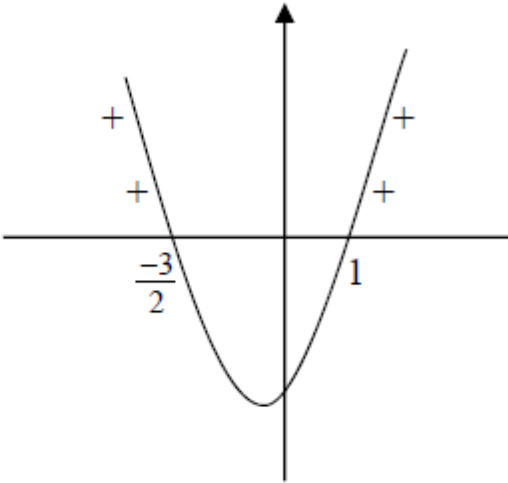
QUESTION 5

5.1.1	$x^2 - x = 12$ $x^2 - x - 12 = 0$ $(x - 4)(x + 3) = 0$ $x = 4$ or $x = -3$ OF $x(x - 1) = 12$ $4(3) = 12$ $(-3)(-4) = 12$ Deur inspeksie $x = 4$ or $x = -3$	
5.1.2	$2x^2 + 3x - 7 = 0$ $x = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-7)}}{2(2)}$ $= \frac{-3 \pm \sqrt{65}}{4}$ $x = 1,27$ of $x = -2,77$	
5.1.3	$7x^2 + 18x - 9 > 0$ $(7x - 3)(x + 3) > 0$ $\begin{array}{ccccccc} + & 0 & - & 0 & + \\ \hline & -3 & & \frac{3}{7} & \end{array}$  $x < -3$ of $x > \frac{3}{7}$ OF $x \in (-\infty; -3) \cup \left(\frac{3}{7}; \infty\right)$	

5.2	$2x - y = 7$ $y = 2x - 7$ Vervang $y = 2x - 7$ in $x^2 + xy = 21 - y^2$ $x^2 + x(2x - 7) = 21 - (2x - 7)^2$ $x^2 + 2x^2 - 7x = 21 - 4x^2 + 28x - 49$ $7x^2 - 35x + 28 = 0$ $x^2 - 5x + 4 = 0$ $(x - 4)(x - 1) = 0$ $x = 4 \text{ of } x = 1$ $y = 1 \text{ of } y = -5$ OR $2x - y = 7$ $x = \frac{7 + y}{2}$ Vervang $x = \frac{7 + y}{2}$ in $x^2 + xy = 21 - y^2$ $\left(\frac{7 + y}{2}\right)^2 + \left(\frac{7 + y}{2}\right)y = 21 - y^2$ $\frac{49 + 14y + y^2}{4} + \frac{7y + y^2}{2} = 21 - y^2$ $49 + 14y + y^2 + 2(7y + y^2) = 84 - 4y^2$ $49 + 14y + y^2 + 14y + 2y^2 = 84 - 4y^2$ $7y^2 + 28y - 35 = 0$ $y^2 + 4y - 5 = 0$ $(y + 5)(y - 1) = 0$ $y = -5 \text{ of } y = 1$ $x = 1 \quad x = 4$	
5.3	$\left(\sqrt[5]{\sqrt{35} + \sqrt{3}}\right)\left(\sqrt[5]{\sqrt{35} - \sqrt{3}}\right)$ $= \sqrt[5]{(\sqrt{35} + \sqrt{3})(\sqrt{35} - \sqrt{3})}$ $= \sqrt[5]{35 - 3}$ $= \sqrt[5]{32}$ $= 2$	

QUESTION 6

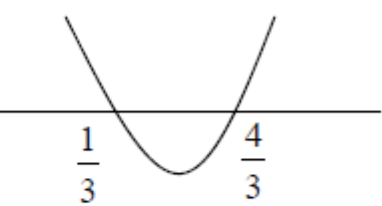
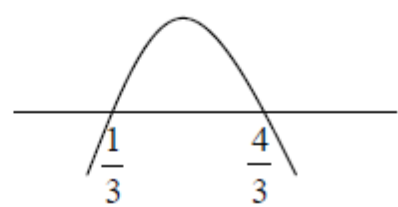
6.1.1	$x^2 = 5x - 4$ $x^2 - 5x + 4 = 0$ $(x - 4)(x - 1) = 0$ $x = 4 \text{ or } x = 1$	
6.1.2	$x(3 - x) = -3$ $3x - x^2 = -3$ $x^2 - 3x - 3 = 0$ $x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-3)}}{2(1)}$ $x = \frac{3 \pm \sqrt{21}}{2}$ $x = 3,79 \text{ or } x = -0,79$ OR $x(3 - x) = -3$ $3x - x^2 = -3$ $-x^2 + 3x + 3 = 0$ $x = \frac{-3 \pm \sqrt{(3)^2 - 4(-1)(3)}}{2(-1)}$ $x = \frac{-3 \pm \sqrt{21}}{-2}$ $x = 3,79 \text{ or } x = -0,79$	

6.1.3	$3 - x < 2x^2$ $-2x^2 - x + 3 < 0$ $2x^2 + x - 3 > 0$ $(2x + 3)(x - 1) > 0$ $x < -\frac{3}{2} \text{ or } x > 1$ OR $x \in (-\infty; -\frac{3}{2}) \cup (1; \infty)$  OR $3 - x < 2x^2$ $0 < 2x^2 + x - 3$ $0 < (2x + 3)(x - 1)$ $x < -\frac{3}{2} \text{ or } x > 1$	
6.2	$y = 3 - 2x$ $x^2 + (3 - 2x) + x = (3 - 2x)^2$ $x^2 + 3 - 2x + x = 9 - 12x + 4x^2$ $3x^2 - 11x + 6 = 0$ $(3x - 2)(x - 3) = 0$ $x = \frac{2}{3} \quad \text{or} \quad x = 3$ $\therefore y = \frac{5}{3} \quad \quad \quad \therefore y = -3$	

6.3	$\frac{x^2 - 4}{x - 2} = \frac{(x + 2)(x - 2)}{(x - 2)} = x + 2$ Therefore when $x = 999\,999\,999\,999$, the value is $999\,999\,999\,999 + 2 = 1\,000\,000\,000\,001$. OR $\frac{x^2 - 4}{x - 2} = \frac{(x + 2)(x - 2)}{(x - 2)} = x + 2$ $999\,999\,999\,999 = 10^{12} - 1$ $x + 2 = 999\,999\,999\,999 + 2$ $= 10^{12} + 1$	
6.4	$\frac{x^4 + 1}{x^4} = 1 + \frac{1}{x^4} > 1 \text{ since } \frac{1}{x^4} > 0$ $\therefore \frac{x^4 + 1}{x^4}$ can never be equal to $\frac{1}{2}$ OR $2x^4 + 2 = x^4$ $\frac{1}{x^4} = -\frac{1}{2}$ Which has no real solution since $\frac{1}{x^4} > 0$ for all $x \in \mathbb{R} - \{0\}$	

QUESTION 7

7.1.1	$x(x-1) = 30$ $x^2 - x = 30$ $x^2 - x - 30 = 0$ $(x-6)(x+5) = 0$ $x = 6 \text{ or } x = -5$ OR $x(x-1) = 30$ $x^2 - x = 30$ $x^2 - x - 30 = 0$ $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-30)}}{2(1)}$ $= \frac{1 \pm \sqrt{121}}{2}$ $= \frac{1 \pm 11}{2}$ $x = 6 \text{ or } x = -5$	
7.1.2	$3x^2 - 5x + 1 = 0$ $a = 3 \quad b = -5 \quad c = 1$ $x = \frac{-(-5) \pm \sqrt{25 - 4(3)(1)}}{2(3)}$ $= \frac{5 \pm \sqrt{13}}{6}$ $x = 1,4 \quad \text{or} \quad x = 0,2$ OR	

	$3x^2 - 5x + 1 = 0$ $x^2 - \frac{5}{3}x = -\frac{1}{3}$ $x^2 - \frac{5}{3}x + \frac{25}{36} = -\frac{1}{3} + \frac{25}{36}$ $\left(x - \frac{5}{6}\right)^2 = \frac{13}{36}$ $x - \frac{5}{6} = \frac{\pm\sqrt{13}}{6}$ $x = \frac{5 \pm \sqrt{13}}{6}$ $x = 1,4 \quad \text{or} \quad x = 0,2$	
7.1.3	$-9x^2 + 15x - 4 < 0$ $9x^2 - 15x + 4 > 0$ $(3x - 4)(3x - 1) > 0$ $\leftarrow \begin{array}{c} + \\ 0 \\ \frac{1}{3} \end{array} \quad - \quad \begin{array}{c} 0 \\ \frac{4}{3} \end{array} \quad + \rightarrow$ OR  $x < \frac{1}{3} \quad \text{or} \quad x > \frac{4}{3}$ Answer can be given as: $x \in \left(-\infty; \frac{1}{3}\right) \cup \left(\frac{4}{3}; \infty\right)$ OR $-9x^2 + 15x - 4 < 0$ $(-3x + 4)(3x - 1) < 0$ $x < \frac{1}{3} \quad \text{or} \quad x > \frac{4}{3}$ 	

7.2	Substitute $x = y + 3$ in $x^2 - xy - 2y^2 - 7$: $(y + 3)^2 - y(y + 3) - 2y^2 - 7 = 0$ $y^2 + 6y + 9 - y^2 - 3y - 2y^2 - 7 = 0$ $2y^2 - 3y - 2 = 0$ $(2y + 1)(y - 2) = 0$ $y = -\frac{1}{2} \text{ or } y = 2$ $x = 2\frac{1}{2} \text{ or } x = 5$ OR $y = x - 3$ $x^2 - x(x - 3) - 2(x - 3)^2 - 7 = 0$ $x^2 - x^2 + 3x - 2(x^2 - 6x + 9) - 7 = 0$ $0 = 2x^2 - 15x + 25$ $0 = (2x - 5)(x - 5)$ $x = 2\frac{1}{2} \text{ or } x = 5$ $y = -\frac{1}{2} \text{ or } y = 2$	
7.3	$\frac{10^{1004}\sqrt{10}}{10^{1005}\sqrt{10} - 10^{1003}\sqrt{10}}$ $= \frac{10^{1004}\sqrt{10}}{\sqrt{10}(10^{1005} - 10^{1003})}$ $= \frac{10^{1004}}{10^{1003}(100 - 1)}$ $= \frac{10}{99}$	

7.4	$\begin{aligned} & \left(1 + \sqrt{2x^2}\right)^2 - \sqrt{8x^2} \\ &= 1 + 2\sqrt{2x^2} + 2x^2 - \sqrt{4 \cdot \sqrt{2x^2}} \\ &= 1 + 2\sqrt{2x^2} + 2x^2 - 2\sqrt{2x^2} \\ &= 1 + 2x^2 \end{aligned}$ OR $\begin{aligned} & \left(1 + \sqrt{2x^2}\right)^2 - \sqrt{8x^2} \\ &= 1 + \sqrt{8x^2} + 2x^2 - \sqrt{8x^2} \\ &= 1 + 2\sqrt{2x^2} + 2x^2 - 2\sqrt{2x^2} \\ &= 1 + 2x^2 \end{aligned}$	
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WEEK 5 – 8 : TOPIC: EUCLIDEAN GEOMETRY

SUB-TOPIC: Circle Geometry

To deal with this topic teachers must ensure:

- ❖ Basic Euclidean Geometry of lines is revised involving fractions and surds.
- ❖ Basic triangle properties is revised
- ❖ Basic terminology on Euclidean Geometry is understood .

The topic is in Paper 2

The topic is used again in Analytical Geometry

The topic accounts for 50 ± 3 Marks in the examination at the end of the year

The following proofs of theorems are examinable (NB. know the proofs by heart)

- ✓ The line drawn from the centre of a circle perpendicular to a chord bisects the chord;
- ✓ The line drawn from the centre of the circle to the midpoint of the chord is perpendicular to the chord
- ✓ The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre);
- ✓ The opposite angles of a cyclic quadrilateral are supplementary;
- ✓ The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment;

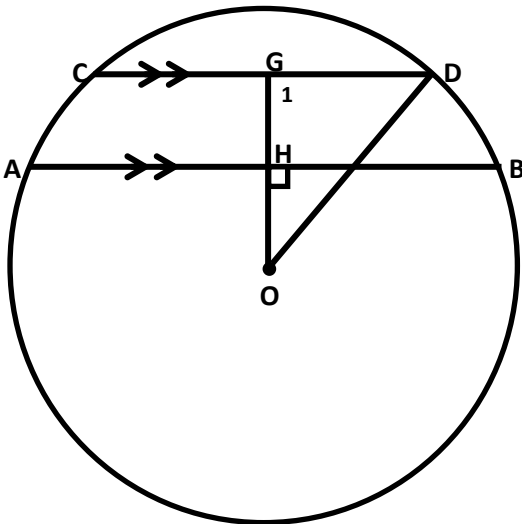
NB : The theorems are applicable even in grade 12.

The theorems are part of Theory allocated in Paper 2

SOLUTIONS

GRADE 11: REASONS WORKSHEET

1. Complete the statement so that it is TRUE:
The line drawn from the centre of the circle to the midpoint of the chord is
Perpendicular to the chord
2. In the diagram, O is the centre of the circle with $AB \parallel CD$ and $OH \perp AB$,
 $AB = 24$ cm, $CD = 10$ cm and $OD = 13$ cm.



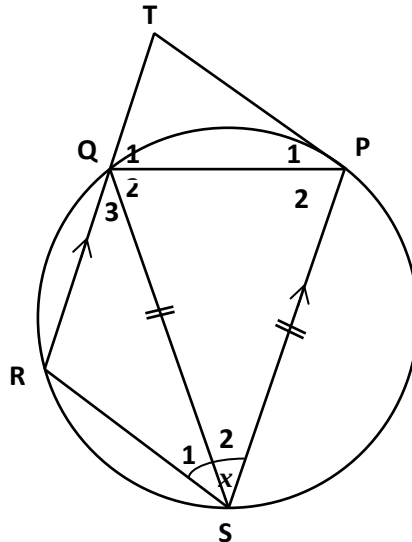
Give reasons for the statements below:

STATEMENT	REASON
2.1 $\hat{G}_1 = 90^\circ$	corresp.angles, $CD \parallel AB$
2.2 $AH = 12$	line from centre $\perp$ to chord
2.3 $OB = 13$	radii of the same circle

3. Complete the statement so that it is TRUE

The angle which an arc subtends at the **centre** of a circle is twice the angle it subtends at the circumference of the circle.

4. In the diagram alongside, $RT \parallel SP$, $SQ = SP$ and TP is a tangent to the circle at P . $\widehat{PSR} = x$. P, Q, R and S are points on the circumference of the circle.



Give reasons for the statements below:

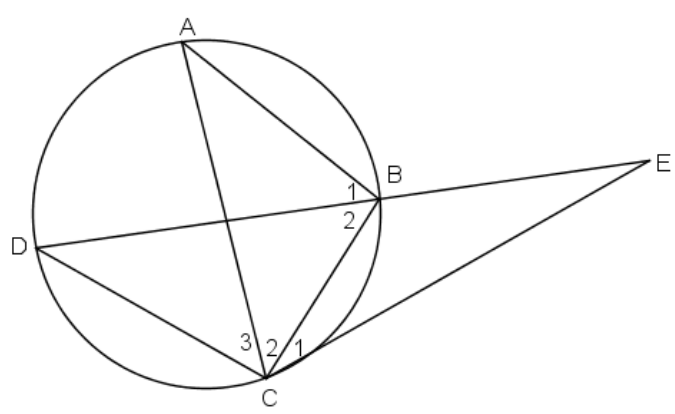
STATEMENT	REASON
4.1 $\hat{Q}_3 = \hat{S}_2$	alt angles, $RQ \parallel PS$
4.2 $\hat{Q}_2 = \hat{P}_2$	angles opp equal sides
4.3 $\hat{R} + \hat{P}_2 = 180^\circ$	opp angles of a cyc quad
4.4 $\hat{Q}_1 = x$	ext angle of a cyc quad

5. Complete the following so that the statements are TRUE:

5.1 The angle between a chord and a tangent at a point of contact **is equal to the angle in the alternate segment**

5.2 The opposite angles of a **cyclic** quadrilateral are supplementary.

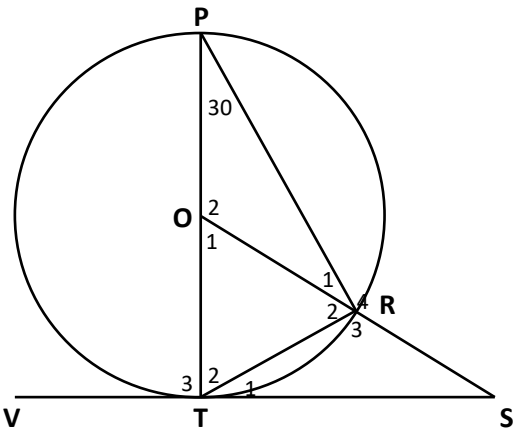
6. In the diagram below circle ABCD is given. CE is a tangent to circle ABCD at C. BD is the diameter to circle ABCD.



Give reasons for the statements below:

STATEMENT	REASONS
6.1 $\hat{A} = \hat{D}$	angles in the same seg
6.2 $\hat{C}_1 = \hat{D}$	tan chord theorem
6.3 $\hat{C}_3 = \hat{B}_1$	angles in the same seg
6.4 $\hat{C}_2 + \hat{C}_3 = 90^\circ$	angle in semi-circle OR Diameter subtend right angle OR angle in the $\frac{1}{2}$ 

7. In the diagram below, PT is a diameter of the circle with centre O and $\hat{TPR} = 30^\circ$. VS is a tangent to the circle at T and meets OR produced at S.

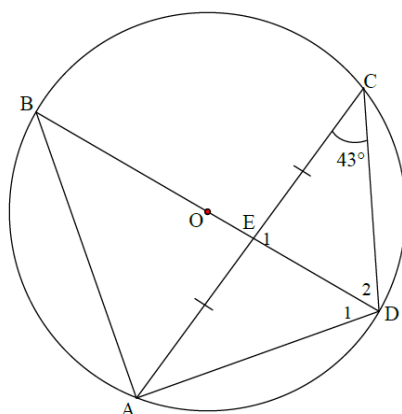


Give reasons for the statements below:

STATEMENT	REASON
7.1 $\hat{P} = \hat{R}_1$	angle opp equal sides
7.2 $\hat{T}_2 = \hat{R}_2$	angle opp equal sides
7.3 $\hat{T}_1 + \hat{T}_2 = 90^\circ$	tan⊥ rad OR tan⊥ diameter
7.4 $\hat{T}_1 = \hat{P}$	tan chord theorem

8. In the diagram, O is the centre of the circle. A, B, C and D are points on the circumference of the circle. Diameter BD bisects chord AC at E. Chords AB, CD and AD are drawn.

$$\hat{C} = 43^\circ$$

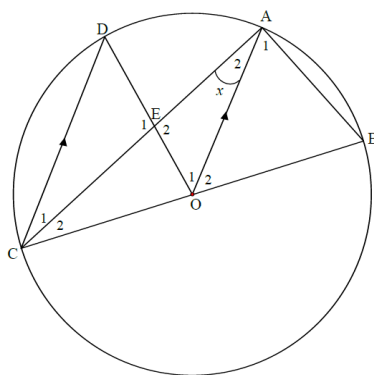


Give reasons for the statements below

STATEMENTS	REASONS
8.1 $DE \perp AC$	line from centre to midpt of chord
8.2 $\hat{B} = 43^\circ$	angles in the same seg

9. In the diagram, O is the centre of the circle. A, B, C and D are points on the circumference of the circle and CB is the diameter of the circle. Chord CA intersect radius OD at E.

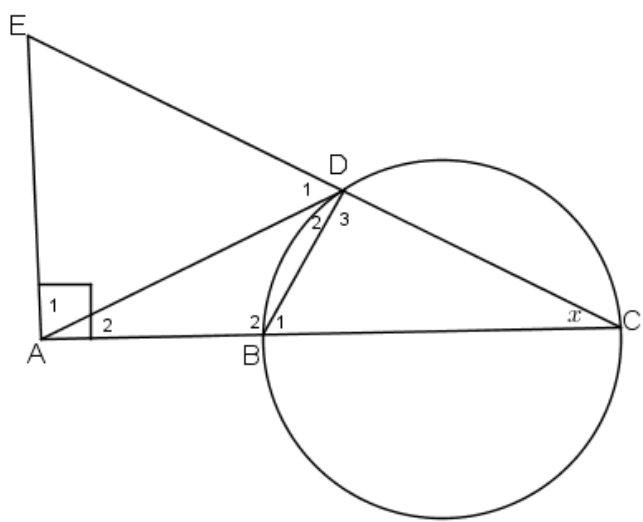
AB is drawn. $CD \parallel OA$ and $\hat{A}_2 = x$.



Give reasons for the statements below

STATEMENTS	REASONS
9.1 $\hat{C}_1 = x$	alt angles, $CD \parallel AO$
9.2 $\hat{C}_2 = x$	angle opp equal sides

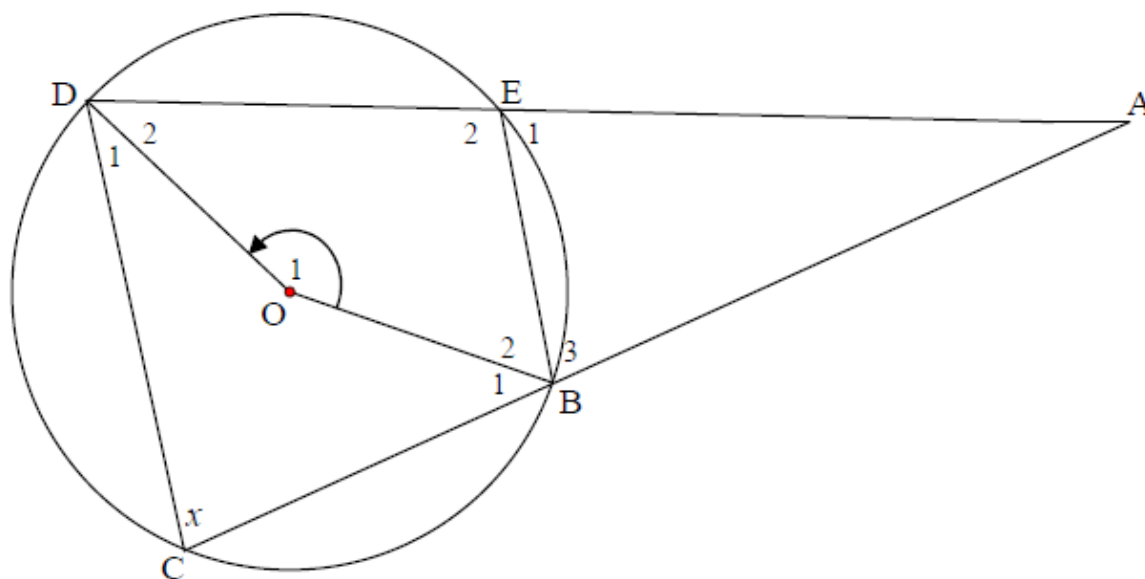
10. In the diagram, BC is a diameter of the circle. The tangent at point D on the circle meets CB produced at A. CD is produced to E such that $EA \perp AC$. Let $\hat{C} = x$



Give reasons for the statements below

STATEMENTS	REASONS
10.1 $\hat{D}_3 = 90^\circ$	angle in semi-circle OR Diameter subtend right angle OR angle in the $\frac{1}{2}$ 
10.2 ABDE is a cyclic quadrilateral	Ext angle = to inter opp angle OR converse ext angle of a cyc quad
10.3 $\hat{D}_2 = x$	tan chord theorem

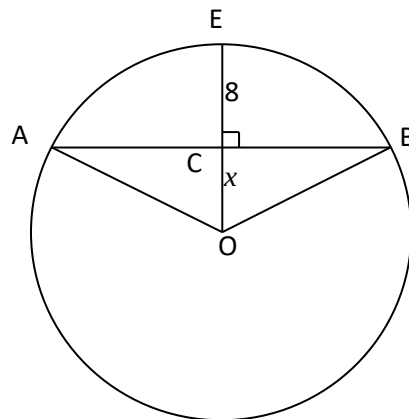
11. In the diagram O is the centre of the circle and BO and OD are drawn. Chords CB and DE are produced to meet in A. Chords BE and CD are drawn. $\hat{BCD} = x$.



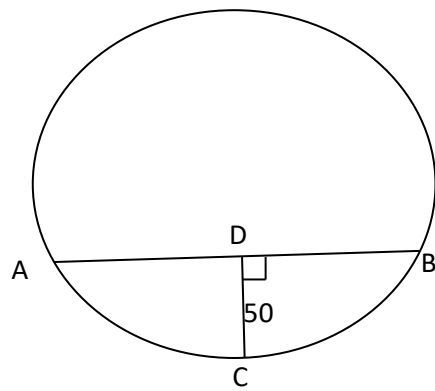
Give reasons for the statements below:

STATEMENTS	REASONS
11.1 $\hat{E}_1 = x$	ext angle of a cyc quad
11.2 $\hat{O}_1 = 2x$	angle at centre = 2 x angle at circumference.

EXERSICE 2
QUESTION 1



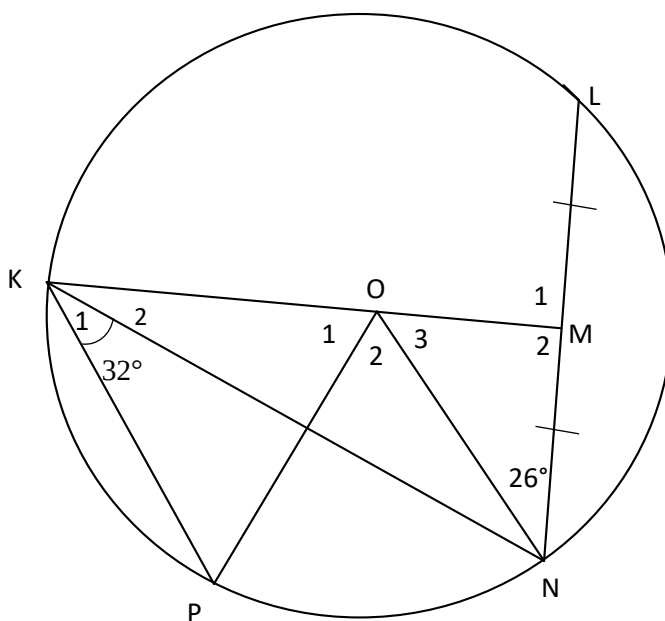
1.1	$OE = x + 8$ $OE = OB$ $AC = CB = 12cm$ line from centre perp to chord $OB^2 = OC^2 + CB^2$ $(x + 8)^2 = x^2 + 12^2$ $x^2 + 16x + 64 = x^2 + 144$ $16x = 80$ $x = 5$	
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1.2	Construct: Centre O and the radius and the line drawn from the centre perpendicular to chord AB Let $OD = x$ $AD = DB = 150mm$ $(OB)^2 = (OD)^2 + (DB)^2$ $(x + 50)^2 = x^2 + 150^2$ $x^2 + 100x + 2500 = x^2 + 22500$ $100x = 20000$ $x = 200$ $\therefore \text{radius} = 250mm$	

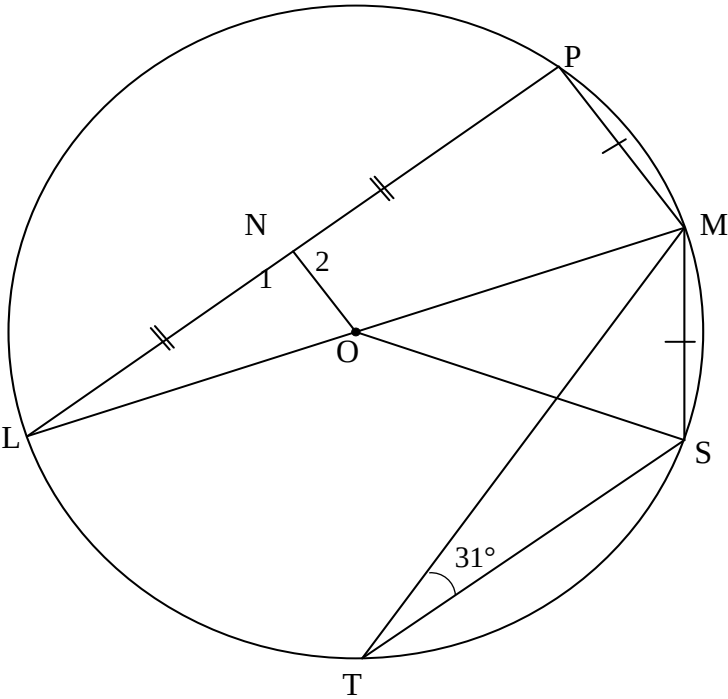
QUESTION 2

O is the centre of the circle.. KOM bisects chord LN and $\widehat{MNO} = 26^\circ$. K and P are points on the circle with $\widehat{NKP} = 32^\circ$. OP is drawn.



2.1.1	$\widehat{O}_2 = 2\widehat{K}_1$ angle at centre is 2 angle at circum. $\widehat{O}_2 = 64^\circ$	
2.1.2	$\widehat{M}_2 = 90^\circ$ line drawn from centre to midpoint of chord is perp to chord $\therefore \widehat{O}_3 = 64^\circ$ sum of angle of Δ $\therefore \widehat{O}_1 = 52^\circ$ angles on the straight line	
2.2	$\widehat{P} = \widehat{K}_1 + \widehat{K}_2 = 64^\circ$ angle opp. Equal sides $\therefore \widehat{K}_1 = \widehat{K}_2 = 32^\circ$ $\therefore$ KN bisects $\widehat{OKP}$.	

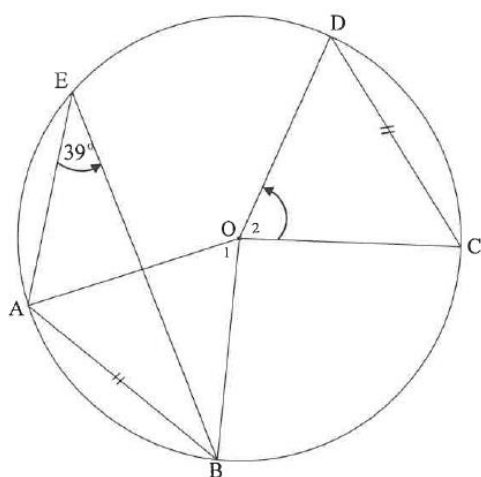
QUESTION 3



3.1.1	$M\hat{O}S = 2\hat{T} = 62^\circ$ angle at centre is twice angle at circum.	
3.1.2	$\hat{L} = \hat{T} = 31^\circ$ Equal chords equal angles	
3.2	$\hat{O}$ is the midpoint of LM $\therefore OL = OM$ $ON = \frac{1}{2}PM$ Midpoint theorem But $PM = MS$ $\therefore ON = \frac{1}{2}MS$	

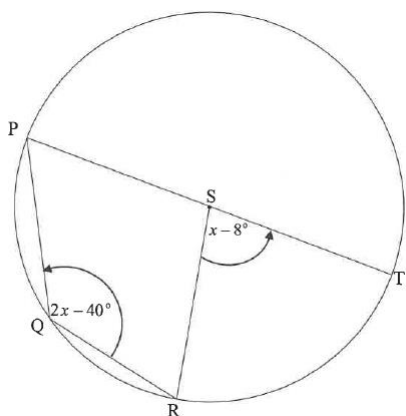
QUESTION 4

- 4.1 In the figure, O is the centre of the circle. A, B, C, D and E lie on the circle such that chord AB and chord DC are equal in length and $\angle AEB = 39^\circ$



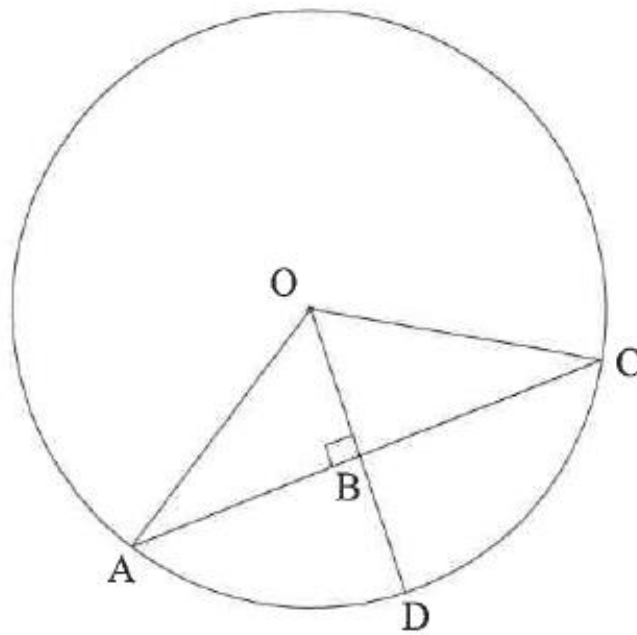
4.1.1	$\hat{O}_1 = 78^\circ$ angle at centre = 2 angle at circum.	
4.1.2	$\hat{O}_2 = 78^\circ$ equal chords, equal angles	

4.2



$$\begin{aligned}
 x - 8^\circ + 180^\circ &= 2(2x - 40^\circ) \quad \left[\begin{array}{l} \text{angle at centre} = 2 \times \angle \text{ at circumference} / \\ \text{middelpuntshoek} = 2 \times \text{omtrekshoek} \end{array} \right. \\
 4x - 80^\circ &= 172^\circ + x \\
 3x &= 252^\circ \\
 x &= 84^\circ
 \end{aligned}$$

4.3



$$AB = BC = 4x \quad \left[\begin{array}{l} \text{line from centre } \perp \text{ to chord /} \\ \text{lyn van middelpunt } \perp \text{ aan koord} \end{array} \right]$$

$$OA^2 = (4x)^2 + (2x)^2 \quad [\text{Pythagoras}]$$

$$OA = \sqrt{16x^2 + 4x^2}$$

$$= \sqrt{20x^2}$$

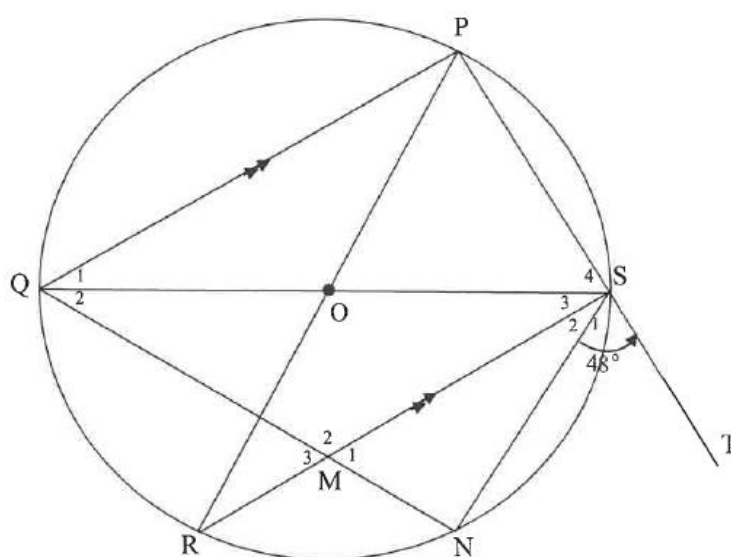
$$= 2\sqrt{5}x$$

$$OD = OA = 2\sqrt{5}x \quad [\text{radii}]$$

$$BD = 2\sqrt{5}x - 2x$$

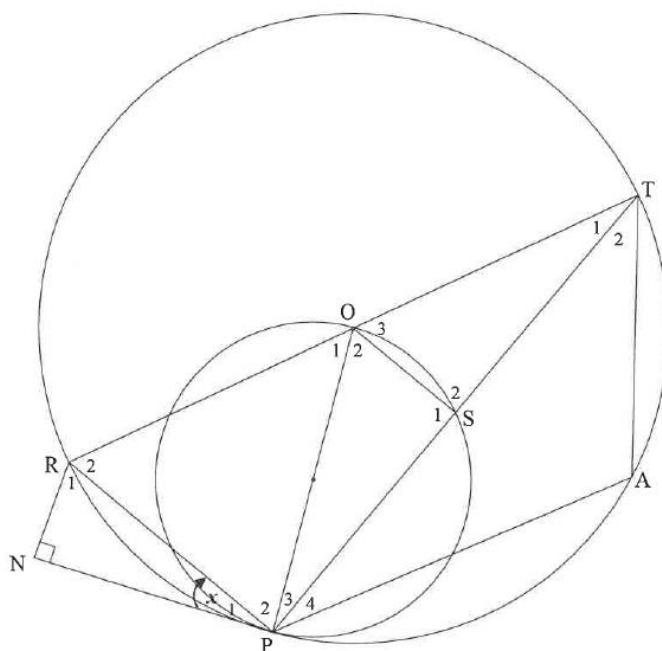
$$= 2x(\sqrt{5} - 1)$$

QUESTION 5



5.1.1	$\hat{Q} = \hat{S}_1 = 48^\circ$ [ext $\angle$ of cyclic quad/ buite $\angle$ van 'n koodervierhoek] $\hat{Q}_1 = \hat{Q}_2 = 24^\circ$ [QS bisects/ halveer $P\hat{Q}N$]	
5.1.2	$\hat{R} = \hat{Q}_1 = 24^\circ$ [$\angle^s$ in the same segment/ in dieselfde segment]	
5.1.3	$\hat{M}_1 = \hat{Q} = 48^\circ$ [corresp/ ooreenkomst $\angle^s$, $PQ \parallel SR$] OR/OF $\hat{S}_3 = \hat{Q}_1 = 24^\circ$ [alt $\angle^s$ / ooreenkomst $\angle^s$, $PQ \parallel SR$] $\hat{M}_1 = 48^\circ$ [ext $\angle$ of Δ / buite $\angle$ van Δ]	
5.2	$\hat{M}_1 = \hat{S}_1 = 48^\circ$ $\therefore ST$ is a tangent to circle MNS. [converse tan – chord theorem] $\therefore ST$ is 'n raaklyn aan MNS [omgekrd raaklyn-koord st.]	

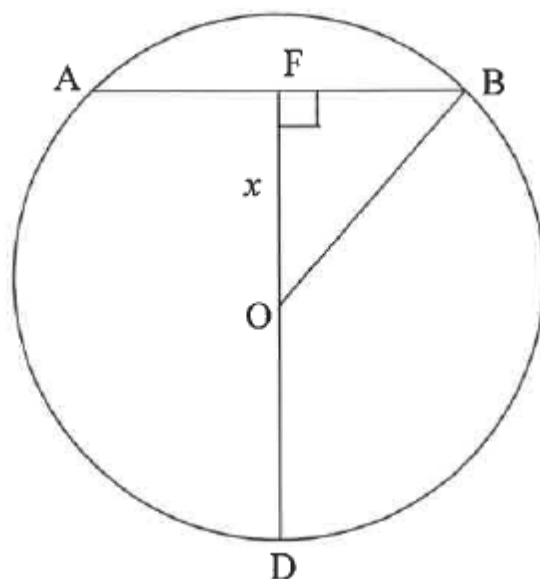
QUESTION 6



6.1	$\hat{T}_1 = x$ [tan – chord theorem / raaklyn-koord st] $\hat{O}_1 = 2x$ $\left[\begin{array}{l} \angle \text{ at centre} = 2 \times \angle \text{ at circumference /} \\ \text{middelpuntshoek} = 2 \text{ keer omtrekshoek} \end{array} \right]$ $\hat{P}_2 = 90^\circ - x$ [tan $\perp$ diameter/ raaklyn $\perp$ middellyn] $\hat{R}_2 = 90^\circ - x$ [$\angle^s$ opp.=sides / $\angle^s$ teenoor gelyke sye] $\therefore \hat{R}_1 = \hat{R}_2$ PR bisects / halveer ORN	
6.2	$\hat{P}\hat{A}\hat{T} = 90^\circ + x$ $\left[\begin{array}{l} \text{opp } \angle^s \text{ of cyclic quad /} \\ \text{teenoorst. hoeke van koordevierhoek} \end{array} \right]$ $\hat{S}_2 = 90^\circ$ [Line from centre $\perp$ to chord / lyn van mdpt $\perp$ aan ko. $\hat{R}\hat{O}\hat{S} = 90^\circ + x$ [ext $\angle$ of Δ / buite $\angle$ van Δ] $\therefore \hat{R}\hat{O}\hat{S} = \hat{P}\hat{A}\hat{T}$	

QUESTION 7

In the diagram, O is the centre of circle ABD. F is a point on chord AB such that $DOF \perp AB$.
 . $AB = FD = 8$ cm and $OF = x$ cm



$$AF = FB = 4\text{cm} \left[\begin{array}{l} \text{line from centre } \perp \text{ to chord /} \\ \text{lyn van mdpt } \perp \text{ aan koord} \end{array} \right]$$

$$OD = OB = 8 - x (\text{radii})$$

$$OB^2 = OF^2 + FB^2 \text{ (Pythagoras)}$$

$$(8 - x)^2 = x^2 + 4^2$$

$$64 - 16x + x^2 = x^2 + 4^2$$

$$48 = 16x$$

$$x = 3$$

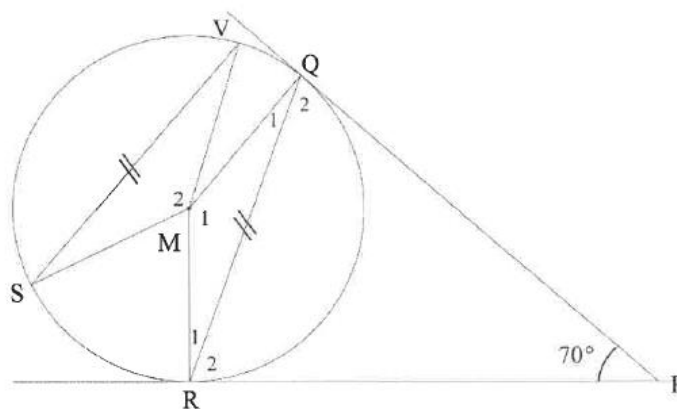
$$\text{length of / lengte van radius} = 8 - x$$

$$= 8 - 5$$

$$= 3 \text{ units / eenh}$$

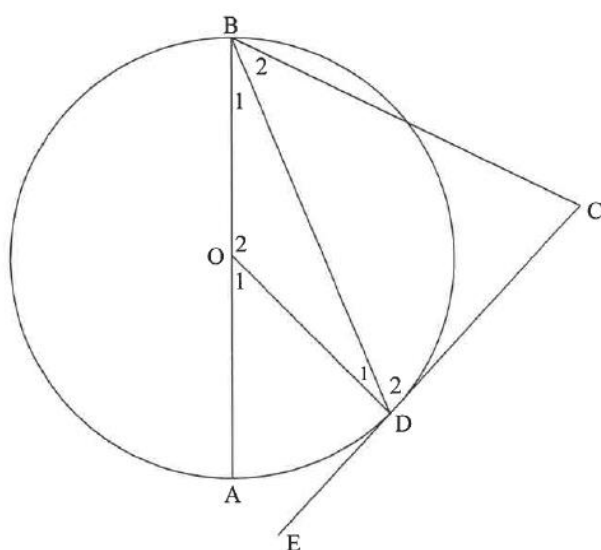
QUESTION 8

M is the centre of the circle SVQR having equal chords SV and QR. RP and QP are tangents to the circle at R and Q respectively such that $\hat{RPQ} = 70^\circ$



8.1	$\hat{Q}_2 = \hat{R}_2$ [tangents from common point/ <i>rk ln e van selfde punt</i> $\hat{Q}_2 + \hat{R}_2 + 70^\circ = 180^\circ$ [sum $\angle \Delta$] $2\hat{R}_2 = 110^\circ$ $\hat{R}_2 = 55^\circ$
8.2	$\hat{Q}_2 + \hat{Q}_1 = 90^\circ$ [tan/rkl $\perp$ rad] $\hat{Q}_1 = 35^\circ$ OR/OF $\hat{R}_1 + \hat{R}_2 = 90^\circ$ [tan/rkl $\perp$ rad] $\hat{R}_1 = 35^\circ$ $\hat{Q}_1 = \hat{R}_1 = 35^\circ$ [OR = OQ]
8.3	$\hat{M}_1 + \hat{R}_1 + \hat{Q}_1 = 180^\circ$ sum of angles of triangle $\hat{M}_1 = 180^\circ - 70^\circ = 110^\circ$ $\therefore \hat{M}_2 = 110^\circ$ equal chords equal angles

QUESTION 9



9.1

$$\hat{B}_1 = \hat{B}_2 = x \text{ [BD bisect/halveer } \angle \hat{ABC} \text{]}$$

$$\hat{ABC} = 2x$$

$$\hat{O}_1 = 2x \text{ [}\angle \text{ at centre} = 2 \text{ times } \angle \text{ at circumference/]}$$

$$\text{[midpts } \angle = 2 \times \text{ omtreks } \angle \text{]}$$

$$\therefore BC \parallel OD \text{ [corresponding } \angle \text{ are equal/ooreenk. } \angle \text{ is gelyk]}$$

OR/OF

$$\hat{B}_1 = \hat{B}_2 = x \text{ [BD bisect/halveer } \angle \hat{ABC} \text{]}$$

$$\hat{D}_1 = x \text{ [angle opp = sides/} \angle \text{e to gelyke sye]}$$

$$\hat{D}_1 = \hat{B}_2 = x$$

$$\therefore BC \parallel OD \text{ [alternate angles are equal/verw } \angle \text{e gelyk]}$$

OR/OF

$$\hat{B}_1 = \hat{B}_2 = x \text{ [BD bisect/halveer } \angle \hat{ABC} \text{]}$$

$$\hat{ABC} = 2x$$

$$\hat{O}_1 = 2x \text{ [angle at centre} = 2 \text{ times angle at circumference]}$$

$$\text{[midpts } \angle = 2 \times \text{ omtreks } \angle \text{]}$$

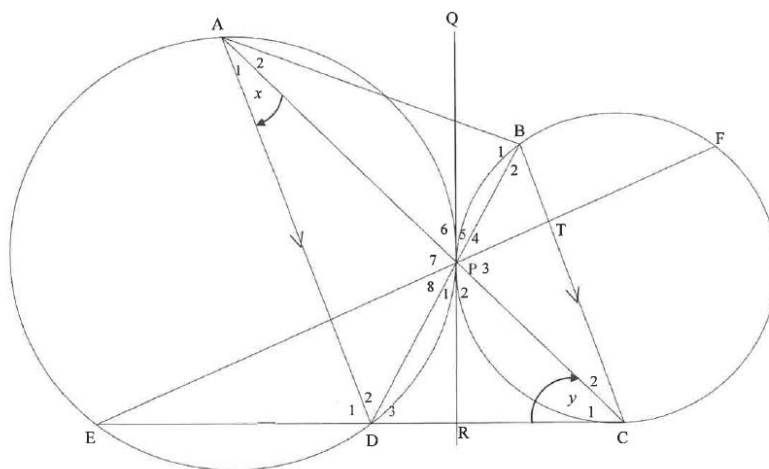
$$\hat{O}_2 = 180^\circ - 2x \text{ [}\angle \text{ on a straight line/} \angle \text{ op reguit lyn]}$$

$$\hat{O}_2 + \hat{ABC} = 180^\circ - 2x + 2x = 180^\circ$$

$$\therefore BC \parallel OD \text{ [co-int angles are suppl/ko-binne } \angle \text{ is suppl}]$$

9.2	$\hat{O}DC = 90^\circ$ [tan/rkl $\perp$ rad] $\hat{C} = 90^\circ$ [co – int / ko-binne $\angle$'s $OD \parallel BC$] OR/OF $\hat{D}_1 = x$ $\hat{D}_2 = 90^\circ - x$ [tan/rkl $\perp$ rad] $\hat{C} = 180^\circ - (90^\circ - x) - x$ [int $\angle$'s of / van Δ] $= 90^\circ$ OR/OF $\hat{E}DC = 90^\circ$ [tan/rkl $\perp$ rad] $\hat{C} = 90^\circ$ [corresp. / ooreenk. $\angle$'s $OD \parallel BC$]
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QUESTION 10



10.1	$\hat{P}_1 = x$ [tan – ch th/ rkl-kdst] $\hat{C}_2 = x$ [alt / verw. $\angle$'s $AD \parallel BC$] $\hat{E} = x$ [$\angle$'s in the same segment / dieselfde segment] $\hat{P}_5 = x$ [vert opp/ reg oorst]
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10.2	$\hat{DCB} = x + y$ $\hat{EDA} = \hat{D}_1 = x + y \text{ [corresp / ooreenk. } \angle^s = , AD \parallel BC \text{]}$ $\therefore \hat{EPA} = x + y \text{ [} \angle^s \text{ in the same segment / dieselfde segment]}$
10.3	$\hat{P}_2 = y \text{ [tan from a commompoint / rklyne v dieselfde pt]}$ $\hat{DPT} = \hat{P}_1 + \hat{P}_2 + \hat{P}_3$ $= x + y + (x + y)$ $= 2x + 2y$ $\hat{C} = x + y$ $\hat{DPT} + \hat{C} = 180^\circ \text{ [opp } \angle^s \text{ of a cyclic quad / teenoorst. } \angle^s \text{ van kvhk]}$ $2x + 2y + x + y = 180^\circ$ $3x + 3y = 180^\circ$ $\therefore x + y = 60^\circ$

WEEK 9 - 10: TOPIC TRIGONOMETRY

SUB-TOPIC:

- Identities
- Reduction Formula
- Determine for which values of a variable an identity holds.

NOTES & EXAM TIPS

Deal with this topic teachers must ensure:

- ❖ Fundamental identities are mastered and used correctly involving fractions and surds.
- ❖ Learners understand that the reduction formulae and identities can be used in trigonometric equations

The topic of Trigonometry is in Paper 2

It accounts for 50±3 in the Paper

ACTIVITIES/WORKSHEETS/EXAM TYPE QUESTIONS

1.

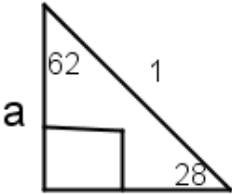
$$\begin{aligned}\frac{1}{1 - \cos(180^\circ - x)} + \frac{1}{1 - \sin(90^\circ - x)} &= \frac{2}{\sin^2 x} \\ LHS &= \frac{1}{1 - \cos(180^\circ - x)} + \frac{1}{1 - \sin(90^\circ - x)} \\ &= \frac{1}{1 + \cos x} + \frac{1}{1 - \cos x} \\ &= \frac{1 - \cos x + 1 + \cos x}{1 - \cos^2 x} \\ &= \frac{2}{\sin^2 x} \\ &= RHS\end{aligned}$$

2.1	$1 - \cos\theta = 0$ $\cos\theta = 1$ Ref angle = 0° $\theta = 0^\circ + k \cdot 360^\circ, k \in \mathbb{Z}$ $\theta = \{-360^\circ; 0^\circ; 360^\circ\}$ $\sin\theta = 0$ $\theta = \{-360^\circ; -180^\circ; 0^\circ; 180^\circ; 360^\circ\}$ $\therefore \theta = \{-360^\circ; -180^\circ; 0^\circ; 180^\circ; 360^\circ\}$
2.2	$\frac{\sin\theta}{1 - \cos\theta} - \frac{\cos\theta}{\sin\theta} = \frac{1}{\sin\theta}$ $LHS = \frac{\sin\theta}{1 - \cos\theta} - \frac{\cos\theta}{\sin\theta}$ $= \frac{\sin^2\theta - \cos\theta(1 - \cos\theta)}{(1 - \cos\theta)(\sin\theta)}$ $= \frac{\sin^2\theta - \cos\theta + \cos^2\theta}{(1 - \cos\theta)(\sin\theta)}$ $= \frac{1 - \cos\theta}{(1 - \cos\theta)(\sin\theta)}$ $= \frac{1}{\sin\theta}$ $= RHS$

QUESTION 3

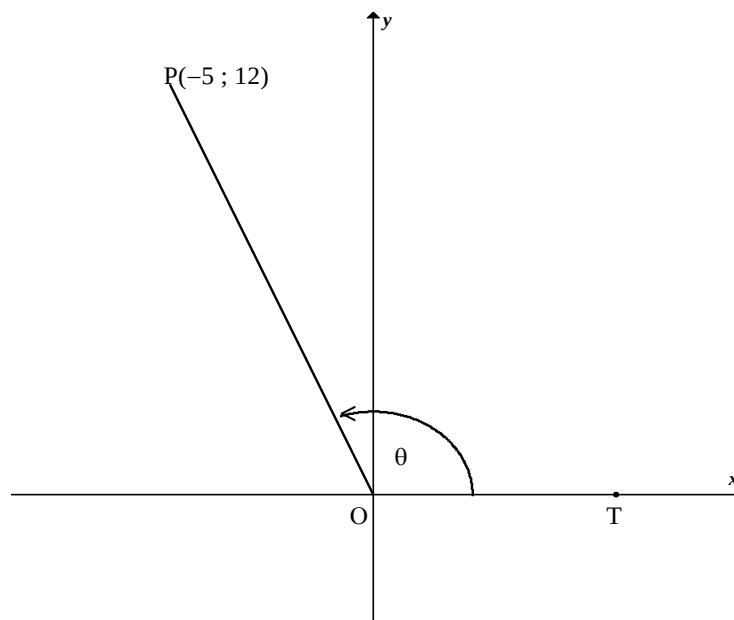
	$\sin 191^\circ$ $= \sin(180^\circ + 11^\circ)$ $= -\sin 11^\circ$
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QUESTION 4

4.1	$\cos 28^\circ$  $\text{adj side} = \sqrt{1 - a^2}$ $\cos 28^\circ = \frac{\sqrt{1 - a^2}}{1}$ $\cos 28^\circ = \sqrt{1 - a^2}$
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QUESTION 5

5.1 In the diagram, P(-5 ; 12) and T lies on the positive x -axis. $\widehat{\text{POT}} = \theta$



5.1.	$\tan \theta = -\frac{12}{5} \quad \text{or} \quad -2\frac{2}{5}$	
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5.2	$(OP)^2 = 5^2 + (-12)^2$ $OP = 13$ $\cos \theta = \frac{5}{13}$	
5.3	$\sin(\theta + 90^\circ) = \frac{b}{6,5}$ $\cos \theta = \frac{b}{6,5}$ $\frac{-5}{13} = \frac{b}{6,5}$ $b = -\frac{5}{2}$	

QUESTION 6

6.1	$\tan \theta + \frac{\cos \theta}{\sin \theta - 1} = -\frac{1}{\cos \theta}$ $LHS = \tan \theta + \frac{\cos \theta}{\sin \theta - 1}$ $= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta - 1}$ $= \frac{\sin \theta (\sin \theta - 1) + (\cos \theta)(\cos \theta)}{(\cos \theta)(\sin \theta - 1)}$ $= \frac{\sin^2 \theta - \sin \theta + \cos^2 \theta}{(\cos \theta)(\sin \theta - 1)}$ $= \frac{1 - \sin \theta}{(\cos \theta)(\sin \theta - 1)}$ $= \frac{-(\sin \theta - 1)}{(\cos \theta)(\sin \theta - 1)}$ $= -\frac{1}{\cos \theta}$ $= RHS$
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6.2	$\frac{\tan \theta \cos \theta \sin \theta}{1 - \cos^2 \theta} = \sin^2 \theta + \cos^2 \theta$ $LHS = \frac{\tan \theta \cos \theta \sin \theta}{1 - \cos^2 \theta}$ $= \frac{\tan \theta \cos \theta \sin \theta}{\sin^2 \theta}$ $= \frac{\frac{\sin \theta}{\cos \theta} \cdot \cos \theta}{\sin \theta}$ $= 1$ $RHS = \sin^2 \theta + \cos^2 \theta$ $= 1$
6.3	$\frac{\sin \theta}{1 + \cos \theta} + \frac{1}{\tan \theta} = \frac{1}{\tan \theta \cos \theta}$ $LHS = \frac{\sin \theta}{1 + \cos \theta} + \frac{1}{\tan \theta}$ $= \frac{\sin \theta}{1 + \cos \theta} + \frac{1}{\frac{\sin \theta}{\cos \theta}}$ $= \frac{\sin \theta}{1 + \cos \theta} + \frac{\cos \theta}{\sin \theta}$ $= \frac{\sin^2 \theta + \cos \theta + \cos^2 \theta}{(1 + \cos \theta)(\sin \theta)}$ $= \frac{1 + \cos \theta}{(1 + \cos \theta)(\sin \theta)}$ $= \frac{1}{\sin \theta}$ $RHS = \frac{1}{\tan \theta \cos \theta}$ $= \frac{1}{\frac{\sin \theta}{\cos \theta} \cdot \cos \theta}$ $= \frac{1}{\sin \theta}$ $= LHS$

6.4	$\left(\frac{1}{\cos \theta} - \tan \theta\right)^2 = \frac{1 - \sin \theta}{1 + \sin \theta}$ $LHS = \left(\frac{1}{\cos \theta} - \tan \theta\right)^2$ $= \left(\frac{1}{\cos^2 \theta} - 2 \frac{1}{\cos \theta} \cdot \tan \theta + \tan^2 \theta\right)$ $= \frac{1}{\cos^2 \theta} - 2 \frac{1}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta} + \frac{\sin^2 \theta}{\cos^2 \theta}$ $= \frac{1 - 2 \sin \theta + \sin^2 \theta}{\cos^2 \theta}$ $= \frac{(1 - \sin \theta)(1 - \sin \theta)}{1 - \sin^2 \theta}$ $= \frac{(1 - \sin \theta)(1 - \sin \theta)}{(1 - \sin \theta)(1 + \sin \theta)}$ $= \frac{1 - \sin \theta}{1 + \sin \theta}$ $= RHS$
6.5	$\frac{\sin \theta - \sin \theta \cos \theta}{\cos \theta - (1 - \sin^2 \theta)} = \tan \theta$ $LHS = \frac{\sin \theta - \sin \theta \cos \theta}{\cos \theta - (1 - \sin^2 \theta)}$ $= \frac{\sin \theta (1 - \cos \theta)}{\cos \theta - \cos^2 \theta}$ $= \frac{\sin \theta (1 - \cos \theta)}{\cos \theta (1 - \cos \theta)}$ $= \frac{\sin \theta}{\cos \theta}$ $= \tan \theta$ $= RHS$

6.6	$\frac{\sin x \cdot \cos x}{1 + \cos^2 x - \sin^2 x} = \frac{1}{2} \tan x$ $LHS = \frac{\sin x \cdot \cos x}{1 + \cos^2 x - \sin^2 x}$ $= \frac{\sin x \cdot \cos x}{\sin^2 x + \cos^2 x + \cos^2 x - \sin^2 x}$ $= \frac{\sin x \cdot \cos x}{2 \cos^2 x}$ $= \frac{\sin x}{2 \cos x}$ $= \frac{1}{2} \tan x$ $= RHS$
6.7	$(\sin P + \cos P)^2 + (\sin P - \cos P)^2 = 2$ $LHS = (\sin P + \cos P)^2 + (\sin P - \cos P)^2$ $= \sin^2 P + 2 \sin P \cos P + \cos^2 P + \sin^2 P - 2 \cos P \sin P + \cos^2 P$ $= 1 + 1$ $= 2$ $= RHS$
6.8	$\cos^2 x (2 + \tan^2 x) = 2$ $LHS = \cos^2 x (2 + 2 \tan^2 x)$ $= 2 \cos^2 x + 2 \cos^2 x \tan^2 x$ $= 2 \cos^2 x + 2 \cos^2 x \cdot \frac{\sin^2 x}{\cos^2 x}$ $= 2 \cos^2 x + 2 \sin^2 x$ $= 2(\cos^2 x + \sin^2 x)$ $= 2(1)$ $= 2$ $= RHS$

6.9	$\frac{2 \tan A - 2 \sin A \cos A}{2 \sin^2 A} = \tan A$ $LHS = \frac{2 \tan A - 2 \sin A \cos A}{2 \sin^2 A}$ $= \frac{2 \tan A}{2 \sin^2 A} - \frac{2 \sin A \cos A}{2 \sin^2 A}$ $= \frac{\frac{\sin A}{\cos A}}{\sin^2 A} - \frac{\cos A}{\sin A}$ $= \frac{1}{\cos A \sin A} - \frac{\cos A}{\sin A}$ $= \frac{1 - \cos^2 A}{\cos A \sin A}$ $= \frac{\sin^2 A}{\cos A \sin A}$ $= \frac{\sin A}{\cos A}$ $= \tan A$ $= RHS$
6.10	$\frac{\cos x (\tan x - \sin x)}{\sin^3 x} = \frac{1}{1 + \cos x}$ $LHS = \frac{\cos x (\tan x - \sin x)}{\sin^3 x}$ $= \frac{\cos x \cdot \tan x - \cos x \cdot \sin x}{\sin^3 x}$ $= \frac{\cos x \cdot \frac{\sin x}{\cos x} - \cos x \cdot \sin x}{\sin^3 x}$ $= \frac{\sin x - \cos x \cdot \sin x}{\sin^3 x}$ $= \frac{\sin x (1 - \cos x)}{\sin^3 x}$ $= \frac{1 - \cos x}{\sin^2 x}$

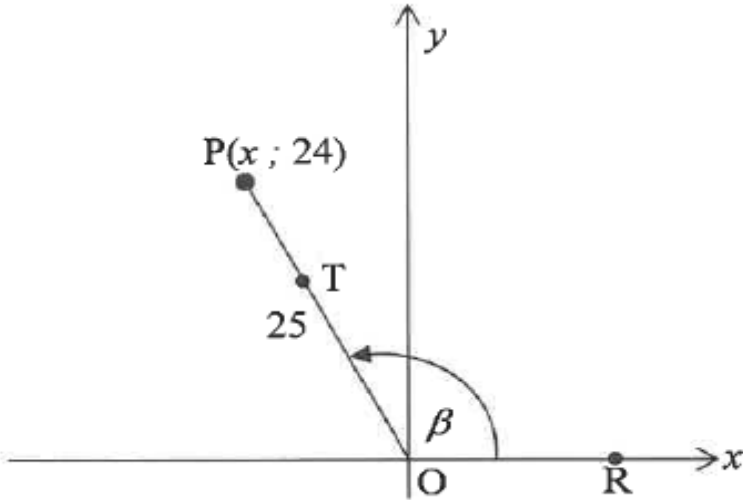
	$= \frac{1 - \cos x}{1 - \cos^2 x}$ $= \frac{1 - \cos x}{(1 + \cos x)(1 - \cos x)}$ $= \frac{1}{1 + \cos x}$ $= RHS$
6.11	$\frac{1}{\cos x} - \cos x = \tan x \sin x$ $LHS = \frac{1}{\cos x} - \cos x$ $= \frac{1 - \cos^2 x}{\cos x}$ $= \frac{\sin^2 x}{\cos x}$ $= \tan x \sin x$ $= RHS$
6.12	$\frac{1}{\tan \alpha} (\sin \alpha \tan \alpha + \cos \alpha) = \frac{1}{\sin \alpha}$ $LHS = \frac{1}{\tan \alpha} (\sin \alpha \tan \alpha + \cos \alpha)$ $= \sin \alpha + \frac{\cos \alpha}{\tan \alpha}$ $= \sin \alpha + \frac{\cos^2 \alpha}{\sin \alpha}$ $= \frac{\sin^2 \alpha + \cos^2 \alpha}{\sin \alpha}$ $= \frac{1}{\sin \alpha}$ $= RHS$
6.13	$\cos x \left(\tan x + \frac{\cos x}{\sin x} \right) = \frac{1}{\sin x}$

	$LHS = \cos x \left(\tan x + \frac{\cos x}{\sin x} \right)$ $= \cos x \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right)$ $= \sin x + \frac{\cos^2 x}{\sin x}$ $= \frac{\sin^2 x + \cos^2 x}{\sin x}$ $= \frac{1}{\sin x}$ $= RHS$
6.14	$\frac{\cos^2 x \cdot \sin x}{\sin x - \sin^2 x} = 1 + \sin x$ $LHS = \frac{\cos^2 x \cdot \sin x}{\sin x - \sin^2 x}$ $= \frac{\cos^2 x \cdot \sin x}{\sin x(1 - \sin x)}$ $= \frac{\cos^2 x}{1 - \sin x}$ $= \frac{1 - \sin^2 x}{1 - \sin x}$ $\frac{(1 - \sin x)(1 + \sin x)}{1 - \sin x}$ $= 1 + \sin x$ $= RHS$
6.15	$\frac{\cos^4 x - \sin^4 x}{(\cos x - \sin x)(1 + \tan x)} = \cos x$ $LHS = \frac{\cos^4 x - \sin^4 x}{(\cos x - \sin x)(1 + \tan x)}$ $= \frac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{(\cos x - \sin x)(1 + \tan x)}$ $= \frac{(\cos x - \sin x)(\cos x + \sin x)(1)}{(\cos x - \sin x)(1 + \tan x)}$

	$= \frac{\cos x + \sin x}{1 + \tan x}$ $= \frac{\cos x + \sin x}{1 + \frac{\sin x}{\cos x}}$ $= \frac{\cos x + \sin x}{\frac{\cos x + \sin x}{\cos x}}$ $= \cos x$ $= RHS$
6.16	$\sin \theta (1 + \tan \theta) + \cos \theta (1 + \frac{\cos \theta}{\sin \theta}) = \frac{1}{\sin \theta} + \frac{1}{\cos \theta}$ $LHS = \sin \theta (1 + \tan \theta) + \cos \theta (1 + \frac{\cos \theta}{\sin \theta})$ $= \sin \theta + \sin \theta \cdot \frac{\sin \theta}{\cos \theta} + \cos \theta + \frac{\cos^2 \theta}{\sin \theta}$ $= \sin \theta + \frac{\sin^2 \theta}{\cos \theta} + \cos \theta + \frac{\cos^2 \theta}{\sin \theta}$ $= \frac{\sin^2 \theta \cos \theta + \sin^3 \theta + \cos^2 \theta \sin \theta + \cos^3 \theta}{\cos \theta \sin \theta}$ $= \frac{\sin^2 \theta (\cos \theta + \sin \theta) + \cos^2 \theta (\sin \theta + \cos \theta)}{\cos \theta \sin \theta}$ $= \frac{(\sin^2 \theta + \cos^2 \theta)(\cos \theta + \sin \theta)}{\cos \theta \sin \theta}$ $= \frac{(1)(\cos \theta + \sin \theta)}{\cos \theta \sin \theta}$ $= \frac{\cos \theta}{\cos \theta \sin \theta} + \frac{\sin \theta}{\cos \theta \sin \theta}$ $= \frac{1}{\sin \theta} + \frac{1}{\cos \theta}$ $= RHS$

6.17	$\frac{\sin x}{1 + \cos x} + \frac{\cos x}{\sin x} = \frac{1}{\sin x}$ $RHS = \frac{\sin x}{1 + \cos x} + \frac{\cos x}{\sin x}$ $= \frac{\sin^2 x + \cos x + \cos^2 x}{(1 + \cos x)(\sin x)}$ $= \frac{1 + \cos x}{(1 + \cos x)(\sin x)}$ $= \frac{1}{\sin x}$ $= RHS$
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QUESTION 7

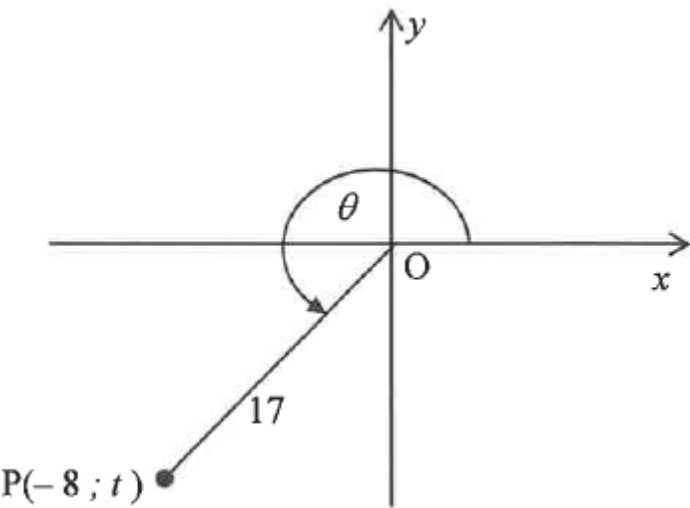


7.1.1	$x^2 + y^2 = r^2$ $x^2 + (24)^2 = 25^2$ $x^2 = 49$ $x = -7$	
7.1.2(a)	$\sin \beta = \frac{24}{25}$	

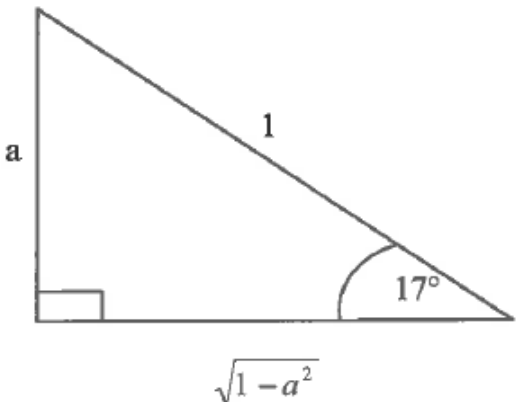
7.1.2(b)	$\cos(180^\circ - \beta)$ $= -\cos \beta$ $= -(-\frac{7}{25})$ $= \frac{7}{25}$	
7.1.2(c)	$\tan(-\beta)$ $= -\tan \beta$ $= -(\frac{24}{-7})$ $= \frac{24}{7}$	
7.1.3	$\sin \beta = \frac{24}{25} = \frac{y_T}{15}$ $\therefore y_T = 14\frac{2}{5}$ $\cos \beta = -\frac{7}{25} = \frac{x_T}{15}$ $\therefore x_T = -4\frac{1}{5}$ $\therefore T(-4\frac{1}{5} ; 14\frac{2}{5})$	

7.2	$\frac{2 \sin x \cdot \cos x \cdot (1 + \frac{\sin^2 x}{\cos^2 x})}{\frac{\sin x}{\cos x}}$ $= 2 \sin x \cdot \cos x \cdot (\frac{\cos^2 x + \sin^2 x}{\cos^2 x}) \times \frac{\cos x}{\sin x}$ $= 2 \cos^2 x (\frac{1}{\cos^2 x})$ $= 2$ OR/OF $\frac{2 \sin x \cdot \cos x \cdot (1 + \frac{\sin^2 x}{\cos^2 x})}{\frac{\sin x}{\cos x}}$ $= 2 \sin x \cdot \cos x \cdot (1 + \frac{\sin^2 x}{\cos^2 x}) \times \frac{\cos x}{\sin x}$ $= 2 \cos^2 x (1 + \frac{\sin^2 x}{\cos^2 x})$ $= 2(\cos^2 x + \sin^2 x)$ $= 2$	
7.3	$\frac{1 - \cos^2 A}{4(-\sin A)}$ $= \frac{\sin^2 A}{-4 \sin A}$ $= -\frac{1}{4} \sin A$	

QUESTION 8



8.1.1	$x^2 + y^2 = r^2$ $(-8)^2 + (t)^2 = 17^2$ $t^2 = 225$ $t = -15$	
8.1.2(a)	$\cos(-\theta)$ $= \cos \theta$ $= \frac{-8}{17}$	
8.1.2(b)	$1 - \sin \theta = 1 - \frac{-15}{17}$ $= \frac{17}{17} + \frac{15}{17}$ $= \frac{32}{17}$	

8.2.1	$\tan 17^\circ = \frac{a}{\sqrt{1-a^2}}$ 	
8.2.2	$\begin{aligned}\sin 107^\circ &= \sin(90^\circ + 17^\circ) \\ &= \cos 17^\circ \\ &= \sqrt{1-a^2}\end{aligned}$ OR/OF $\begin{aligned}\sin 107^\circ &= \sin(180^\circ - 73^\circ) \\ &= \sin 73^\circ \\ &= \sqrt{1-a^2}\end{aligned}$	
8.2.3	$\begin{aligned}\cos^2 253^\circ + \sin^2 557^\circ &= (-\cos 73^\circ)^2 + (-\sin 17^\circ)^2 \\ &= (-a)^2 + (-a)^2 \\ &= 2a^2\end{aligned}$	

8.3

$$\begin{aligned}
 RHS &= \frac{-1}{\tan^2 x \cdot \cos^2 x} \\
 &= \frac{-1}{\frac{\sin^2 x}{\cos^2 x} \times \cos^2 x} \\
 &= \frac{-1}{\sin^2 x} \\
 &= \frac{-1}{1 - \cos^2 x} \\
 &= \frac{1}{\cos^2 x - 1} \\
 &= \frac{1}{(\cos x + 1)(\cos x - 1)} \\
 &= LHS
 \end{aligned}$$

OR

$$\begin{aligned}
 LHS &= \frac{1}{(\cos x + 1)(\cos x - 1)} \\
 &= \frac{1}{\cos^2 x - 1} \\
 &= \frac{1}{-\sin^2 x} \\
 &= \frac{-1}{\sin^2 x} \\
 &= \frac{-1}{\frac{\sin^2 x}{\cos^2 x} \times \frac{\cos^2 x}{1}} \\
 &= \frac{-1}{\tan^2 x \cdot \cos^2 x}
 \end{aligned}$$

OR

	$RHS = \frac{-\cos^2 x}{\sin^2 x \cdot \cos^2 x}$ $= \frac{-1}{\sin^2 x}$ $LHS = \frac{1}{\cos^2 x - 1}$ $= \frac{1}{-\sin^2 x}$ $RHS = LHS$	
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C. TERM 2

WEEK	TOPIC	SUBTOPIC	PAGE
1	Trig equations and general solutions	4. Determine the general solutions of trigonometric equations. Also, determine solutions in specific intervals	4 - 10
2	Quadrilaterals.	Teach past grade content on Quadrilaterals Parallelogram, square, rectangle etc.	11 - 21
3	Analytical Geometry	Revise 1. distance between the two points; 2. gradient of the line segment connecting the two points (and from that identify parallel and perpendicular lines); and 3. Coordinates of the mid-point of the line segment joining the two points. Derive and apply: 1. the equation of a line through two given points	
4		Derive and apply: 2. the equation of a line through one point and parallel or perpendicular to a given line; and 3. The inclination (θ) of a line, where $m = \tan\theta$ is the gradient of the line ($0^\circ \leq \theta \leq 180^\circ$) Applications in complex diagrams.	
5	Number Patterns	- Revise linear number patterns. - Investigate number patterns leading to those where there is a constant second difference between consecutive terms, and the general term is therefore quadratic.	22 - 29
6		Investigate number patterns leading to those where there is a constant second difference between consecutive terms, and the general term is therefore quadratic	
7	Functions and Graphs	1. Revise the effect of the parameters a and q and investigate the effect of p on the graphs of the functions defined by: $y = f(x) = a(x + p)^2 + q$	30 - 48
8		1.1. $y = f(x) = \frac{a}{x+p} + q$ 1.2. $y = f(x) = a \cdot b^{x+p} + q$ where $b > 0, b \neq 1$ 2. Investigate numerically the average gradient between two points on a curve and develop an intuitive understanding of the concept of the gradient of a curve at a point. NB: Apply nature of roots with Functions	
9		Point by point plotting of basic graphs defined by $y = \sin\theta$, $y = \cos\theta$ and $y = \tan\theta$ for $\theta \in [-360^\circ; 360^\circ]$ 4. Investigate the effect of the parameter k on the graphs of the functions defined by $y = \sin(kx)$, $y = \cos(kx)$ and $y = \tan(kx)$	
10		5. Investigate the effect of the parameter p on the graphs of the functions defined by $y = \sin(x + p)$, $y = \cos(x + p)$ and $y = \tan(x + p)$ 6. Draw sketch graphs defined by: $y = a \sin k(x + p)$, $y = a \cos k(x + p)$ and $y = a \tan k(x + p)$ at most two parameters at a time.	

WEEK 1 – TOPIC:TRIGONOMETRY

SUB TOPIC

1.Trigonometric Equations and General Solutions

To deal with this topic teachers must ensure:

- ❖ Basic fundamental identities
- ❖ Basic reduction formula
- ❖ Basic definitions of trigonometric ratios
- ❖ All concepts covered in grade 10 trigonometry.
 - Trigonometric ratios.
 - Special angles.
- ❖ All concepts covered in grade 11 (term 1) trigonometry.
 - Trigonometric Identities & Reduction formulae.

Teachers must know the Common misconceptions:

- Some learners might struggle when proving certain identities. Give them extra time and guide them to a full understanding of what they are doing.
- Some learners may have issues understanding the identities. Guide them to a full understanding of what they are doing and give them extra time to learners that need it.
- Some learners may have issues understanding negative angles. Guide them to a full understanding of what they are doing and give them extra time to learners that need it.
- Some learners may have issues understanding with $\tan x$, $\sin x$ and $\cos x$ in the same equation. Guide them to a full understanding of what they need to accomplish.

SOLUTIONS TO LEARNERS' ACTIVITY

QUESTION	ANSWERS	Marks
1.	$\tan \frac{x}{2} = \sqrt{3}$ $\frac{x}{2} = 60^\circ + k \cdot 180^\circ$ $x = 120^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$	✓ Equation ✓ General solutions (2)
2.1	$\sin x(\tan x + 1) = 0$ $\sin x = 0 \quad \text{or} \quad \tan x = -1$ $x = 0^\circ + k \cdot 180^\circ; k \in \mathbb{Z} \quad \text{or} \quad x = -45^\circ + k \cdot 180^\circ; k \in \mathbb{Z}$	✓ Factors ✓ Value sin x ✓ Value tan x ✓ $0^\circ + k \cdot 180^\circ$ ✓ $-45^\circ + k \cdot 180^\circ$; $k \in \mathbb{Z}$ (5)
2.2	$3\sin x - 3 + 2(1 - \sin^2 x) = 0$ $3\sin x - 3 + 2 - 2\sin^2 x = 0$ $2\sin^2 x - 3\sin x + 1 = 0$ $(2\sin x - 1)(\sin x - 1) = 0$ $\sin x = \frac{1}{2} \quad \text{or} \quad \sin x = 1$ $x = 30^\circ + k \cdot 360^\circ \quad \text{or} \quad x = 150^\circ + k \cdot 360^\circ \quad \text{or} \quad x = 90^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$	✓ $1 - \sin^2 x = \cos^2 x$ ✓ expansion ✓ factors ✓ both values of sin x ✓ $90^\circ + k \cdot 360^\circ$ ✓ $30^\circ + k \cdot 360^\circ$ ✓ $150^\circ + k \cdot 360^\circ$; $k \in \mathbb{Z}$ (7)
3.	$\frac{\sin y}{\cos y} = -1$ $\therefore \tan y = -1$ $\tan y = -1$ $\therefore \text{ref } \angle = \tan^{-1}(1)$ $= 45^\circ$ $y = (180^\circ - 45^\circ) + k \cdot 180^\circ$ $\therefore y = 135^\circ + k \cdot 180^\circ, \quad k \in \mathbb{Z}$ $y = 135^\circ + k \cdot 180^\circ \text{ where } k \in \mathbb{Z}.$	✓ $\tan y = \frac{\sin y}{\cos y}$ ✓ Referent < ✓ Reduction formulae. ✓ $135^\circ + k \cdot 180^\circ$; $k \in \mathbb{Z}$ (4)

4.	$x - 20^\circ = 2x + 10^\circ + k.180^\circ; k \in \mathbb{Z}$ $-x = 30^\circ + k.180^\circ$ $x = -30^\circ + k.180^\circ; k \in \mathbb{Z}$	✓ ✓ Equation General solutions (2)
5.	$3(\sin^2 x + \cos^2 x) - 5\sin x \cos x - 5\cos^2 x = 0$ $3\sin^2 x - 5\sin x \cdot \cos x - 2\cos^2 x = 0$ $(3\sin x + \cos x)(\sin x - 2\cos x) = 0$ $3\sin x = -\cos x \text{ or } \sin x = 2\cos x$ $\frac{3\sin x}{3\cos x} = \frac{-\cos x}{\cos x} \text{ or } \frac{\sin x}{\cos x} = \frac{2\cos x}{\cos x}$ $\tan x = \frac{-1}{3} \text{ or } \tan x = 2$ $x = 180^\circ - 18,43^\circ + k.180^\circ \text{ or } x = 63,43^\circ + k.180^\circ; k \in \mathbb{Z}$ $x = 161,57^\circ + k.180^\circ; k \in \mathbb{Z}$	✓ ✓ ✓ ✓ ✓ ✓ ✓ $\sin^2 x + \cos^2 x = 1$ expansion factorization both equations /cos Referent < Reduction formulae General solutions (8)
6.1	$2\cos x + 6 = 3\tan x + 6$ $2\cos x = 3\tan x$ $2\cos x = \frac{3\sin x}{\cos x}$ $2\cos^2 x = 3\sin x$ $2(1 - \sin^2 x) = 3\sin x$ $2 - 2\sin^2 x = 3\sin x$ $2\sin^2 x + 3\sin x - 2 = 0$ $2\sin^2 x + 3\sin x - 2 = 0$ $(2\sin x - 1)(\sin x + 2) = 0$ $\sin x = \frac{1}{2} \text{ or } \sin x = -2 \text{ (no solution)}$ $x = 30^\circ + k.360^\circ \text{ or } x = 150^\circ + k.360^\circ; k \in \mathbb{Z}$	✓ expansion ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ multiplying by $\cos \theta$ ✓ $\cos^2 x = 1 - \sin^2 x$ ✓ factors ✓ both values of $\sin x$ ✓ no solution ✓ $30^\circ + k.360^\circ$ ✓ $150^\circ + k.360^\circ; k \in \mathbb{Z}$ (9)
6.2	$3y = 30^\circ + k.360^\circ \text{ or } 3y = 150^\circ + k.360^\circ$ $y = 10^\circ + k.120^\circ \text{ or } y = 50^\circ + k.120^\circ$ $\therefore y = 100^\circ + 10^\circ \text{ or } y = 100^\circ + 50^\circ$ $y = 100^\circ \text{ or } y = 150^\circ$	✓ $y = 6^\circ + k.72^\circ$ ✓ $y = 30^\circ + k.72^\circ$ ✓ 150° ✓ 174° (4)

WEEK 2 – 4 ANALYTICAL GEOMETRY

SUB-TOPIC:

- 1.Distance between the two points.
- 2-The coordinates of the mid-point of the line segment joining the points.
- 3-The gradient of the line segment joining the point.
- 4-Conditions for parallel and perpendicular lines.
- 5-The equation of a line through two given points.
- 6-The equation of a line through one point and parallel or perpendicular to a given line.
- 7-The inclination (θ) of a line, where $m = \tan\theta$ is the gradient of the line ($0^\circ \leq \theta \leq 180^\circ$)

OVERVIEW:

Paper 2

This chapter should take about 3 weeks.

- ✦ Revision: 3 .
- ✦ The equation of a straight line (perpendicular and parallel lines): 6 lessons.
- ✦ The inclination of a line:.

These topics deals with where things are in space and comparing them in term position. As example of where analytical geometry is used in real life is in the technology of GPS (Global Positioning System) or GNSS (global Navigation Satellite System).

Common misconceptions:

1. Calculator work for finding an angle or the ratio involving tan.
2. Learners get confuse when alternative words are used in description.
3. Learner find it difficult to select the correct method to match the information given.
4. Positive and negative gradients resulting in an angle $<$ or $> 90^\circ$
5. Make sure learners know when to subtract from 180° or what it means when they get a negative angle on the calculator.

Teacher must guide learners taking into consideration the following learners' hints

HINTS

1. Mathematical language and terminology must be learnt in more detail.
2. *Learners must learn which formula is to be used to prove the most basic aspects of Analytical Geometry.*
 - Select the correct formula from the data sheet
 - Label the ordered pairs using the correct two points, eg A and C.

eg. *Learners should then follow the method laid out below:* Bisect is 2 mid-points
Perpendicular is the product of 2 gradients = - 1

 - Substitute correctly and accurately into your chosen formula
 - Perform the arithmetic, preferably *without* a calculator
3. Often Analytical Geometry questions *follow on*, (scaffolding). Look out for that, as you might have already proven an aspect above, that you will require for the next sub-question.
4. Use the diagram more effectively.

e.g. Highlight the sides you are going to use for proving perpendicular, so you can see clearly which points you are going to use for the substitution.
5. You must answer the question, and remember to conclude, exactly what you were asked to show / prove / conclude. Use wording to do this.
6. Learners need to know the 2 forms of the equations of a circle, by alluding to several different types of examples and exercises. Furthermore, this formula needs to be shown to the candidates by training them to use the Data Sheets. Completing the square in terms of x and y need to be emphasized.
7. Where questions required learners to prove or show a certain expected response, you must convince the marker of their proof.
8. Such questions often require more calculations than those expecting mere calculations, to be given at the end.
9. Learners need to remember that the product of gradients equals -1, is accepted to prove $\perp$. If asked to prove $\perp$, every effort must be made to show that this product = -1. Repetition of exam type questions such as this must be practiced with learners.
10. With regards to determining the equation of a line, educators must cover all aspects of the equation of a line including those passing through the origin, as well as the equations of horizontal and vertical lines in their teaching.
11. Practice exercises are often required to teach the above points.
 12. Grade 11 work must **NOT** be ignored. Circle center (0; 0) was no problem in Grade 11, but can be completely forgotten in Grade 12.

SOLUTIONS TO LEARNERS' ACTIVITY

QUESTION	ANSWERS	MARKS
1.	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $d = \sqrt{(2 - (-3))^2 + (7 - 5)^2}$ $d = \sqrt{(5)^2 + (2)^2}$ $d = \sqrt{29}$	✓ Formula ✓ substitution ✓ length of AB (3)
2.	$M_{MN} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ $M_{MN} \left(\frac{2 - 4}{2}, \frac{3 - 1}{2} \right)$ $M_{MN} \left(\frac{-2}{2}, \frac{2}{2} \right)$ $M_{MN}(-1; 1)$	✓ Formula ✓ substitution ✓ midpoint of MN (3)
3	$m_{MN} = \frac{y_1 - y_2}{x_1 - x_2}$ $m_{MN} = \frac{-1 - 3}{-4 - 2}$ $m_{MN} = \frac{-4}{-6}$ $m_{MN} = \frac{2}{3}$	✓ Formula ✓ substitution ✓ Gradient MN (3)
4.1	$m_{AB} = \frac{y_B - y_A}{x_B - x_A}$ $m_{AB} = \frac{0 - 3}{5 - (-1)} = \frac{-3}{6} = -\frac{1}{2}$ $y - y_1 = m(x - x_1) \text{ using } (x_1; y_1) = (5; 0)$ $y - 0 = -\frac{1}{2}(x - 5)$ $y = -\frac{1}{2}x + \frac{5}{2}$	✓ Formula gradient AB ✓ substitution ✓ gradient ✓ equation of line (5)

4.2	$y - y_1 = m(x - x_1)$ using $(x_1; y_1) = (2; -3)$ and $m = -\frac{1}{2}$ $y + 3 = -\frac{1}{2}(x - 2)$ $y = -\frac{1}{2}x - 1 + 3$ $y = -\frac{1}{2}x - 2$	Formula gradient substitution of $m = -\frac{1}{2}$ (parallel lines have equal gradients) Substitution of point equation of line (4)
4.3	$y - y_1 = m(x - x_1)$ using $(x_1; y_1) = (0; 0)$ $-\frac{1}{2} \times 2 = -1$ then $m = 2$ $y - 0 = 2(x - 0)$ $y = 2x$	✓ Formula gradient ✓ perpendicular line gradients: $m_1 \times m_2 = -1$ ✓ substitution of $m = 2$ ✓ Substitution of point ✓ equation of line (5)
5	$\tan \theta = m_{AB}$ $\tan \theta = \frac{y_2 - y_1}{x_2 - x_1}$ $\tan \theta = \frac{-1 - 3}{-4 - (-1)} = \frac{4}{3}$ $\theta = 53,1^\circ$	✓ Formula gradient ✓ Substitution ✓ tan-1 (3)
6.1	$m_{PQ} = \frac{y_Q - y_P}{x_Q - x_P}$ $m_{PQ} = \frac{1 - (-2)}{1 - 0} = 3$	✓ Formula gradient ✓ Substitution ✓ Gradient (3)
6.2	QR: $y = -\frac{1}{3}x - 2$ $\therefore m_{QR} = -\frac{1}{3}$	✓ Gradient of the equation ✓ $m_{PQ} \times m_{QR} = -1$ (perpendicular lines) ✓ Answer.

	$m_{PQ} \times m_{QR} = 3 \times -\frac{1}{3}$ $= -1$ $\therefore PQ \perp QR \quad \therefore \hat{PQR} = 90^\circ$	(3)
6.3	$-\frac{1}{3}x - 2 = -x + 2$ $\frac{2}{3}x = 4$ $x = 6$ $y = -4$ $\therefore R(6; -4)$	✓ Intersection of sides(equation) ✓ Find y ✓ Find x ✓ Find R (4)
6.4	$d_{PR} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $PR = \sqrt{(1 - 6)^2 + (1 - (-4))^2}$ $= \sqrt{50} = 5\sqrt{2}$	✓ Formula ✓ substitution ✓ length of PR (3)
6.5	m of PR = -1 $\therefore m$ of tangent = 1 Equation of tangent: $y - y_1 = (x - x_1)$ $y - 1 = x - 1$ $\therefore y = x$	✓ Formula gradient ✓ perpendicular line gradients: ✓ substitution of $m = 1$ ✓ Substitution of point ✓ equation of line (5)

6.6	$\tan \hat{PNT} = m_{PR} = -1$ $\therefore \hat{PNT} = 135^\circ$ $\tan \hat{PMT} = m_{PQ} = 3$ $\therefore \hat{PMT} = 71,57^\circ$ $\hat{P} = 63,43^\circ$ [ext $\angle$ of Δ] $\therefore \theta = 26,57^\circ$ [sum of $\angle$ s in Δ]	✓ gradient mPR ✓ tan-1 ✓ gradient mPQ ✓ tan-1 ✓ Ext $\angle$ of Δ for (P) ✓ sum of $\angle$s in Δ for θ (6)
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WEEK 5 - 6: TOPIC: NUMBER PATTERNS

SUB-TOPIC:

- Linear number pattern
- Quadratic number pattern

Overview:

Paper 1

This chapter should take about 2 weeks.

Revise linear number patterns: 3 lessons.

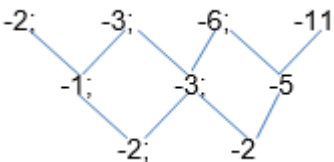
- ✦ Investigate number patterns leading to those where there is a constant second difference between consecutive terms, and the general term is therefore quadratic: 4 lessons

Common misconceptions:

- ❖ Some learners find the topic of numbers patterns, with a constant second difference, difficult. Give learners more time on practical work to develop a better understanding of the concepts discussed. The strategy should be on:
 - Identifying the pattern.
 - Using diagram to calculate first and second differences.
 - Extending the sequence with a few terms and describing in algebraically.
 - Finding any term in the sequence or finding the positioning of a term in the sequence.
 - Learners must able to display a true understanding of linear and quadratic number patterns.
 - Make sure learners know how to differentiate between the value of the term (T_n) and the positioning (n) of a term.

Teacher must guide learners taking into consideration the following learners' hints

- Since some of the contexts covered in this unit may be unfamiliar to all or some of learners, be sure to explain the contexts clearly, and to explain the meaning of all new terms and concepts clearly.
- Spend more time on explaining and teaching the concepts to the weaker learners, while the others can be provided with additional exercises.

SOLUTIONS TO LEARNERS' ACTIVITY		
QUESTION	ANSWERS	MARKS
1.1	$d = T_2 - T_1$ $= 16 - 13$ $= 3$ $\therefore T_n = 10 + 3n$ $T_n = 10 + 3n$ $\therefore T_{10} = 10 + 3(10)$ $= 40$ $\therefore T_{15} = 10 + 3(15)$ $= 55$ $\therefore T_{30} = 10 + 3(30)$ $= 100$	
1.2	$d = T_2 - T_1$ $= -15 - (-10)$ $= -5$ $\therefore T_n = -5 - 5n$ $T_n = -5 - 5n$ $\therefore T_{10} = -5 - 5(10)$ $= -55$ $\therefore T_{15} = -5 - 5(15)$ $= -80$ $\therefore T_{30} = -5 - 5(30)$ $= -155$	✓ Substitute ✓ d ✓ T_n ✓ T_{10} T_{15} T_{30} (4)
2.1	 The 2nd differences are equal.	✓ 1 st ✓ 2 nd ✓ Answer (3)
2.2	$2a = -2$ $a = -1$ $3a + b = -1$ $3(-1) + b = -1$ $b = -1 + 3$ $b = 2$ $a + b + c = -2$ $(-1) + 2 + c = -2$ $c = -2 + 1 - 2$ $c = -3$ $\therefore T_n = -n^2 + 2n - 3$	✓ Substitute ✓ a ✓ Substitute ✓ b ✓ Substitute ✓ c ✓ substitute ✓ T_n (8)

3.1	1st differences: $-5; -4; -3; -2; \dots$ 2nd differences: $+1; +1; +1; \dots$ Second differences are equal then the sequence is a quadratic sequence. substitute a, b and c into the general term formula:	✓ 1st ✓ 2nd ✓ Answer (3)
3.2	$2a = 1$ $\therefore a = \frac{1}{2}$ Substitute for b: $T_3 - T_2 = 5a + b = -4$ $\therefore b = -4 - 5\left(\frac{1}{2}\right)$ $\therefore b = \frac{-13}{2}$ Substitute for c: $T_1 = a + b + c = 18$ $\therefore c = 18 - \left(\frac{1}{2} - \frac{13}{2}\right)$ $\therefore c = 24$ $T_n = an^2 + bn + c$ $T_n = \left(\frac{1}{2}\right)n^2 + \left(-\frac{13}{2}\right)n + 24$ $T_n = \frac{1}{2}n^2 - \frac{13}{2}n + 24$	✓ Substitute ✓ a ✓ Substitute ✓ b ✓ Substitute ✓ c ✓ substitute ✓ expand ✓ T_n (9)
3.3	$T_6 = \frac{1}{2}6^2 - \frac{13}{2}6 + 24$ $T_6 = \frac{1}{2}36 - \frac{13}{2}6 + 24$ $T_6 = 18 - 39 + 24$ $T_6 = 3$	✓ Substitute ✓ Expand ✓ T_6 (3)

4.1	8	(1)
4.2	$2a = 1$ $a = \frac{1}{2}$ $3a + b = 0$ $3\left(\frac{1}{2}\right) + b = 0$ $b = -\frac{3}{2}$ $\frac{1}{2} - \frac{3}{2} + c = 2$ $-\frac{2}{2} + c = 2$ $-1 + c = 2$ $c = 2 + 1$ $c = 3$ $T_n = an^2 + bn + c$ $T_n = \frac{1}{2}n^2 - \frac{3}{2}n + 3$	✓ Substitute ✓ a ✓ Substitute ✓ b ✓ Substitute ✓ c ✓ substitute ✓ expand ✓ T_n (9)

4.3	$\frac{1}{2}n^2 - \frac{3}{2}n + 3 = 1083$ $n^2 - 3n + 6 = 2166$ $n^2 - 3n - 2160 = 0$ $(n - 48)(n + 45) = 0$ $n = 48$	✓ Equation ✓ Factorization ✓ Factor ✓ n=48 (4)
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WEEK 7 - 10: TOPIC: FUNCTIONS AND GRAPHS

SUB-TOPIC:

$$f(x) = a(x + p) + q, \quad f(x) = a(x + p)^2 + q, \quad f(x) = \frac{a}{x+p} + q \text{ and} \\ f(x) = ab^{x+p} + q; b > 0; b \neq 1$$

Overview:

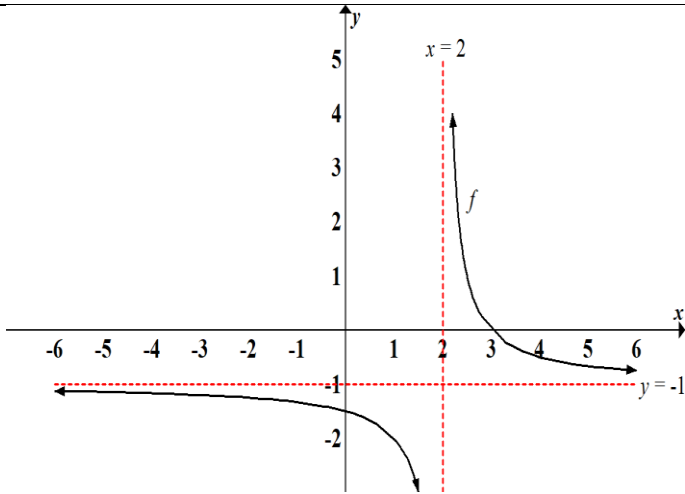
Paper 1: Marks :45±3

TEACHING APPROACH_HYPERBOLIC FUNCTIONS

- ❖ Use table method to illustrate the effect of "a", "p" and "q". Encourage self-discovery and explain asymptotes.
- ❖ Let learners draw rough sketches (If they know the values of a, p and q. at this stage ignore the intercepts).
- ❖ Introduce then intercepts $\left(0 = \frac{3}{x-1} + 2\right)$ and draw the graph.
- ❖ Write asymptotes in equation form.
- ❖ Lines of symmetry: $y = x + c$ and $y = -x + c$.
- ❖ Domain and range.
- ❖ Finding the equation of the graph.
- ❖ Making deductions from the given sketch.
- ❖ Combination of the two or more graphs – focus on the points of intersection.
- ❖ Hyperbola has two arms at all times if is complete and show at least one point on the other arm with intercepts on the other, but there some cases where it has one arm when the constrain is $x < 0$ or $x > 0$

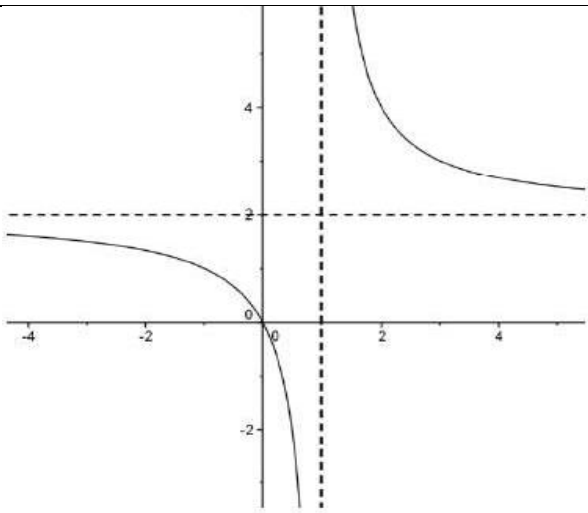
SOLUTIONS TO LEARNERS' ACTIVITY

QUESTION 1

$f(x) = \frac{1}{x-2} - 1$			[15]
No.	Calculations	Marking Descriptors	Marks
1.1	$x=2$ $y=-1$	✓ Horizontal asymptote ✓ Vertical asymptote	2
1.2	$\frac{1}{x-2} - 1 = 0$ $1 = x - 2$ $x = 3$	✓ Method $x=0$ and $y=0$ ✓ $x=3$ ✓ $y=-\frac{3}{2}$	3
1.3		✓ Both intercepts ✓ Shape ✓ Both asymptote	3
1.4	$y = \frac{1}{x-2} - 1$ $2 = \frac{1}{k-2} - 1$ $3k - 6 = 1$ $3k = 7$ $k = \frac{7}{3}$	✓ Substitution ✓ Answer	2

1.5	$y = \frac{1}{x-2} - 1$ $= \frac{1-(x-2)}{x-2}$ $= \frac{1-x+2}{x-2}$ $= \frac{-x+3}{x-2}$	✓ LCD ✓ Simplification	2
1.6	2 units to the left and 1 unit up.	✓✓ Answer	2
1.7	$x \in \mathbb{R}, \quad x \neq 0$	✓ Answer	2

QUESTION 2

$f(x) = \frac{2}{x-1} + 2$			
No	Calculations	Marking Descriptors	Marks
2.1	$x=1$ $y=2$	✓ $x=1$ ✓ $y=2$	2
2.2		✓ Intercept of f at $(0;0)$ ✓ Shape of f ✓ Asymptotes	3
2.3	$y-2 = -1(x-1)$ $y = -x+3$ OR	✓ $m = -1$ ✓ Substitute $(2;-1)$ ✓ Equation	3

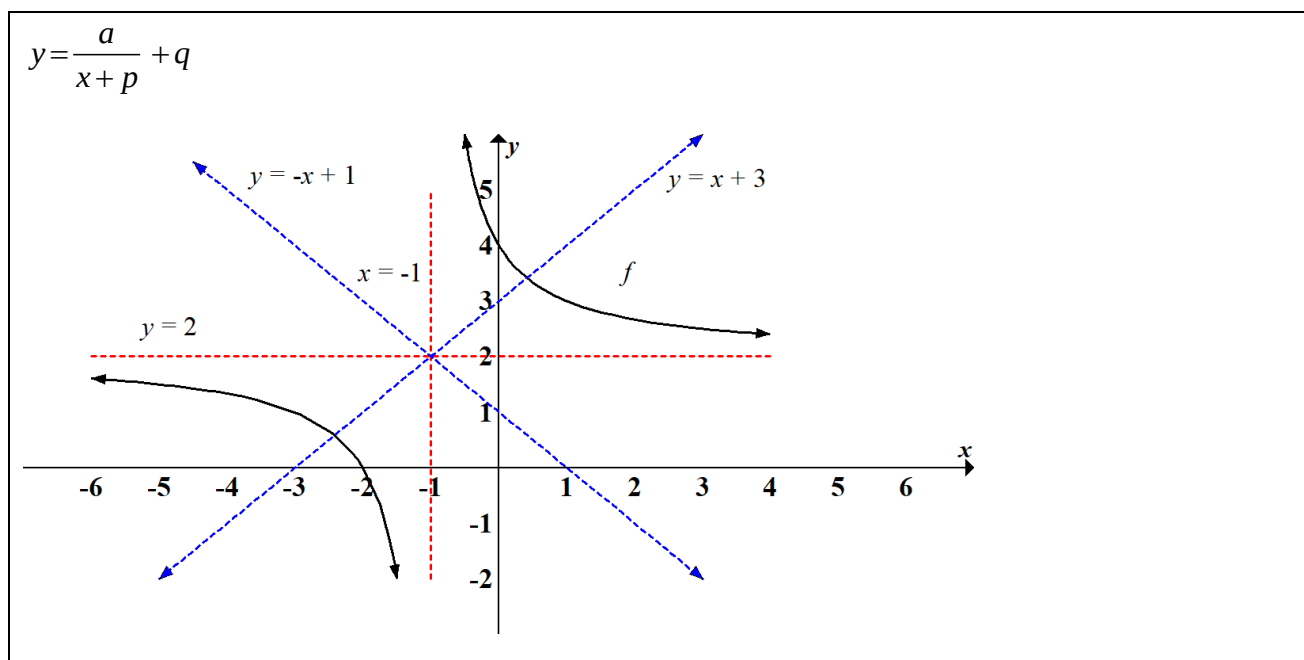
	$y = -x$ that has been moved 1 unit to the right and 2 units upwards $y = -(x-1)+2$ $y = -x+3$	OR ✓ $-(x-1)$ ✓ $+2$ ✓ Equation	
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QUESTION 3

$f(x)-4=\frac{p}{x-2}$			
No	Calculations	Marking Descriptors	Marks
3.1	$x=2$ $y=4$	✓ $x=2$ ✓ $y=4$	2
3.2	$f(3)-4=\frac{p}{3-2}$ $7-4=p$ $p=3$	✓ Substitution ✓ Answer	2

3.3	Decreasing function because $p > 0$ OR Decreasing, as x -increases, y -decreases	✓ Decreasing ✓ Reason OR ✓ Decreasing ✓ Reason	2
3.4	(2;4) $y = -x + c$ $y = x + c$ $4 = -2 + c$ $4 = 2 + c$ $c = 6$ $c = -2$ $LHS : -1 + 6$ $= 5$ $= RHS$ $\therefore (1;5)$ lies on the line	✓ Equation of axis of symmetry ✓ Equation of axis of symmetry ✓ Testing coordinate	3
3.5	$2 < x \leq 3$	✓ Endpoints ✓ Notation	2

QUESTION 4

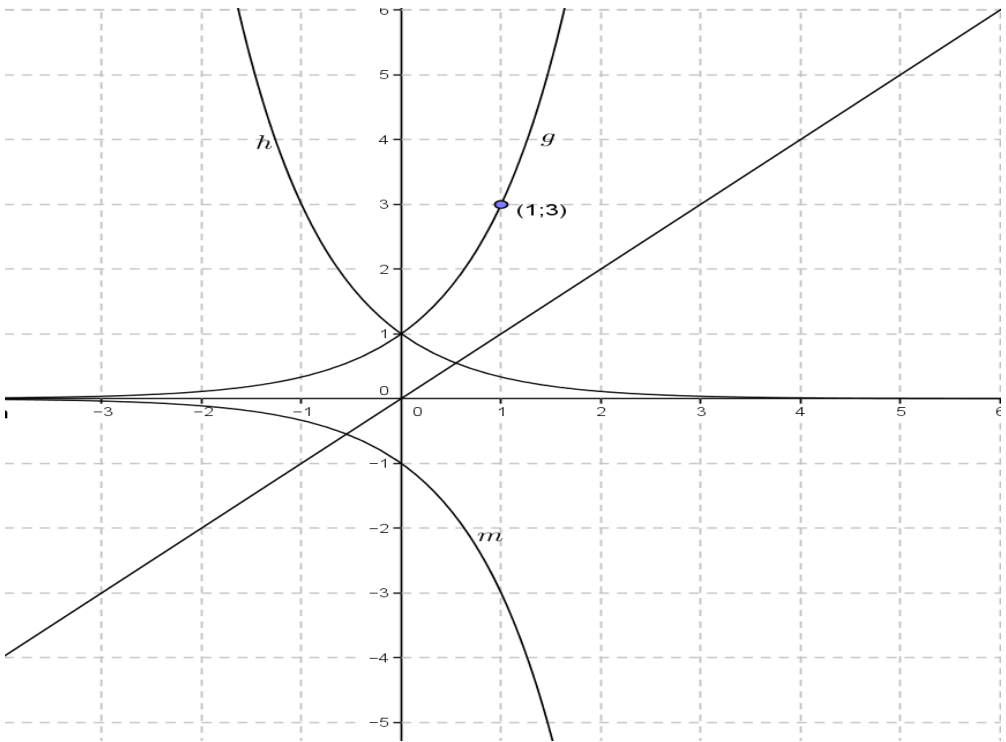


No	Calculations	Marking Descriptors	Marks
4.1	$p=1$ $q=2$	✓ Value of p ✓ Value of q	2
4.2	$y = \frac{a}{x+1} + 2$ $4 = \frac{a}{0+1} + 2$ $a = 2$ $\therefore y = \frac{2}{x+1} + 2$	✓ Substitution of point ✓ Value of a ✓ Equation	3
4.3	Point of intersection of axes of symmetry of f is $(-1; 2)$ Point of intersection of axes of symmetry of g is: $x - 3 = -x + 1$ $2x = 4$ $x = 2$ $y = -1$ Transformation is from $(-1; 2) \rightarrow (2; -1)$ $\therefore$ 3 units to the right and 3 units down	✓ Point of intersection ✓ Equating ✓ Both x - and y -values ✓ Method ✓ Answer	5

EXERSICE2

QUESTION 1

On the Cartesian plane below is the graph of $g(x) = 3^x$.
1.1 , 1.4 and 1.6

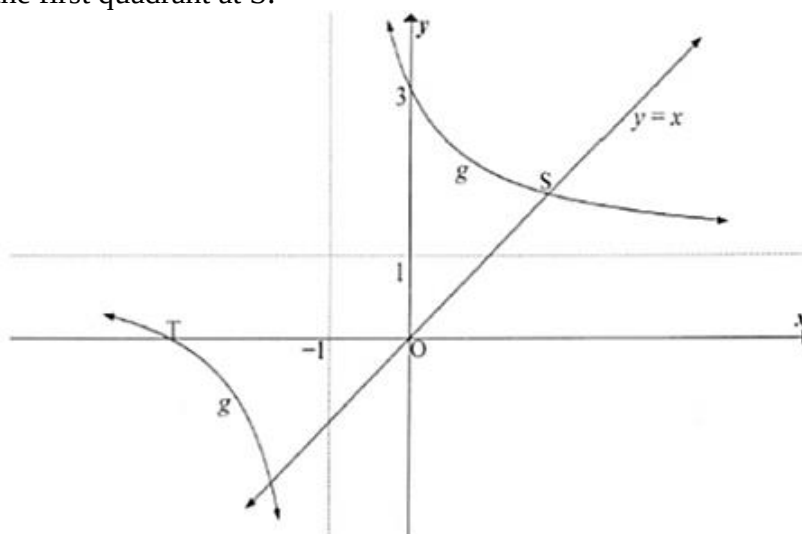


1.2	h is the reflection of g in the y -axis.
1.3	$h(x) = g(-x) = 3^{-x}$
1.5	$m(x) = -g(x) = -3^x$

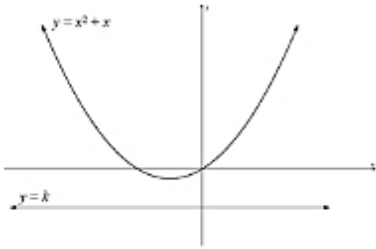
QUESTION 2

The diagram below shows hyperbola g defined by the equation $g(x) = \frac{2}{x+1} + 1$.

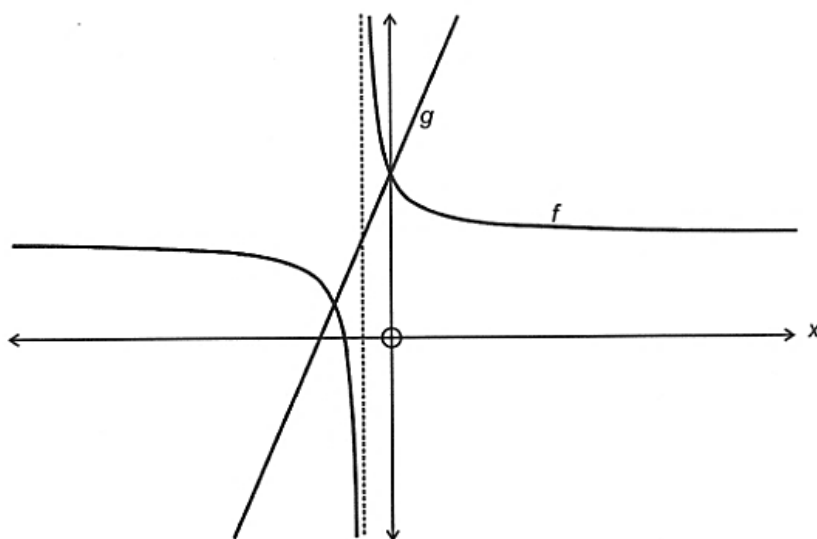
The graph of g intersects the x -axis at T and the y -axis at (0;3). The line $y = x$ intersects the hyperbola in the first quadrant at S.



2.1	$0 = \frac{2}{x+1} + 1$ $-x - 1 = 2$ $x = -3$ OR/OF Reflect (0 ; 3) across $y = -x$ to get T(-3 ; 0) $x = -3$ Reflekteer (0 ; 3) om $y = -1$ om T(-3 ; 0) te kry $x = -3$	$\checkmark 0 = \frac{2}{x+1} + 1$ $\checkmark x = -3$ (2) $\checkmark \text{ reflect across/reflekteer om } y = -x$ $\checkmark x = -3$ (2)	
2.2	$\frac{2}{x+1} + 1 = x$ $2 + x + 1 = x^2 + x$ $x^2 = 3$ $\therefore x = \sqrt{3} \text{ since at S, } x > 0$ $y = \sqrt{3} = 1,73\dots$ $OS^2 = x^2 + y^2 = 3 + 3 = 6$ $\therefore OS = \sqrt{6} = 2,45 \text{ units/eenhede}$ OR/OF	$\checkmark \text{ equating both graphs/stel grafieke gelyk}$ $\checkmark x^2 = 3$ $\checkmark x = \sqrt{3} \text{ and } y = \sqrt{3}$ $\checkmark OS^2 = 6$ $\checkmark \text{ answer/antwoord} \quad (5)$	

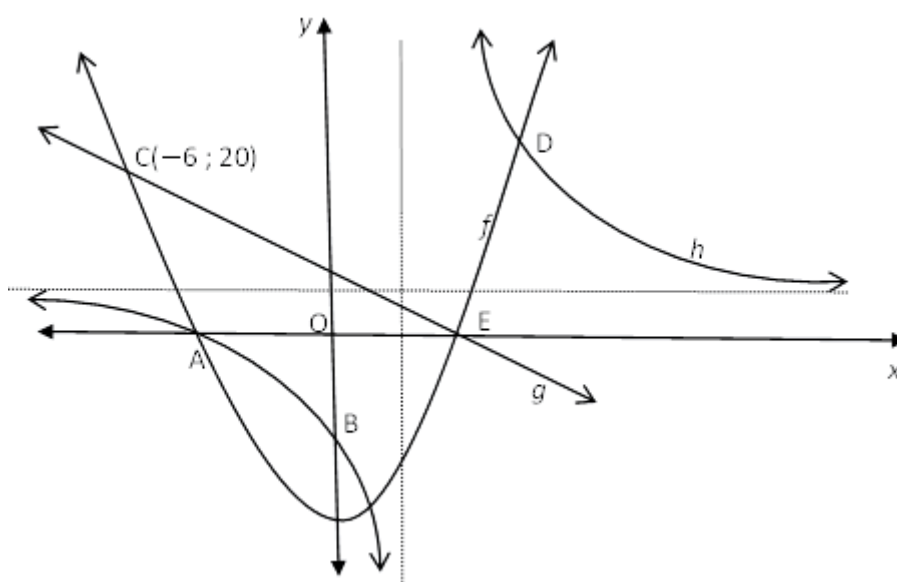
	Translate g one unit down and one unit to the right/<i>Transleer g een eenheid af en een eenheid na regs</i> The new equation/<i>Die nuwe vergelyking</i> : $p(x) = \frac{2}{x}$ Therefore the image of S is $S'(\sqrt{2}; \sqrt{2})$/ <i>Daarom is die beeld van S nou $S'(\sqrt{2}; \sqrt{2})$</i> Now translate p back to g/<i>Transleer p terug na g</i>: $S(\sqrt{2}-1; \sqrt{2}+1)$ $OS^2 = (\sqrt{2}-1)^2 + (\sqrt{2}+1)^2 = 2 - 2\sqrt{2} + 1 + 2 + 2\sqrt{2} + 1$ $\therefore OS = \sqrt{6} = 2,45 \text{ units/eenhede}$	$\checkmark p(x) = \frac{2}{x}$ $\checkmark\checkmark$ coord. of/koörd. van S' $\checkmark$ coord. of/koörd. van S $\checkmark$ answer/antwoord (5)	
2.3	$k < 3$ will give roots with opposite signs/ <i>$k < 3$ sal wortels met teenoorgestelde tekens gee</i> Need 2 x-intercepts. When $q = 1$ there is only one x-intercept By shifting the graph down 3 units i.e. making $k = -3$ or -2 etc the y-intercept becomes 0. I think this should be the answer: $k < 3$	$\checkmark k < 3$ (1) ---	
2.4	Solution 1 of 3 $x^2 + x = k$ $x^2 + x - k = 0 \checkmark \text{ standard form}$ $\Delta < 0 \checkmark$ $b^2 - 4ac < 0$ $1^2 - 4(1)(-k) \checkmark < 0$ $1 + 4k < 0$ $k < \frac{-1}{4} \checkmark$ OR Solution 2 of 3 $x^2 + x = k$ $x^2 + x + \frac{1}{4} = k + \frac{1}{4} \checkmark \text{ add } \frac{1}{4} \text{ to both sides}$ $\left(x + \frac{1}{2}\right)^2 = k + \frac{1}{4} \checkmark$ For non-real roots $k + \frac{1}{4} < 0 \checkmark$ $k < \frac{-1}{4} \checkmark\checkmark$ OR Solution 3 of 3 Consider the functions $y = x^2 + x$ and $y = k$  $\checkmark$ sketch or explanation Turning point of $y = x^2 + x$ is $\left(\frac{-1}{2} \checkmark; \frac{-1}{4} \checkmark\right)$ $x^2 + x = k$ does not have real roots when the line $y = k$ does not intersect $y = x^2 + x$ Therefore $k < \frac{-1}{4} \checkmark$	(4)	

QUESTION 3



3.1	For a vertical asymptote at $x = -1$, $b = 1$ $g(-1) = 3$ so we know that the horizontal asymptote of f is $y = 3$ $\therefore c = 3$ finally we know that $(0;5)$ lies on f $\therefore 5 = \frac{a}{0+1} + 3$ $\therefore a = 2$	
3.2.1	for x-intercepts let $y = 0$ $\frac{2}{x+1} + 3 = 0$ $\therefore \frac{2}{x+1} = -3$ $\therefore x+1 = -\frac{2}{3}$ $\therefore x = -\frac{5}{3}$ is x-intercept of f and $-\frac{5}{2}$ is x-intercept of g	
3.2.2	$x \leq -\frac{5}{2}$ or $-\frac{5}{3} \leq x < -1$	

QUESTION 4

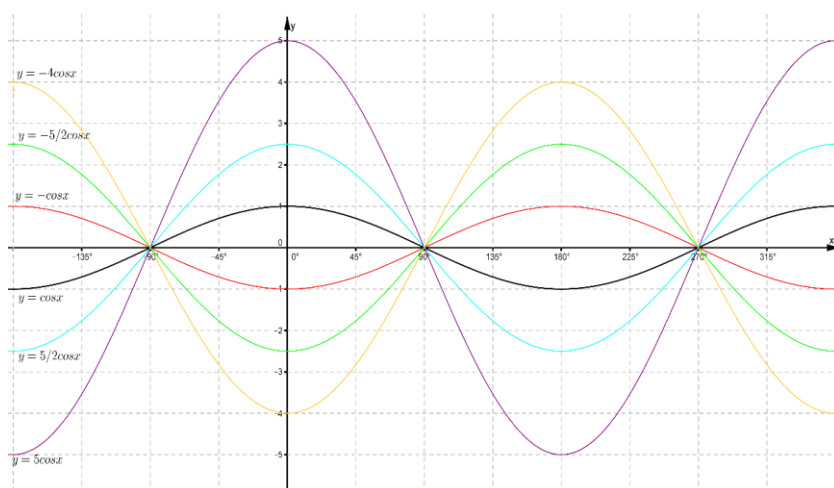


4.1	at A, $y = 0 \therefore \frac{6}{x-2} + 1 = 0$ $6 = -x + 2$ $4 = -x$ $\therefore x = -4$ Thus A(-4;0) $x = 0 \therefore y = \frac{6}{-2} + 1$ $y = -3 + 1$ $\therefore y = -2$ Thus B(0;-2) At E, $y = 0; \therefore 0 = -2x + 8$ $2x = 8$ $x = 4$ Thus E(4;0)	
4.2	$f(x) = x^2 + k$ and $C(-6; 20)$ is on f substitute the coordinates of C into $y = x^2 + k$ $(20) = (-6)^2 + k$ $20 - 36 = k$ $\therefore k = -16$ Write $f(x) = x^2 - 16$ and -16 on the diagram	

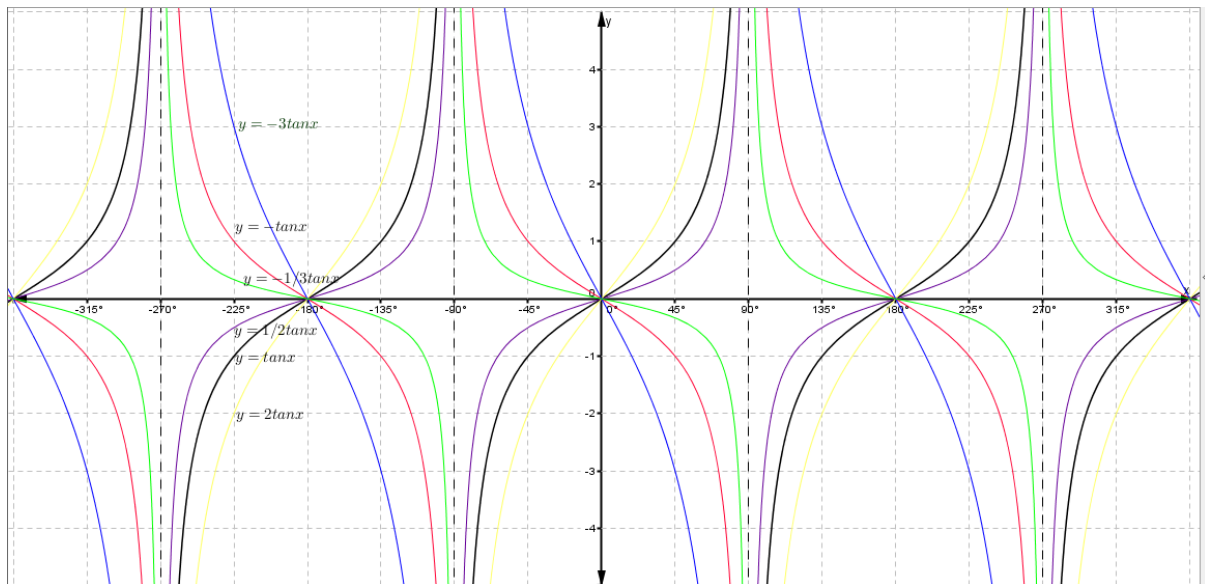
4.3	Domain of f is $x \in \mathbb{R}$	
4.4	Range of f is $y \geq -16$ or $y \in [-16; \infty)$	
4.5	$g(x) - f(x) \geq 0$ for $-6 \leq x \leq 4$ or $x \in [-6; 4]$	
4.6	$h(x) = \frac{6}{x-2} + 1$ Lines of symmetry are $y = \pm(x - 2) + 1$ so use $y = -(x - 2) + 1$ for the axis of symmetry with a negative slope OR PREFERABLY USE: $y = -x$ has been shifted up 1 unit and right 2 units $y = -(x - 2) + 1$ $y = -x + 2 + 1$ $y = -x + 3$	
4.7	$s(x) = f(x) + 2$ $= x^2 - 16 + 2$ $= x^2 - 14$ $y \geq -16 + 2 \text{ i.e. } y \geq -14$ or $y \in [-14; \infty)$	
4.8	$t(x) = h(x) + 2$ $= \frac{6}{x-2} + 1 + 2$ The horizontal asymptote is now at $1 + 2$. The range of t is $y \in \mathbb{R}; y \neq 3$ or $y \in (-\infty; 3) \cup (3; \infty)$	

SOLUTIONS ON SKETCHING

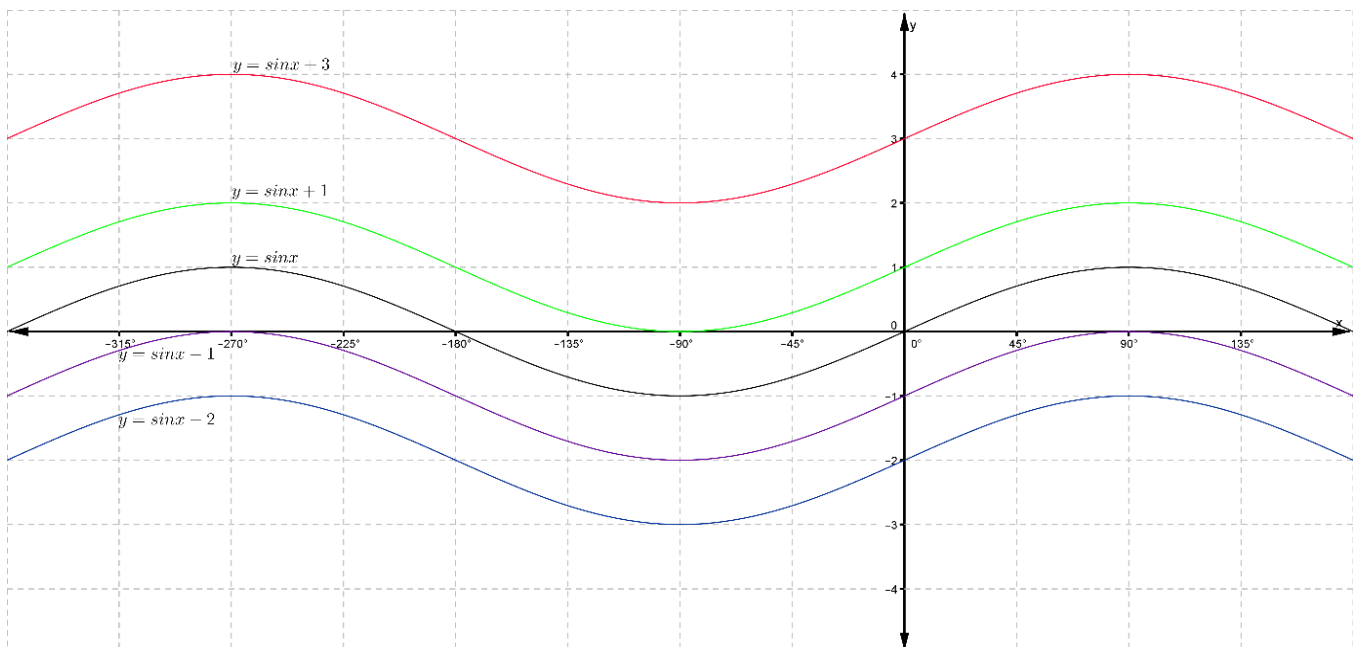
1. $y = \cos x, y = -4 \cos x, y = -\frac{5}{2} \cos x, y = -\cos x, y = \frac{5}{2} \cos x$ AND $y = 5 \cos x$ when $x \in [-180^\circ; 360^\circ]$



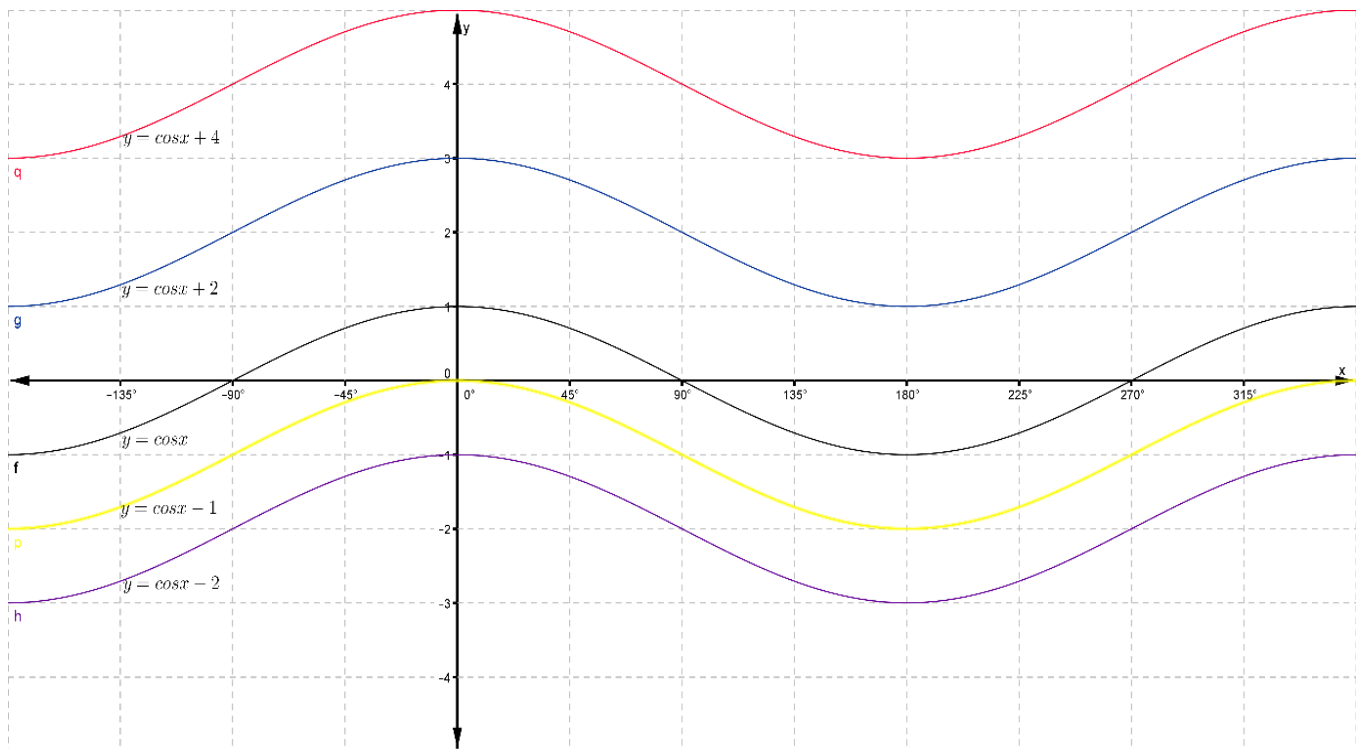
2. $y = \tan x, y = -3 \tan x, y = -\frac{1}{3} \tan x, y = \frac{1}{2} \tan x$ and $y = 2 \tan x, x \in [-360^\circ; 360^\circ]$



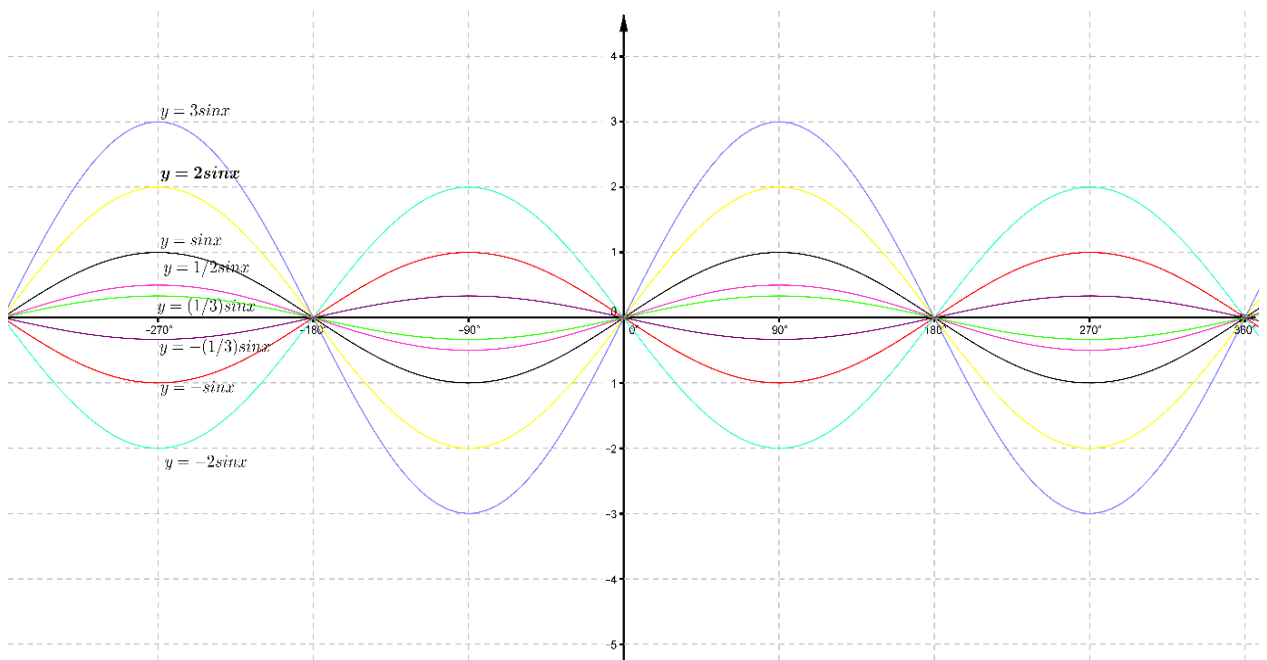
3. $y = \sin x, y = \sin x + 1, y = \sin x + 3, y = \sin x - 1, y = \sin x - 2$ when $-360^\circ \leq x \leq 180^\circ$.



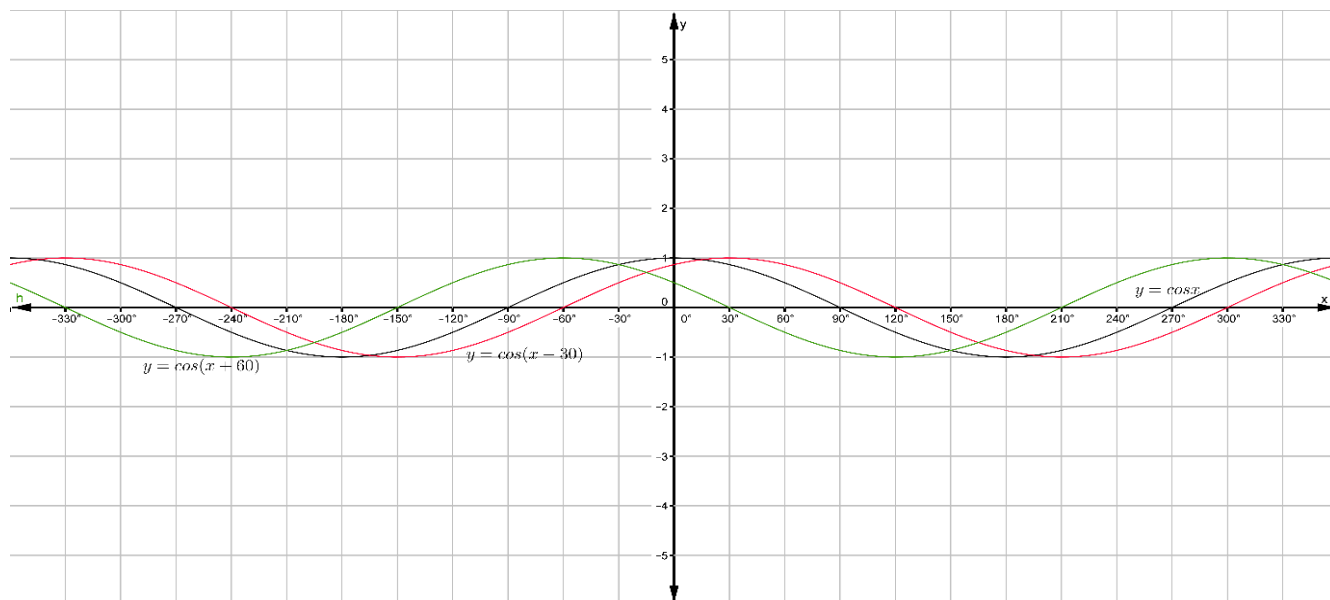
4. $y = \cos x, y = \cos x + 2, y = \cos x - 1, y = \cos x - 2, y = \cos x + 4, x \in [-180^\circ; 360^\circ]$



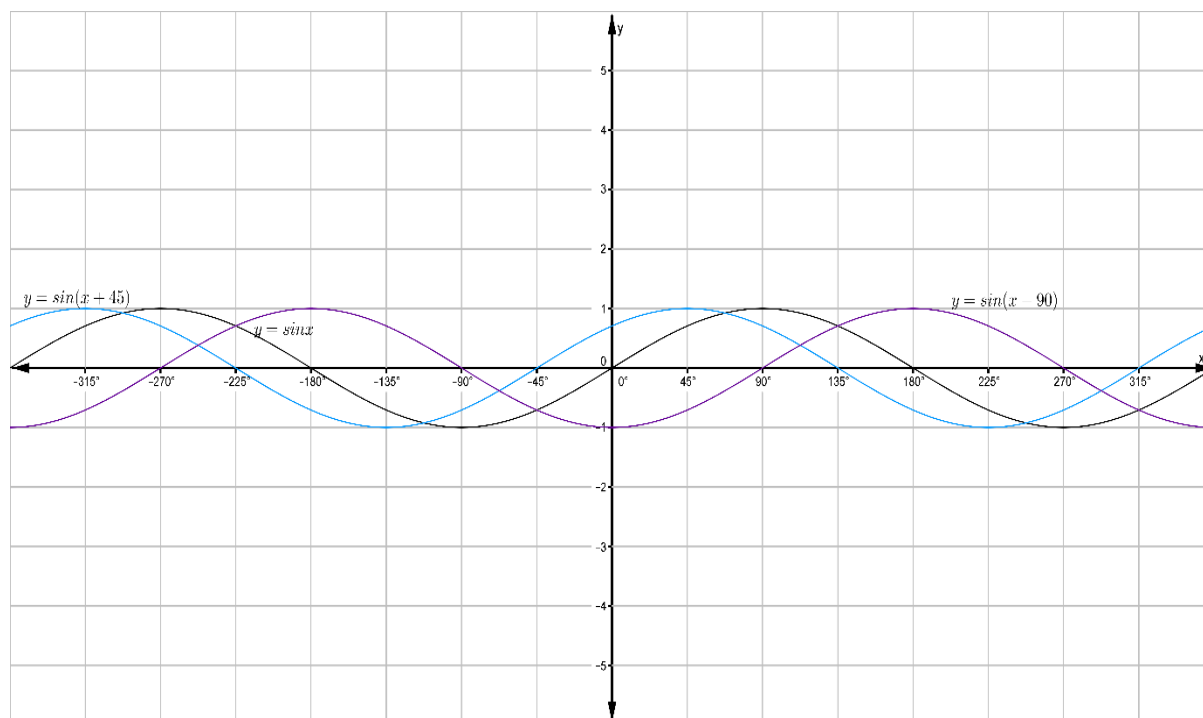
5. $y = \sin x, y = 2 \sin x, y = \frac{1}{2} \sin x, y = -2 \sin x, y = -\frac{1}{3} \sin x, y = \frac{1}{3} \sin x, y = -\sin x$
and $y = 3 \sin x, x \in [-360^\circ; 360^\circ]$



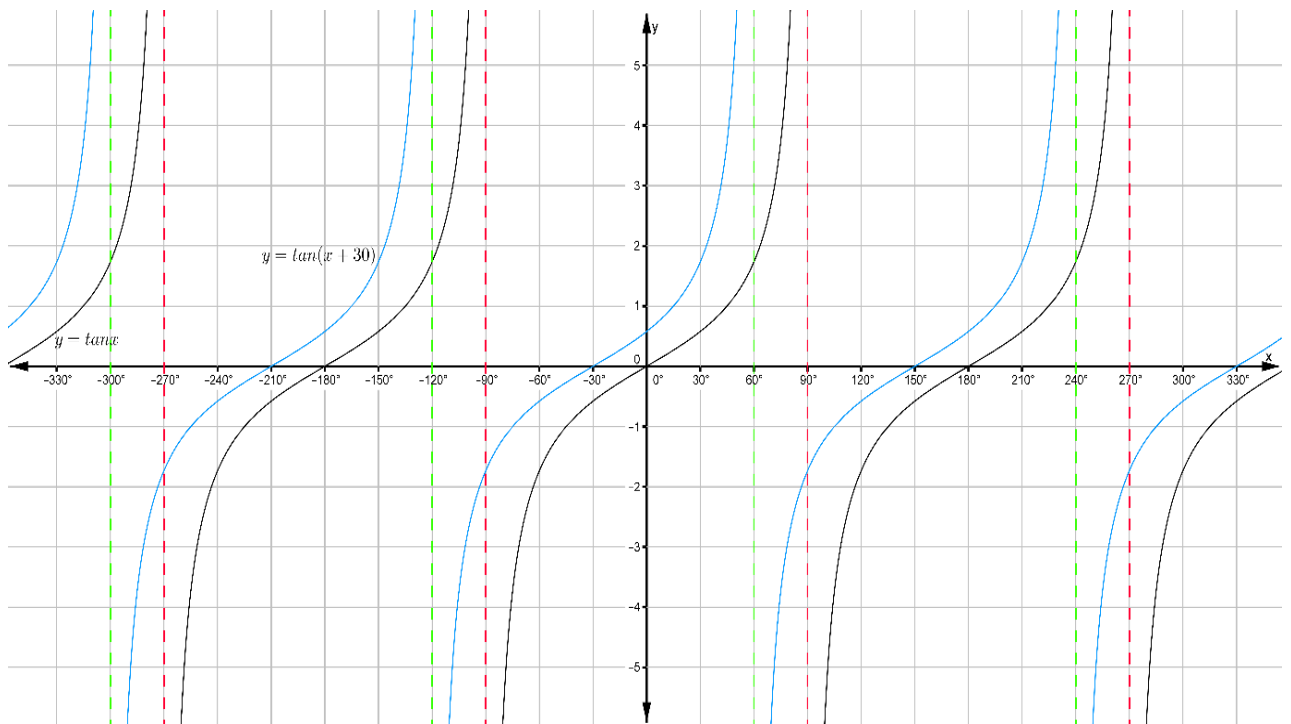
6 $y = \cos x$, $y = \cos(x - 60^\circ)$, $y = \cos(x + 30^\circ)$, when $-360^\circ \leq x \leq 360^\circ$



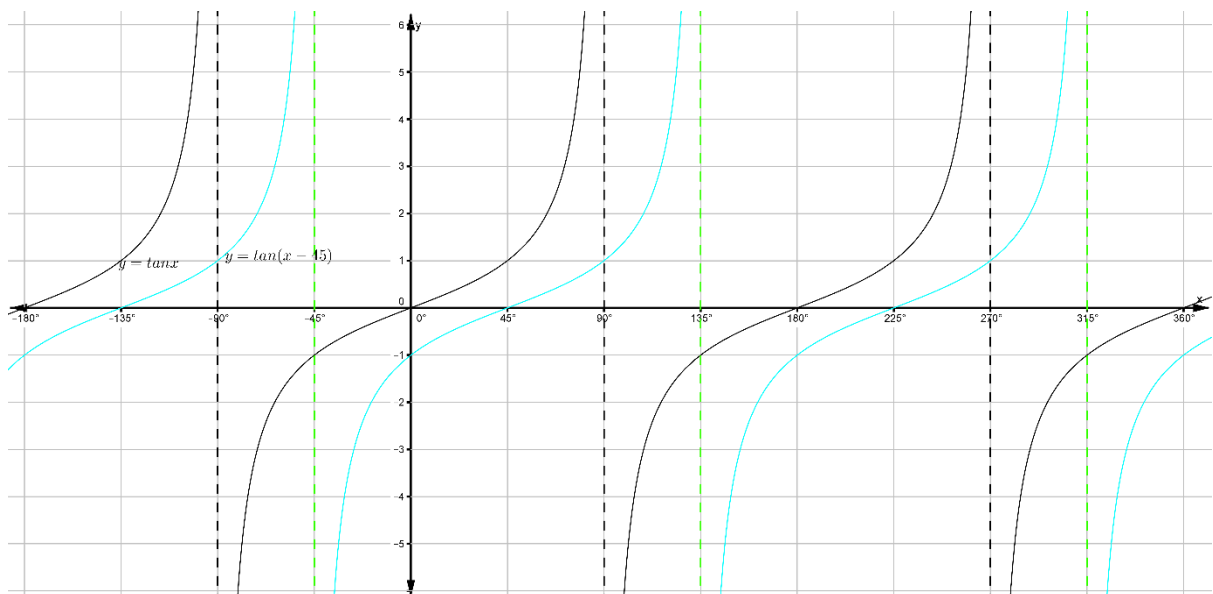
7 $y = \sin x$, $y = \sin(x + 45^\circ)$ and $y = \sin(x - 90^\circ)$ for $x \in [360^\circ; 360^\circ]$



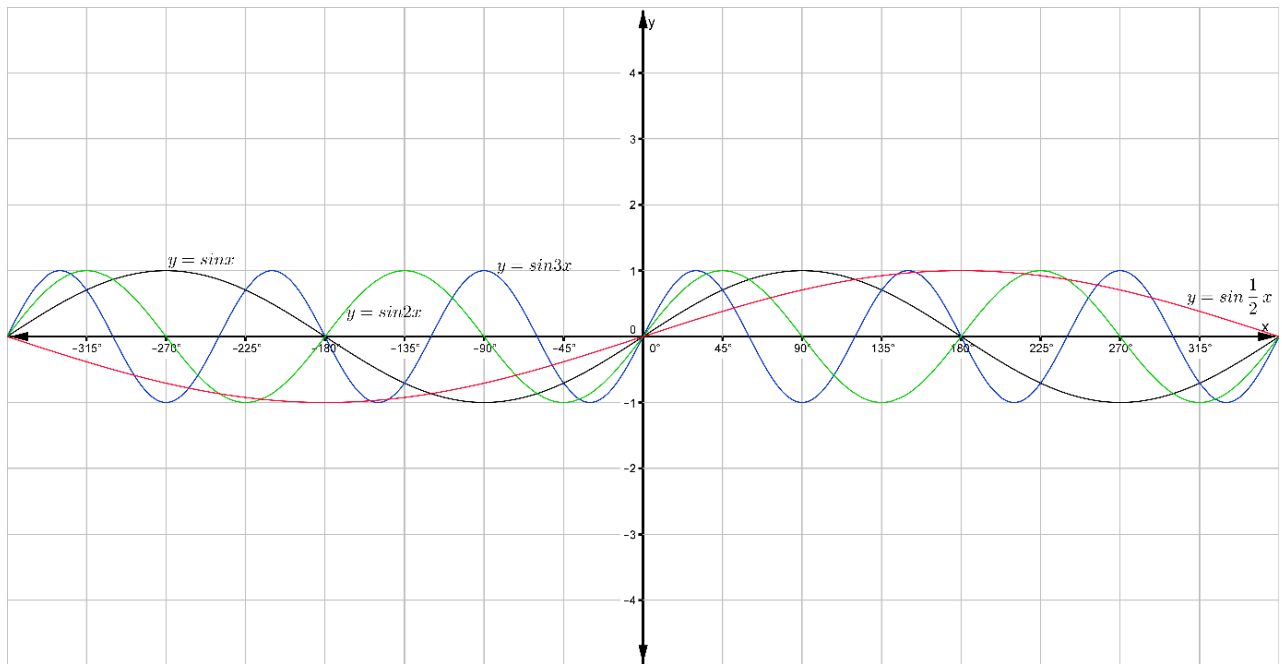
8 $y = \tan x$ and $y = \tan(x + 30^\circ)$ for $x \in [-360^\circ; 360^\circ]$



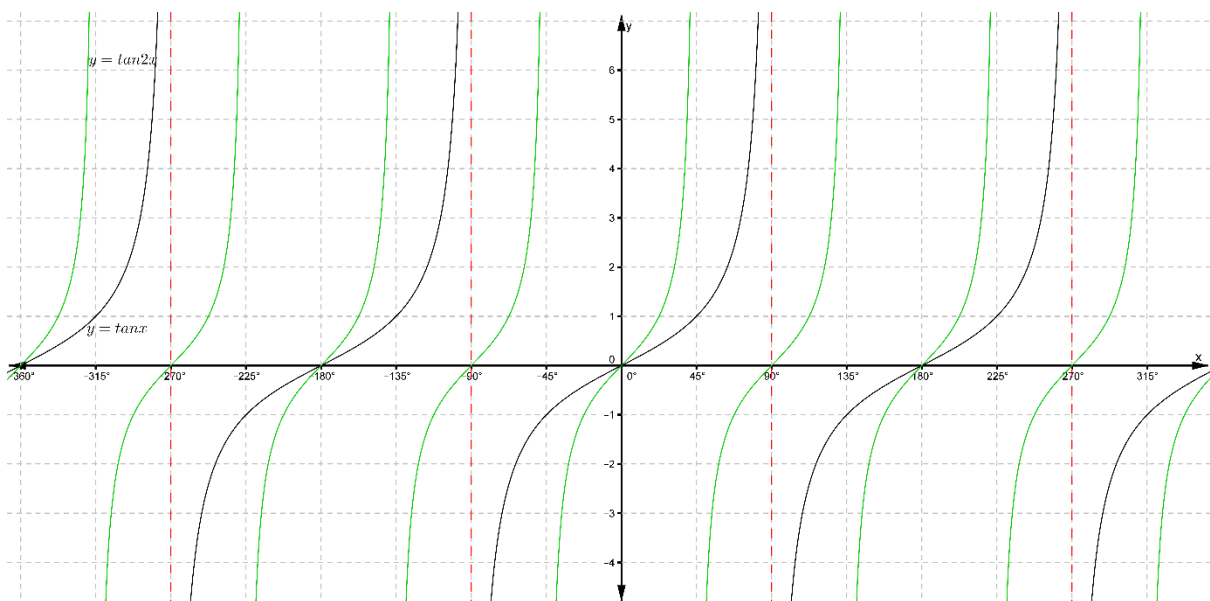
9 $y = \tan x$ and $y = \tan(x - 45^\circ)$ when $-180^\circ \leq x \leq 360^\circ$



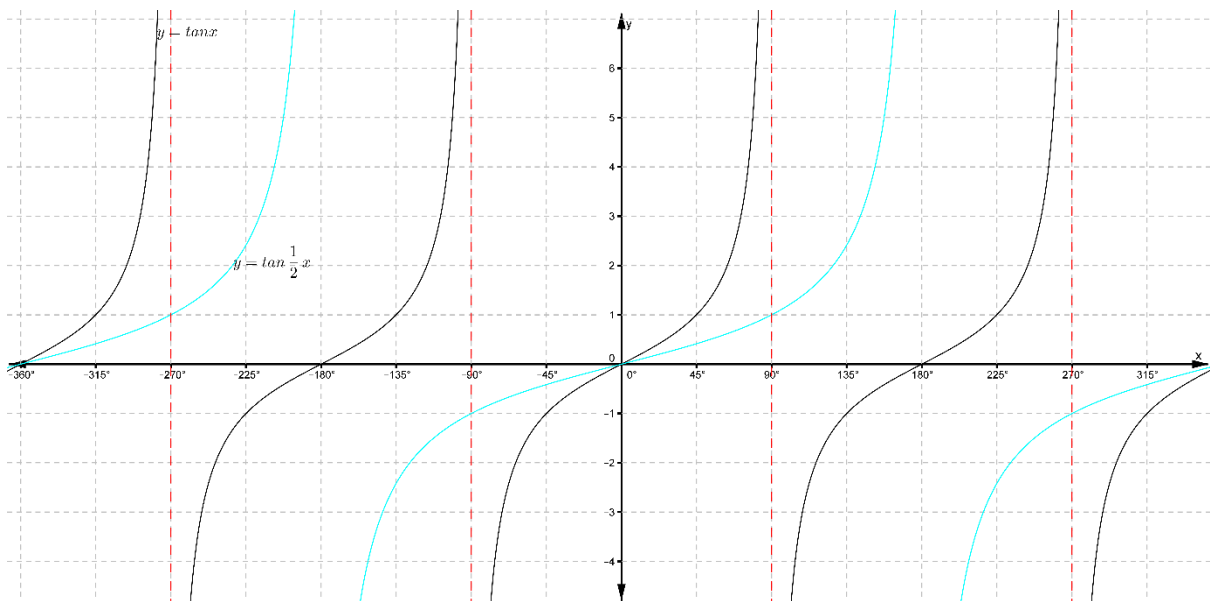
10 $y = \sin x$, $y = \sin 2x$, $y = \sin 3x$ and $y = \sin \frac{1}{2}x$ for $x \in [-360^\circ; 360^\circ]$



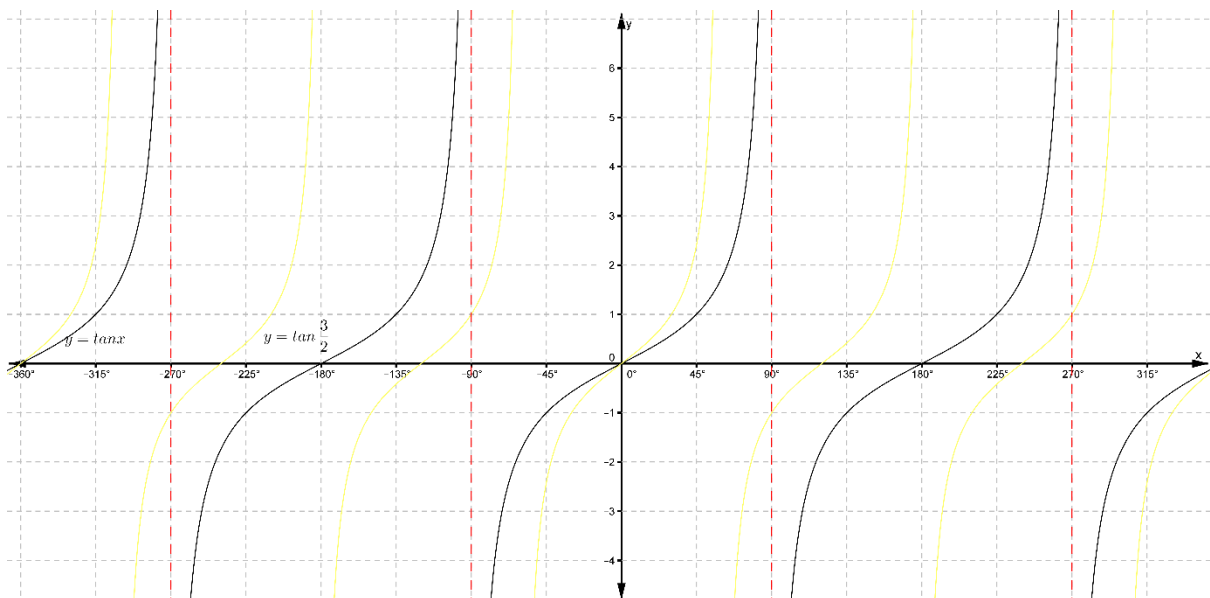
11 $y = \tan x$ and $y = \tan 2x$ when $-360^\circ \leq x \leq 360^\circ$



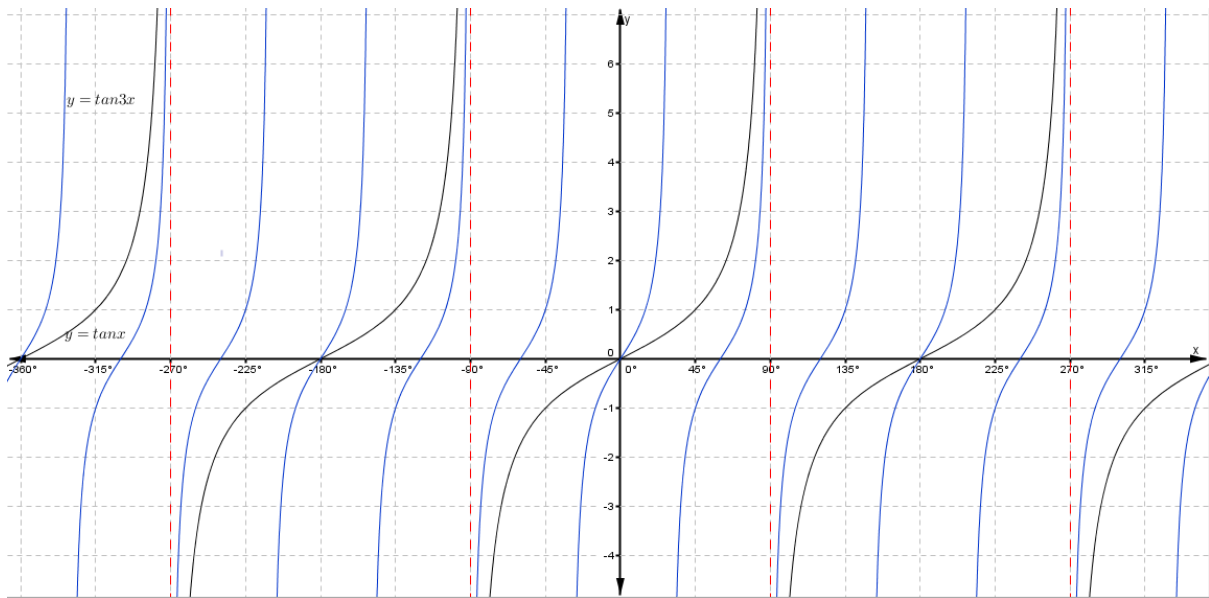
12 $y = \tan x$ and $y = \tan \frac{1}{2}x$ for $x \in [-360^\circ; 360^\circ]$



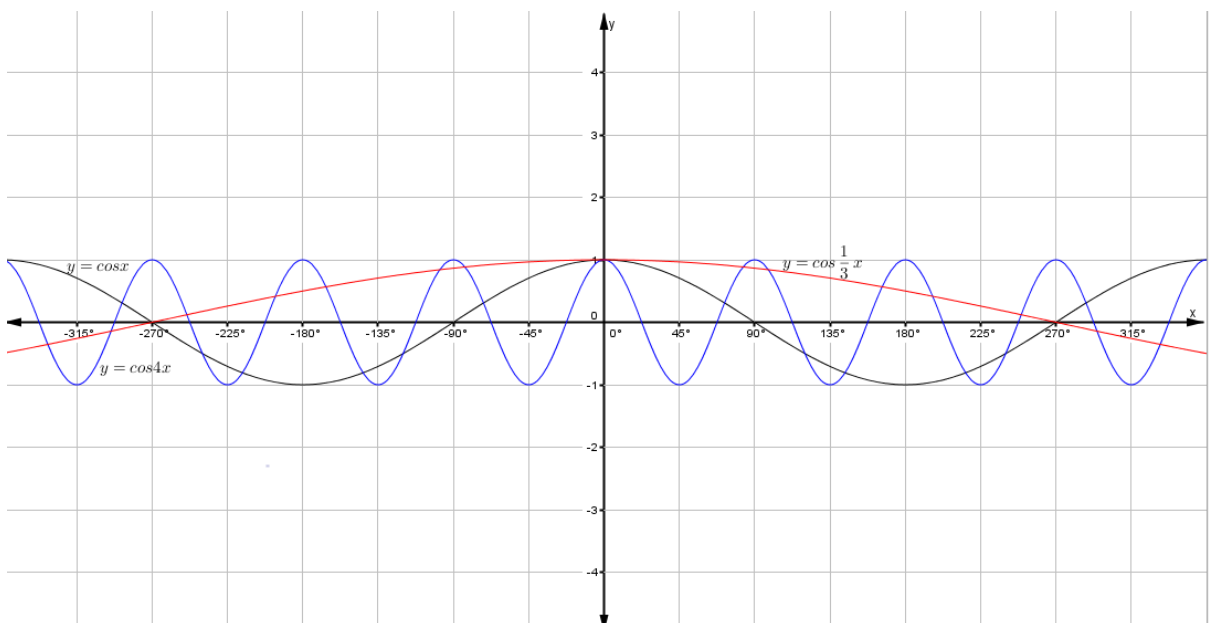
13 $y = \tan x$ and $y = \tan \frac{3}{2}x$, $x \in [360^\circ; 360^\circ]$



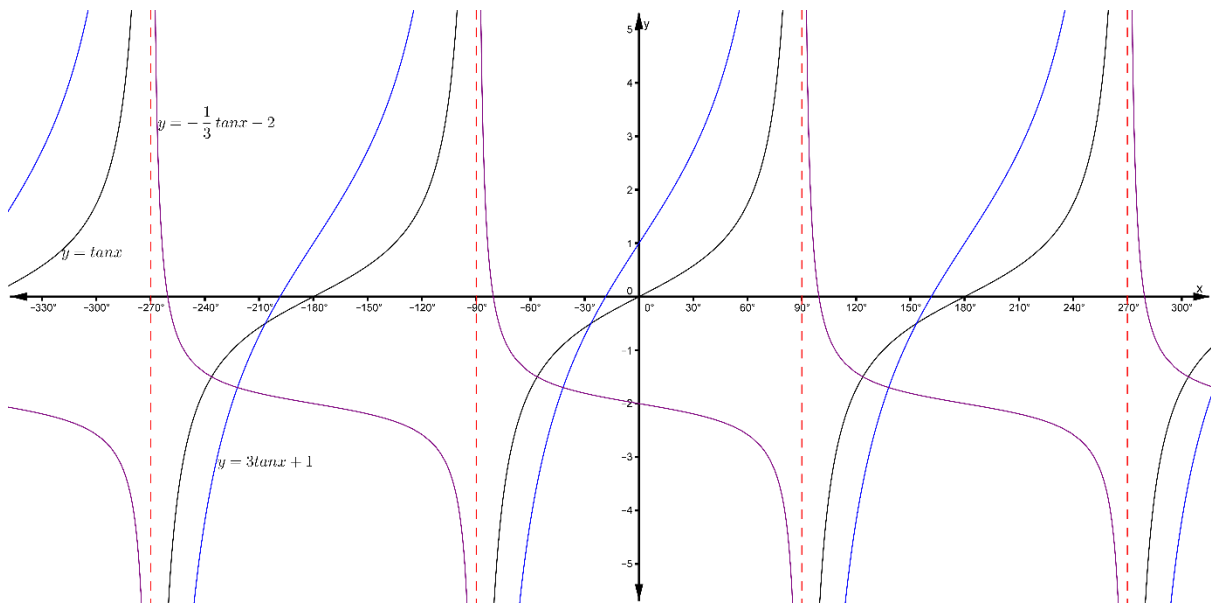
14 $y = \tan x$ and $y = \tan 3x$, $x \in [-360^\circ; 360^\circ]$



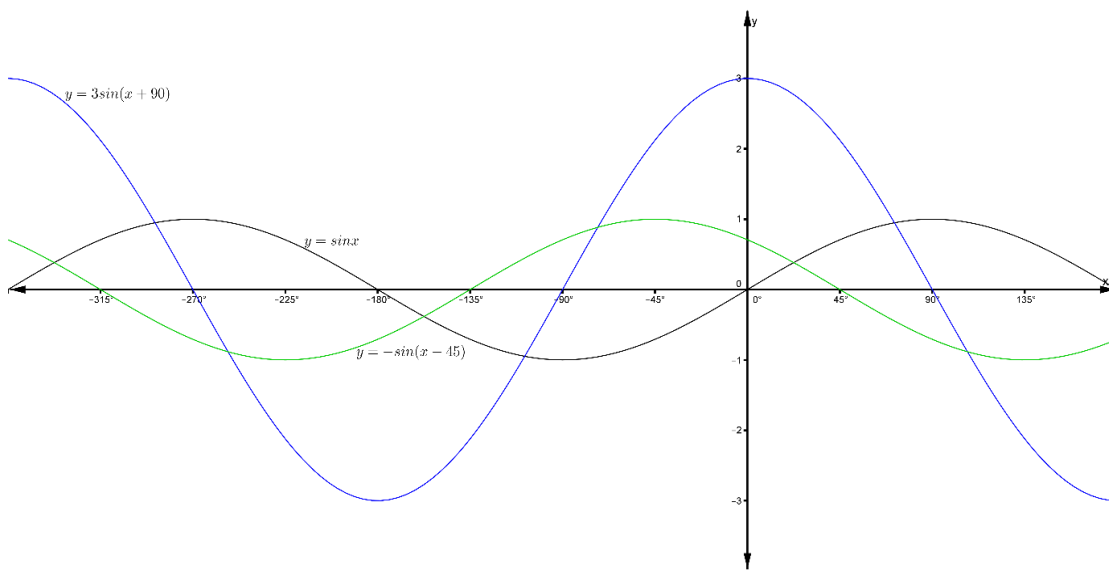
15 $y = \cos x$, $y = \cos \frac{1}{3}x$ and $y = \cos 4x$, $x \in [360^\circ; 360^\circ]$



16 $y = \tan x$, $y = -\frac{1}{3}\tan x - 2$ and $y = 3\tan x + 1$, when $-360^\circ \leq x \leq 360^\circ$

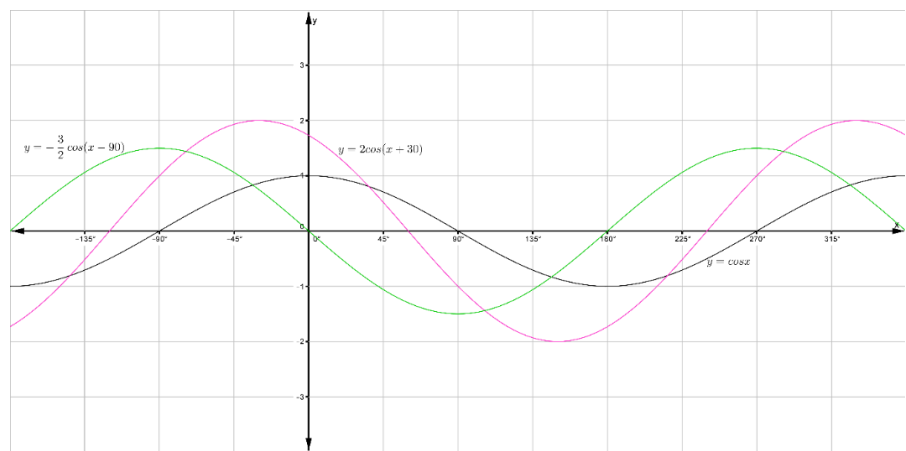


17 $y = \sin x$, $y = -\sin(x - 45^\circ)$ and $y = -3\tan(x + 90^\circ)$ for $x \in [-360^\circ; 180^\circ]$



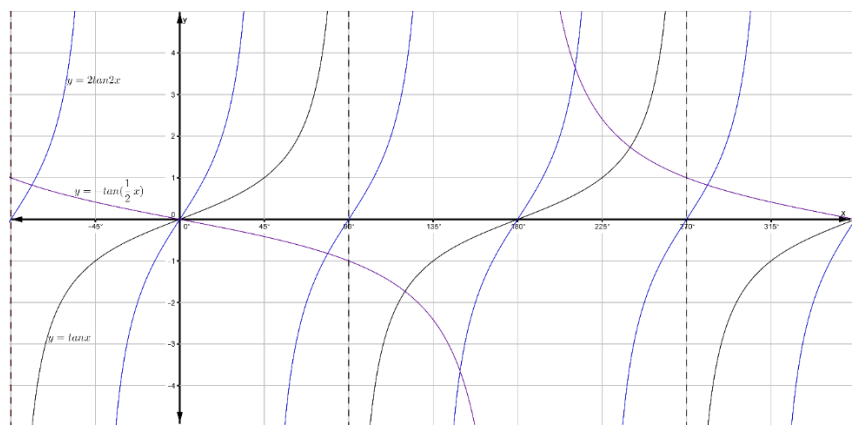
SOLUTIONS – DESCRIPTION OF GRAPHS -SOLUTIONS

1



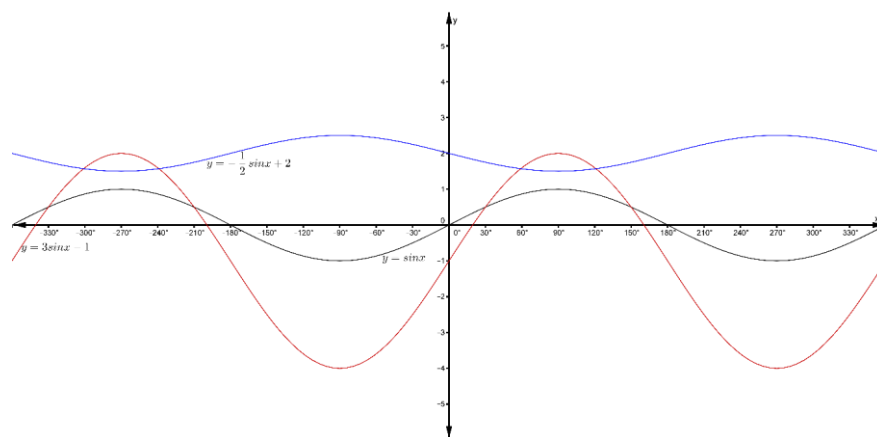
Function	Domain	Range	Amplitude	Period
$y = \cos x$	$x \in [-360^0; 360^0]$	$y \in [-1; 1]$	1	360^0
$y = -\frac{3}{2} \cos(x - 90^0)$	$x \in [-360^0; 360^0]$	$y \in [-1.5; 1.5]$	1.5	360^0
$y = 2 \cos(x + 30^0)$	$x \in [-360^0; 360^0]$	$y \in [-2; 2]$	2	360^0

2



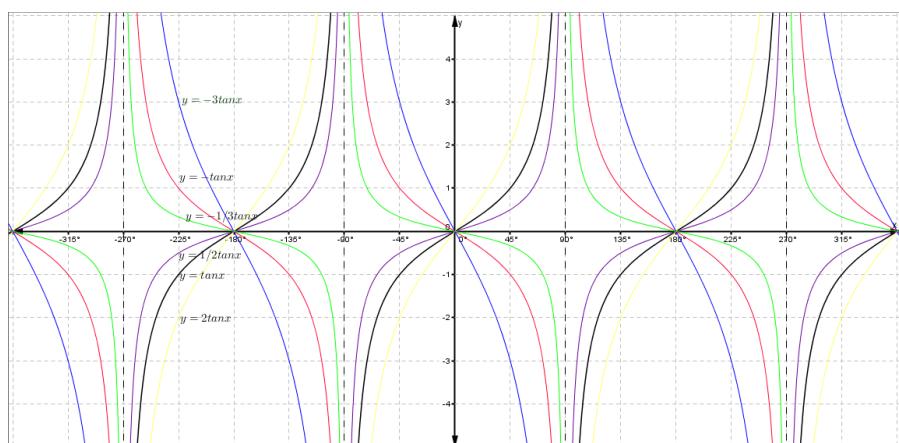
Function	Domain	Range	Amplitude	Period
$y = \tan x$	$x \in [-90^0; 360^0]$	$y \in \mathbb{R}$	No amplitude because the maximum and the minimum values do not exist.	180^0
$y = -\tan\left(\frac{1}{2}x\right)$	$x \in [-90^0; 360^0]$	$y \in \mathbb{R}$		360^0
$y = 2 \tan 2x$	$x \in [-90^0; 360^0]$	$y \in \mathbb{R}$		90^0

3



Function	Domain	Range	Amplitude	Period
$y = \sin x$	$x \in [-360^0; 360^0]$	$y \in [-1; 1]$	1	360^0
$y = -\frac{1}{2}\sin x + 2$	$x \in [-360^0; 360^0]$	$y \in [1.5; 2.5]$	0.5	360^0
$y = 3 \sin x - 1$	$x \in [-360^0; 360^0]$	$y \in [-4; 2]$	3	360^0

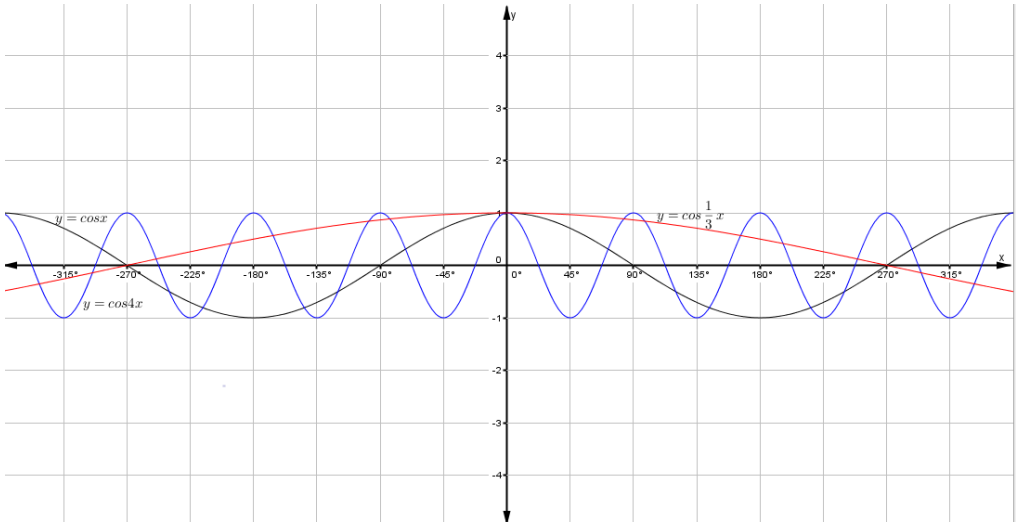
4



Function	Domain	Range	Amplitude	Period
$y = \tan x$	$x \in [-360^0; 360^0]$	$y \in \mathbb{R}$	No amplitude because the maximum and the minimum values do not exist.	180^0
$y = 2 \tan x$	$x \in [-360^0; 360^0]$	$y \in \mathbb{R}$		180^0

$y = \frac{1}{2} \tan x$	$x \in [-360^0; 360^0]$	$y \in \mathbb{R}$		180^0
$y = -\tan x$	$x \in [-360^0; 360^0]$	$y \in \mathbb{R}$		180^0
$y = -3 \tan x$	$x \in [-360^0; 360^0]$	$y \in \mathbb{R}$		180^0
$y = -\frac{1}{3} \tan x$	$x \in [-360^0; 360^0]$	$y \in \mathbb{R}$		180^0

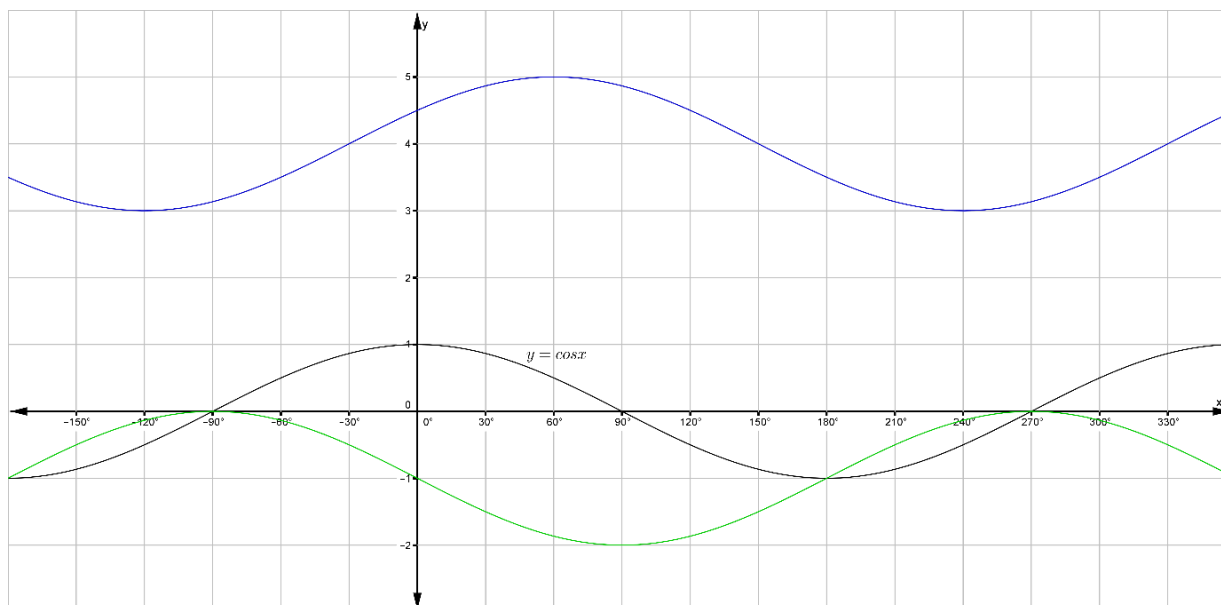
5



Function	Domain	Range	Amplitude	Period
$y = \cos x$	$x \in [-360^0; 360^0]$	$y \in [-1; 1]$	1	360^0
$y = \cos 4x$	$x \in [-360^0; 360^0]$	$y \in [-1; 1]$	1	360^0
$y = \cos \frac{1}{3}x$	$x \in [-360^0; 360^0]$	$y \in [-1; 1]$	1	360^0

SOLUTIONS – EQUATIONS OF DRAWN GRAPHS

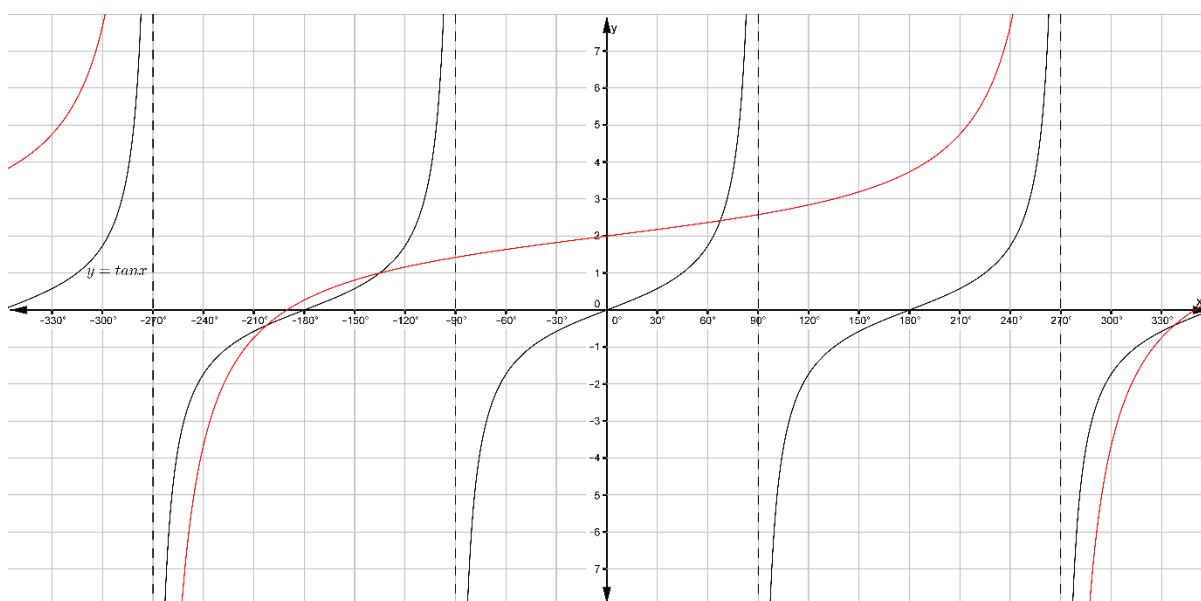
1



Blue: $y = \cos(x - 60^\circ) + 4$

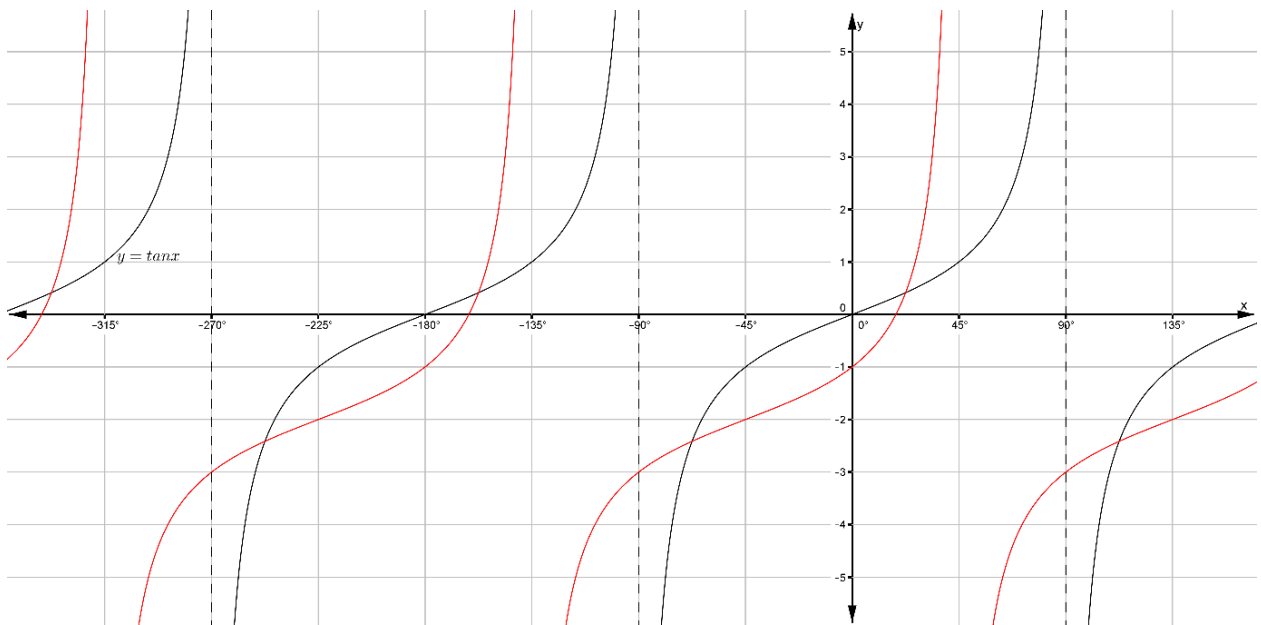
Green: $y = \cos(x + 90^\circ) - 1$

2



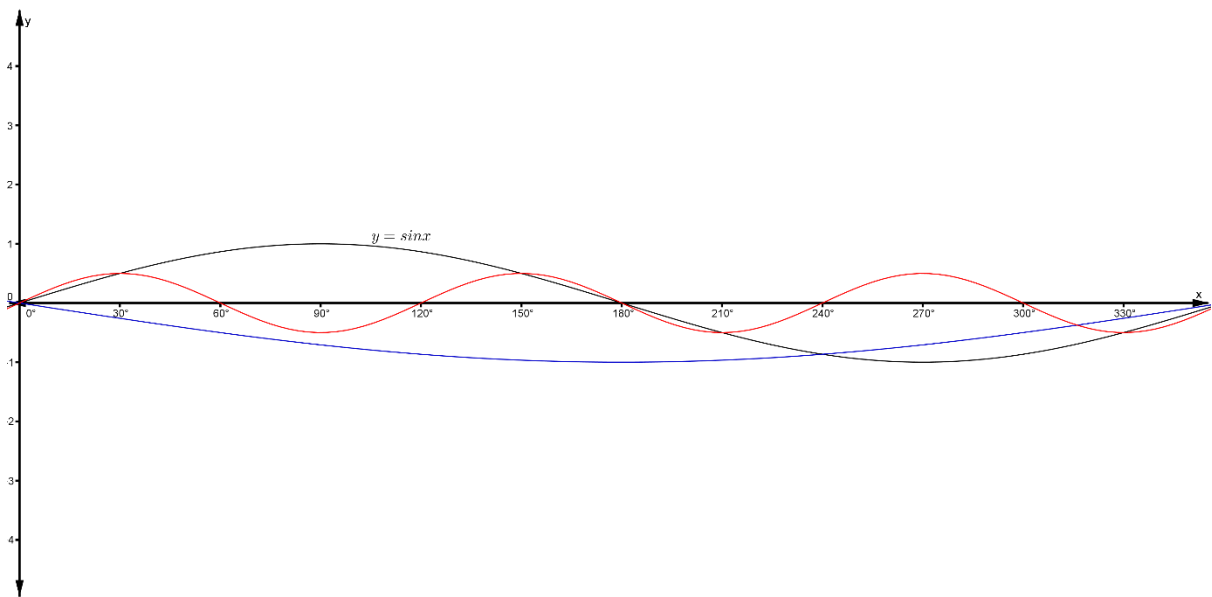
Red: $y = \tan \frac{1}{3}x + 2$

3



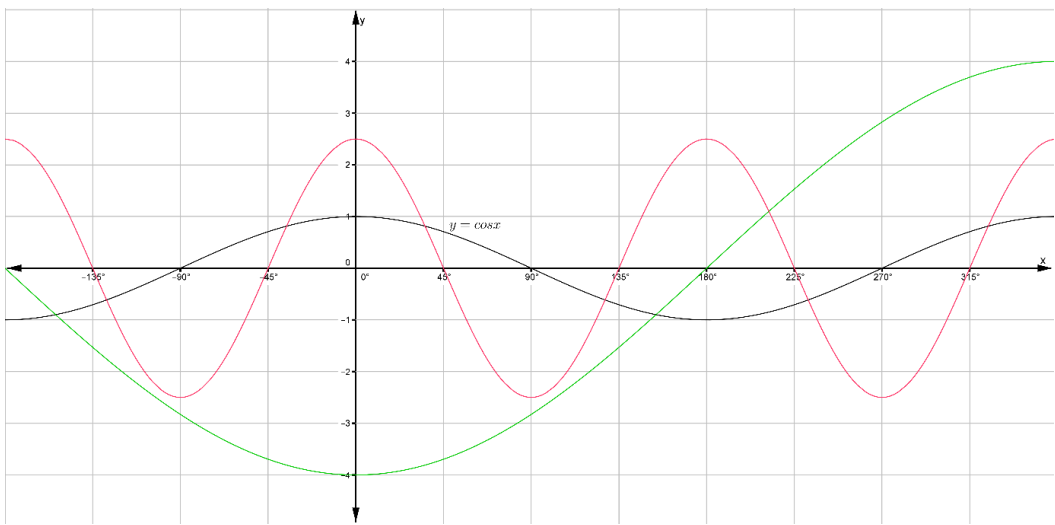
Red: $y = \tan(x + 45^\circ) - 1$

4



Red: $y = \frac{1}{2} \sin 3x$

Blue: $y = -\sin \frac{1}{2}x$

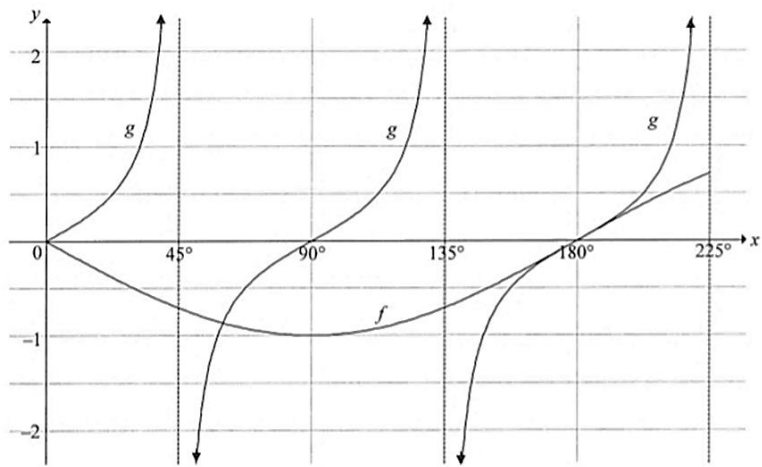


Green: $y = -4 \cos \frac{1}{2}x$

Pink: $y = \frac{5}{2} \cos 2x$

SOLUTIONS EXAMINATION TYPE

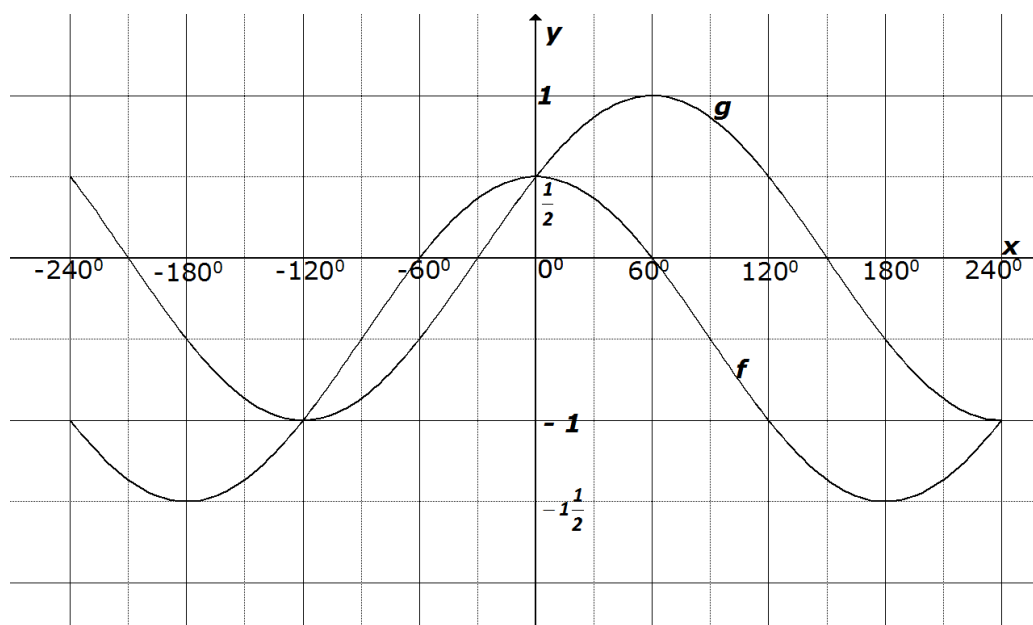
QUESTION 1



1.1	For a look at the amplitude of $f(x)$ it is 1 but $f(x)$ is a sine graph reflected about the x-axis $a = -1$ b indicates a change of period of a tangent graph. There are 2 complete tangent graphs in 180° so $b = 2$.	✓ $a = -1$ ✓ $b = 2$ (2)
1.2	There will be one complete sine graph in 360° so the period is 360°.	✓ 360° (2)

1.3	$f(x) < g(x)$ means we need x - values where $f(x)$ is below $g(x)$ between and including 90° and 225° . Look from left to right from 90° to 225° . $x \in (90^\circ; 135^\circ)$ or $90^\circ < x < 135^\circ$	✓Values ✓notation (2)
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QUESTION 2



2.1	On $g(x) = \sin(x + p)$, when $y = 0$, $x = -30^\circ$ and multiples thereof (these are x-intercepts) so the basic graph $y = \sin x$ has been moved 30° to the left and so $p = +30^\circ$. On $f(x) = \cos x + q$, when $x = 0^\circ$, $y = \frac{1}{2}$ so the basic graph $y = \cos x$ has been moved $\frac{1}{2}$ a unit down and so $q = -\frac{1}{2}$. The equations of the graphs are $f(x) = \cos x - \frac{1}{2}$ and $g(x) = \sin(x + 30^\circ)$.
2.2	We are looking for values of x where $f(x)$ lies above $g(x)$ across the whole domain of the graphs. Work from left to right across the x-axis. There is only one section where $f(x)$ lies above $g(x)$: $x \in (-120^\circ; 0^\circ)$ OR $-120^\circ < x < 0^\circ$.
2.3	We need $g(x) = \sin(x + 30^\circ)$ to become $h(x) = -\cos x$. To become a cosine graph shift $g(x)$ 60° to the left. It then lines up with $f(x)$ which is a cosine graph. To become negative $\cos x$, reflect the graph about the x-axis. The graph of g has to shift 60° to the left and then be reflected about the x-axis.

2.4	The graphs can be used to find the solutions to $f(x) = g(x)$ for $-240^\circ \leq x \leq 240^\circ$. If you are given the equations of the graphs, the question could say “Solve for x where $-240^\circ \leq x \leq 240^\circ$ and $\sin(x + 30^\circ) = \cos x - \frac{1}{2}$”. You would look from left to right across the graphs and read off the x-values of their points of intersection namely $x \in \{-120^\circ; 0^\circ; 240^\circ\}$ which can also be written as $x = -120^\circ$ or $x = 0^\circ$ or $x = 240^\circ$.
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D. TERM 3

WEEK	TOPIC	CONTENT
WEEK 1	Trigonometry Sin/Cos/Area rules	1. Prove and apply the sine, cosine and area rules.
WEEK 2		2. Solve problems in two dimensions using the sine, cosine and area rules.
WEEK 3		2. Solve problems in two dimensions using the sine, cosine and area rules.
WEEK 4	Measurements	Revise the volume and surface areas of right-prisms and cylinders. TRIMMED
WEEK 5		- Study the effect on volume and surface areas when multiplying any dimension by a constant factor k. - Calculate volume and surface areas of spheres, right prisms, right cones and combination of those objects (figures).
WEEK 6	Statistics	1. Revise measures of central tendency in ungrouped data. 2. Measures of central tendency in grouped data: calculation of mean estimate of grouped and ungrouped data and identification of modal interval and interval in which the median lies. 3. Revision of range as a measure of dispersion and extension to include percentiles, quartiles, inter-quartile and semi-inter-quartile range.
WEEK 7		4. Five number summary (maximum, minimum and quartiles) and box and whisker diagram. 5. Use the statistical summaries (measures of central tendency and dispersion), and graphs to analyse and make meaningful comments on the context associated with the given data. 6. Histograms
WEEK 8		7. Frequency polygons 8. Ogives (cumulative frequency curves) 9. Variance and standard deviation of ungrouped data 10. Symmetric and skewed data 11. Identification of outliers.
WEEK 9	Probability	1. The use of probability models to compare the relative frequency of events with the theoretical probability. 2. The use of Venn diagrams to solve probability problems, deriving and applying the following for any two events in a sample space S: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ A and B are Mutually exclusive if $P(A \text{ and } B) = 0$; A and B are complementary if they are mutually exclusive; and $P(A) + P(B) = 1.$ Then $P(B) = P(\text{not}(A)) = 1 - P(A)$ 3. Revised the addition rule for mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$

	The complementary rule: $P(\text{not } A) = 1 - P(A)$ and the identity $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
WEEK 10	4. Identify dependents and independent events and the product rule for independent events: $P(A \text{ and } B) = P(A) \times P(B)$ 5. The use of Venn diagrams to solve probability problems, Deriving and applying formulae for any three events A, B and C in a sample space S. 6. Use tree diagrams for the probability of consecutive or simultaneous events which are not necessarily independent

WEEK 1 – 3: TRIGONOMETRY

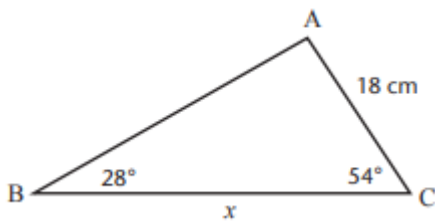
CONTENT

. Prove and apply the sine, cosine and area rules.
Solve problems in two dimensions using the sine, cosine and area rules.

SOLUTIONS

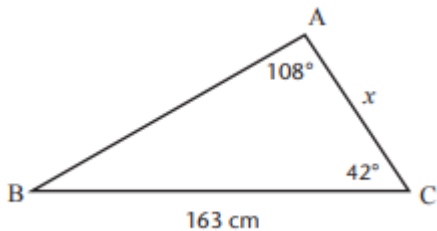
Activity 1

1.



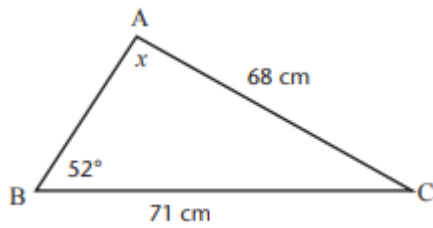
$$\begin{aligned} 1. \quad \frac{x}{\sin 98^\circ} &= \frac{18}{\sin 28^\circ} \\ x &= \frac{18 \sin 98^\circ}{\sin 28^\circ} \\ x &= 38,97 \end{aligned}$$

2.



$$\begin{aligned} \frac{x}{\sin 30^\circ} &= \frac{163}{\sin 108^\circ} \\ x &= \frac{163 \sin 30^\circ}{\sin 108^\circ} \\ x &= 85,69 \end{aligned}$$

3



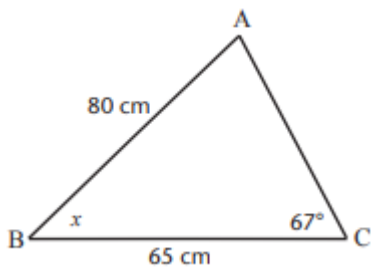
$$\frac{\sin x}{71} = \frac{\sin 52^\circ}{68}$$

$$\sin x = \frac{71 \sin 52^\circ}{68}$$

$$\sin x = 0,82$$

$$x = 55,1^\circ$$

4.



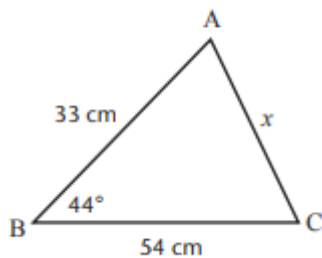
$$\frac{\sin A}{65} = \frac{\sin 67^\circ}{80}$$

$$\sin A = 0,7479$$

$$A = 48,41^\circ$$

$$x = 64,59$$

5.

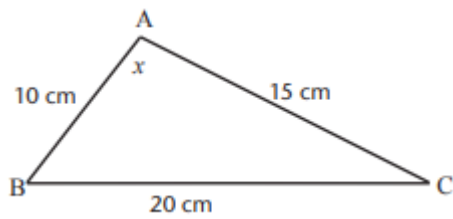


$$2. \quad x^2 = 33^2 + 54^2 - 2(33)(54)\cos 44$$

$$x^2 = 1441,27$$

$$x = 37,96$$

6

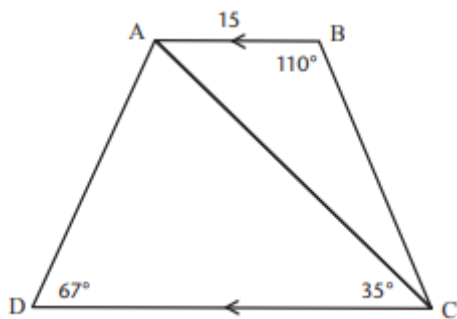


$$\begin{aligned}
 3. \quad 20^2 &= 10^2 + 15^2 - 2(10)(15)\cos x \\
 2(10)(15)\cos x &= 10^2 + 15^2 - 20^2 \\
 \cos x &= \frac{10^2 + 15^2 - 20^2}{2 \times 10 \times 15} \\
 \cos x &= -0,25
 \end{aligned}$$

$$x = 180^\circ - 75,52 = 104,48$$

Activity 2

1.



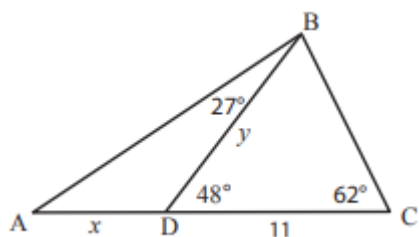
$$\frac{AC}{\sin 110^\circ} = \frac{15}{\sin 35^\circ}$$

$$AC = 24,57$$

$$\frac{DC}{\sin 78} = \frac{24,57}{\sin 67}$$

$$DC = 26,11$$

2



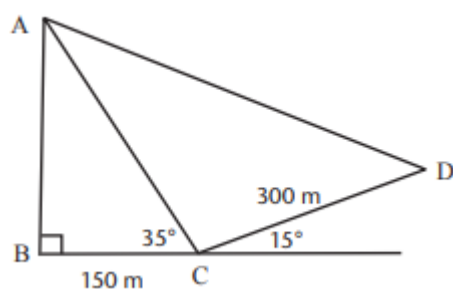
$$\frac{y}{\sin 62^\circ} = \frac{11}{\sin 70^\circ}$$

$$y = 10,34$$

$$\frac{x}{\sin 27^\circ} = \frac{10,34}{\sin 21^\circ}$$

$$x = 13,10$$

3



$$\cos C = \frac{BC}{AC}$$

$$\cos 35^\circ = \frac{150}{AC}$$

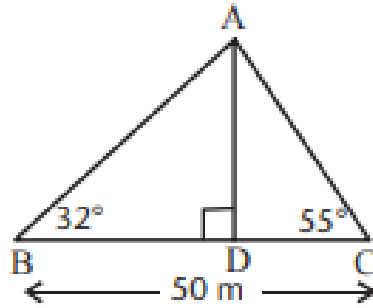
$$AC = \frac{150}{\cos 35^\circ} = 183,11$$

$$AD^2 = 183,11^2 + 300^2 - 2 \times 183,11 \times 300 \times \cos 130^\circ$$

$$AD^2 = 194149,78$$

$$AD = 440,62$$

4



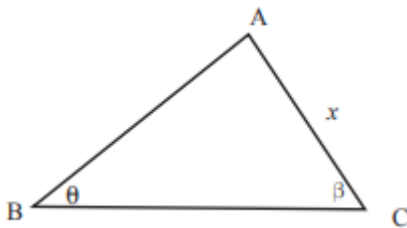
$$4. \frac{AB}{\sin 55^\circ} = \frac{50}{\sin 93^\circ}$$

$$AB = 41,01$$

$$\frac{AD}{\sin 32^\circ} = \frac{41,01}{\sin 90^\circ}$$

$$AD = 21,73$$

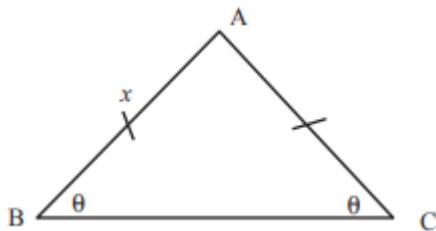
5.1



$$\frac{BC}{\sin(\theta + \beta)} = \frac{x}{\sin \theta}$$

$$BC = \frac{x \sin(\theta + \beta)}{\sin \theta}$$

5.2

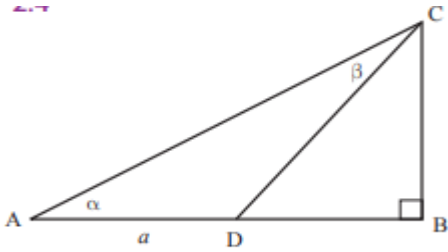


$$BC^2 = x^2 + x^2 - 2xx \cos 2\theta$$

$$BC^2 = 2x^2 - 2x^2 \cos \theta$$

$$BC = \sqrt{2x^2 - 2x^2 \cos \theta}$$

5.3



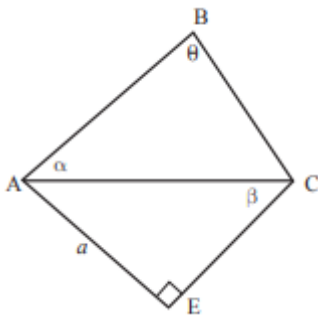
$$5.3 \quad \frac{DC}{\sin \alpha} = \frac{a}{\sin \beta}$$

$$DC = \frac{a \sin \alpha}{\sin \theta}$$

$$\frac{BC}{\sin (\alpha + \beta)} = \frac{DC}{\sin 90^\circ}$$

$$BC = \frac{s \sin \alpha \sin (\alpha + \beta)}{\sin \theta}$$

5.4



$$\frac{AC}{\sin 90} = \frac{s}{\sin \beta}$$

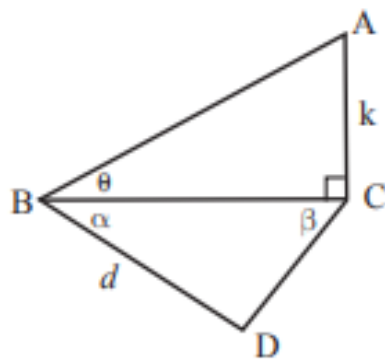
$$AC = \frac{a}{\sin \beta}$$

$$\frac{BC}{\sin \alpha} = \frac{AC}{\sin \theta}$$

$$BC = \frac{AC \sin \alpha}{\sin \theta}$$

$$BC = \frac{a \sin \alpha}{\sin \beta \sin \theta}$$

6.1



$$\frac{BC}{\sin(90-\theta)} = \frac{k}{\sin\theta}$$

$$BC = \frac{k\cos\theta}{\sin\theta}$$

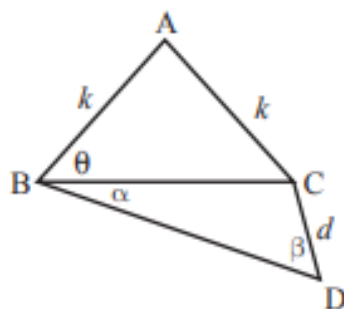
$$BC = \frac{k}{\tan\theta}$$

$$\frac{d}{\sin\beta} = \frac{BC}{\sin(\alpha+\beta)}$$

$$d = \frac{BC\sin\theta}{\sin(\alpha+\beta)}$$

$$d = \frac{k\sin\theta}{\sin(\alpha+\beta)\tan\theta}$$

6.2



$$\frac{BC}{\sin 2\theta} = \frac{k}{\sin\theta}$$

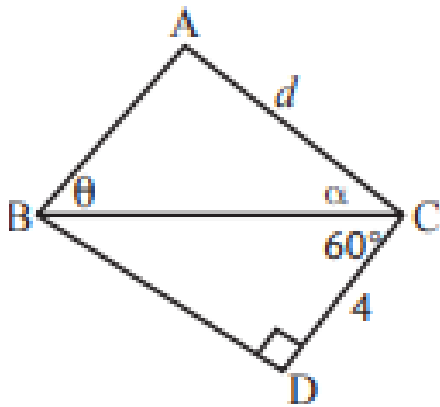
$$BC = \frac{k\sin 2\theta}{\sin\theta}$$

$$\frac{d}{\sin\alpha} = \frac{BC}{\sin\beta}$$

$$d = \frac{BC \sin\alpha}{\sin\beta}$$

$$d = \frac{k\sin 2\theta \sin\alpha}{\sin\beta \sin\theta}$$

6.3



$$\frac{BC}{\sin 90^\circ} = \frac{4}{\sin 30^\circ}$$

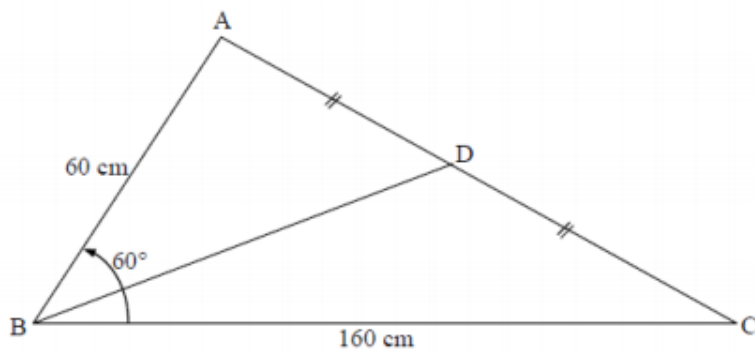
$$BC = \frac{4}{\frac{1}{2}} = 8$$

$$\frac{d}{\sin \theta} = \frac{BC}{\sin (\alpha + \theta)}$$

$$d = \frac{8 \sin \theta}{\sin (\alpha + \theta)}$$

Activity 3

1.1



$$\begin{aligned}
 AC^2 &= AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos B \\
 &= (60)^2 + (160)^2 - 2(60)(160) \cos 60^\circ \\
 &= 3600 + 25600 - 9600 \\
 &= 19600 \\
 AC &= 140 \text{ mm}
 \end{aligned}$$

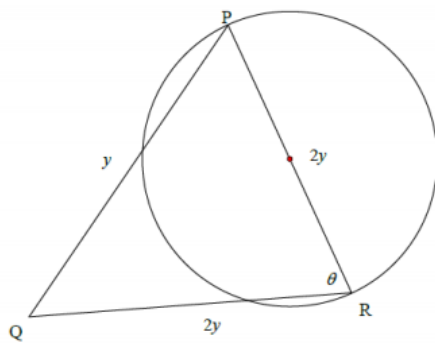
1.2

$$\begin{aligned}
 \frac{\sin A}{BC} &= \frac{\sin B}{AC} \\
 \frac{\sin A}{160} &= \frac{\sin 60^\circ}{140} \\
 \sin A &= \frac{160 \sin 60^\circ}{140} \\
 &= \frac{4\sqrt{3}}{7}
 \end{aligned}$$

1.3

$$\begin{aligned}
 &\text{Area of } \triangle ABC \\
 &= \frac{1}{2} \cdot AB \cdot AD \cdot \sin A \\
 &= \frac{1}{2} (60)(70) \left(\frac{4\sqrt{3}}{7} \right) \\
 &= 1200\sqrt{3} \text{ units}^2 \\
 &= 2078,5 \text{ units}^2
 \end{aligned}$$

2.1



$$PQ^2 = QR^2 + PR^2 - 2QR \cdot PR \cdot \cos \theta$$

$$y^2 = (2y)^2 + (2y)^2 - 2(2y)(2y) \cdot \cos \theta$$

$$\cos \theta = \frac{4y^2 + 4y^2 - y^2}{2(2y)(2y)}$$

$$= \frac{7y^2}{8y^2}$$

$$= \frac{7}{8}$$

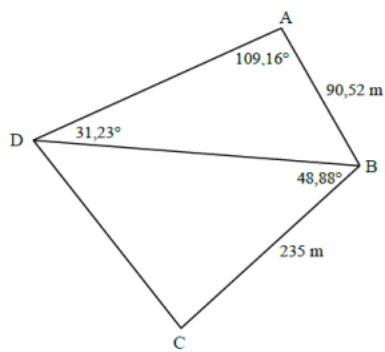
2.2

$PT \perp QR$ [$\angle$ in semi-circle/halbsirkel]

$$\sin \theta = \frac{PT}{PR} = \frac{PT}{2y}$$

$$\therefore PT = 2y \sin \theta$$

3.1



In $\triangle ABD$

$$\frac{BD}{\sin 109,16^\circ} = \frac{90,52}{\sin 31,23^\circ}$$

$$\begin{aligned} BD &= \frac{90,52 \times \sin 109,16^\circ}{\sin 31,23^\circ} \\ &= 164,92 \text{ m} \end{aligned}$$

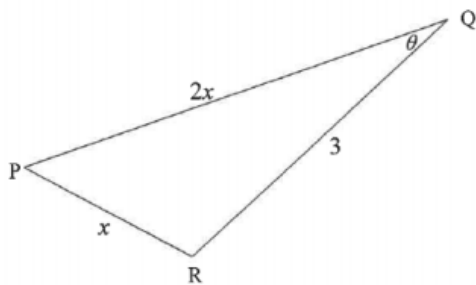
3.2

$$CD^2 = 164,92^2 + 235^2 - 2 \times 164,92 \times 235 \times \cos 48,88^\circ$$

$$CD^2 = 31448,4874$$

$$CD = 177,34 \text{ m}$$

4.1



$$x^2 = (2x)^2 + (3)^2 - 2(2x)(3)\cos \theta$$

$$12x \cos \theta = 3x^2 + 9$$

$$\cos \theta = \frac{3x^2 + 9}{12x}$$

$$\cos \theta = \frac{3(x^2 + 3)}{12x}$$

$$\cos \theta = \frac{x^2 + 3}{4x}$$

4.2.1

$$\cos \theta = \frac{(2,4)^2 + 3}{4(2,4)}$$

$$\cos \theta = 0,9125$$

$$\theta = 24,15^\circ$$

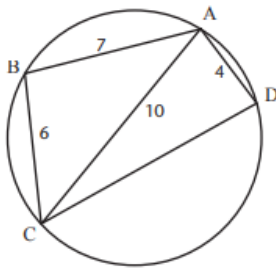
4.2.2

$$\text{Area of van } \triangle PQR = \frac{1}{2} \times PQ \times QR \times \sin \hat{Q}$$

$$= \frac{1}{2} \times 4,8 \times 3 \times \sin 24,15$$

$$= 2,95 \text{ units/eenh}^2$$

5.1



$$10^2 = 7^2 + 6^2 - 2 \times 7 \times 6 \times \cos B$$

$$2 \times 6 \times 7 \cos B = 7^2 + 6^2 - 10^2$$

$$\cos B = \frac{7^2 + 6^2 - 10^2}{2 \times 6 \times 7}$$

$$\cos b = -0,1785$$

$$B = 180^\circ - 79,71^\circ$$

$$\beta = 100,29^\circ$$

5.2

$$D = 79,71$$

$$\frac{\sin C}{4} = \frac{\sin 79,71}{10}$$

$$\sin C = 0,394$$

$$C = 66,80^\circ$$

WEEK 4 – 5 :TOPIC : MEASUREMENT

CONTENT:

- Study the effect on volume and surface areas when multiplying any dimension by a constant factor k .

Calculate volume and surface areas of spheres, right prisms, right cones and combination of those objects (figures).

SOLUTIONS

To deal with this topic teachers must ensure:

- ❖ Basic formula and properties of area and perimeter of 2D shape.
- ❖ Pythagoras's theorem.
- ❖ Volume and surface area (3D shapes) covered in grade 10.
- ❖ key vocabulary terms and the characteristics of each shape. The learners must understand the formulas for areas and volumes; volume, area, radius, height, length, width, prism, cylinder, cone, sphere, total surface area.

Some key recommendation about how to teach measurements:

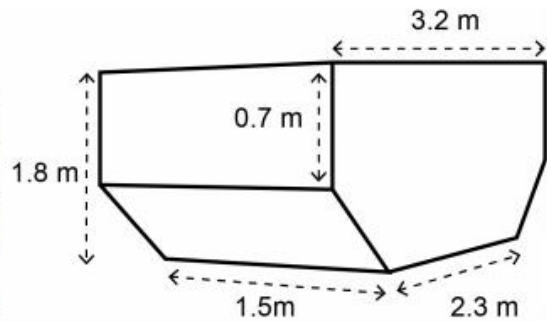
1- Diagrams are important to represent the figures(3D) and its parts (2D).

2- Explain that the outside length is the outline of the shape; therefore, the diameter of the semi-circle is not included. If the learners do not understand the concept, draw the outline on the board; start and end at the same corner. Let them identify the dimensions that contribute to the circumference. Let the learners discuss how they will calculate the circumference of half a circle.

3- Explain to the learners that the best approach is to identify familiar shapes. Let the learners identify and name the shapes and formulas they will use to calculate the volume and surface area.

4- Motivate the lesson using example base on real life situation, as:

- An airline container is in the shape of a prism. Calculate the volume of the airline container correct to 2 significant figures.

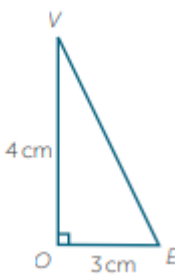


- The picture shows the Great Pyramid of Giza in Egypt. It is a square-based pyramid of length 230.4 m and height 146.4 m. Calculate the volume of the Great Pyramid of Giza, correct to 3 significant figures.



- Encourage learners to solve situations in their house, using formula for 3D shape.

QUESTION	ANSWERS	
1.	Solution: Here $r = 6.6\text{cm}$, $h = 12.5\text{cm}$ $\begin{aligned}\text{Volume} &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(6.6)^2 \times 12.5 \\ &= 570. \text{ cm}^3\end{aligned}$ Since $s^2 = h^2 + r^2$ $\begin{aligned}&= 12.5^2 + 6.6^2 \\ &= 156.25 + 43.56 \\ s^2 &= 199.81\end{aligned}$ $s = 14.14\text{cm}$ Total surface area $= \pi r(s + r)$ Total surface area $= \pi 6.6(14.14 + 6.60)$ Total surface area $= \pi 6.6(20.74)$ Total surface area $= 430 \text{ cm}^2$	
2	Solutions a Here, $r = 3$ and $h = 8$. $\begin{aligned}\text{Surface area} &= 2\pi rh + 2\pi r^2 \\ &= 2 \times \pi \times 3 \times 8 + 2 \times \pi \times 3^2 \\ &= 66\pi \text{ cm}^2 \\ &\approx 207.35 \text{ cm}^2 \quad (\text{to 2 decimal places}).\end{aligned}$	

	b Here, $r = 10$ and $h = 15$. Area of curved section $= \frac{1}{2} \times 2\pi rh = 150\pi \text{ cm}^2$ Area of top rectangle $= 300 \text{ cm}^2$ Area of two semicircles $= \pi r^2$ $= 100\pi \text{ cm}^2$ Surface area $= (250\pi + 300) \text{ cm}^2$ $\approx 1085.40 \text{ cm}^2$ (to 2 decimal places).	
3	Solution: height VE of triangle VBC: using Pythagoras' Theorem. $\begin{aligned} VE^2 &= VO^2 + OE^2 \\ &= 4^2 + 3^2 \\ &= 25 \end{aligned}$ Hence $VE = 5 \text{ cm}$.  Area of $VCB = \frac{1}{2} \times CB \times VE$ $= \frac{1}{2} \times 6 \times 5$ $= 15 \text{ cm}^2$ Area of base $= 6 \times 6$ $= 36 \text{ cm}^2$ Surface area $= 4 \times 15 + 36$ $= 96 \text{ cm}^2$ The surface area of the pyramid is 96 cm^2.	

4.	$d = 30; d=2r$ $r = \frac{30}{2} = 15; \text{ radius is } 15\text{m}$ $V = \frac{4}{3}\pi r^3$ $= \frac{4}{3} \times \pi \times 15^3$ $= 4500\pi$ $= 14,130 \text{ m}^3$	
5.	$V = \frac{4}{3}\pi r^3$ $2800 = \frac{4}{3}\pi r^3$ so $r^3 = \frac{2800 \times 3}{4\pi}$ $r = \sqrt[3]{\frac{2800 \times 3}{4\pi}}$ $\approx 8.7 \text{ cm}$ (correct to 1 decimal place). The radius is approximately 87 mm.	
6.1	Height of cone = 8 cm Radius of cone (and cylinder) = 6 cm Volume of solid = Volume of cylinder + 2 X volume of cone $= \pi r^2 h + 2 \left[\frac{1}{3} \times \pi r^2 h_1 \right]$ $= \pi (6\text{cm})^2 (20\text{cm}) + \frac{2}{3} (\pi) (6\text{cm})^2 (8\text{cm})$ $= 36\pi \left[20 + \frac{16}{3} \right] \text{ cm}^3$ $= 2865,13 \text{ cm}^3$	
7.	Volume of prism: V = area of base X height of prism $V = 3 (4) (4.3)$ $V = 51,6 \text{ cm}^3$ Volume of cylinder:	

	$V = \pi r^2 h$ $h = 12 - 4,3 - 2,4$ $h = 5,3$ $V = \pi (1,1)^2 (5,3)$ $V = 20,15 \text{ cm}^3$ Volume of cone: $V = \frac{1}{3} \pi r^2 h$ $V = \frac{1}{3} \pi (1,1)^2 2,4$ $V = 3,04 \text{ cm}^3$ volume of wood needed to make the toy: $V = V(\text{prism}) + V(\text{cylinder}) + V(\text{cone})$ $V = 51,6 + 20,15 + 3,04$ $V = 74,79 \text{ cm}^3$ $V = 74,8 \text{ cm}^3$	
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WEEK 6 - 8: TOPIC:STATISTICS

CONTENT::
Measure of central

NOTES & EXAM TIPS

QUESTION 1.

55	34	28	55	72	30	3	68	44	29
37	63	71	43	87	55	26	53	59	51

- 1.1 Find the mean of the set of data in the table above.
(48,15)
- 1.2 Calculate the median of the set of data in the table above.
(52)
- 1.3 Find the mode of the set of data in the table above.
(55)
- 1.4 Determine the lower quartile (Q1) of the above data.
(32)
- 1.5 Determine the upper quartile (Q3) of the above data.
(61)
- 1.6 Determine the interquartile range.
(29)
- 1.7 Draw a box and whisker graph to represent the data in the table above
. (3---32□52□61'---87)
- 1.8 What observation can you make about the spread of the data using the box and whisker graph in question 1.7
(Negatively skewed or skewed to the left)
- 1.9 Can you possibly identify an outlier and provide a reason for your answer?
(3)
- 1.10 Calculate the standard deviation of this data set.
(19,37)

QUESTION 2.

Time(t) in seconds(s)	Count (Frequency)
$60 < t \leq 65$	12
$65 < t \leq 70$	13
$70 < t \leq 75$	17
$75 < t \leq 80$	20
$80 < t \leq 85$	29
$85 < t \leq 90$	7
$90 < t \leq 95$	2

2.1 Find the estimated mean of this grouped data.

(76)

2.2 Determine the modal group.

(80<t≤85)

2.3 Determine the median group.

(75<t≤80)

QUESTION 3

3.

19	27	22	17	13	30	26	18
22	10	21	32	23	20	13	22

3.1 What is the median for the above-mentioned data.

(21,5)

3.2 Write down the upper and lower quartiles.

(24,5 and 17,5)

3.3 Determine the standard deviation of the scores.

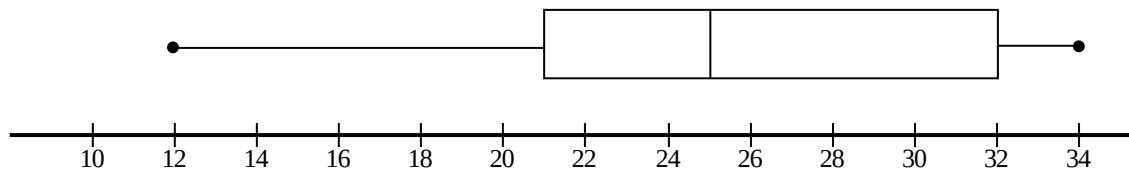
(5,86)

3.4 Draw a box and whisker diagram of the scores recorded in the table above for player A.

(10---17,5□21,5□24,5---32)

3.5 Another player B, playing in the same competition had his 15 scores recorded.

The five number summary of his scores is **(12 ; 21 ; 25 ; 32 ; 34)**. This box and whisker diagram is showed below. How does player A and player B compare regarding their performance? Give reasons for your decision. (Player B is better, has a higher average score.)



4.

27	28	30	32	34	38	41	42	43	43	44	46	53	56	62
----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

4.1 Determine the median of the given data.

(42)

4.2 Determine the interquartile range for the data.

(12)

4.3 Draw a box and whisker diagram to represent the data.

(27--33□42□45—32)

4.3.1 By referring to the box and whisker diagram, comment on the skewness of the learners in this Olympiad.

(Skewed to the left or negatively skewed)

5.

Number of visits to the movies in a year	Frequency	Cumulative Frequency
0 - 5	8	8
6 - 10	10	18
11- 15	6	24
16 - 20	6	B
A	4	34
26 -30	4	38
31 - 35	2	C
36 - 40	0	40

5.1 Complete the table above by giving the values for A, B and C
A=21-25,B=30,C=40)

5.2 Draw the ogive of the data given in the table.

5.3 Estimate the number of visits to the movies made by the top 50% of the learners last year.
(28)

6.

6.1 Complete the cumulative frequency table above.
(5,20,55,105,140,150)

6.2 Draw the Histogram of the info above with the Interval on the horizontal axis and the Frequency on the vertical axis.

6.3 Draw an ogive (cumulative frequency curve) to represent the data.

6.4 Estimate the percentage of the learners who sent more than 50 messages in one day.

(74%)

6.5 In which direction is the data skewed?

(To the left)

WEEK 9 - 10: TOPIC:PROBABILITY

SUB-TOPIC:

- Identities
- Reduction Formula
- Determine for which values of a variable an identity holds.

SOLUTIONS

SOLUTIONS

1.1	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $0,625 = 0,25 + 0,5 - P(A \text{ and } B)$ $P(A \text{ and } B) = 0,125$
-----	---

1.2.1	Not mutually exclusive $P(A \text{ and } B) \neq 0$
-------	--

1.2.2	Yes, they are inclusive $P(A \text{ and } B) = 0,125$
-------	--

1.2.3	$P(A) + P(B)$ $= 0,25 + 0,5$ $= 0,75$ The events are not complementary $P(A) + P(B) \neq 1$
-------	--

1.2.4	$P(A) \times P(B)$ $= 0,25 \times 0,5$ $= 0,125$
-------	--

	$P(A \text{ and } B) = P(A) \times P(B) = 0,125$
--	--

ACTIVITIES

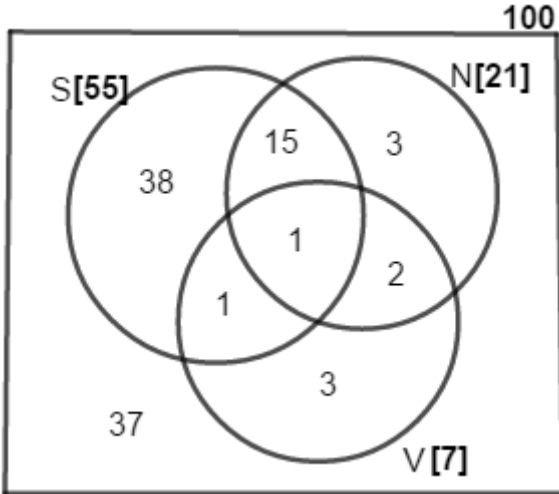
1.1	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $0,6 = 0,2 + 0,5 - P(A \text{ and } B)$ $P(A \text{ and } B) = 0,1$
1.2	$P(A) \times P(B)$ $= 0,2 \times 0,5$ $= 0,1$ $P(A \text{ and } B) = P(A) \times P(B) = 0,1$ Events A and B are independent
2.1	$P(A \text{ and } B) = 0,12 \neq 0$ Events A and B not mutually exclusive
2.2	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $0,83 = \frac{1}{4}P(B) + P(B) - 0,12$ $0,95 = \frac{5}{4}P(B)$ $\frac{4}{5} \times \frac{5}{4}P(B) = 0,95 \times \frac{4}{5}$ $P(B) = 0,76$
2.3	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $0,83 = P(A) + 0,76 - 0,12$ $P(A) = 0,19$

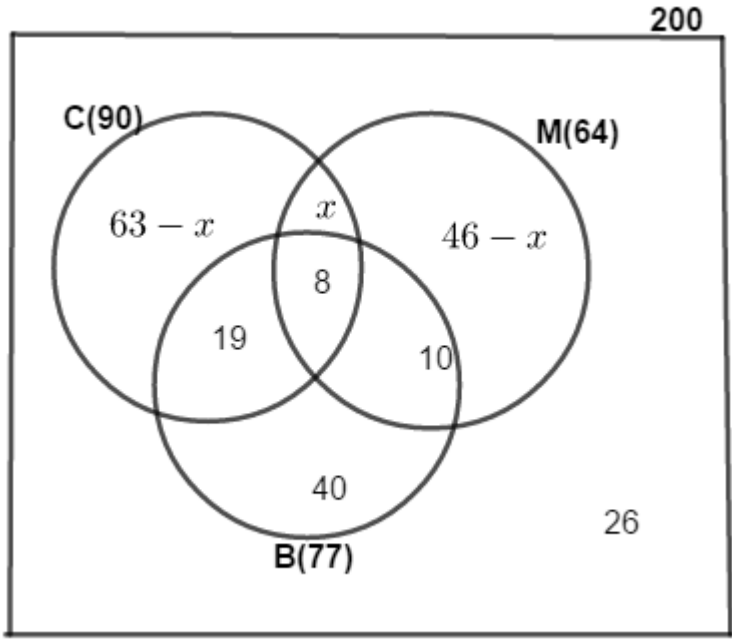
		$P(A) + P(A') = 1$ $0,19 + P(A') = 1$ $\therefore P(A') = 0,81$
	3.1	$P(A) + P(A') = 1$ $0,35 + P(A') = 1$ $P(A') = 0,65$
	3.2	$P(A \text{ and } B) = 0$
	3.3	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $P(A \text{ or } B) = 0,35 + 0,52 - 0$ $P(A \text{ or } B) = 0,87$
4		$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $0,8 = 0,6 + 0,3 - P(A \text{ and } B)$ $P(A \text{ and } B) = 0,1$ $P(A \text{ and } B) \neq 0$ $\therefore$ the events are not mutually exclusive
5.		$P(A) \times P(B) = 0.12 \times 0.35$ $= 0,042$ $P(A \text{ and } B) = 0,428$ $P(A \text{ and } B) \neq P(A) \times P(B)$ Events A and B are not independent
	6.1	$P(T \text{ and } W) = 0,14 \neq 0$ The events are NOT mutually exclusive

	6.2	$P(T) \times P(BW) = 0.4 \times 0.35$ $= 0,14$ $P(T \text{ and } W) = 0,14$ $P(T \text{ and } W) = P(T) \times P(W)$ $\therefore$ Events T and W are independent
--	-----	--

ACTIVITIES

1	
5 customers	
$P(B \text{ and } C) = \frac{17}{240} + \frac{12}{240} = \frac{29}{240}$ $P(B \text{ and } C) \neq 0$ $\therefore$ events A and B are not mutually exclusive	
1.3.1	$P(V \text{ only}) = \frac{58}{240} = \frac{29}{120}$
1.3.2	$P(B \text{ and } C) = \frac{29}{240}$
1.3.3	$P(\text{not } C) = \frac{52 + 3 + 58 + 5}{240}$ $= \frac{118}{240} = \frac{59}{120}$ OR

	$P(\text{not } C) = 1 - P(C)$ $= 1 - \frac{122}{240} = \frac{59}{120}$	
1.3.4	$P(B \text{ or } V) = P(B) + P(V) - P(B \text{ and } V)$ $P(B \text{ or } V) = \frac{17 + 12 + 3 + 52}{240} + \frac{9 + 12 + 3 + 58}{240} - \frac{12 + 3}{240}$ $P(B \text{ or } V) = \frac{84}{240} + \frac{82}{240} - \frac{15}{240}$ $P(B \text{ or } V) = \frac{151}{240}$ OR $P(B \text{ or } V) = \frac{17 + 12 + 3 + 52 + 9 + 58}{240}$ $P(B \text{ or } V) = \frac{151}{240}$	
2.1		
2.2.1	$P(\text{plays netball or volleyball}) = \frac{15 + 3 + 1 + 2 + 1 + 3}{100} = \frac{25}{100}$ $= \frac{1}{4}$	(
2.2.2	$P(\text{plays atleast two sporting codes}) = \frac{15 + 1 + 1 + 2}{100} = \frac{19}{100}$	

	2.2.3	$P(\text{plays only two sporting codes}) = \frac{15 + 1 + 2}{100} = \frac{18}{100} = \frac{9}{50}$	
3.1	 A Venn diagram within a rectangle labeled 200. Three circles are labeled C(90), M(64), and B(77). The regions are labeled as follows: C only: 63 - x; M only: 46 - x; B only: 40; C ∩ M: x; C ∩ B: 19; M ∩ B: 10; C ∩ M ∩ B: 8; Outside all circles: 26.		
3.2	$63 - x + x + 8 + 19 + 46 - x + 10 + 40 + 26 = 200$ $212 - x = 200$ $x = 12$ $n(\text{only chicken}) = 63 - x = 63 - 12 = 51$		(4)
3.3	$P(\text{only two types}) = \frac{x + 19 + 10}{200} = \frac{12 + 19 + 10}{200} = \frac{41}{200}$		(3)

ACTIVITIES

SOLUTIONS

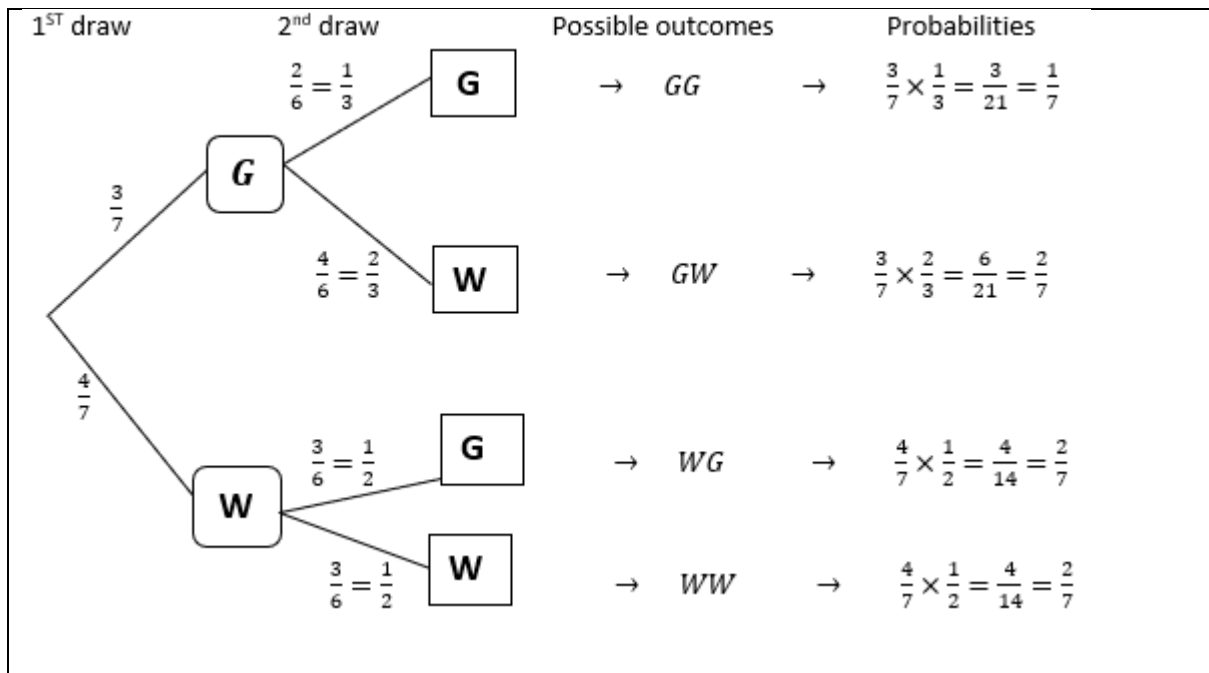
1.1		
1.2.1	$P(N) = \frac{4 + 2 + 3 + 4}{33} = \frac{13}{33}$	
1.2.2	$P(H \text{ or } N) = \frac{4 + 4 + 2 + 3 + 5 + 8}{33} = \frac{26}{33}$ OR $P(H \text{ or } N) = \frac{33 - 1 - 6}{33} = \frac{26}{33}$	
1.2.3	$P(H') = \frac{1 + 4 + 4 + 6}{33} = \frac{15}{33} = \frac{5}{11}$ OR $P(H') = 1 - P(H) = 1 - \frac{5 + 2 + 3 + 8}{33} = \frac{15}{33} = \frac{5}{11}$	
2.1	$P(\text{Eat Out nor Go to Movies}) = \frac{48}{150} = \frac{8}{25}$	
2.2	$P(\text{Eat Out}) = \frac{23 + 8 + 15 + 4}{150} = \frac{50}{150} = \frac{1}{3}$	

	$P(\text{Go to Movies}) = \frac{4 + 15 + 11 + 41}{150} = \frac{71}{150}$ $P(\text{Eat Out}) \times P(\text{Go to Movies}) = \frac{1}{3} \times \frac{71}{150} = \frac{71}{450} = 0,157777 \dots$ $P(\text{Eat Out and Go to Movies}) = \frac{4 + 15}{150} = \frac{19}{150} = 0,12666 \dots$ $P(\text{Eat Out and Go to Movies}) \neq P(\text{Eat Out}) \times P(\text{Go to Movies})$ $\therefore$ the Events are NOT independent	
3.1	$80 - 69 = 11$	
3.2	A Venn diagram with three overlapping circles labeled E(44), A(33), and S(39) inside a rectangular universal set of 80. The regions are labeled as follows: E only is $44 - 23 - x$; A only is $33 - 19 - x$; S only is 6; the intersection of E and A is x; the intersection of E and S is 14; the intersection of A and S is 10; the intersection of all three sets E, A, and S is 9; and the region outside all three sets is 11. The total number of elements in the universal set is 80.	
3.3	$44 - 23 - x + x + 9 + 14 + 33 - 19 - x + 10 + 6 + 11 = 80$ $85 - x = 80$ $x = 5$ OR $21 - x + x + 9 + 14 + 14 - x + 10 + 6 = 69$ $74 - x = 69$ $x = 5$	

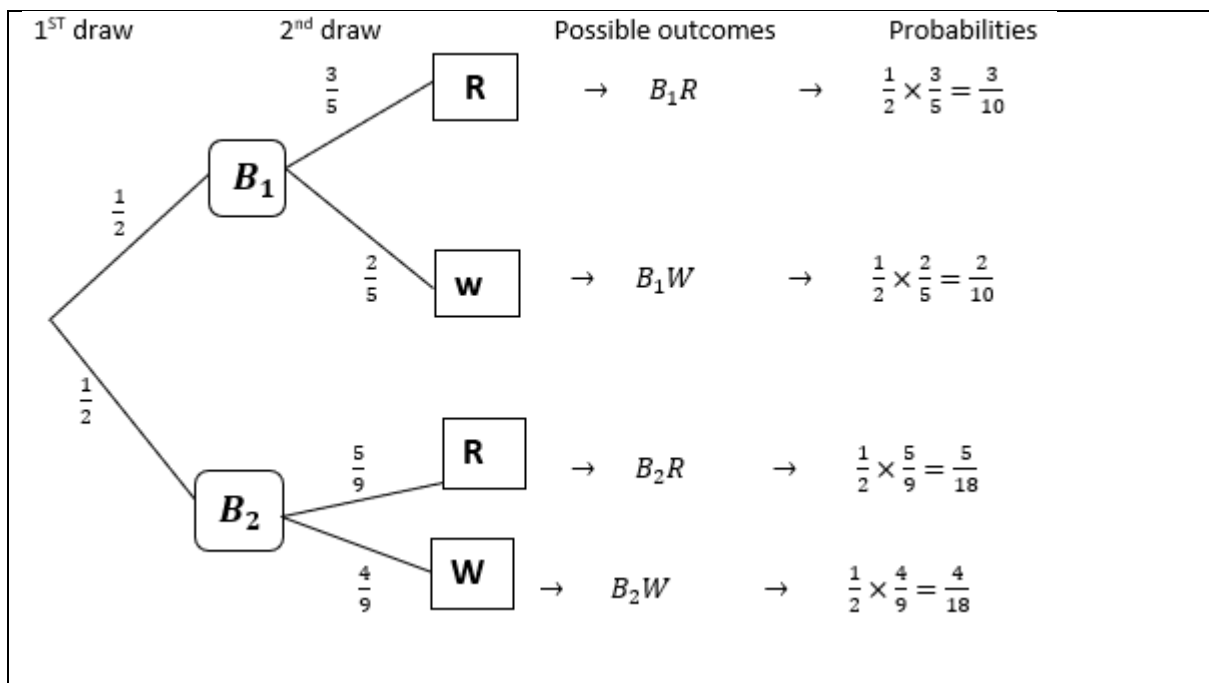
3.4	At least Two means the student reads 2 or 3 Magazine.	
	$P(\text{at least 2}) = \frac{5 + 9 + 10 + 14}{80} = \frac{38}{80} = \frac{19}{40} = 0,475$	

ACTIVITIES

1.



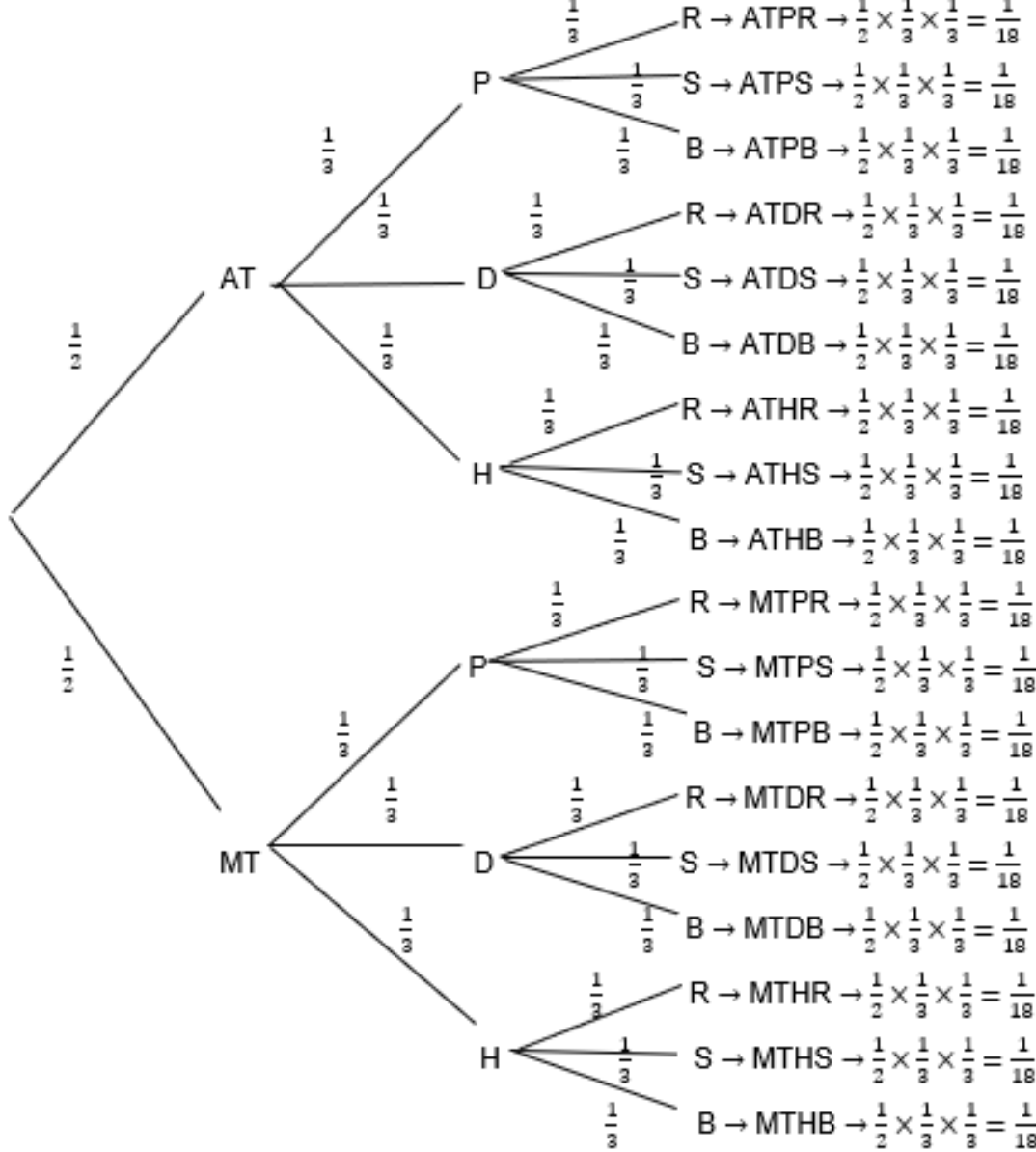
2.



3.

4.1	1ST draw 2nd draw 48% F 52% NF 95% W 5% D Possible outcomes → WF → 95% × 48% = 0,456 → WNF → 95% × 52% = 0,494 → DF → 5% × 32% = 0,016 → DNF → 5% × 68% = 0,034 Probabilities
3.2.1	P(F) = 0,456 + 0,016 = 0,472 = 47,2% (2)
3.2.2	P(W and NF) = 0,494 = 49,4% (2)

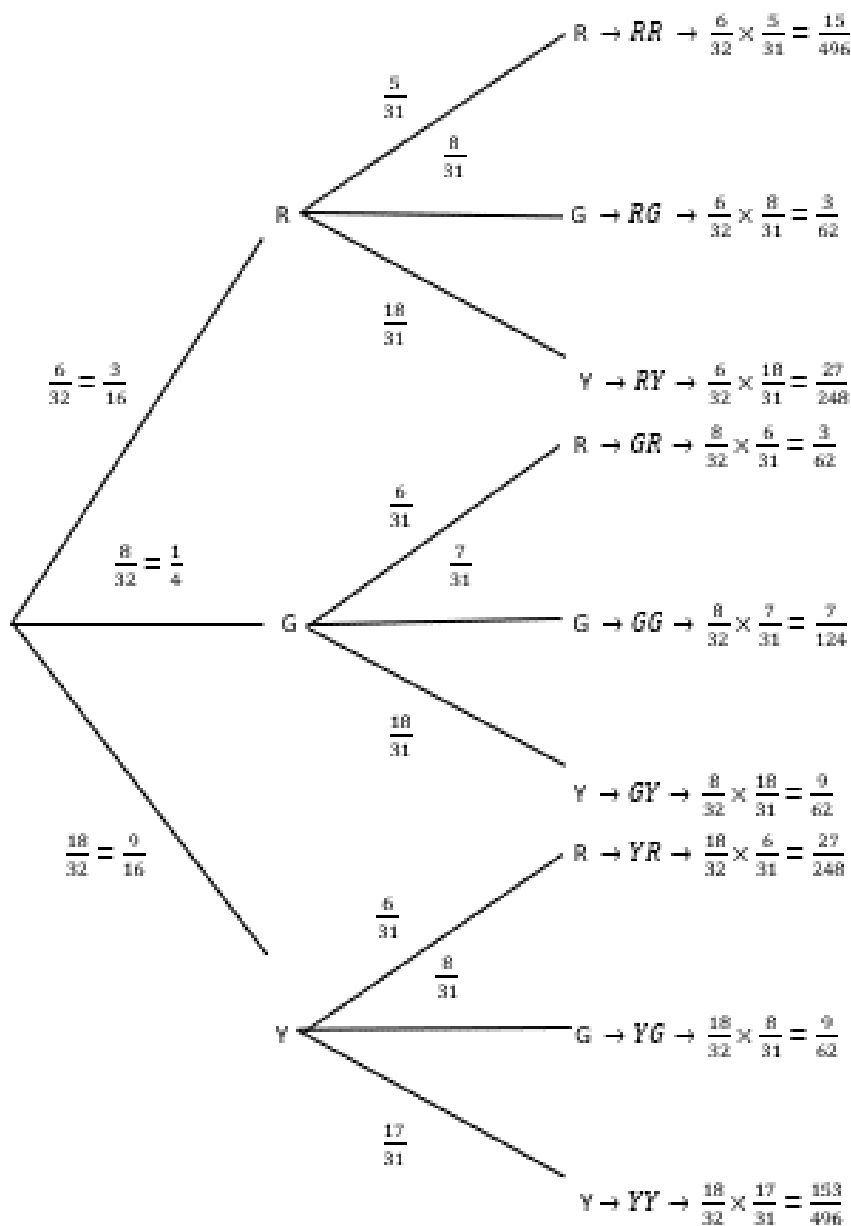
ACTIVITIES

1.1			
1.2.1	$P(APB) + P(ADB) + P(AHB) + P(MPB) + P(MDB) + P(MHB)$ $= 6\left(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{3}\right) = \frac{1}{3}$		
1.2.2	$P(AHR) + P(AHS) + P(AHB) = 3\left(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{3}\right) = \frac{1}{6}$		
1.2.3	$P(MDR) = 3\left(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{3}\right) = \frac{1}{6}$		

	1.2.4	$P(P \text{ and } S) = P(APS) + P(MPS)$ $P(P \text{ and } S) \left(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{3} \right) + \left(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{3} \right) = \frac{1}{9}$	
2.1		$m = \frac{10}{24} = \frac{5}{12}$ $n = \frac{10}{24} \times \frac{3}{10} = \frac{30}{240} = \frac{1}{8}$	
2.2		$\frac{14}{24} \times \frac{9}{x+9} = \frac{7}{20}$ $\frac{9}{x+9} = \frac{3}{5}$ $45 = 3x + 27$ $18 = 3x$ $x = 6$ OR $\frac{14}{24} \times \frac{x}{x+9} = \frac{7}{30}$ $\frac{x}{x+9} = \frac{2}{5}$ $5x = 2x + 18$ $3x = 18$ $x = 6$	
	2.3	$P(G) = \frac{7}{24} + \frac{7}{30} = \frac{21}{40} = 0,525$	

3.1	$n(R) = 80 - \left(\frac{3}{5} \times 80 + 10\% \times 80 \right) = 24$
3.2	$\frac{48}{80} = \frac{3}{5}$ $\frac{8}{80} = \frac{1}{10}$ $\frac{24}{80} = \frac{3}{10}$ $\frac{48}{80} = \frac{3}{5}$ $\frac{8}{80} = \frac{1}{10}$ $\frac{24}{80} = \frac{3}{10}$ $\frac{48}{80} = \frac{3}{5}$ $\frac{8}{80} = \frac{1}{10}$ $\frac{24}{80} = \frac{3}{10}$ $\frac{48}{80} = \frac{3}{5}$ $\frac{8}{80} = \frac{1}{10}$ $\frac{24}{80} = \frac{3}{10}$ $G \rightarrow GG \rightarrow \frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$ $Y \rightarrow GY \rightarrow \frac{3}{5} \times \frac{1}{10} = \frac{3}{50}$ $R \rightarrow GR \rightarrow \frac{3}{5} \times \frac{3}{10} = \frac{9}{50}$ $G \rightarrow YG \rightarrow \frac{1}{10} \times \frac{3}{5} = \frac{3}{50}$ $Y \rightarrow YY \rightarrow \frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$ $R \rightarrow YR \rightarrow \frac{1}{10} \times \frac{3}{10} = \frac{3}{100}$ $G \rightarrow RG \rightarrow \frac{3}{10} \times \frac{3}{5} = \frac{9}{50}$ $Y \rightarrow RY \rightarrow \frac{3}{10} \times \frac{1}{10} = \frac{3}{100}$ $R \rightarrow RR \rightarrow \frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$
3.3	$P(GY) + P(YG) = \frac{48}{80} \times \frac{56}{80} + \frac{56}{80} \times \frac{48}{80} = \frac{21}{25}$
4.1	$\frac{8}{x} = 25\%$ $x = 32$

4.2

4.3 $P(\text{same colour}) = P(RR) + P(GG) + P(YY)$

$$= \left(\frac{6}{32} \times \frac{5}{31} \right) + \left(\frac{8}{32} \times \frac{7}{31} \right) + \left(\frac{18}{32} \times \frac{17}{31} \right)$$

$$= \frac{15}{496} + \frac{7}{124} + \frac{153}{496}$$

$$= \frac{49}{124}$$

5.1	<pre> graph LR Root(()) --- 0,4 C1[C] Root --- 0,6 W1[W] C1 --- 0,5 CC[CC] C1 --- 0,5 CW[CW] W1 --- 0,3 WC[WC] W1 --- 0,7 WW[WW] CC --- P1["→ 0,4 × 0,5 = 0,2"] CW --- P2["→ 0,4 × 0,5 = 0,2"] WC --- P3["→ 0,6 × 0,3 = 0,18"] WW --- P4["→ 0,6 × 0,7 = 0,42"] </pre>
5.2	$P(\text{second answer correct}) = 0,2 + 0,18 = 0,38$

ACTIVITY

SOLUTIONS		
1.1	$P(\text{disabled})$ $= \frac{35}{400} = \frac{7}{80} = 0,0875$	(2)
1.2	$P(\text{disabled person who uses the disabled parking bays})$ $= \frac{12}{400} = \frac{3}{100} = 0,03$	(2)
1.3	$P(\text{not disabled and does not use th disabled parking bays})$ $= \frac{60}{400} = \frac{3}{20} = 0,15$	(2)
2.4	$P(\text{Uses the disabled parking bays and is not disabled})$ $= \frac{305}{400} = \frac{61}{80} = 0,76$	(2)

ACTIVITIES

1.1	A = 672 B = 2237 C = 1173 D = 2738
1.2.1	$P(F \text{ and More than 5 cups}) = \frac{672}{4895} = 0,14$
1.2.2	$P(F) = \frac{2237}{4895} = 0,46$
1.3	$P(F) \times P(\text{More than 5 cups})$ $= \frac{2237}{4895} \times \frac{2157}{4895}$ $= 0,20$ $P(F \text{ and More than 5 cups}) \neq P(F) \times P(\text{More than 5 cups})$ $\therefore$ The events are not independent
2.1	A = 50 B = 250 C = 100 D = 150 E = 100 F = 400
2.2	$P(\text{More than 2 and older than 40 years})$ $= \frac{50}{400} = 0,125$

	$P(\text{More than 2}) \times (\text{older than 40 years})$ $= \frac{150}{400} \times \frac{100}{400} = 0,09375$ Events are not independent
3.1	$\frac{a}{140} = \frac{21}{140} \times \frac{80}{140}$ $a = \frac{21}{140} \times \frac{80}{140} \times 140$ $a = 12$
3.2	$b = 9$ $c = 40$ $d = 30$ $P(\text{Gym or 35 years and younger})$ $= P(\text{Gym}) + P(35 \text{ years and younger}) - P(\text{Gym and 35 years and younger})$ $= \frac{70}{140} + \frac{80}{140} - \frac{40}{140}$ $= \frac{110}{140}$ $= \frac{11}{14}$

E. TERM 4

WEEK	TOPIC	CONTENT
WEEK 2	Financial Mathematics	1. Use the simple and compound growth formulae to solve problems, including interest, hire purchase, inflation, population growth and other real-life problems.
WEEK 3		2. Understand the implication of fluctuating foreign exchange rates (e.g. on the petrol price, imports, exports, overseas travel). 3. Use simple and compound decay formulae: $A = (1 - in)$ and $A = (1 - i)^n$ to solve problems (including straight line depreciation and depreciation on a reducing balance).
WEEK 4		4. The effect of different periods of compound growth and decay, including nominal and effective interest rates

WEEK 2 – 4: FINANCIAL MATHEMATICS

1. Use the simple and compound growth formulae to solve problems, including interest, hire purchase, inflation, population growth and other real-life problems.
2. Understand the implication of fluctuating foreign exchange rates (e.g. on the petrol price, imports, exports, overseas travel).
3. Use simple and compound decay formulae:
 $A = (1 - in)$ and
 $A = (1 - i)^n$
to solve problems (including straight line depreciation and depreciation on a reducing balance).
4. The effect of different periods of compound growth and decay, including nominal and effective interest rates

SOLUTIONS

1.1	$1 + i_e = \left(1 + \frac{i_{nom}}{m}\right)^m$ $1 + i_e = \left(1 + \frac{0,1025}{4}\right)^4$ $1 + i_e = 1,1065....$ $i_e = 0,1065...$ $r = 10,65\%$	4
1.2	R25 000 R15 000 7,5% p.a. A = P(1 + i)ⁿ = 25 000 $\left(1 + \frac{0,075}{12}\right)^{18}$ = R27 967,01 P = 27 967,01 + R15 000 = R42 967,01 1,5 years 9,7% 1,5 years	8

	$A = P(1+i)^n$ $= 42\,967,01 \left(1 + \frac{0,097}{2}\right)^3$ $= R49\,526,82$	
1.3	Jojo, a taxi owner, buys a new taxi for R280 000.	
1.3.1	$A = P(1-i)^n$ $\text{Value} = 280\,000(1-0,18)^4$ $= R126\,594,09$	
1.3.2	Cost of new taxi $= P(1+i)^n$ $= 280\,000(1+0,069)^4$ $= R365\,652,76$	
1.3.3	Additional money $= R365\,652,76 - R126\,594,09$ $= R239\,058,67$	
2	Sheena receives R1 500 as a gift. She invests her money in a savings account, earning interest at 15% per annum compounded semi-annually	
2.1	$A = P(1+i)^n$ $= 1500 \left(1 + \frac{0,15}{2}\right)^{5 \times 2}$ $= R\,3091,55$	4
2.2	$1 + i_{\text{eff}} = \left(1 + \frac{0,15}{2}\right)^2$ $i_{\text{eff}} = 0,1556 = 15,56\%$	3
3		
3.1	$A = P(1-i)^n$ $= 23000(1-0,135)^5$ $= R11\,138,03$	2

3.2	$A = P(1+i)^n$ $= 23\,000(1+0,066)^5$ $= R31\,660,21$	2
3.3	Value of fund needed = R 31 660,21 – R 11 138,03 <i>Waarde van die fonds benodig</i> $= R20\,522,18$ $20522,18 = P\left(1 + \frac{0,047}{12}\right)^{60}$ $P = R\,16231,69$	6
4.1	$A = P(1-i)^n$ $11090,41 = 120000(1-i)^{12}$ $\therefore i = 1 - \sqrt[12]{\frac{11090,41}{120000}}$ Thus $i = 0,179999\dots$ Rate of Depreciation = 18%	4
4.2	$i_{\text{eff}} = \left(1 + \frac{i}{m}\right)^m - 1$ $= \left(1 + \frac{0,098}{12}\right)^{12} - 1$ $= 0,10252\dots$ rate = 10,25%	3
4.3	$A = P(1+i_1)^{n_1}(1+i_2)^{n_2}$ $= 80\,000\left(1 + \frac{0,075}{4}\right)^{16}\left(1 + \frac{0,092}{12}\right)^{36}$ $= R141768,60$	4
4.4	Exactly 8 years ago Tashil invested R30 000 in an account earning 6,5% per annum, compounded monthly.	
4.4.1	Investment : end of third year : $A = P(1+i)^n$ $= 30\,000\left(1 + \frac{0,065}{12}\right)^{96}$ $= R50390,07$	3

4.4.2	$\begin{array}{cccc} T_0 & T_3 & T_5 & T_8 \\ \hline 30000 & -10000 & +10000 & \end{array}$ $A = 30000\left(1 + \frac{0,65}{12}\right)^{96} - 10000\left(1 + \frac{0,65}{12}\right)^{60} + 10000\left(1 + \frac{0,65}{12}\right)^{36}$ $A = R48708,61$ $\therefore \text{difference} = 48708,61 - 50390,07$ $= -R1681,46$	
5.1	$A = P(1+i)^n$ $= 140\,000(1+0,061)^4$ $= R177\,414,69$	
5.2	$1 + i_{eff} = \left(1 + \frac{0,07}{2}\right)^2$ $1 + i_{eff} = (1 + 0,035)^2$ $i_{eff} = (1 + 0,035)^2 - 1$ $= 0,071225$ $= 7,12\%$	
5.3	$A = 24000\left(1 + \frac{0,105}{12}\right)^{48} - 7000\left(1 + \frac{0,105}{12}\right)^{30}$ $= R27\,369,56$	
5.4	$102\,755,34 = 198\,000\left(1 - \frac{r}{100}\right)^3$ $\sqrt[3]{\frac{102\,755,34}{198\,000}} = 1 - \frac{r}{100}$ $1 - \frac{r}{100} = 0,8036119818$ $- \frac{r}{100} = -0,1963880182$ $r = 19,64\%$	

6.1	$A = P(1 - i)^n$ $A = 635000 \left(1 - \frac{15}{100}\right)^5$ $A = 281\,752,87$			
6.2.1	$P_v = \frac{x[1 - (1+i)^{-n}]}{i}$ $50000 = \frac{x \left[1 - \left(1 + \frac{16.75}{1200}\right)^{-48}\right]}{\frac{16.75}{1200}}$ $x = R\,1\,436,29$			
6.2.2	$P_v = \frac{x[1 - (1+i)^{-n}]}{i}$ $P_v = \frac{1436.29 \left[1 - \left(1 + \frac{16.75}{1200}\right)^{-18}\right]}{\frac{16.75}{1200}}$ $P_v = R22\,721,97704$ $P_v = R22\,722$	$OB = 50000 \left(1 + \frac{16.75}{1200}\right)^{30} - \left[\frac{1436.29 \left[\left(1 + \frac{16.75}{1200}\right)^{30} - 1 \right]}{\frac{16.75}{1200}} \right]$ $OB = R\,22722,14$ $OB = R22722$		
6.3	$A = P(1 + i)^n$ $A = 2x \text{ and/en } P = x$ $2x = x \left(1 + \frac{14.75}{100}\right)^n$ $n = \frac{\log 2}{\log \left(1 + \frac{14.75}{100}\right)}$ $n = 5.04 \text{ years}$			

7.1	$A = P(1 - i)^n$ $= 500000(1 - 0,2)^6$ $= 131\,072$
7.2	$A_e = P(1 + i)^n$ $= 500000(1 + 0,07)^6$ $= 750\,365,1759$ Cost of the new server: $750\,365,18 - 131\,072,00 = 619\,293,18$

