



GAUTENG PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

REMOTE LEARNING ACTIVITY BOOKLET

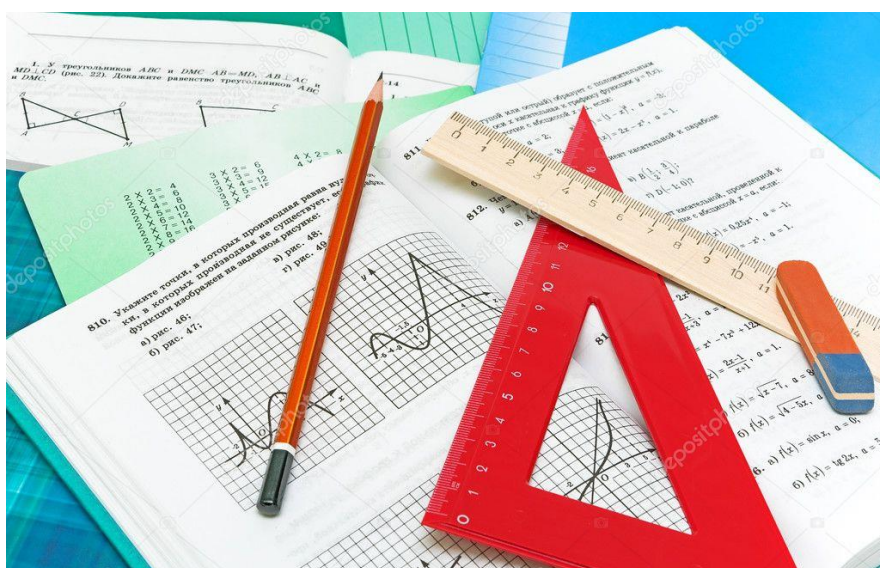
(RELAB)

SUBJECT: MATHEMATICS

GRADE: 11

TERM 1 – TERM 4

LEARNER



GGT2030
GROWING GAUTENG TOGETHER

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B.INTRODUCTION AND PURPOSE OF THE RELAB

The pandemic has forced schools to resort to the implementation of rotational timetables-where learners who are at home during normal schooling must continue learning. Hence RELAB as a strategy towards the deployment of remote learning.

The RELAB is underpinned by the following Legislative demands:

- a) Responding to GDE Strategic goal 2 promoting quality education across all classrooms and schools
- b) **DBE Circular S13 of 2020** the requires the GDE to support the implementation of the Recovery Annual Teaching Plan (RATP)
- c) **GDE Circular 11 of 2020** requiring districts to issue Learning Activity Packs to support schools for lockdown learning. Understanding learning constraints at home as majority of learners do not have access to devices or data to use for online learning. Many households are depending on schools to provide them with learning resources packs

RELAB is designed in a study guide format, where the content is briefly explained with related concepts as revision, in the form of e.g. notes, mind-maps, concept progression from the previous grade/s followed by exemplar exercises then practice exercises/problems . The exercises are pitched at different cognitive levels to expose learners at Grade 10 & 11 to these different cognitive levels of questioning. The NSC diagnostic reports in different subjects have revealed that learners fail to analyse questions and as a result fail to respond accordingly.

The RELAB is intended to ensure that learners work on exercises as per topics taught while at school. These exercises must be completed at home, fully and learners will receive feedback as groups or individually at school. It is therefore of paramount importance that teachers mark the work with learners in class, as a way of providing feedback. Educators must diagnose learner responses, remediate where necessary and plan further intervention.

Educators are encouraged to create WhatsApp groups to remind learners on what is expected of them in a particular week/ day(s). There shouldn't be a backlog on curriculum coverage as content will be covered simultaneously. Feedback from learners at home will confirm usage of the RELAB material.

RELAB further prepares learners for formal assessment.

C.TERM 1

WEEK	TOPIC	SUBTOPIC	PAGE
1	Exponents and Surds	1. Simplify expressions and solve equations using the laws of exponents for rational exponents where $x^{\frac{p}{q}} = \sqrt[q]{x^p}; x > 0; q > 0$	4 - 12
2		2. Add, subtract, multiply and divide simple surds.	
3		3. Solve simple equations involving surds.	
4			
2	Equations and Inequalities	1. Complete the square	
3		2. Solve Quadratic equations (by factorization and by using the quadratic formula) ; K- Method	
3		3. Solve Quadratic inequalities in one unknown (Interpret solutions graphically.) NB: It is recommended that the solving of equations in two unknowns is important to be used in other equations like hyperbola-straight line as this is normal in the case of graphs	
4		4. Equations in two unknowns, one of which is linear and the other quadratic 5. Nature of roots	
5	Euclidean Geometry	Teach past grades concepts required for smooth content progression into Gr 11 such as: Angles, lines, triangles, etc.	13 - 36
6		Accept results established in earlier grades as axioms and also that a tangent to a circle is perpendicular to the radius, drawn to the point of contact. Then investigate and prove the theorems of the geometry of circles: - The line drawn from the centre of a circle perpendicular to a chord bisects the chord. - The line drawn from the centre of the circle to the midpoint of the chord is perpendicular to the chord - The perpendicular bisector of a chord passes through the centre of the circle; - An angle subtend by the arc at the centre of the circle is two times an angle subtended by the same arc at the circumference of the circle	
7		- Angles subtended by a chord of the circle, on the same side of the chord, are equal; - The opposite angles of a cyclic quadrilateral are supplementary; - An exterior angle of a cyclic quad is equal to the interior opposite angle - Two tangents drawn to a circle from the same point outside the circle are equal in length;	
8		- The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment. - Use the above theorems and their converses, where they exist, to solve riders.	
9	Trigonometry	1. Derive and use the identities: $\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\theta \neq k \cdot 90^\circ, k$ an odd integer; and $\sin^2 \theta + \cos^2 \theta = 1$. 2. Derive and use reduction formulae to simplify the following expressions: 2.1. $\sin(90^\circ \pm \theta)$; $\cos(90^\circ \pm \theta)$; 2.2. $\sin(180^\circ \pm \theta)$; $\cos(180^\circ \pm \theta)$ and $\tan(180^\circ \pm \theta)$; 2.3. $\sin(360^\circ \pm \theta)$; $\cos(360^\circ \pm \theta)$ and $\tan(360^\circ \pm \theta)$; 	37 - 44
10		2.4. $\sin(-\theta)$; $\cos(-\theta)$ and $\tan(-\theta)$; 3. Determine for which values of a variable an identity holds.	

WEEK 1 – 4: EXPONENTS, SURDS AND EQUATIONS

SUB TOPIC

1. Simplify expressions and solve equations using the laws of exponents for rational exponents

$$\text{where } x^{\frac{p}{q}} = \sqrt[q]{x^p}; x > 0; q > 0$$

2. Add, subtract, multiply and divide simple surds.

3. Solve simple equations involving surds.

QUADRATIC EQUATIONS, SIMULTANEOUS

1. Complete the square

2. Solve Quadratic equations (by factorization and by using the quadratic formula) ; K- Method

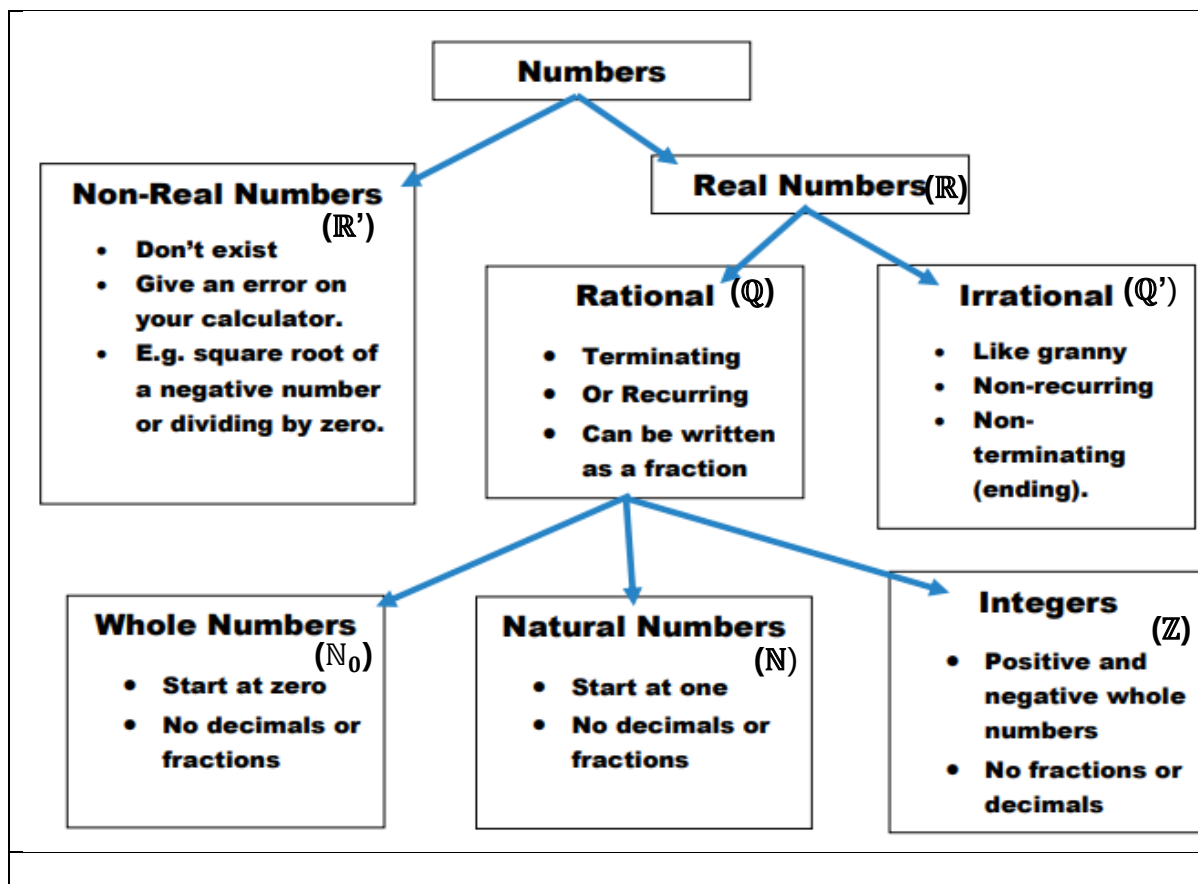
3. Solve Quadratic inequalities in one unknown (Interpret solutions graphically.)

NB: It is recommended that the solving of equations in two unknowns is important to be used in other equations like hyperbola-straight line as this is normal in the case of graphs

4. Equations in two unknowns, one of which is linear and the other quadratic

NATURE OF ROOTS

NOTES & EXAM TIPS



Exponents and Surds

- (a) Simplify expressions using the laws of exponents for rational exponents.
(b) Add, subtract, multiply and divide simple surds (e.g. see subject statement)

$$x^{\frac{p}{q}} = \sqrt[q]{x^p}; x > 0; q > 0$$

- (b) Change from exponents to surds and surds to exponents

NATURE OF ROOTS

ROOTS: the values of the variable that makes the equation true i.e.. the answers, are called the roots of the equation.

CLASSIFICATION OF NUMBERS: Real or Non Real (Imaginary); Rational or Irrational

QUADRATIC FORMULA: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

DISCRIMINANT (Δ):

The terms included under the square root sign in the Quadratic Formula:

i.e. $\Delta = b^2 - 4ac$

THE NATURE OF THE ROOTS OF THE QUADRATIC EQUATION IS DETERMINED BY THE VALUE OF THE DISCRIMINANT (GREEK LETTER CALLED DELTA)

$\Delta = b^2 - 4ac$	Nature of roots
$\Delta \geq 0$	Real
$\Delta = 0$	Real , Rational ,Equal
$\Delta = \textit{Perfect square}$	Real , Rational ; Unequal
$\Delta \neq \textit{Perfect square}$	Real ; Irrational ; Unequal
$\Delta < 0$	Non - Real

VALUE OF THE DISCRIMINANT	TYPE AND NUMBER OF SOLUTIONS	EXAMPLE OF GRAPH
Positive Discriminant $b^2 - 4ac > 0$	Two Real Solutions If the discriminant is a perfect square the roots are Rational. If the discriminant is not a perfect square, then the roots are Irrational.	
Discriminant is Zero $b^2 - 4ac = 0$	One Real Solution	
Negative Discriminant $b^2 - 4ac < 0$	No Real Solutions Two <u>Imaginary</u> Solutions	

TIPS AND HINTS

HINTS TO LEARNERS: QUADRATIC EQUATIONS/INEQUALITIES

- If a quadratic equation is already factorized and one side is equal to zero, then there is no need to expand the equation just write the answers from the given factors
- If one side is factorized and the other side is not equal to zero, then first write the question in standard form and factorize
- Simplify until the right-hand side is 0
- Then factorize the trinomial
- You will get at most two solutions
- You might want to check your answers
- When the question says correct to one OR two decimal places. You are expected to solve the quadratic equation using the quadratic formula
- For the inequality, simplify so that the right-hand side is 0

- Then use the graphic (draw the sketch of the parabola) or number line Method
- Don't cross multiply against an inequality sign. You will lose roots

HINTS TO LEARNERS: SIMULTANEOUS EQUATIONS

- Make x or y (**Any variable that is given in the equation**) the subject of the formula in the linear equation.
- Avoid solving for x or y if it has a co-efficient other than 1
- Ensure after substitution you have an equation in 1 variable x or y not both.
- Then solve the quadratic equation.
- Substitute these values in either equation to get the corresponding values

HINTS TO LEARNERS: NATURE OF ROOTS

To answer questions on the questions on the Nature of the Roots, we go through the following steps:

1. **Write the equation in standard form:** $ax^2 + bx + c = 0$ and get the values for $a; b$ and c .
2. **Calculate the value of Δ**
3. Three types of questions may be asked:
 - 3.1 **You have to determine the nature of the roots** from the value of Δ that you already obtained.
 - 3.2 You have to **determine the value** of an **unknown** while the nature of the roots is given.
 - 3.3 You have to **prove / show** that the roots have a certain given nature.

EXAMPLE

1.

$$(x - 2)(x + 1) = 0$$

$$x = 2 \text{ or } x = -1$$

2

$$(x - 10)(x + 4) = 3 \quad (\text{Leave the answer correct two decimal places})$$

$$x^2 - 5x + 4 - 3 = 0$$

$$x^2 - 5x + 1 = 0$$

$$a = 1, b = -5, c = 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{21}}{2}$$

$$x = 0,21 \quad \text{or} \quad x = 4,79$$

3.

EXAMPLE**Determine the nature of the roots of the following equations:**

$$2x^2 - 7x + 3 = 0$$

SOLUTION

$$a = 2, b = -7, c = 3$$

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= (-7)^2 - 4(2)(3) \\ &= 49 - 16 \\ &= 25\end{aligned}$$

 Δ is a perfect square **$\therefore$ the roots real; rational; unequal****ACTIVITY 1****Tick the correct columns in the table below:**

Number	Real ($\mathbb{R}$)	Non- Real ($\mathbb{R}'$)	Rational ($\mathbb{Q}$)	Irrational ($\mathbb{Q}'$)	Whole ($\mathbb{N}_0$)	Natural ($\mathbb{N}$)	Integer ($\mathbb{Z}$)
0							
-9							
$\sqrt{3}$							
$\frac{13}{16}$							
-1,45							
$\sqrt{-27}$							
$(\sqrt{-3})^2$							
2π							
$\sqrt[3]{-343}$							

ACTIVITY 2

QUESTION 1

For which of the value of x :

$$x \in \{-11; -5; 0; 15\} \text{ is } \sqrt{\frac{9}{11-x}}$$

1.1. a Rational number (2)

1..2 an Irrational number (3)

1.3 not defined (2)

QUESTION 2

2.1 Solve for x :

2.1.1 $x^2 - 6x = 0$ (2)

2.1.2 $x^2 + 10x + 8 = 0$ (correct to TWO decimal places) (3)

2.1.3 $(1-x)(x+2) < 0$ (3)

2.1.4 $\sqrt{x+18} = x-2$ (5)

2.2 Solve simultaneously for x and y :

$$x + y = 3 \quad \text{and} \quad 2x^2 + 4xy - y = 15 \quad (6)$$

2.3 If n is the largest integer for which $n^{200} < 5^{300}$, determine the value of n . (3)
[22]

QUESTION 3

3.1 Solve for x :

3.1.1 $x^2 - 5x - 6 = 0$ (2)

3.1.2 $(3x-1)(x-4) = 16$ (correct to TWO decimal places) (4)

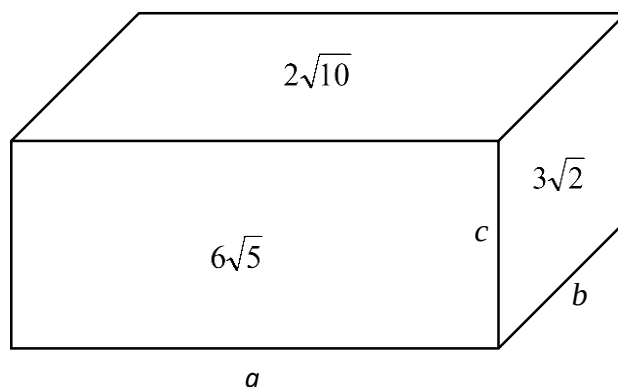
3.1.3 $4x - x^2 \geq 0$ (3)

3.1.4 $\frac{5^{2x} - 1}{5^x + 1} = 4$ (3)

3.2 Solve simultaneously for x and y :

$$x + 3y = 2 \quad \text{and} \quad x^2 + 4xy - 5 = 0 \quad (5)$$

3.3 A rectangular box has dimensions a , b and c . The area of the surfaces are $2\sqrt{10}$; $3\sqrt{2}$ and $6\sqrt{5}$, as shown in the diagram below.



Calculate, **without using a calculator**, the volume of the rectangular box. (5)
[22]

QUESTION 4

4.1 Solve for x :

4.1.1 $x(x-1) = 0$ (2)

4.1.2 $2x^2 - 4x - 5 = 0$ (correct to TWO decimal places) (3)

4.1.3 $5^x = \frac{1}{125}$ (2)

4.1.4 $(x-3)(2-x) > 0$ (3)

4.2 Given: $f(x) = x + 1$ and $g(x) = \frac{-4}{x-3}$

4.2.1 For which values of x will $g(x)$ be undefined? (1)

4.2.2 Solve for x if $f(x) = g(x)$. (4)

- 4.3 The distance between Joe's house and the supermarket is x km. He drives from his house to the supermarket at an average speed of y km/h. From the supermarket Joe returns to his house, along the same route, at an average speed that is one and a half times faster than the original average speed of y km/h. Calculate the overall average speed at which Joe drove from his house to the supermarket and back. Leave your answer in terms of y . (6)
[21]

QUESTION 5

- 5.1 Solve for x , correct to TWO decimal places, where necessary:
- 5.1.1 $x(x-1) = 12$ (3)
- 5.1.2 $2x^2 + 3x - 7 = 0$ (4)
- 5.1.3 $7x^2 + 18x - 9 > 0$ (4)
- 5.2 Solve for x and y simultaneously:
- $$\begin{aligned} 2x - y &= 7 \\ x^2 + xy &= 21 - y^2 \end{aligned} \quad (7)$$
- 5.3 Simplify completely, without the use of a calculator:
- $$\left(\sqrt[5]{\sqrt{35} + \sqrt{3}} \right) \left(\sqrt[5]{\sqrt{35} - \sqrt{3}} \right) \quad (3)$$
- [21]

QUESTION 6

- 6.1 Solve for x , rounded off TWO decimal places where necessary
- 6.1.1 $x^2 = -5x - 4$ (3)
- 6.1.2 $x(3-x) = -3$ (5)
- 6.1.3 $3-x < 2x^2$ (5)
- 6.2 Determine the value of x and y if they satisfy both the following equations (8)
- $$\begin{aligned} 2x + y &= 3 \\ x^2 + y + x &= y^2 \end{aligned}$$

6.3 Given : $x = 999\,999\,999\,999$ determine the exact value of $\frac{x^2-4}{x-2}$ (3)

6.4 Explain why the equation $\frac{x^4+1}{x^4} = \frac{1}{2}$ has no real roots (2)

[26]

QUESTION 7

7.1 Solve for x

7.1.1 $x(x - 1) = 30$ (3)

7.1.2 $3x^2 - 5x + 1 = 0$ (Correct to one decimal place) (4)

7.1.3 $15x - 4 < 9x^2$ (4)

7.2 Solve for x and y in the following set of equations

$$\begin{aligned} x - y &= 3 \\ x^2 - xy - 2y^2 - 7 &= 0 \end{aligned} \quad (5)$$

7.3 Calculate the exact value of:

$$\frac{\sqrt{10^{2009}}}{\sqrt{10^{2011}} - \sqrt{10^{2007}}} \quad (\text{Show ALL the calculations}) \quad (3)$$

7.4 Simplify completely without the use of a calculator:

$$\left(1 + \sqrt{2x^2}\right)^2 - \sqrt{8x^2} \quad (3)$$

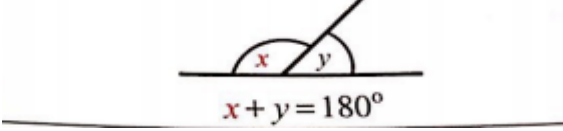
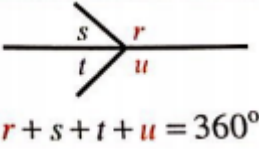
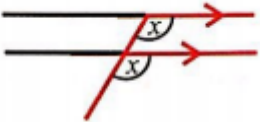
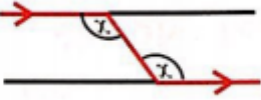
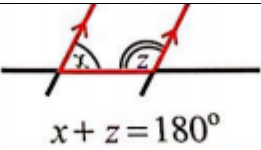
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WEEK 5 – 8 : TOPIC: EUCLIDEAN GEOMETRY

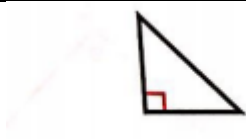
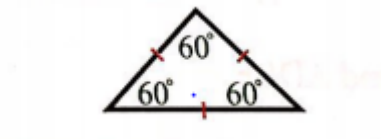
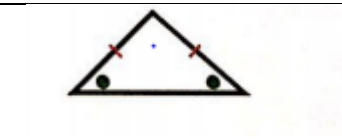
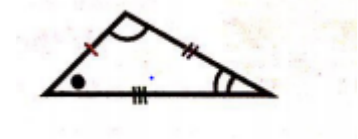
SUB-TOPIC: Circle Geometry

NOTES & EXAM TIPS

A: LINES

PROPERTY/DEFINITION	ILLUSTRATION
Adjacent angles on a line are supplementary (adding to 180°)	
Vertically opposite angles are equal.	
Angles around a point add up to 360°	
Corresponding angles of parallel lines are equal (F- Shape)	
Alternate angles of parallel lines are equal. (Z – Shape)	
Co-interior angles of parallel lines are supplementary (U -Shape)	

B: TRIANGLES AND PROPERTIES

PROPERTY/DEFINITION	ILLUSTRATION
A right-angled triangle has one angle that is equal to 90°	
An equilateral triangle has equal sides and equal angles. All angles in an equilateral triangle measure 60°	
An isosceles triangle has two sides equal and the angles opposite these equal sides are also equal.	
A Scalene triangle has no sides or angles that are equal.	

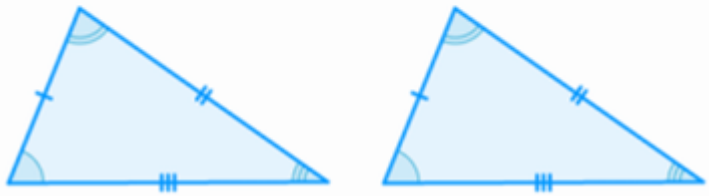
CONGRUENCY OF TRIANGLES

Definition of congruency: Two figures having same size and shape or one has the same size and shape as the mirror image of the other.

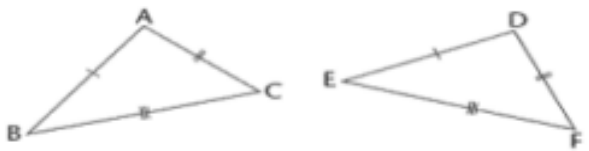
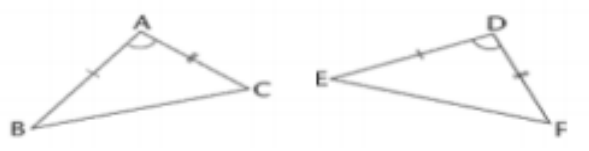
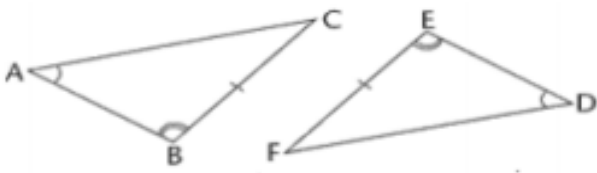
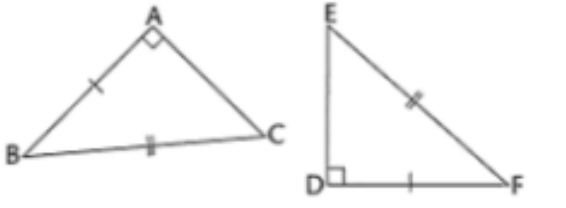
If the two triangles are congruent we use this sign $\equiv$

CONGRUENT TRIANGLES

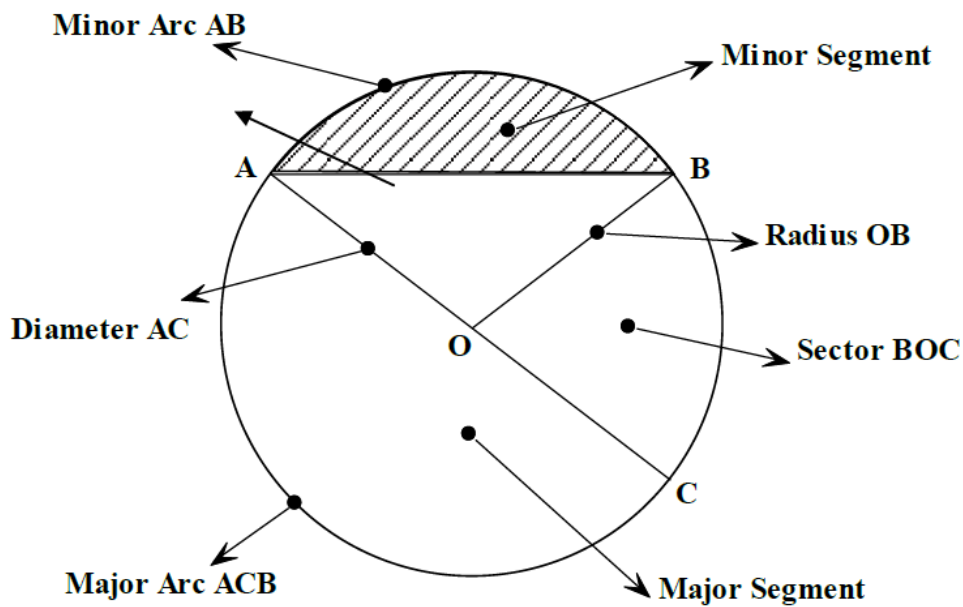
Same Shape & Size



FOUR CASES OF CONGRUENCY:

THREE SIDES $\triangle BAC \equiv \triangle DEF$ (SSS)	
TWO SIDES AND THE INCLUDED ANGLE $\triangle ABC \equiv \triangle DEF$ (SAS)	
TWO ANGLES AND THE CORRESPONDING SIDES $\triangle BAC \equiv \triangle DEF$ (AAS)	
A RIGHT ANGLES, HYPOTENUSE AND SIDE $\triangle ABC \equiv \triangle DEF$ (RHS)	

CIRCLES AND LINES



Circle

The set of points equidistant from the centre

Radius

A straight line segment that joins the centre of the circle to any point on the circle. All radii are equal in length.

Diameter

A straight line segment that joins 2 points on the circle and passes through the centre of a circle. $Diameter = 2 \times radius$

Circumference

A measure of the length of the “perimeter” of the circle: $C = 2\pi r$

Chord

A line segment that joins any 2 points on a circle. A diameter is a chord that passes through the centre of a circle.

Arc

A part of the circle. A chord cuts the circle into a minor and major arc.

Segment

A part of the interior of a circle cut off by a line intersecting. A chord cuts the interior of a circle into a minor and major segment

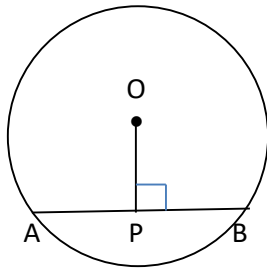
Tangent

A straight line that touches the circle at one point only.

GROUP 1 : CENTRE THEOREMS

THEOREM 1A

The line drawn from the centre of the circle perpendicular to the chord bisect the chord

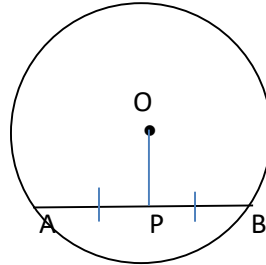


Given : $OP \perp AB$

RTP: $AP = PB$

CONVERSE OF THEOREM 1A

The line drawn from the centre of the circle to the midpoint of the chord is perpendicular to the chord.

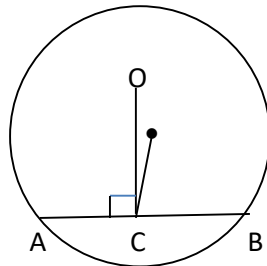


Given: $AP = PB$

RTP: $OP \perp AB$

THEOREM 1B

The perpendicular bisector of a chord passes through the centre of the circle.

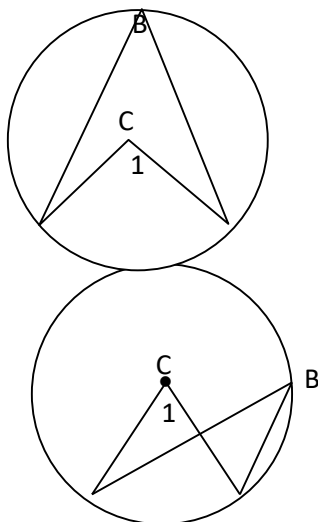


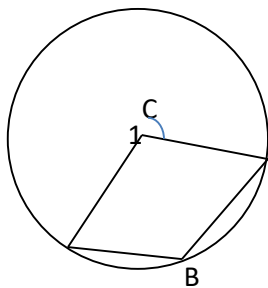
Given: $OC \perp AB$, $AC = CB$

RTP: OC passes through the centre of the circle.

THEOREM 2

The angle subtended by an arc at the centre of the circle is twice at the angle subtended by the same arc at the circumference of the circle.

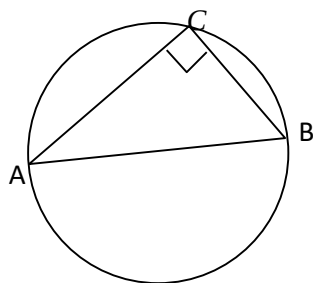




RTP: $\hat{C}_1 = 2B$

THEOREM 3

The angle in the semi-circle is a right angle

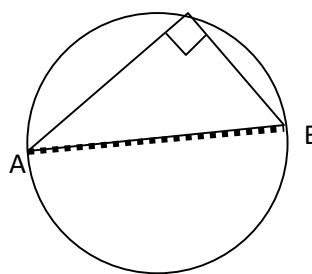


Given: Diameter AB

RTP: $\hat{C} = 90^\circ$

CONVERSE OF THEOREM 3

If the chord of a circle subtends a right angle at a point on the circumference then the chord is a diameter of the circle.

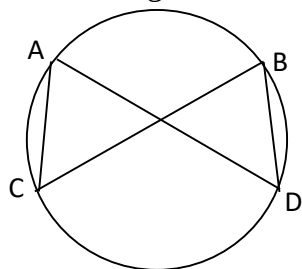


RTP : AB is a diameter

GROUP 2 : CYCLIC QUADRILATERAL THEOREMS

THEOREM 4

An arc or a chord of a circle subtends equal angles at the circumference of the circle. (We say angles in the same segment are equal.)

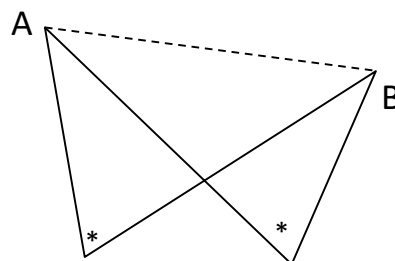


Given : Circle centre O

RTP: $\hat{A} = \hat{B}$ and $\hat{C} = \hat{D}$

CONVERSE OF THEOREM 4

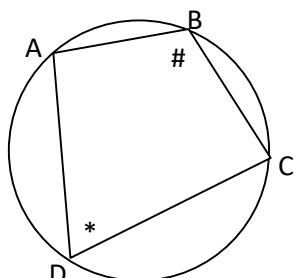
If two equal angles are subtended by the same line then the four points are concyclic (lie on the circumference).



RTP : ABCD lie on the circle

THEOREM 5

The opposite angles of a cyclic quadrilateral are supplementary

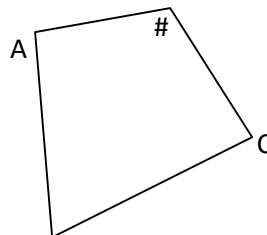


Given : ABCD lie on the circumference of the circle

RTP : $\hat{B} + \hat{D} = 180^\circ$

CONVERSE OF THEOREM 5

If a pair of opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic

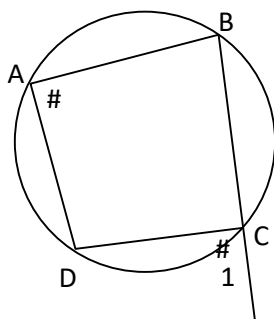


Given : $\hat{B} + \hat{D} = 180^\circ$

RTP: ABCD is a cyclic quad

THEOREM 6

An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

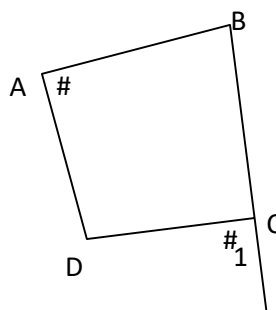


Given: ABCD are points that lie on the circumference of the circle

RTP: $\hat{C}_1 = \hat{A}$

CONVERSE OF THEOREM 6

If an exterior angle of a quadrilateral is equal to the interior opposite angle then the quadrilateral is cyclic.

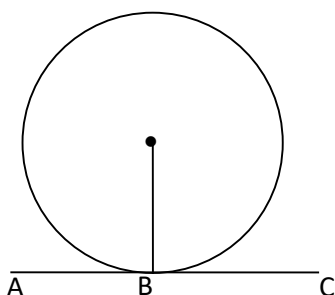


If $\hat{C}_1 = \hat{A}$

RTP: then ABCD is a cyclic quadrilateral

GROUP 3 TANGENTS THEOREM**THEOREM 7**

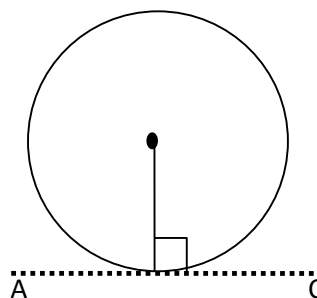
A tangent to a circle is perpendicular to the radius at the point of contact



If AC is a tangent to the circle at B then the radius $OB \perp AC$, i.e. $\hat{B}_1 = \hat{B}_2 = 90^\circ$

CONVERSE OF THEOREM 7

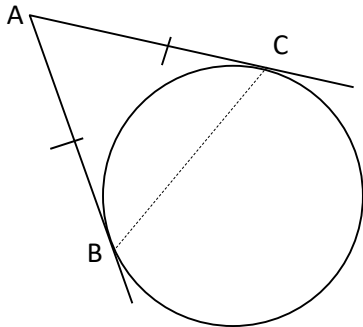
If a line is drawn perpendicular to the radius at a point of contact then the line is a tangent to a circle.



RTP: AC is a tangent.

THEOREM 8

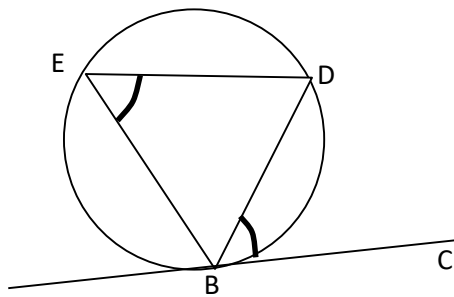
Two tangents drawn to a circle from the same point outside the circle are equal in length.



Given AB and AC as tangents
RTP: $AB = AC$

THEOREM 9

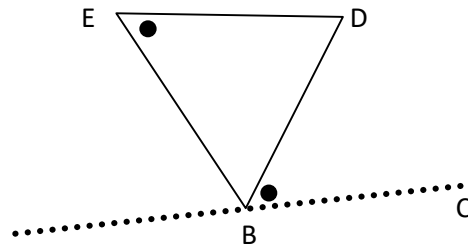
The angle between a chord and a tangent to a circle at a point of contact is equal to the angle in the alternate segment.



Given : ABC as tangent at B
RTP: $\hat{DBC} = \hat{E}$

CONVERSE OF THEOREM 9

If an angle between a chord and line at the point of contact is equal to the angle in the alternate segment then the line is a tangent.



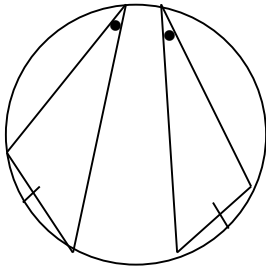
Given : : $\hat{DBC} = \hat{E}$
RTP: ABC is a tangent

The following proofs of theorems are examinable (NB. know the proofs by heart)

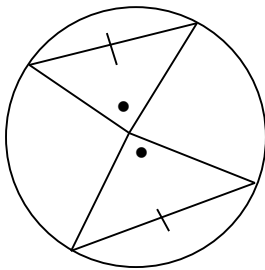
- ✓ The line drawn from the centre of a circle perpendicular to a chord bisects the chord;
- ✓ The line drawn from the centre of the circle to the midpoint of the chord is perpendicular to the chord
- ✓ The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre);
- ✓ The opposite angles of a cyclic quadrilateral are supplementary;
- ✓ The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment;

COROLLARIES ON THEOREM 4:

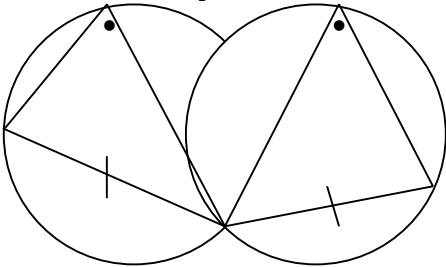
1. Equal chords subtend equal angles at the circumference



2. Equal chords subtend equal angles at the center.



3. Equal chords of equal circles subtend equal angles at the circumference



STEPS OF SOLVING A GEOMETRIC RIDER

STEP 1

- **Analyse** the diagram

STEP 2

- Analyse the diagram by paying attention to **key words**
- Look for information the diagram which can be **helpful and useful**
- **Use colours** to mark off equal angles /sides
- Look for **implied information**

STEP 3

- Brainstorm and develop a **rough proof**
- **Link** information you have acquired

STEP 4

- Rewrite a **formal proof**

Examples of **KEY** words

- Parallel lines
- Triangle information
- parallelogram
- Intersecting lines
- square
- rectangle

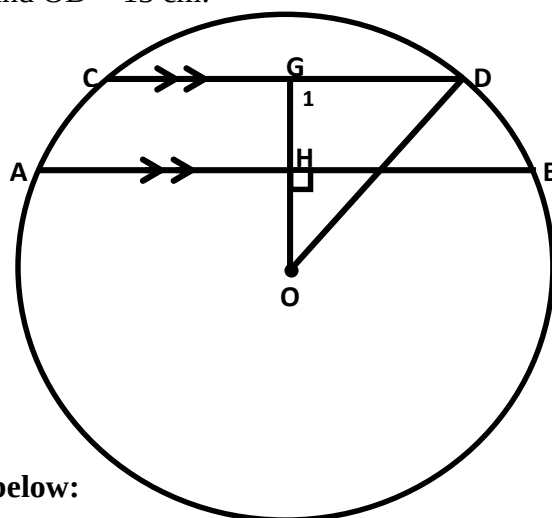
EXERCISE 1

GRADE 11: REASONS WORKSHEET

1. Complete the statement so that it is TRUE:

The line drawn from the centre of the circle to the midpoint of the chord.....

2. In the diagram, O is the centre of the circle with $AB \parallel CD$ and $OH \perp AB$, $AB = 24$ cm, $CD = 10$ cm and $OD = 13$ cm.



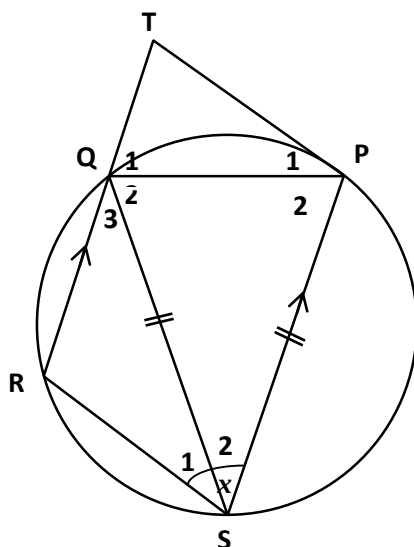
Give reasons for the statements below:

STATEMENT	REASON
2.1 $\hat{G}_1 = 90^\circ$	
2.2 $AH = 12$ cm	
2.3 $OB = 13$ cm	

3. Complete the statement so that it is TRUE

The angle which an arc subtends at the of a circle is twice the angle it subtends at the circumference of the circle.

4. In the diagram alongside, $RT \parallel SP$, $SQ = SP$ and TP is a tangent to the circle at P . $\hat{PSR} = x$. P, Q, R and S are points on the circumference of the circle.



Give reasons for the statements below:

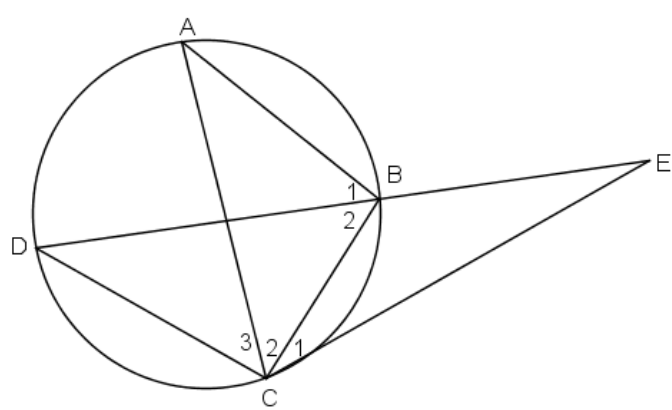
STATEMENT	REASON
4.1 $\hat{Q}_3 = \hat{S}_2$	
4.2 $\hat{Q}_2 = \hat{P}_2$	
4.3 $\hat{R} + \hat{P}_2 = 180^\circ$	
4.4 $\hat{Q}_1 = x$	

5. Complete the following so that the statements are TRUE:

5.1 The angle between a chord and a tangent at a point of contact is

5.2 The opposite angles of a Quadrilateral are supplementary.

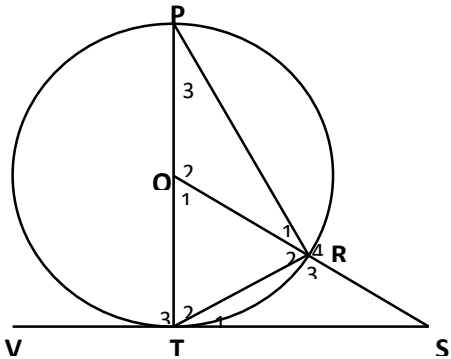
6. In the diagram below circle ABCD is given. CE is a tangent to circle ABCD at C. BD is the diameter to circle ABCD.



Give reasons for the statements below:

STATEMENT	REASONS
6.1 $\hat{A} = \hat{D}$	
6.2 $\hat{C}_1 = \hat{D}$	
6.3 $\hat{C}_3 = \hat{B}_1$	
5.4 $\hat{C}_2 + \hat{C}_3 = 90^\circ$	

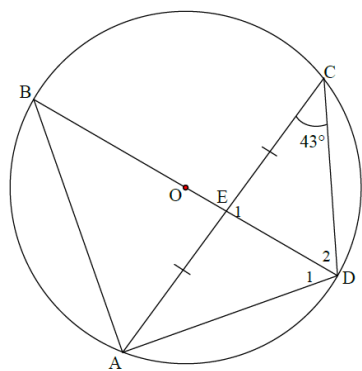
7. In the diagram below, PT is a diameter of the circle with centre O and $\angle TPR = 30^\circ$. VS is a tangent to the circle at T and meets OR produced at S.



Give reasons for the statements below:

STATEMENT	REASON
7.1 $\hat{P} = \hat{R}_1$	
7.2 $\hat{T}_2 = \hat{R}_2$	
7.3 $\hat{T}_1 + \hat{T}_2 = 90^\circ$	
7.4 $\hat{T}_1 = \hat{P}$	

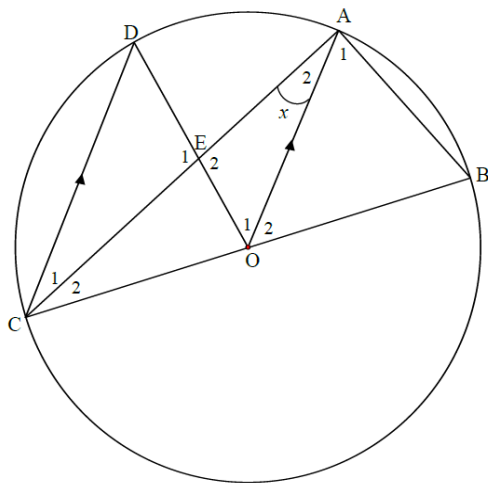
8. In the diagram, O is the centre of the circle. A, B, C and D are points on the circumference of the circle. Diameter BD bisects chord AC at E. Chords AB, CD and AD are drawn. $\hat{C} = 43^\circ$



Give reasons for the statements below

STATEMENTS	REASONS
8.1 $DE \perp AC$	
8.2 $\hat{B} = 43^\circ$	

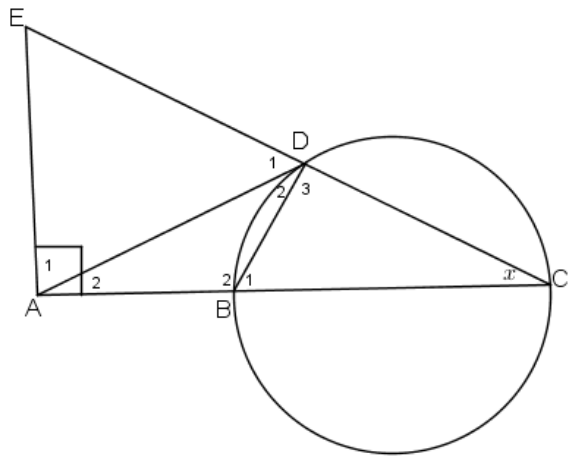
9. In the diagram, O is the centre of the circle. A, B, C and D are points on the circumference of the circle and CB is the diameter of the circle. Chord CA intersect radius OD at E. AB is drawn. $CD \parallel OA$ and $\hat{A}_2 = x$.



Give reasons for the statements below

STATEMENTS	REASONS
9.1 $\hat{C}_1 = x$	
9.2 $\hat{C}_2 = x$	

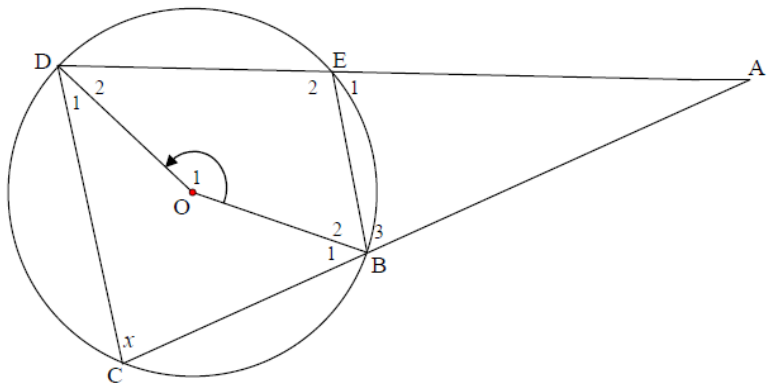
10. In the diagram, BC is a diameter of the circle. The tangent at point D on the circle meets CB produced at A. CD is produced to E such that $EA \perp AC$. Let $\hat{C} = x$



Give reasons for the statements below

STATEMENTS	REASONS
10.1 $\hat{D}_3 = 90^\circ$	
10.2 ABDE is a cyclic quadrilateral	
10.3 $\hat{D}_2 = x$	

11. In the diagram O is the centre of the circle and BO and OD are drawn. Chords CB and DE are produced to meet in A. Chords BE and CD are drawn. $\hat{BCD} = x$.



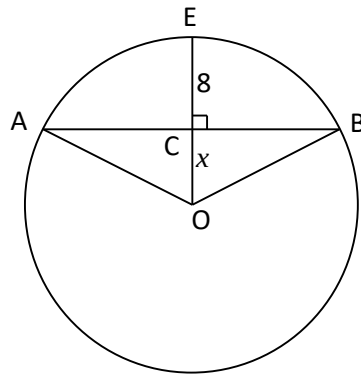
Give reasons for the statements below:

STATEMENTS	REASONS
11.1 $\hat{E}_1 = x$	
11.2 $\hat{O}_1 = 2x$	

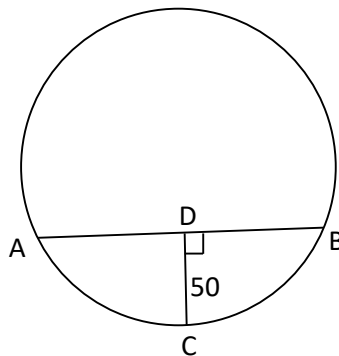
EXERSICE 2

QUESTION 1

1.1	AB is the chord of the circle with centre O and is 24cm long. C is the midpoint of AB. $CE \perp AB$ cuts the circle at E. Calculate the value of x if $CE = 8$ cm. $AC = \dots\dots\dots$ cm
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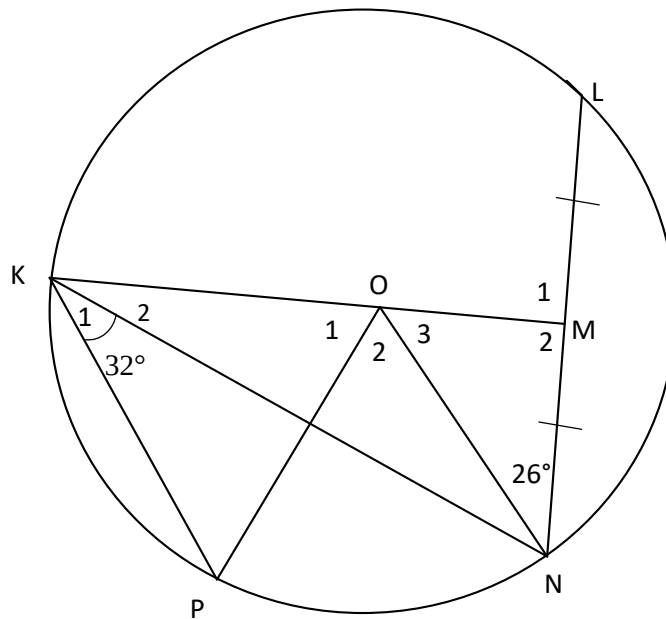


- 1.2 D is the midpoint of the chord AB and $DC \perp AB$ with C on the circle. If $AB = 300$ mm, and $CD = 50$ mm, calculate the radius of the circle.



QUESTION 2

O is the centre of the circle.. KOM bisects chord LN and $\hat{MNO} = 26^\circ$. K and P are points on the circle with $\hat{NKP} = 32^\circ$. OP is drawn.



2.1 Determine, giving reasons, the size of:

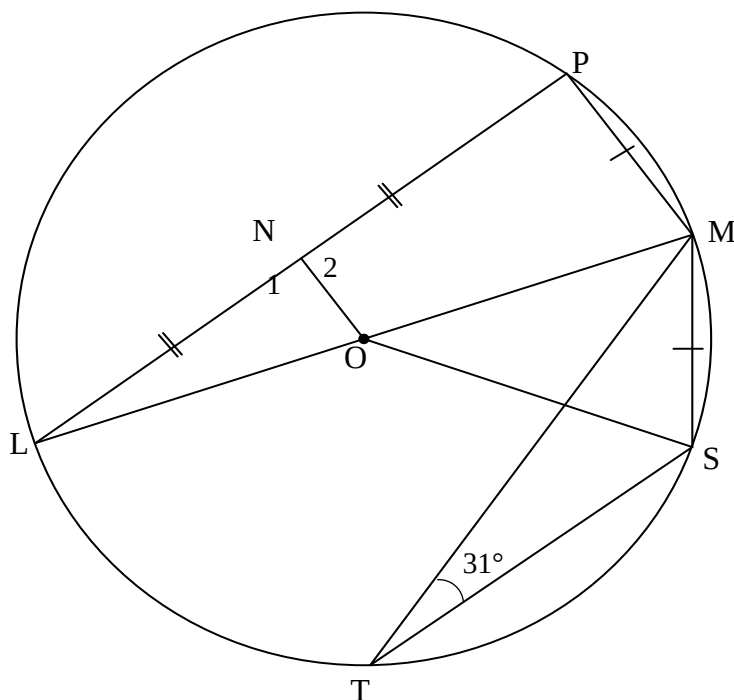
2.1.1 $\hat{O}_2$ (2)

2.1.2 $\hat{O}_1$ (4)

2.2 Prove, giving reasons, that KN bisects $\hat{OKP}$. (3)

QUESTION 3

In the diagram, O is the centre of the circle and LOM is a diameter of the circle. ON bisects chord LP at N. T and S are points on the circle on the other side of LM with respect to P. Chords PM, MS, MT and ST are drawn. $PM = MS$ and $\hat{M}TS = 31^\circ$



3.1 Determine, with reasons, the size of each of the following angles:

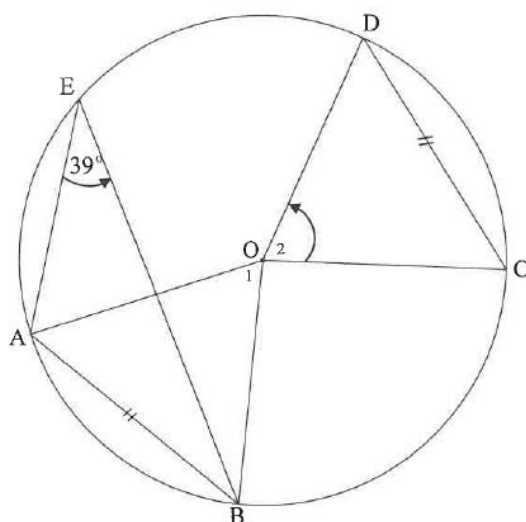
3.1.1 $\hat{M}OS$ (2)

3.1.2 $\hat{L}$ (2)

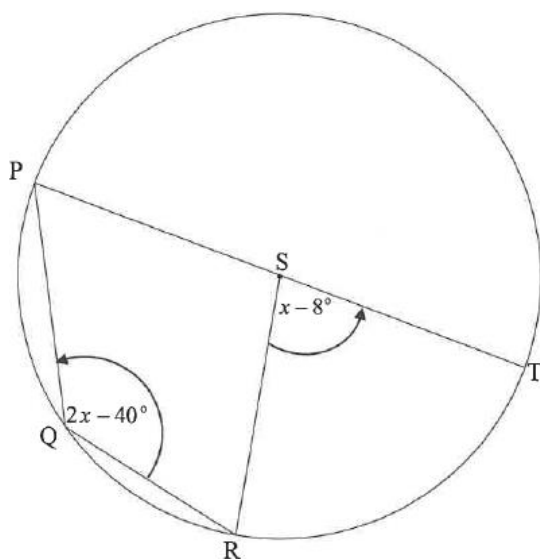
3.2 Prove that $ON = \frac{1}{2} MS$. (4)

QUESTION 4

- 4.1 In the figure, O is the centre of the circle. A, B, C, D and E lie on the circle such that chord AB and chord DC are equal in length and $\hat{AEB} = 39^\circ$

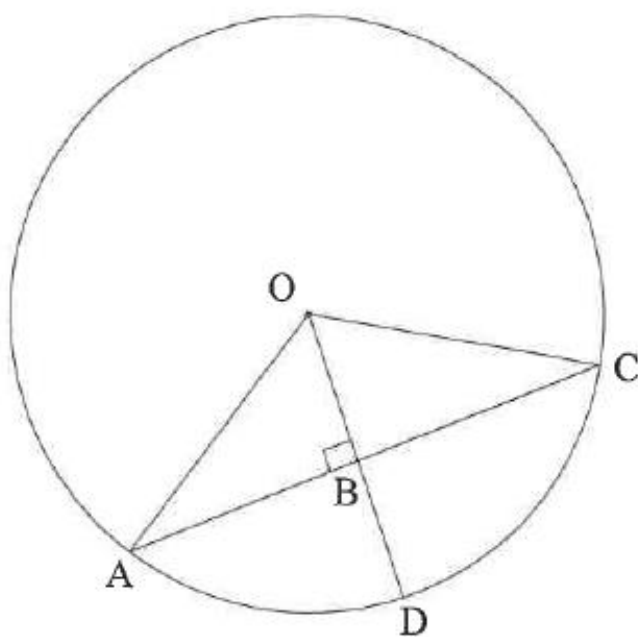


- 4.1.1 Determine the size of $\hat{O}_1$ (2)
- 4.1.2 Determine the size of $\hat{O}_2$ (2)
- 4.2 In the diagram, S is the centre of the circle PQRT. PT is a diameter $\hat{RST} = x - 8^\circ$ and $\hat{PQR} = 2x - 40^\circ$



Determine the value of x (4)

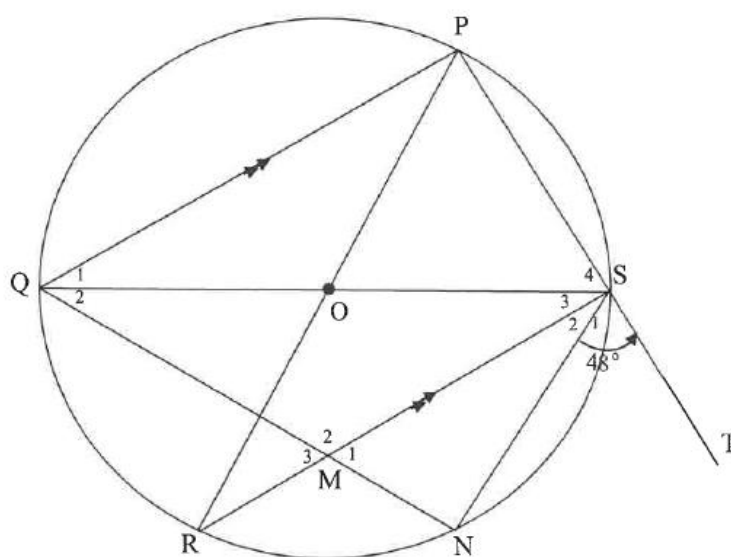
- 4.3 In the diagram, O is the centre of the circle. Chord AC is perpendicular to radius OD at B . $OB = 2x$ units and $AC = 8x$ units



Show that the length of BD is $2x(\sqrt{5} - 1)$ units. (5)

QUESTION 5

In the figure , QS and PR are diameters of the circle with centre O such that $PQ \parallel SR$. PS is produced to T. N is a point on the circle such that $\hat{Q}_1 = \hat{Q}_2$. SN is drawn. RS intersect QN at M. $\hat{S} = 48^\circ$



5.1 Determine , with reasons , the size of:

5.1.1 $\hat{Q}_1$ (3)

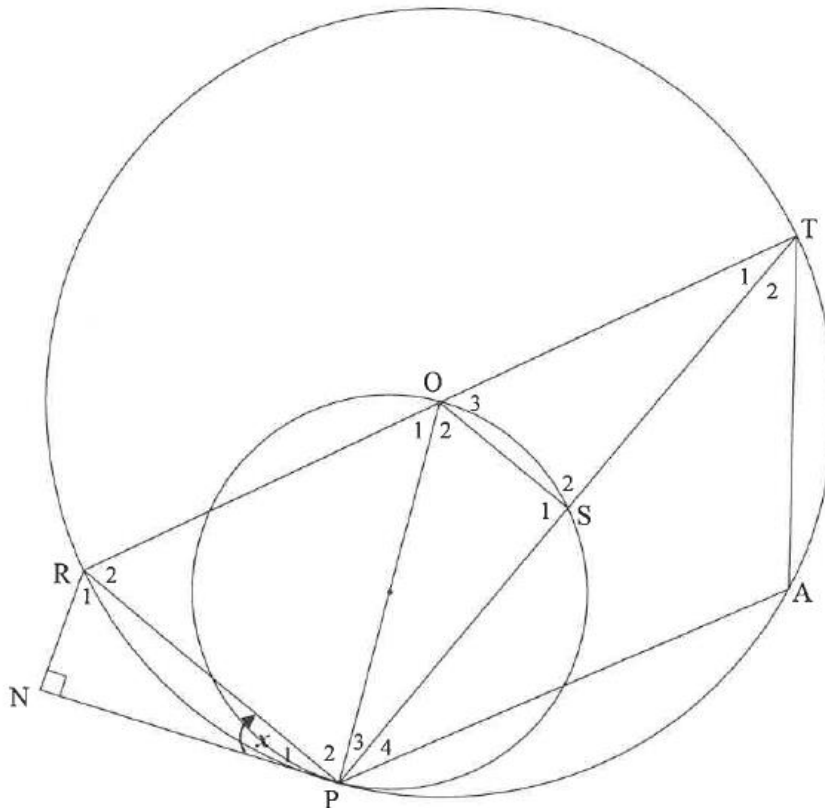
5.1.2 $\hat{R}$ (2)

5.1.2 $\hat{M}_1$ (2)

5.2 Prove that ST is a tangent to the circle passing through M , N and S (2)

QUESTION 6

O is the centre of the larger circle RTAP, OP is a diameter of the smaller circle PSO. NP is a tangent to both circles at P. $RN \perp NP$. Let $\hat{P}_1 = x$

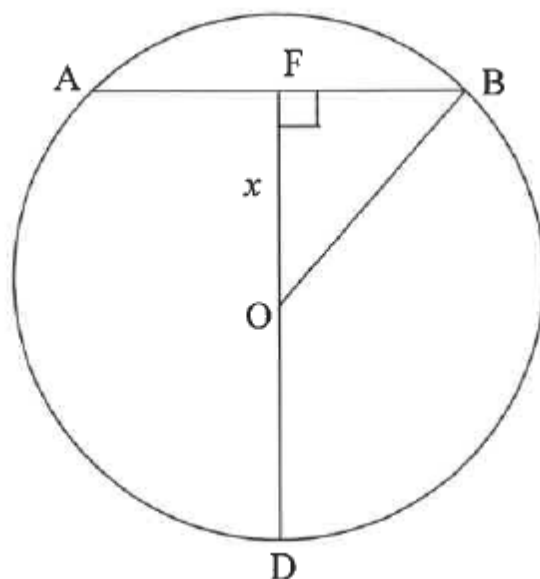


6.1 Prove that PR bisects $\hat{ORN}$ (5)

6.2 Prove that $\hat{ROS} = \hat{PAT}$ (5)

QUESTION 7

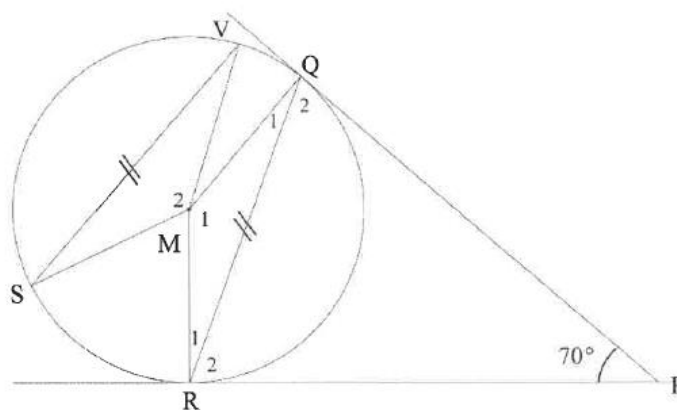
In the diagram, O is the centre of circle ABD. F is a point on chord AB such that $DOF \perp AB$. $AB = FD = 8$ cm and $OF = x$ cm



Determine the length of the radius of the circle (5)

QUESTION 8

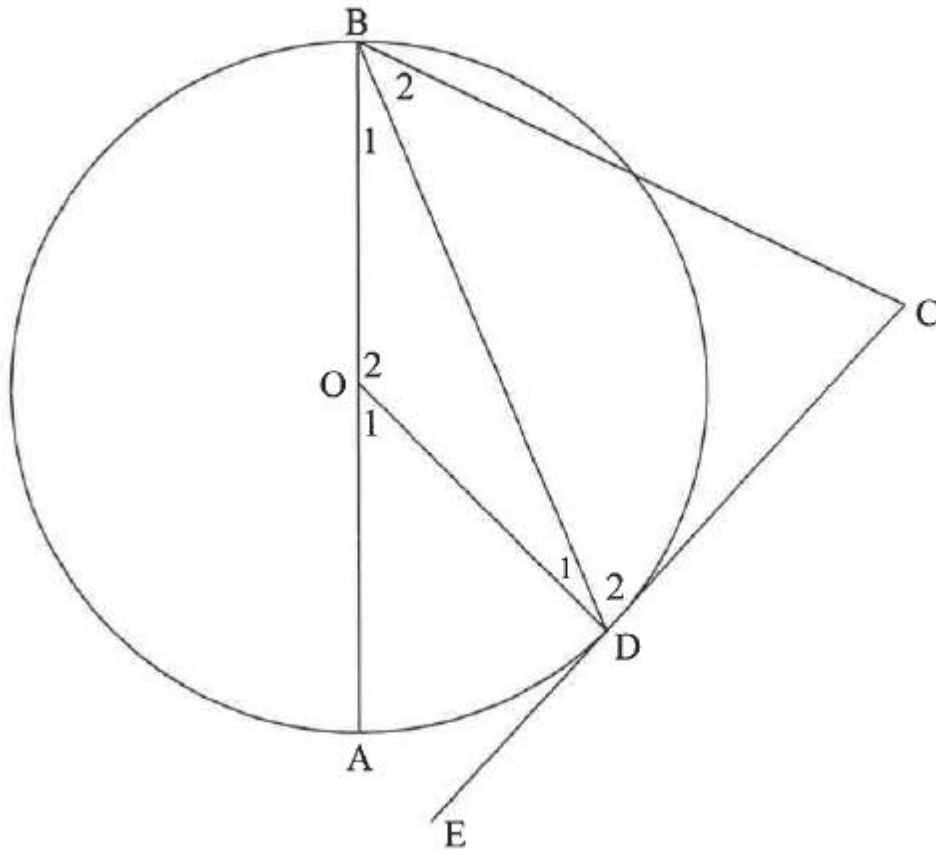
M is the centre of the circle SVQR having equal chords SV and QR. RP and QP are tangents to the circle at R and Q respectively such that $\hat{R}PQ = 70^\circ$



- | | | |
|-----|-----------------------------------|-----|
| 8.1 | Calculate the size of $\hat{R}_2$ | (4) |
| 8.2 | Calculate the size of $\hat{Q}_1$ | (2) |
| 8.3 | Determine the size of $\hat{M}_2$ | (3) |

QUESTION 9

In the diagram below, O is the centre of the circle. CDE is a tangent to the circle at D . DB bisects $\hat{ABC}$. Let $\hat{B}_1 = x$



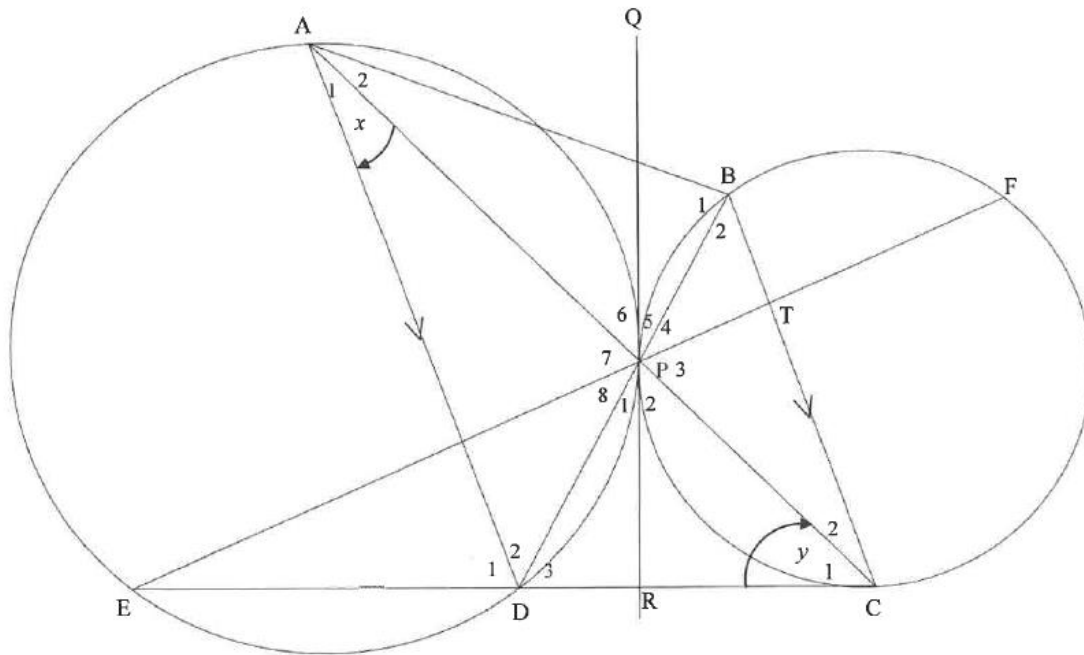
9.1 Prove that $BC \parallel OD$ (4)

9.2 Show that $\hat{C} = 90^\circ$ (3)

QUESTION 10

In the diagram below , two circles touch each other externally at point P.

QPR is a common tangent to both circles at P .EDRC is a tangent to circle PBFC at C.

$$R\hat{C}A = y \text{ and } D\hat{A}C = x. AD \parallel BC$$


- 10.1 Name , with reasons ,FOUR other angles equal to x . (7)
- 10.2 Show that $E\hat{P}A = x + y$ (4)
- 10.3 Determine the numerical value of $x + y$, if it is given that DCTP is a cyclic quadrilateral. (4)

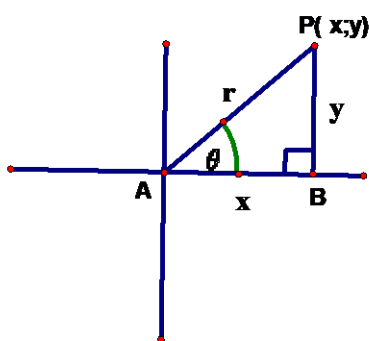
WEEK 9 - 10: TOPIC TRIGONOMETRY

SUB-TOPIC:

- Identities
- Reduction Formula
- Determine for which values of a variable an identity holds.

NOTES & EXAM TIPS

Trigonometric ratios



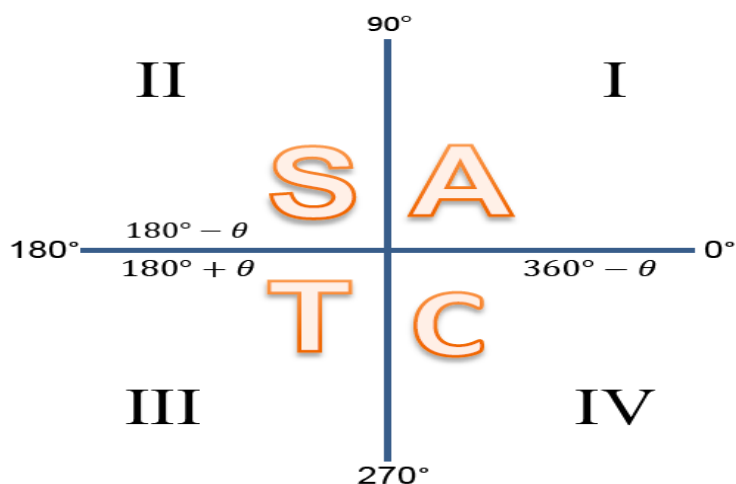
$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

REDUCTION FORMULAE

Summary:



$180^\circ - \theta$

$$\sin(180^\circ - \theta) = \sin \theta$$

$$\cos(180^\circ - \theta) = -\cos \theta$$

$$\tan(180^\circ - \theta) = -\tan \theta$$

$180^\circ + \theta$

$$\sin(180^\circ + \theta) = -\sin \theta$$

$$\cos(180^\circ + \theta) = -\cos \theta$$

$$\tan(180^\circ + \theta) = \tan \theta$$

$360^\circ - \theta$

$$\sin(360^\circ - \theta) = -\sin \theta$$

$$\cos(360^\circ - \theta) = \cos \theta$$

$$\tan(360^\circ - \theta) = -\tan \theta$$

$$\tan(360^\circ + \theta) = \tan \theta$$

$$\cos(360^\circ + \theta) = \cos \theta$$

$$\sin(360^\circ + \theta) = \sin \theta$$

Always remember that negative angles are in the fourth quadrant

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\cos(90^\circ - \theta) = \sin \theta$$

$$\sin(90^\circ - \theta) = \cos \theta$$

$$\cos(90^\circ + \theta) = -\sin \theta$$

$$\sin(90^\circ + \theta) = \cos \theta$$

FUNDAMENTAL IDENTITIES

SQUARE IDENTITY

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\cos^2 x = 1 - \sin^2 x$$

QUOTIENT IDENTITY

$$\tan x = \frac{\sin x}{\cos x}$$

Simplifying or Proving identities

- Choose the complicated side
- Change in terms of sin and cos if there is a tan ratio.

TRIGONOMETRIC IDENTITIES (work with the LHS and RHS separately)

- Choose most difficult side and use identities to simplify it.
- Look for square identities
- Change functions to $\sin x$ and $\cos x$
- If there are fractions: Get LCM
- Factorise or simplify if necessary
- If the expression contains $\cos x - 1$ and cannot be solved, then multiply above and below by $\cos x + 1$
- See which value(s) of x the expression is not defined.

NB: “*hence*” means that the previous answer must be used

RESTRICTIONS FOR $\theta \in [0^\circ; 360^\circ]$

- $\tan 90^\circ$ and $\tan 270^\circ$ are undefined
- $-1 \leq \sin \theta \leq 1$, For all values of θ and $-1 \leq \cos \theta \leq 1$ for all values of θ .
- Division by 0 is undefined. Thus $\frac{1}{f(\theta)}$ is undefined if $f(\theta) = 0$.

e.g.

- $\frac{1}{\cos \theta}$, is undefined if $\cos \theta = 0 \Rightarrow \theta = 90^\circ$ or $\theta = 270^\circ$.
- $\frac{1}{\sin \theta}$, is undefined if $\sin \theta = 0 \Rightarrow \theta = 180^\circ$ or $\theta = 360^\circ$ or $\theta = 0^\circ$
- $\frac{1}{\tan \theta}$, is undefined if $\tan \theta = 0 \Rightarrow \theta = 0^\circ$ or $\theta = 180^\circ$ or $\theta = 360^\circ$.

- $\sqrt{f(\theta)}$ Is undefined if $f(\theta) < 0$.

E.g. $\sqrt{\sin \theta}$ is undefined if $\sin \theta < 0 \Rightarrow 180^\circ < \theta < 360^\circ$.

- $\sqrt{f(\theta)} < 0$ is meaningless because $\sqrt{f(\theta)}$ is a positive value

e.g. $\sqrt{\sin \theta} = -0,6$, means that there is no value for θ .

ACTIVITIES/WORKSHEETS/EXAM TYPE QUESTIONS

1. Prove that

$$\frac{1}{1-\cos(180^\circ-x)} + \frac{1}{1-\sin(90^\circ-x)} = \frac{2}{\sin^2 x}$$

2. Consider the following identity:

$$\frac{\sin \theta}{1-\cos \theta} - \frac{\cos \theta}{\sin \theta} = \frac{1}{\sin \theta}$$

2.1 For which values of θ , where $-360^\circ \leq \theta \leq 360^\circ$ is the identity not defined?
(4)

2.2 Now prove the identity.

QUESTION 3

3. **Without using a calculator**, write the following expressions in terms of $\sin 11^\circ$:

$$\sin 191^\circ \quad (1)$$

QUESTION 4

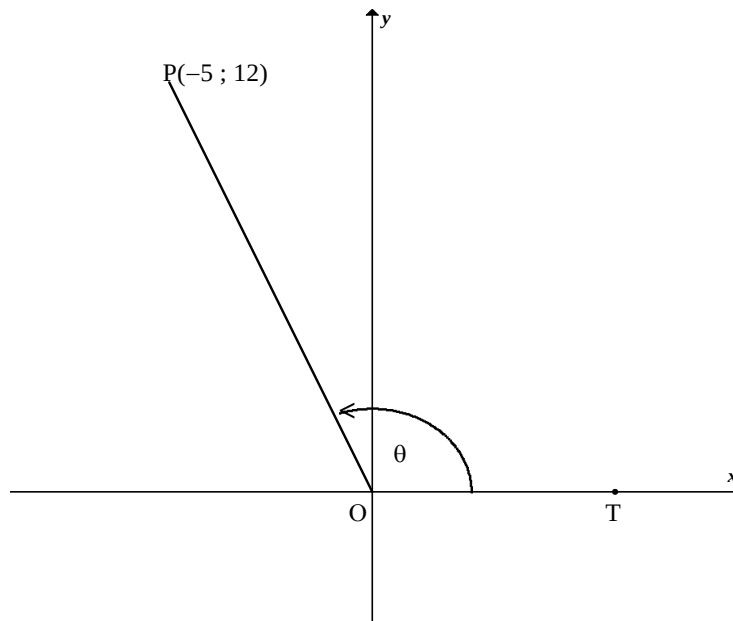
4. If $\sin 28^\circ = a$ and $\cos 32^\circ = b$, determine the following in terms of a and/or b :

$$4.1. \quad \cos 28^\circ \quad (2)$$

$$4.2 \quad \cos 64^\circ \quad (3)$$

QUESTION 5

5. In the diagram, $P(-5 ; 12)$ and T lies on the positive x -axis. $\widehat{POT} = \theta$



Answer the following **without using a calculator**:

- 5.1. Write down the value of $\tan \theta$ (1)
- 5.2 Calculate the value of $\cos \theta$ (3)
- 5.3 $S(a ; b)$ is a point in the third quadrant such that $\widehat{TOS} = \theta + 90^\circ$ and $OS = 6,5$ units. Calculate the value of b . (4)

QUESTION 6

Proof the following identities

$$6.1 \quad \tan \theta + \frac{\cos \theta}{\sin \theta - 1} = -\frac{1}{\cos \theta}$$

$$6.2 \quad \frac{\tan \theta \cos \theta \sin \theta}{1 - \cos^2 \theta} = \sin^2 \theta + \cos^2 \theta$$

$$6.3 \quad \frac{\sin \theta}{1 + \cos \theta} + \frac{1}{\tan \theta} = \frac{1}{\tan \theta \cos \theta}$$

$$6.4 \quad \left(\frac{1}{\cos \theta} - \tan \theta \right)^2 = \frac{1 - \sin \theta}{1 + \sin \theta}$$

$$6.5 \frac{\sin \theta - \sin \theta \cos \theta}{\cos \theta - (1 - \sin^2 \theta)} = \tan \theta$$

$$6.6 \frac{\sin x \cdot \cos x}{1 + \cos^2 x - \sin^2 x} = \frac{1}{2} \tan x$$

$$6.7 (\sin P + \cos P)^2 + (\sin P - \cos P)^2 = 2$$

$$6.8 \cos^2 x (2 + 2 \tan^2 x) = 2$$

$$6.9 \frac{2 \tan A - 2 \sin A \cos A}{2 \sin^2 A} = \tan A$$

$$6.10 \frac{\cos x (\tan x - \sin x)}{\sin^3 x} = \frac{1}{1 + \cos x}$$

$$6.11 \frac{1}{\cos x} - \cos x = \tan x \sin x$$

$$6.12 \frac{1}{\tan \alpha} (\sin \alpha \tan \alpha + \cos \alpha) = \frac{1}{\sin \alpha}$$

$$6.13 \cos x \left(\tan x + \frac{\cos x}{\sin x} \right) = \frac{1}{\sin x}$$

$$6.14 \frac{\cos^2 x \cdot \sin x}{\sin x - \sin^2 x} = 1 + \sin x$$

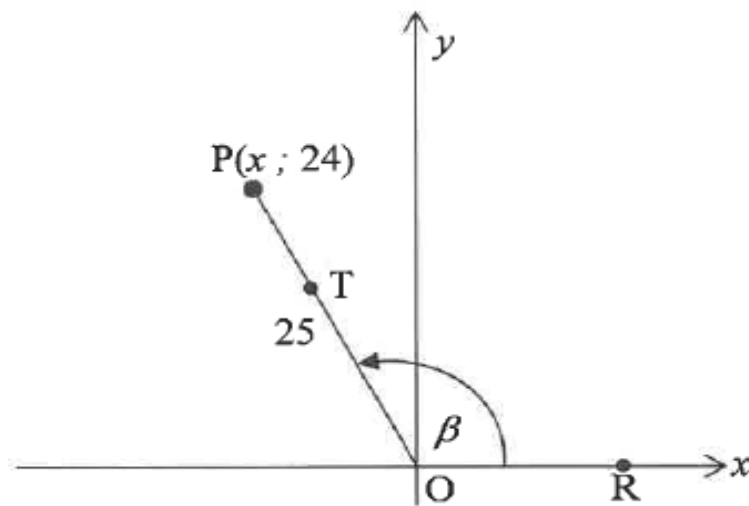
$$6.15 \frac{\cos^4 x - \sin^4 x}{(\cos x - \sin x)(1 + \tan x)} = \cos x$$

$$6.16 \sin \theta (1 + \tan \theta) + \cos \theta \left(1 + \frac{\cos \theta}{\sin \theta} \right) = \frac{1}{\sin \theta} + \frac{1}{\cos \theta}$$

$$6.17 \frac{\sin x}{1 + \cos x} + \frac{\cos x}{\sin x} = \frac{1}{\sin x}$$

QUESTION 7

7.1 In the diagram below, $P(x; 24)$ is a point such that $OP = 25$ and $\widehat{ROP} = \beta$, where β is an obtuse angle.



7.1.1 Calculate the value of x (2)

7.1.2 Determine the value of each of the following WITHOUT the use of the calculator

(a) $\sin \beta$ (1)

(b) $\cos(180^\circ - \beta)$ (2)

(c) $\tan(-\beta)$ (2)

7.1.3 T is a point on OP such that $OT = 15$. (4)

Determine the coordinates of T WITHOUT the use of the calculator

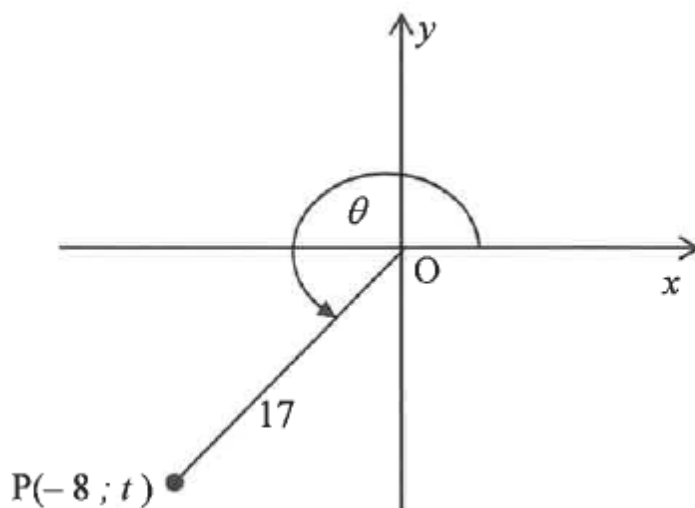
7.2 Determine the value of the following expression $\frac{2\sin x \cdot \cos x (1 + \tan^2 x)}{\tan x}$ (4)

7.3 Consider : $\frac{1 - \cos^2 A}{4\cos(90^\circ + A)}$

Simplify the expression to a single trigonometric term. (3)

QUESTION 8

8.1 In the diagram below , $P(-8; t)$ is a point in the cartesian plane such that $OP = 17$ units and reflex $X\hat{O}P = \theta$.



8.1.1 Calculate the value of t (2)

8.1.2 Determine the value of each of the following WITHOUT the using the calculator

(a) $\cos(-\theta)$ (2)

(b) $1 - \sin\theta$ (2)

8.2 If $\sin 17^\circ = a$, WITHOUT using a calculator, express the following in term of a

8.2.1 $\tan 17^\circ$ (3)

8.2.2 $\sin 107^\circ$ (2)

8.2.3 $\cos^2 253 + \sin^2 557$ (4)

8.3 Prove the identities $\frac{1}{(\cos x + 1)(\cos x - 1)} = \frac{-1}{\tan^2 x \cdot \cos^2 x}$ (4)

[19]

D.TERM 2

WEEK	TOPIC	SUBTOPIC	PAGE
1	Trig equations and general solutions	4. Determine the general solutions of trigonometric equations. Also, determine solutions in specific intervals	4 - 10
2	Quadrilaterals. Analytical Geometry	Teach past grade content on Quadrilaterals Parallelogram, square, rectangle etc.	11 - 21
3		Revise 1. distance between the two points; 2. gradient of the line segment connecting the two points (and from that identify parallel and perpendicular lines); and 3. Coordinates of the mid-point of the line segment joining the two points. Derive and apply: 1. the equation of a line through two given points	
4		Derive and apply: 2. the equation of a line through one point and parallel or perpendicular to a given line; and 3. The inclination (θ) of a line, where $m = \tan\theta$ is the gradient of the line ($0^\circ \leq \theta \leq 180^\circ$) Applications in complex diagrams.	
5	Number Patterns	- Revise linear number patterns. - Investigate number patterns leading to those where there is a constant second difference between consecutive terms, and the general term is therefore quadratic.	22 - 29
6		Investigate number patterns leading to those where there is a constant second difference between consecutive terms, and the general term is therefore quadratic	
7	Functions and Graphs	1. Revise the effect of the parameters a and q and investigate the effect of p on the graphs of the functions defined by: $y = f(x) = a(x + p)^2 + q$	30 - 58
8		1.1. $y = f(x) = \frac{a}{x+p} + q$ 1.2. $y = f(x) = a \cdot b^{x+p} + q$ where $b > 0, b \neq 1$ 2. Investigate numerically the average gradient between two points on a curve and develop an intuitive understanding of the concept of the gradient of a curve at a point. NB: Apply nature of roots with Functions	
9		Point by point plotting of basic graphs defined by $y = \sin\theta$, $y = \cos\theta$ and $y = \tan\theta$ for $\theta \in [-360^\circ; 360^\circ]$ 4. Investigate the effect of the parameter k on the graphs of the functions defined by $y = \sin(kx)$, $y = \cos(kx)$ and $y = \tan(kx)$	
10		5. Investigate the effect of the parameter p on the graphs of the functions defined by $y = \sin(x + p)$, $y = \cos(x + p)$ and $y = \tan(x + p)$ 6. Draw sketch graphs defined by: $y = a \sin k(x + p)$, $y = a \cos k(x + p)$ and $y = a \tan k(x + p)$ at most two parameters at a time.	
	[P-O0I9U =		

WEEK 1 – TOPIC:TRIGONOMETRY

SUB TOPIC

1.Trigonometric Equations and General Solutions

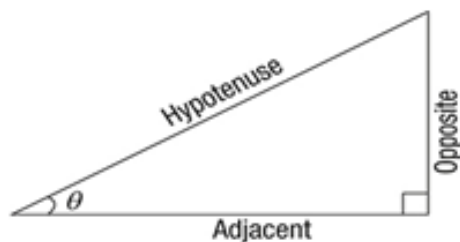
NOTES & EXAM TIPS

Follow these steps for solving trigonometric equations with one ratio:

1. Solve for the given ratio.
2. Determine in which two quadrants the solution lies.
 - ✓ Use the CAST diagram.
 - ✓ This will be where the given ratio is either positive or negative.
3. Draw a diagram in the Cartesian plane to represent this.
4. determine the reference angle by ($\sin^{-1}$ or $\cos^{-1}$ or $\tan^{-1}$) on the positive ratio.
5. Solve for the unknown angle:
 - ✓ Quadrant 1:
 - ✓ $x = \text{reference angle}$
 - ✓ Quadrant 2:
 - ✓ $x_1 = 180^\circ - \text{reference angle}$
 - ✓ Quadrant 3:
 - ✓ $x_2 = 180^\circ + \text{reference angle}$
 - ✓ Quadrant 4:
 - ✓ $x_3 = 360^\circ - \text{reference angle}$

Key Concepts: GRADE

Trigonometric ratio in the Cartesian plane:

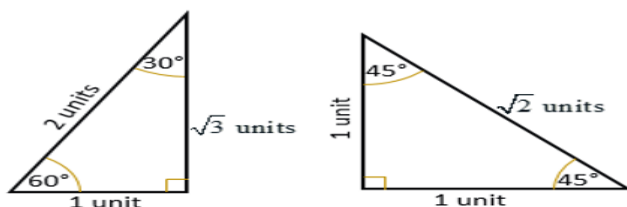


$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Values of the trigonometric ratios for the special angles:



	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Key Concepts grade 11 term 1:

Identities:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

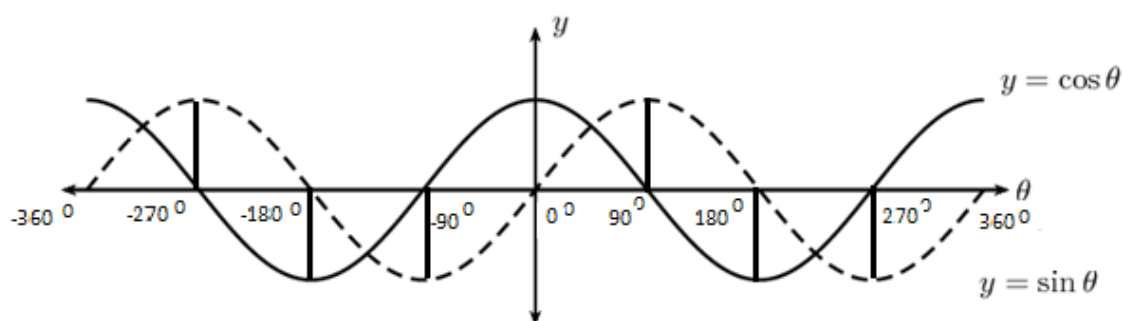
Reduction formulae:

second quadrant ($180^\circ - \theta$) or ($90^\circ + \theta$)	first quadrant (θ) or ($90^\circ - \theta$)
$\sin(180^\circ - \theta) = +\sin \theta$	all trig functions are positive
$\cos(180^\circ - \theta) = -\cos \theta$	$\sin(360^\circ + \theta) = \sin \theta$
$\tan(180^\circ - \theta) = -\tan \theta$	$\cos(360^\circ + \theta) = \cos \theta$
$\sin(90^\circ + \theta) = +\cos \theta$	$\tan(360^\circ + \theta) = \tan \theta$
$\cos(90^\circ + \theta) = -\sin \theta$	$\sin(90^\circ - \theta) = \cos \theta$
	$\cos(90^\circ - \theta) = \sin \theta$
third quadrant ($180^\circ + \theta$)	fourth quadrant ($360^\circ - \theta$)
$\sin(180^\circ + \theta) = -\sin \theta$	$\sin(360^\circ - \theta) = -\sin \theta$
$\cos(180^\circ + \theta) = -\cos \theta$	$\cos(360^\circ - \theta) = +\cos \theta$
$\tan(180^\circ + \theta) = +\tan \theta$	$\tan(360^\circ - \theta) = -\tan \theta$

Trigonometric graphs:

Solving trigonometric equations requires that we find the value of the angles that satisfy the equation.

If a specific interval for the solution is given, then we need only find the value of the angles within the given interval that satisfy the equation. If no interval is given, then we need to find the general solution. The periodic nature of trigonometric functions means that there are many values that satisfy a given equation, as shown in the diagram below.



The periodicity of the trigonometric functions means that there are an infinite number of positive and negative angles that satisfy an equation. If we do not restrict the solution, then we need to determine the general solution to the equation. We know that the sine and cosine functions have a period of 360° and the tangent function has a period of 180° .

General solutions:

If $\sin \theta = x$

$$\theta = \sin^{-1} x + k \cdot 360^\circ$$

$$\text{or } \theta = (180^\circ - \sin^{-1} x) + k \cdot 360^\circ$$

If $\cos \theta = x$

$$\theta = \cos^{-1} x + k \cdot 360^\circ$$

$$\text{or } \theta = (360^\circ - \cos^{-1} x) + k \cdot 360^\circ$$

If $\tan \theta = x$

$$\theta = \tan^{-1} x + k \cdot 180^\circ$$

for $k \in \mathbb{Z}$.

Worked Example:

Example 1:

Determine the general solution of the following : $2\cos^2\theta - \cos\theta - 1$ for

Method for finding the general solutions in trigonometric equations:

1. Simplify the equation using algebraic methods and trigonometric identities.
2. Determine the reference angle (use a positive value).
3. Use the CAST diagram to determine where the function is positive or negative (depending on the given equation/information).
4. Restricted values: find the angles that lie within a specified interval by adding/subtracting multiples of the appropriate period.
5. General solution: find the angles in the interval $[0^\circ; 360^\circ]$ that satisfy the equation and add multiples of the period to each answer.
6. Check answers using a calculator.

Solution:

Step 1: substitute ($1 - \sin^2 \theta = \cos^2 \theta$) and Factorise the equation:

$$2(1 - \sin^2 \theta) - \cos \theta - 1 = 0$$

$$2\cos^2 \theta - \cos \theta - 1 = 0$$

$$(2\cos \theta + 1)(\cos \theta - 1) = 0$$

$$\therefore 2\cos \theta + 1 = 0 \text{ or } \cos \theta - 1 = 0$$

Step 2: Simplify the equations and solve for θ :

$$2 \cos \theta + 1 = 0$$

$$2 \cos \theta = -1$$

$$\cos \theta = -\frac{1}{2}$$

$$\therefore \text{ref } \angle = 60^\circ$$

$$\begin{aligned} \text{II quadrant: } \theta &= 180^\circ - 60^\circ + k \cdot 360^\circ \\ &= 120^\circ + k \cdot 360^\circ \end{aligned}$$

$$\begin{aligned} \text{III quadrant: } \theta &= 180^\circ + 60^\circ + k \cdot 360^\circ \\ &= 240^\circ + k \cdot 360^\circ \end{aligned}$$

$$\cos \theta - 1 = 0$$

$$\cos \theta = 1$$

$$\therefore \text{ref } \angle = 0^\circ$$

$$\text{II and IV quadrants: } \theta = k \cdot 360^\circ$$

Example 2:

Determine α if $2 - 2\cos^2 \alpha = \sin \alpha \cos \alpha$ for $\alpha \in [0^\circ; 360^\circ]$.

Solution:

Step 1: Factorise the equation by taking out a common factor 2:

$$2(1 - \cos^2 \alpha) = \sin \alpha \cos \alpha$$

Step 2: Substitute: $1 - \cos^2 \alpha = \sin^2 \alpha$

$$2(\sin^2 \alpha) = \sin \alpha \cos \alpha$$

$$2\sin^2 \alpha - \sin \alpha \cos \alpha = 0$$

Step 3: Factorise the equation by taking out a common factor:

$$2\sin^2 \alpha - \sin \alpha \cos \alpha = 0$$

$$\sin \alpha (2\sin \alpha - \cos \alpha) = 0$$

$$\therefore \sin \alpha = 0 \text{ or } 2\sin \alpha - \cos \alpha = 0$$

Step 4: Simplify the equations and solve for α :

$$\begin{aligned}\sin \alpha &= 0 \\ \therefore \text{ref } \angle &= 0^\circ \\ \therefore \alpha &= 0^\circ + k \cdot 360^\circ \\ \text{or } \alpha &= 180^\circ + k \cdot 360^\circ \\ \text{and since } 360^\circ &= 2 \times 180^\circ \\ \text{we therefore have } \alpha &= k \cdot 180^\circ\end{aligned}$$

$$\begin{aligned}2 \sin \alpha - \cos \alpha &= 0 \\ 2 \sin \alpha &= \cos \alpha\end{aligned}$$

To simplify further, we divide both sides of the equation by $\cos \alpha$

$$\begin{aligned}\frac{2 \sin \alpha}{\cos \alpha} &= \frac{\cos \alpha}{\cos \alpha} \quad (\cos \alpha \neq 0) \\ 2 \tan \alpha &= 1 \\ \tan \alpha &= \frac{1}{2} \\ \therefore \text{ref } \angle &= 26,6^\circ \\ \therefore \alpha &= 26,6^\circ + k \cdot 180^\circ\end{aligned}$$

where $k \in \mathbb{Z}$.

Step 5: Substitute suitable values of k :

Determine the values of α that lie within the given interval $\alpha \in [0^\circ; 360^\circ]$ by substituting suitable values of k .

If $k=0$:

$$\begin{aligned}\alpha &= 0^\circ \\ \text{or } \alpha &= 26,6^\circ\end{aligned}$$

If $k=1$:

$$\begin{aligned}\alpha &= 180^\circ \\ \text{or } \alpha &= 26,6^\circ + 180^\circ \\ &= 206,6^\circ\end{aligned}$$

If $k=2$:

$$\alpha = 360^\circ$$

Step 6: Write the final answer:

$$\alpha = 0^\circ; 26,6^\circ; 180^\circ; 206,6^\circ; 360^\circ$$

EXERCISE

1. Determine the general solutions of: $\tan \frac{x}{2} = \sqrt{3}$

2. Solve for x :

2.1 $\sin x \tan x + \sin x = 0$

2.2 $3\sin x - 3 = -2\cos^2 x$

3. Solve the following equation for y , without using a calculator:

$$\frac{\sin y}{\cos y} = -1$$

4. Determine the general solutions of the equation $\tan(x - 20^\circ) = \tan(2x + 10^\circ)$

5. Solve the following equation for x : $3 - 5\sin x \cos x - 5\cos^2 x = 0$

6.1. Determine the general solution of x if $2(\cos x + 3) = 3(\tan x + 2)$

6.2. hence, determine two values of y , $100^\circ < y < 200^\circ$, that are solutions of:

$$2(\cos 3y + 3) = 3(\tan 3y + 2)$$

WEEK 2 – 4 ANALYTICAL GEOMETRY

SUB-TOPIC:

- 1.Distance between the two points.
- 2-The coordinates of the mid-point of the line segment joining the points.
- 3-The gradient of the line segment joining the point.
- 4-Conditions for parallel and perpendicular lines.
- 5-The equation of a line through two given points.
- 6-The equation of a line through one point and parallel or perpendicular to a given line.
- 7-The inclination (θ) of a line, where $m = \tan\theta$ is the gradient of the line ($0^\circ \leq \theta \leq 180^\circ$)

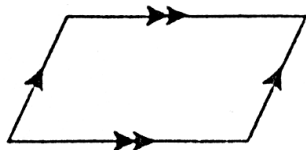
NOTES & EXAM TIPS

QUADRILATERALS

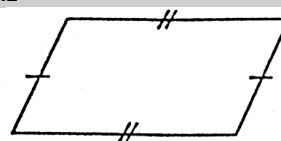
PROPERTIES OF THE SPECIAL QUADRILATERALS

1. THE PARALLELOGRAM

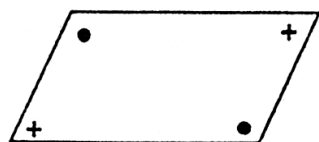
Opposite
sides
parallel



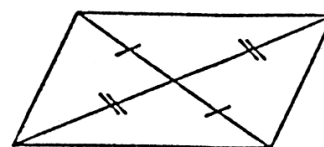
Opposite
sides
equal in
length



Opposite
angles equal
in size

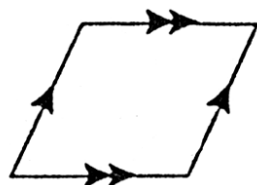


Diagonals
bisect
each
other

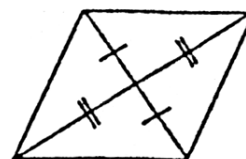


2. THE RHOMBUS

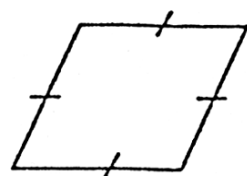
Opposite
sides
parallel



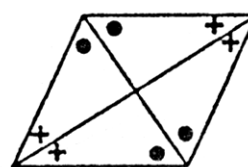
Diagonals
bisect
each
other



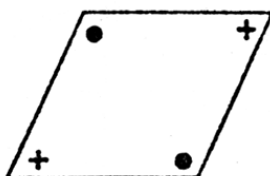
All sides
equal in
length



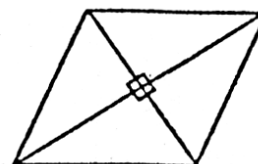
Diagonals
bisect
corner
angles

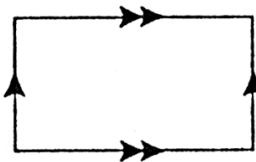
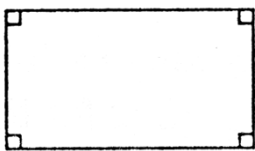
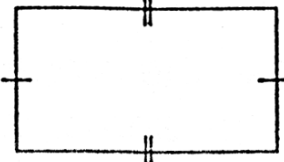
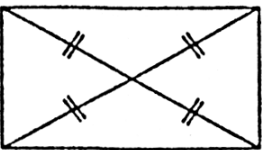


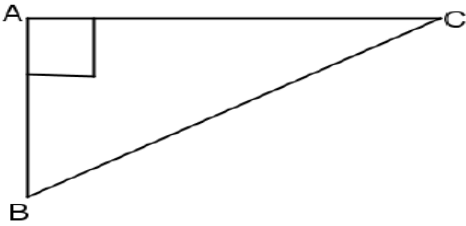
Opposite
angles equal
in size



Diagonals
cross at
right
angles



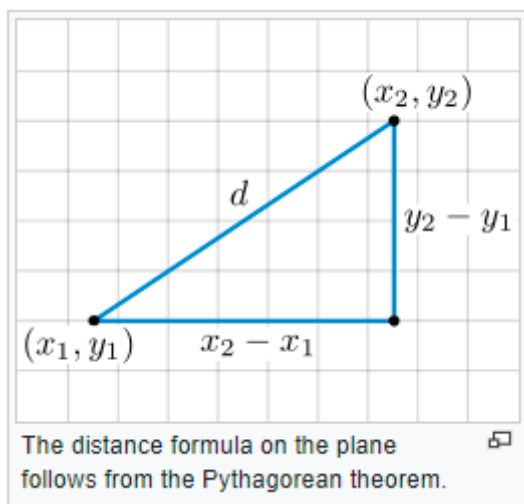
3. THE RECTANGLE			
Opposite sides parallel		All angles equal in size (90°)	
Opposite sides equal in length		Diagonals equal in length and bisect each other	

Pythagoras Theorem
 Given: $\triangle ABC$ with $\hat{A} = 90^\circ$ RTP: $BC^2 = AB^2 + AC^2$

Previous Knowledge from grade 10:

1. Represent geometric figures in a Cartesian coordinate system, and derive and apply, for any two points $(x_1; y_1)$ and $(x_2; y_2)$, a formula for calculating:
 - 1.1. The distance between the two points;
 - 1.2. The coordinates of the mid-point of the line segment joining the points;
 - 1.3. The gradient of the line segment joining the point; and
 - 1.4. conditions for parallel and perpendicular lines.

1. THE DISTANCE BETWEEN THE TWO POINTS:

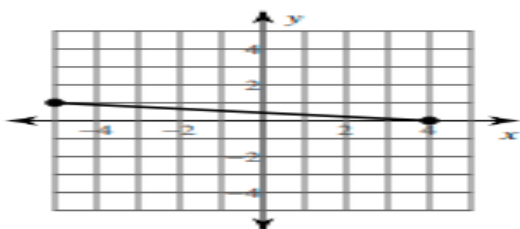


$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

WORKED EXAMPLE:

Example 1:

Determine the distance between the two points.



SOLUTION:

Points:

$(x_1; y_1): (-5; 1)$

$(x_2; y_2): (4; 0)$

The distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

substitution of the points:

$$d = \sqrt{(4 - (-5))^2 + (0 - 1)^2}$$

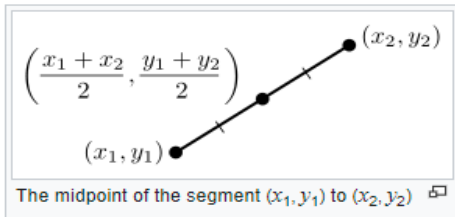
$$d = \sqrt{9^2 + (-1)^2}$$

$$d = \sqrt{82}$$

distance between $(-5; 1)$ and $(4; 0)$

2-The coordinates of the mid-point of the line segment joining the points:

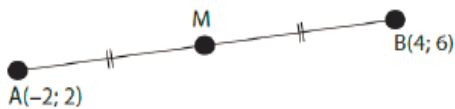
MIDPOINT FORMULA:



WORKED EXAMPLE:

Example 2:

Line segment AB is given below with A and B as shown:



Determine the coordinates of M, the midpoint of AB.

SOLUTION:

Midpoint formula:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Substitute A and B into the midpoint formula:

$$\begin{aligned} M & \left(\frac{-2+4}{2}, \frac{2+6}{2} \right) \\ & = (1; 4) \end{aligned}$$

3-The gradient of the line segment joining the point:

Standard form of line equation:

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

gradient formula: $m = \frac{y_2 - y_1}{x_2 - x_1} ; \quad x_2 \neq x_1$

GRADIENT – THE SLOPE OF A LINE

Example 3:

Given the points P (-2;1), Q (2;5), S (-2;3)

3.1. Determine the gradient of PQ

3.2. Determine the gradient of PS .What can be said about the gradient of PS?

SOLUTION:

To find the gradient we take into regard the points in the question and substitute their coordinates into the gradient formula.

Gradient formula:

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A}$$

3.1

$$m_{PQ} = \frac{y_Q - y_P}{x_Q - x_P}$$

$$\begin{aligned} m_{PQ} &= \frac{5-1}{2-(-2)} \\ &= \frac{4}{4} = 1 \end{aligned}$$

$$3.2 \quad m_{PS} = \frac{y_S - y_P}{x_S - x_P}$$

$$m_{PS} = \frac{3-1}{-2-(-2)} = \frac{2}{0}$$

$$\frac{2}{0} \Rightarrow \text{UNDEFINED}$$

Line PS is perpendicular to the x-axis.

4- Conditions for parallel and perpendicular lines:

- Parallel lines have the same gradient. $m_1 = m_2$
- The product of the gradient of perpendicular lines is equal to -1: $m_1 \times m_2 = -1$

WORKED EXAMPLE:

Example 4:

Consider the points A (-4; 1), B (-1; -3), C (3; 0) and D (0; 4).

4.1. Prove that: AB and DC are parallel lines.

4.2. Prove that: $\angle BAD = 90^\circ$.

SOLUTION:

4.1.

gradient formula: $m = \frac{y_2 - y_1}{x_2 - x_1} ; \quad x_2 \neq x_1$

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A}$$

$m_{AB} = \frac{y_B - y_A}{x_B - x_A}$	$m_{DC} = \frac{y_C - y_D}{x_C - x_D}$
$m_{AB} = \frac{-3 - 1}{-1 - (-4)} = -\frac{4}{3}$	$m_{DC} = \frac{-4 - 0}{0 - 3} = -\frac{4}{3}$

thus, $m_{AB} = m_{DC}$ then: AB and DC are parallel lines.

Recommendation for going through 4.2.

Determine the gradient of m_{AD} and multiply $m_{AD} \times m_{AB}$, if the result is -1 then $AD \perp AB$ and $\hat{BAD} = 90^\circ$.

5-The equation of a line through two given points

Steps:

1. Find the Gradient.
2. -Select any of the two points and substitute it into the gradient formula.
3. -Determine the equation in the form $y = mx + c$

WORKED EXAMPLE:

Example 5:

5.1. Find the equation of the line that goes through the points A (2; 3) and B (4; -5)

SOLUTION:

First, we find the gradient:

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A}$$

$$m_{AB} = \frac{-5 - 3}{4 - 2} = -4$$

Now select any one of the two points:

$$y - y_A = m(x - x_A)$$

$$y - 3 = -4(x - 2)$$

$$y - 3 = -4x + 8$$

$$y = -4x + 8 + 3$$

$$y = -4x + 11$$

6-The equation of a line through one point and parallel or perpendicular to a given line:

STEPS:

- 1- Put line in standard form.
- 2- Parallel lines have equal gradients and perpendicular lines $m_1 \times m_2 = -1$ then we can find the gradient of parallel or perpendicular from the standard form.
- 3- Substitute the calculated gradient(m) and the given point into the equation: $y - y_1 = m(x - x_1)$
- 4- Find the standard form of parallel or perpendicular line.

WORKED EXAMPLES:

Example 6 :

6.1. Determine the equation of the line PQ. PQ is parallel to MF, which has an equation of $y - 3x = 5$ And goes through the point A (-3;2).

SOLUTION:

First put line MF in standard form:

$$y - 3x = 5$$

$$y = 3x - 5$$

$$m_{MF} = 3 \quad (\text{gradient})$$

Parallel line has equal gradients then,

$$m_{PQ} = 3$$

$$y - y_A = m_{PQ} (x - x_A)$$

$$y - 2 = 3(x - (-3))$$

$$y - 2 = 3(x + 3)$$

$$y - 2 = 3x + 9$$

$$y = 3x + 11$$

6.2. Find the equation of the line PG. PG is Perpendicular to line RS, which has an equation of:

$2y - 8x + 10 = 0$ and passes through the point A (8; -3).

SOLUTION:

First put RS in standard form:

$$2y - 8x + 10 = 0$$

$$y = 4x - 5$$

$$m_{RS} = 4 \quad m_{RS} \times m_{PG} = -1 \quad (\text{perpendicular lines})$$

$$4 m_{PG} = -1$$

$$m_{PG} = -\frac{1}{4}$$

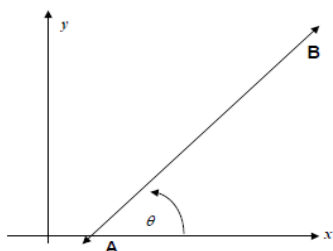
$$y - y_A = m_{PG} (x - x_A)$$

$$y - y_A = -\frac{1}{4} (x - x_A)$$

$$y - (-3) = -\frac{1}{4}(x - 8)$$

$$y = -\frac{1}{4}x - 1$$

7-The INCLINATION (θ) of a line, where $m = \tan\theta$ is the gradient of the line ($0^\circ \leq \theta \leq 180^\circ$)



The angle of inclination of the straight line AB is the angle, θ , made by the line with the positive direction of the x axis, measured in an anti-clockwise direction.

If θ is the angle of inclination of the line AB, then $0^\circ \leq \theta \leq 180^\circ$

To calculate the angle of inclination, we can use the formulas:

$$\tan \theta = m = \text{gradient}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \tan \theta$$

WORKED EXAMPLE:

Example 7:

Calculate the angle of inclination of a line passing through the points: A (1 ; 5) and B (-2;-1)

Firstly, calculate the gradient of the straight line passing through the points A and B. Use the formula:

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \tan \theta$$

$$m_{AB} = \frac{-1-5}{-2-1} = \tan \theta \quad \text{Substitute in the coordinates of the points A and B:}$$

Then using the formula for the angle of inclination: $\tan \theta = m$ and Substitute in the gradient already calculated:

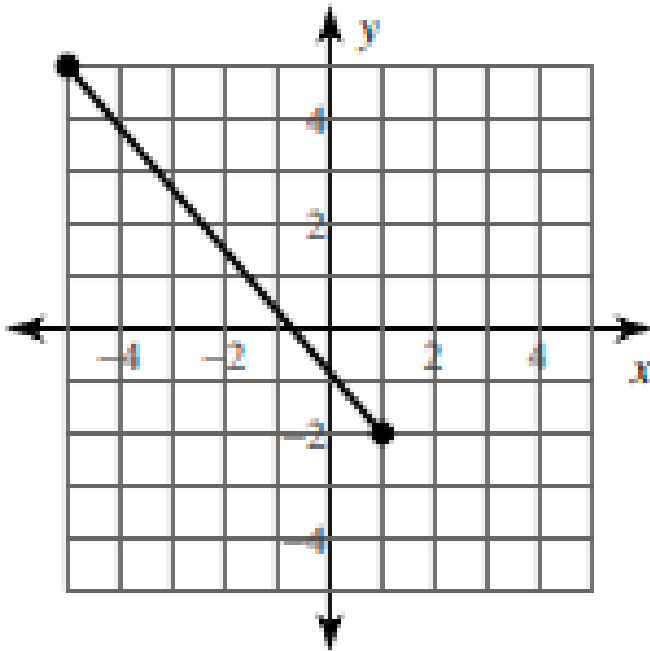
$$\tan \theta = 2$$

Using a calculator, type in $\tan^{-1}(2) = 63,43^\circ$:

$\theta = 63,43^\circ$ (correct to 2 decimal places).

QUESTION: 1

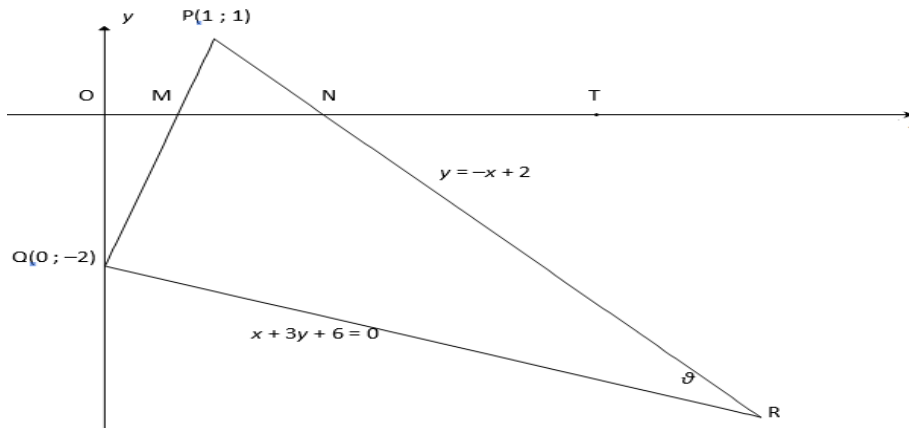
1.1



Determine the length of AB if $A(2;7)$ and $B(-3;5)$ are given.

2. Determine the coordinates of the midpoint of $M(2;3)$ and $N(-4;-1)$.
3. Determine the gradient of $M(2;3)$ and $N(-4;-1)$.
4. A line passes through the points $A(-1; 3)$ and $B(5; 0)$
 - 4.1. Determine the equation of line AB
 - 4.2. Determine the equation of a line parallel to AB and passing through the point $(2;-3)$
 - 4.3. Determine the equation of the line perpendicular to AB and passing through the origin.
5. Calculate the angle of inclination of a line passing through the points: $A(-1; 3)$ and $B(-4;-1)$

6. In the diagram below, $P(1; 1)$, $Q(0; -2)$ and R are the vertices of a triangle and $\hat{P}RQ = \theta$. The x -intercepts of PQ and PR are M and N respectively. The equations of the sides PR and QR are $y = -x + 2$ and $x + 3y + 6 = 0$ respectively. T is a point on the x -axis, as shown.



6.1. Determine the gradient of QP .

6.2. Prove that $\hat{P}QR = 90^\circ$.

6.3. Determine the coordinates of R .

6.4. Calculate the length of PR . Leave your answer in surd form.

6.5. Determine the equation of the line (s) in the form $y = mx + c$. (s) is perpendicular to line $y = -x + 2$ and passes through the point P .

6.6. Calculate the size of θ .

WEEK 5 - 6: TOPIC: NUMBER PATTERNS

- SUB-TOPIC:
- Linear number pattern
 - Quadratic number pattern

NOTES & EXAM TIPS

Previous Knowledge:

Terminology:	
Sequence/pattern	A sequence or pattern is an ordered set of numbers or variables.
Successive/consecutive	Successive or consecutive terms are terms that directly follow one after another in a sequence.
Common difference	The common or constant difference (d) is the difference between any two consecutive terms in a linear sequence.
General term	A mathematical expression that describes the sequence and that generates any term in the pattern by substituting different values for n.
Conjecture	A statement, consistent with known data, that has not been proved true nor shown to be false.

Describing patterns:

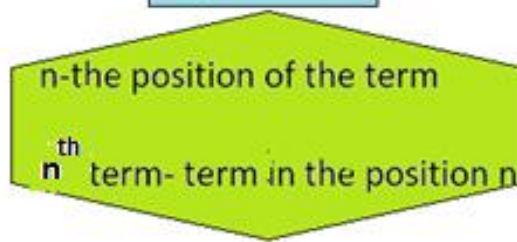
To describe terms in a pattern we use the following notation:

T_1 is the first term of a sequence.

T_3 is third term of a sequence.

T_n is the general term and is often expressed as the n^{th} term of a sequence.

Key words:



A sequence does not have to follow a pattern but when it does, we can write an equation for the general term. The general term can be used to calculate any term in the sequence. For example, consider the following linear sequence: 3;9;15;21; ... The n^{th} term is given by the equation $T_n = 6n - 3$.

You can check this by substituting values for n :

$$T_2 = 6(2) - 3 = 9$$

$$T_3 = 6(3) - 3 = 15$$

$$T_4 = 6(4) - 3 = 21$$

•
•
•

$$T_n = 6n - 3$$

• The general term can be used to find the value of a specific term.

The general term

• The general term can be used to find the term number or the position of a specific term.

The general formula , the of the n^{th} : The formula given below can be used

$$T_n = dn + q$$

Linear sequence: A sequence of numbers in which there is a common difference (d) between any consecutive terms .

Important: $d = T_{n+1} - T_n = T_{10} - T_9 = T_2 - T_1$

Worked example: 1

1- Consider the pattern: 6; 22; 38; 54...

1.1 Write the next two terms if the pattern is a linear sequence:

1.2 Write a formula for the n^{th} term in the form $T_n = \dots$

1.3 Determine the 100th term.

1.4 which term has a value of 790?

Solutions:

$$1.1 \ d = T_4 - T_3 = T_3 - T_2 = T_2 - T_1 = 16$$

There is a constant difference of 16 between the consecutive terms then, the terms increase by 16 each time, so the next two terms are 70 and 86.

1.2 The general formula:

$$T_n = d(n-1) + T_1$$

$$d = 16$$

$$T_1 = 6$$

substituting d and T_1 :

$$T_n = 16(n-1) + 6$$

$$T_n = 16n - 16 + 6$$

$$T_n = 16n - 10$$

1.3 substituting $n = 100$ into the general term formula:

$$T_n = 16n - 10$$

$$T_{100} = 16(100) - 10$$

$$T_{100} = 1600 - 10$$

$$T_{100} = 1590$$

1.4 set the general term equal to 790

$$16n - 10 = 790$$

$$16n = 790 + 10$$

$$16n = 800$$

$$n = 50$$

$$\text{then: } T_{50} = 790$$

Quadratic patterns:

We now know that linear patterns have a constant first difference between consecutive terms. Now we are going to investigate sequences that have a constant second difference. This means there is a constant difference between the consecutive terms of the sequence of the first differences.

A linear pattern has a constant first difference, but a **quadratic pattern** has a **constant second difference**.

Illustration:

Sequence: 2; 8; 18; 32; 50; ...

1st differences: 6; 10; 14; 18; ...

2nd differences: 4; 4; 4;

In the above illustration, we notice that the 2nd differences are constant. Thus, in general, if the 2nd differences are constant, then the general term of the sequence will be quadratic.

You often refer to quadratic patterns as second difference patterns that have the following general formula:

$$T_n = an^2 + bn + c$$

2.1 Finding the general term for a quadratic sequence:

To work out the values for a, b and c use:

$$a + b + c = \text{first term}$$

$$3a + b = \text{first term of the first difference}$$

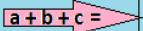
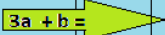
$$2a = \text{first term of the second difference}$$

Worked example: 2

Find the general term of the quadratic sequence:

2; 8; 18; 32; 50; ...

Solution:

Term number (n)	1		2		3		4		5
Patterns:	 $a + b + c =$ 2		8;		18;		32;		50
1 st difference:	 $3a + b =$ 6			10		14		18	
2 nd difference:	 $2a =$ 4		4		4		4		

$$2a = 4$$

$$a = 2$$

substitute for $a = 2$

$$3a + b = 6$$

$$3(2) + b = 6$$

$$b = 6 - 6$$

$$b = 0$$

Substitute for a = 2 and b = 0

$a + b + c = 2$

$2 + 0 + c = 2$

$C = 2 - 2$

$C = 0$

substitute for a, b and c: $T_n = an^2 + bn + c$

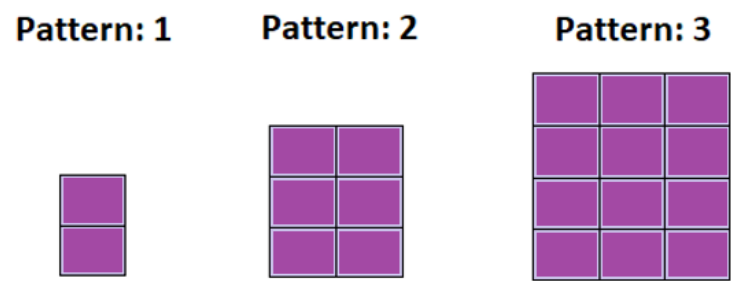
$T_n = an^2 + bn + c$

$T_n = 2n^2 + (0)n - 0$

$T_n = 2n^2$

Worked example: 2

Yamaris and Heyner are talking about the following pattern:



Heyner says that he can work out the general formula for the number of purple squares by taking the number of columns in that pattern and multiplying that by the number of rows.

Yamaris says that if she increases the pattern number by one, squares that and then subtracts the pattern number increased by one, he will get the general formula.

- 1.1. Check who of the two learners is correct.
- 1.2. How many purple squares will be in Pattern 4?
- 1.3. Use algebraic methods to determine the values of a, b and c, and hence the formula for the general term.
- 1.4. Use this formula to determine the number of purple squares in Pattern 20.

Solutions:

1.1

Heyner strategy:

Pattern	rows	columns	Rows X columns
1	2	1	$2 \times 1 = 2$
2	3	2	$3 \times 2 = 6$
3	4	3	$4 \times 3 = 12$
n	$n + 1$	n	$(n + 1)n = n^2 + n$

General term: $n^2 + n$

Sequence: 2; 6; 12; ...; $n^2 + n$

Yamaris Strategy:

- ❖ she increases the pattern number by one: $n + 1$
- ❖ square of $n + 1$ is $(n + 1)^2$
- ❖ subtracts the pattern number increased by one: $(n + 1)^2 - (n + 1)$

expand:

$$(n + 1)^2 - (n + 1) = n^2 + 2n + 1 - (n + 1)$$

$$(n + 1)^2 - (n + 1) = n^2 + 2n + 1 - (n + 1)$$

$$(n + 1)^2 - (n + 1) = n^2 + 2n + 1 - n - 1$$

$$(n + 1)^2 - (n + 1) = \mathbf{n^2 + n}$$

General term: $n^2 + n$

Sequence: 2; 6; 12; ...; $n^2 + n$

Conclusion:

Yamaris and Heyner got the same general term so they both have a correct strategy to get it.

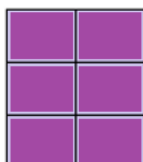
1.2

Way 1: draw pattern 4 and then count:

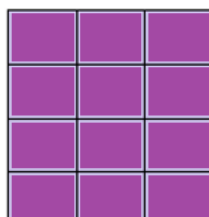
Pattern: 1



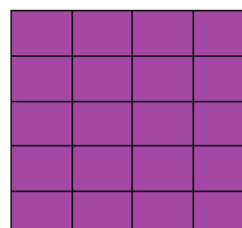
Pattern: 2



Pattern: 3



Pattern: 4



Answer: in Pattern 4, there are 20 purple squares.

Way 2: using the general term formula:

$$T_n = n^2 + n$$

$$T_4 = 4^2 + 4$$

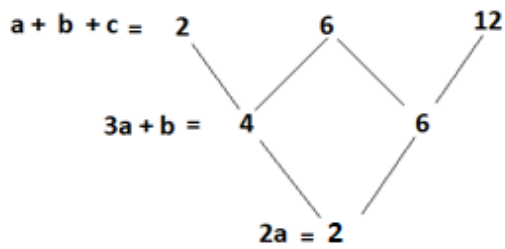
$$T_4 = 16 + 4$$

$$T_4 = 20$$

Answer: in Pattern 4, there are 20 purple squares.

1.3. algebraic methods to determine the values of a, b and c, and hence the formula for the general term.

Sequence: 2; 6; 12; ...



$$2a = 2$$

$$a = 1;$$

substitute a

$$3a + b = 4$$

$$3(1) + b = 4$$

$$3 + b = 4$$

$$b = 4 - 3$$

$$b = 1;$$

substitute a and b

$$a + b + c = 2$$

$$1 + 1 + c = 2$$

$$c = 0$$

substitute a, b and c into the general term formula:

$$T_n = an^2 + bn + c$$

$$T_n = (1)n^2 + (1)n + 0$$

$$T_n = n^2 + n$$

1.4

$$T_{20} = (20)^2 + 20$$

$$T_{20} = (20)^2 + 20$$

Another General formula for T_n : Can be used

$$T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)d_2}{2}$$

ACTIVITIES/WORKSHEETS/EXAM TYPE QUESTIONS

1-Determine the general formula for the following sequences and then calculate T_{10} , T_{15} and T_{30} :

1.1 18; 24; 30; 36; ...

1.2 13; 16; 19; 22; ...

2 Given the sequence: -2; -3; -6; -11; ...

2.1 Prove that the sequence is a quadratic sequence pattern.

2.2 Determine the general formula.

3 $S(n) = 6; 11; 18; 27; 38; \dots$ is a sequence:

3.1 Prove that $S(n)$ is a quadratic sequence.

3.2 Determine the general term for the sequence.

3.3 Determine the 6th term of the sequence.

4 2; 2; 3; 5; ... are the first four terms of a quadratic number pattern.

4.1 Determine the value of the fifth term of the pattern.

4.2 Determine a formula to represent the general term of the pattern.

4.3 which term of the pattern has a value of 1083?

WEEK 7 - 10: TOPIC: FUNCTIONS AND GRAPHS

SUB-TOPIC:

$$f(x) = a(x + p) + q, \quad f(x) = a(x + p)^2 + q, \quad f(x) = \frac{a}{x+p} + q$$

and

$$f(x) = ab^{x+p} + q; b > 0; b \neq 1$$

$$y = a \sin k(x + p),$$

$$y = a \cos k(x + p) \text{ and}$$

$$y = a \tan k(x + p) \text{ at most two parameters at a time.}$$

NOTES & EXAM TIPS

What learners must know:

- Revise the functions discussed in earlier grades:

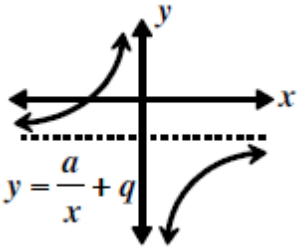
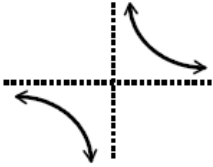
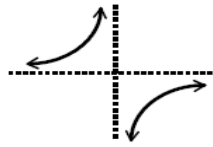
$$\text{Hyperbola: } y = \frac{k}{x} \text{ or } xy = k \quad y = \frac{a}{x} + q$$

- Effects of a, p and q on the graphs of the functions: Hyperbola: $y = \frac{a}{x+p} + q$
- To differentiate between different shapes of graphs based on the equation
- To draw graphs
- To analyze and interpret graphs, make deductions from the given graphs
- How to recognize the domain and the range
- Identify the asymptote
- How to find the point (s) of intersection

That hyperbola must have arms at all times, and show at least one point on the other arm;

Key Concepts

A. EQUATIONS OF FUNCTIONS

EQUATIONS OF FUNCTIONS		
	Determine the Shape of hyperbolic graph	
Characteristics/Properties of a function: - Equations of functions: Shape - Intercepts with the axes - Axis of symmetry - Domain & Range - Equations of asymptotes - Increasing/decreasing function - Transformations of Hyperbolic functions: ❖ Effects of parameter p ❖ Effects of parameter q	$y = \frac{a}{x} + q$, where $a \neq 0$; $y \neq 0$; $x \neq 0$ Examples: $y = \frac{5}{x}$, $y = \frac{4}{x} + 2$; $y = -\frac{2}{x} - 1$	If $a > 0$ then the graph decreases for all $x < 0$ or $x > 0$. 
	$y = \frac{a}{x+p} + q$, where $a \neq 0$; $y \neq 0$; $x \neq 0$ e.g. $y = \frac{2}{x-1} + 2$; $y = \frac{-1}{x-1} - 2$	If $a < 0$ then the graph increases for all $x < 0$ or $x > 0$. 

B. INTERCEPTS WITH THE AXES:

y-intercept: y- intercept at c :let $x = 0$ (if necessary)

x-intercepts: x-intercept: let $y = 0$ (if necessary)

C. AXES OF SYMMETRY

The hyperbola has TWO (2) axes of symmetry.

- $y = x - p + q$
- $y = -(x - p) + q$
 $= -x + p + q$
- The **axes of symmetry** of the shifted hyperbola $y = \frac{a}{x+p} + q$ intersect at the point $(-p; q)$

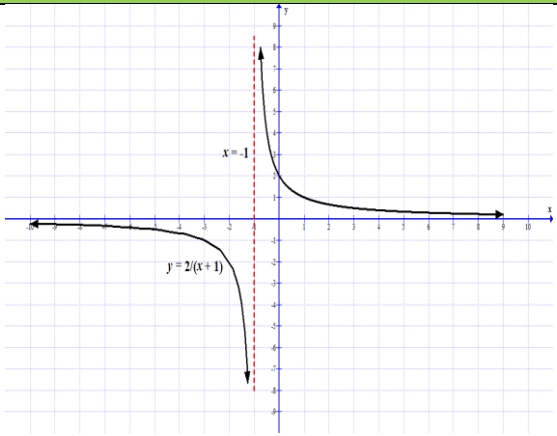
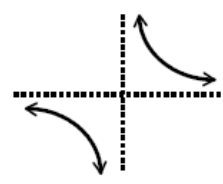
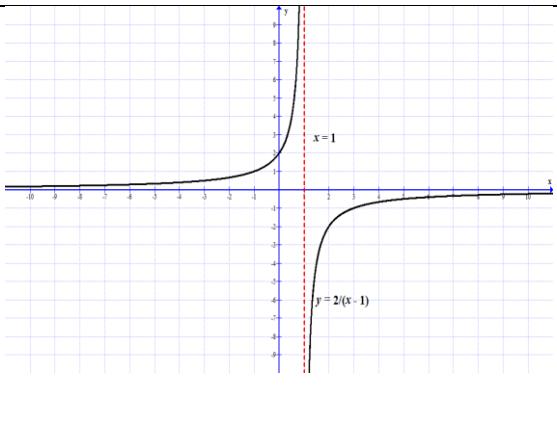
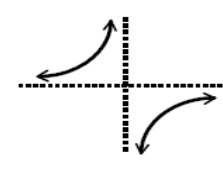
D. DOMAIN & RANGE

- The **domain** of a hyperbola is $x \in \mathbb{R}, x \neq p$ or $\{x : x \neq 0, x \in \mathbb{R}\}$
- The **range** of a hyperbola is $y \in \mathbb{R}, y \neq q$ or $\{y : y \neq 0, y \in \mathbb{R}\}$

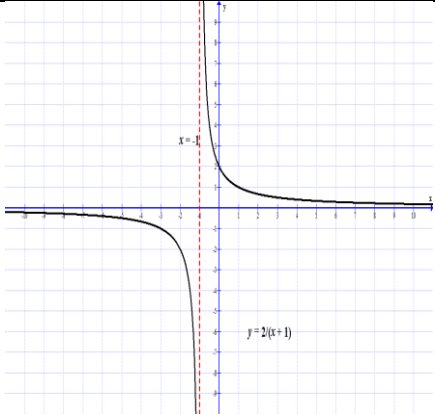
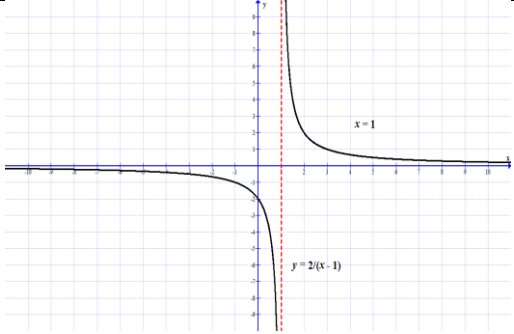
E. EQUATIONS OF ASYMPTOTES

- The **vertical asymptote** of the graph is the line $x = p$
- The **horizontal asymptote** of the graph is the line $y = q$

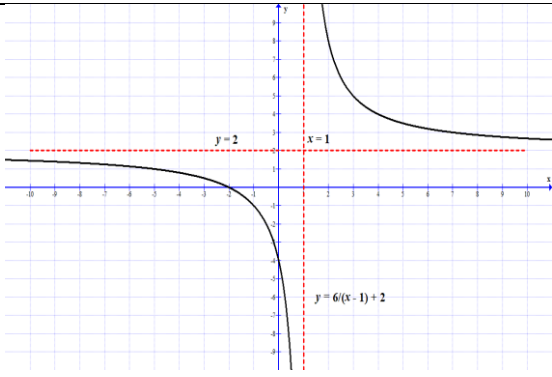
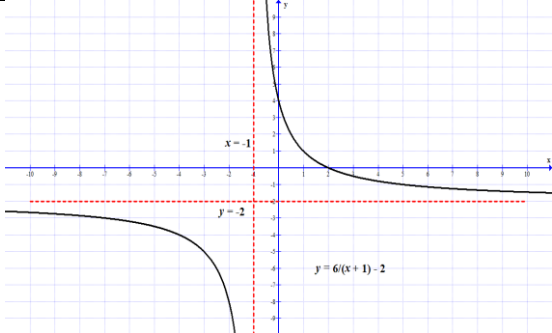
F. Determine whether the function is increasing /decreasing:

Increasing/Decreasing functions:		
$a > 0$ (positive) The hyperbola is a decreasing function. NB: as values increase, the values decrease		Decreasing function: The hyperbola will lie in the 1st and 3rd quadrants If $a > 0$ then the graph decreases for all $x < 0$ or $x > 0$. 
$a < 0$ (negative) The hyperbola is an increasing function. NB: as values increase, the values increase		Increasing function: The hyperbola will lie in the 2nd and 4th quadrants If $a < 0$ then the graph increases for all $x < 0$ or $x > 0$. 

G. Transformations of hyperbolic functions (effect of p).

Transformations of hyperbolic functions (effect of p)		
$p > 0$ (positive) p affects the horizontal shift of the graph and gives the position of the vertical asymptote ($x = -p$).		In general, for $y = \frac{a}{x+p} + q$ P has the effect of moving the graph of $y = \frac{a}{x}$ to the left of p units if p is positive.
$p < 0$ (negative) p affects the horizontal shift of the graph and gives the position of the vertical asymptote ($x = -p$).		In general, for $y = \frac{a}{x+p} + q$ P has the effect of moving the graph of $y = \frac{a}{x}$ to the right of p units if p is negative.

H. Transformations of hyperbolic functions (effect of q)

$q > 0$ (positive) q affects the vertical shift of the graph. Also gives the horizontal asymptote ($y = q$)		In general, for $y = \frac{a}{x+p} + q$ q has the effect of moving the graph of $y = \frac{a}{x}$, q units upwards if q is positive.
$q < 0$ (negative) q affects the vertical shift of the graph. Also gives the horizontal asymptote ($y = q$)		In general, for $y = \frac{a}{x+p} + q$ q has the effect of moving the graph of $y = \frac{a}{x}$, q units downwards if q is negative.

WORKED-OUT EXAMPLES:

EXAMPLE 1:

Given: $f(x)=\frac{2}{x-3}+1$

- 1.1. Calculate the x - and y - intercepts of f . (3)
- 1.2. Write down the equations of the asymptote of f . (2)
- 1.3. Draw a sketch of f and show all intercepts with the axes and the asymptotes (3)
- [8]

SOLUTION:

$$f(x)=\frac{2}{x-3}+1$$

- 1.1. Calculate the x - and y - intercepts of f .

Calculations	Marking descriptor
For x-intercept, let $y=0$ $\frac{2}{x-3}+1=0$ $2+(x-3)=0$ $x-1=0$ $x=1$ $(1;0)$ For y-intercept, let $x=0$ $y=\frac{2}{0-3}+1$ $y=-\frac{1}{3}$	✓ substitution ✓ answer ✓ answer (3)

- 1.2. Write down the equations of the asymptote of f .

Calculations	Marking descriptor
Horizontal asymptote: $y=1$ Vertical asymptote: $x=3$	✓ answer ✓ answer (2)

1.3. Sketch graph of $f(x)=\frac{2}{x-3}+1$

Calculations	Marking descriptor
	✓ shape ✓ asymptotes ✓ both intercepts (3)

EXAMPLE 2:

Given: $f(x)=\frac{3}{x-1}-2$

- 2.1. Determine the equation of the asymptotes of h .

(2)
- 2.2. Determine the x - and y - intercepts of h .

(3)
- 2.3. Draw a sketch of h clearly showing all relevant points.

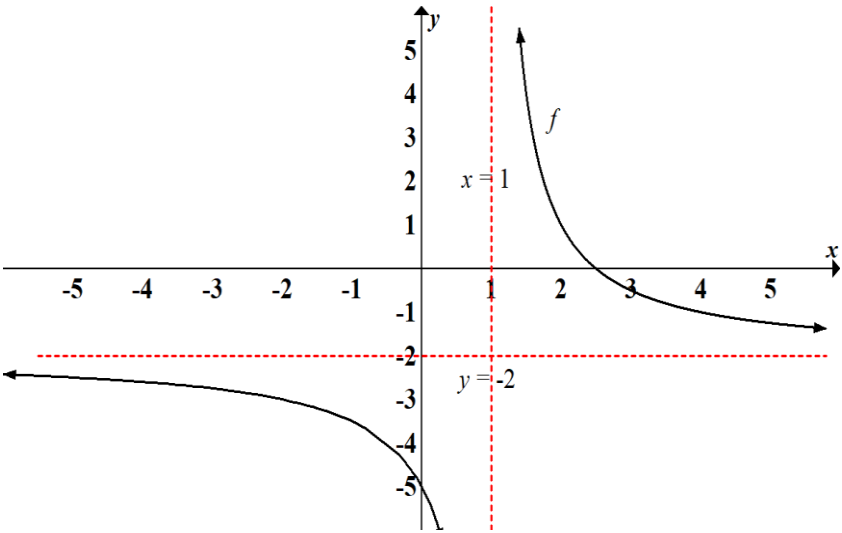
(3)
- 2.4. Write the range of $y=-f(x)$.

(1)
- 2.5. Describe, in words, the transformation of f to g if $g(x)=\frac{-3}{x+1}-2$

(2)
- 2.6. Determine the equation of the axis of symmetry of h .

(2)
- [13]

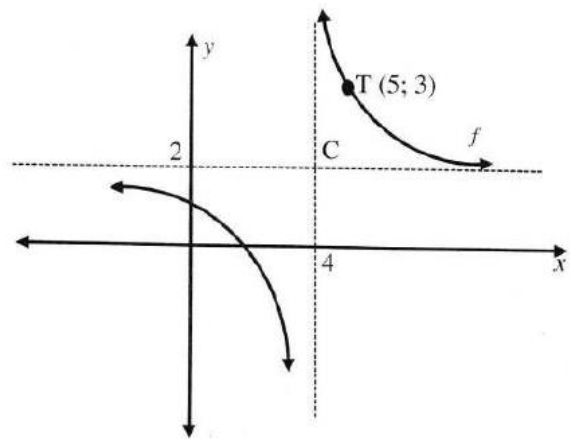
SOLUTION: $f(x)=\frac{3}{x-1}-2$

Calculations	Marking descriptor
2.1. Vertical: $x=1$ Horizontal: $y=-2$	✓ $x=1$ ✓ $y=-2$ (2)
2.2. x-intercept $0=\frac{3}{x-1}-2$ $\therefore 0=3-2(x-1)$ $\therefore 0=3-2x+2$ $\therefore 2x=5$ $\therefore x=\frac{5}{2}$ y-intercept $y=\frac{3}{0-1}-2=-5$	✓ $0=\frac{3}{x-1}-2$ ✓ $0=\frac{5}{2}$ ✓ $y=-5$ (3)
2.3. 	✓ shape ✓ asymptotes ✓ y-intercept (3)
2.4. $-f(x)=\frac{3}{x-1}-2$ Range is: $y\in\mathbb{R}$ where $y\neq 2$	✓ answer (1)

2.5. $f(x) = \frac{3}{x-1} - 2$ Reflection about y-axis: $g(x) = \frac{3}{-x-1} - 2$ $\therefore g(x) = \frac{3}{-(x+1)} - 2$ $\therefore g(x) = \frac{-3}{x+1} - 2$	✓ manipulation ✓ answer (2)
2.6. $f(x) = \frac{3}{x-1} - 2$ $y = x - p + q$ $y = x - 1 - 2$ $y = x - 3$ $y = -x + p + q$ $y = -x + 1 - 2$ $y = -x - 1$	✓ $y = x - 3$ ✓ $y = -x - 1$ (2)

EXAMPLE 3:

The graph of $f(x) = \frac{a}{x+p} + q$ is sketched below with asymptotes $x=4$ and $y=2$. $T(5;3)$ is a position of f and C is the point of intersection of the asymptotes.



- 3.1. Determine the values of a , p and q . (3)
- 3.2. Give the equation of h , the reflection of f in the y-axis. (1)
- 3.3. If the graph of f is symmetrical about the line $y = -x + c$, determine the value of c . (2)

SOLUTION:

No.	Calculations	Marking descriptor
3.1	$p=4$ and $q=2$ $T(5;3): y = \frac{a}{x-4} + 2$ $3 = a + 2$ $a = 1$	✓ $p=4$ ✓ $q=2$ ✓ $a=1$ (3)
3.2	$h(x) = \frac{1}{-x-4} + 2$ or $h(x) = \frac{-1}{x+4} + 2$	✓ answer (1)
3.3	$y = -x + c$ $2 = -(4) + c$ $c = 6$	✓ correct substitution of $(4; 2)$ ✓ answer (2)

EXERSICE 1

QUESTION 1

Given: $f(x) = \frac{1}{x-2} - 1$

- 1.1. Write down the equation of the horizontal and vertical asymptotes of f . (2)
- 1.2. Determine the x and y intercepts of f . (3)
- 1.3. Draw a neat sketch of f on the set of axes. Show all asymptotes and intercepts with the axes. (3)
- 1.4. Calculate the value of k if $(k; 2)$ is a point on f . (2)
- 1.5. Show that f can be rewritten as $y = \frac{-x+3}{x-2}$ (2)
- 1.6. How would you translate f to obtain $h(x) = \frac{1}{x}$. (2)
- 1.7. Write down the domain of $h(x)$. (1)

[15]

QUESTION 2

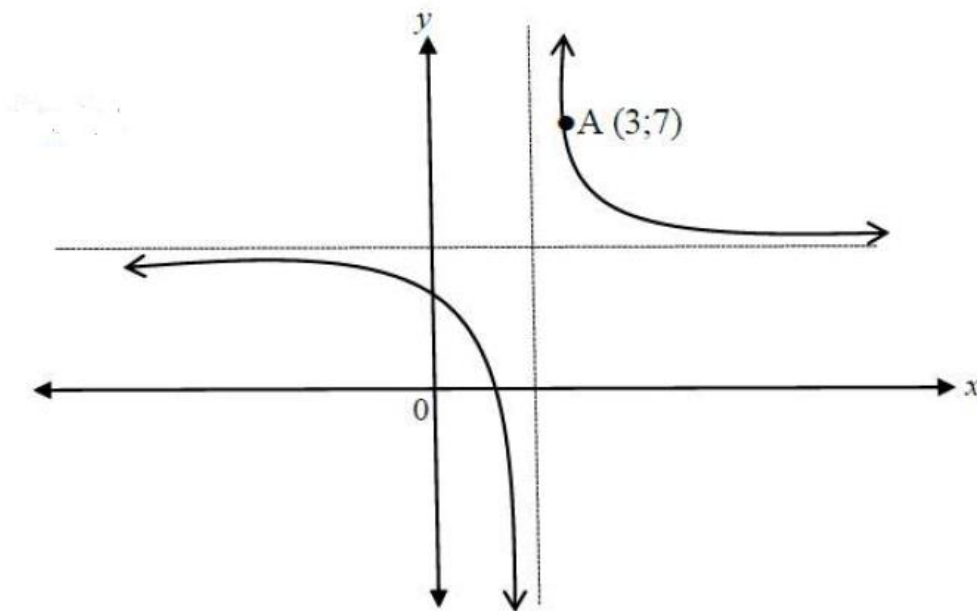
Given: $f(x) = \frac{2}{x-1} + 2$

- 2.1. Write down the equations of the asymptotes of f . (2)
- 2.2. Sketch the graph of f and clearly show all intercepts and asymptotes. (3)
- 2.3. Determine the equation of the axis of symmetry which has a negative gradient. (3)
- [8]**

QUESTION 3

The given diagram shows the graph of $f(x) - 4 = \frac{p}{x-2}$.

The point $A(3;7)$ lies on the graph of f .

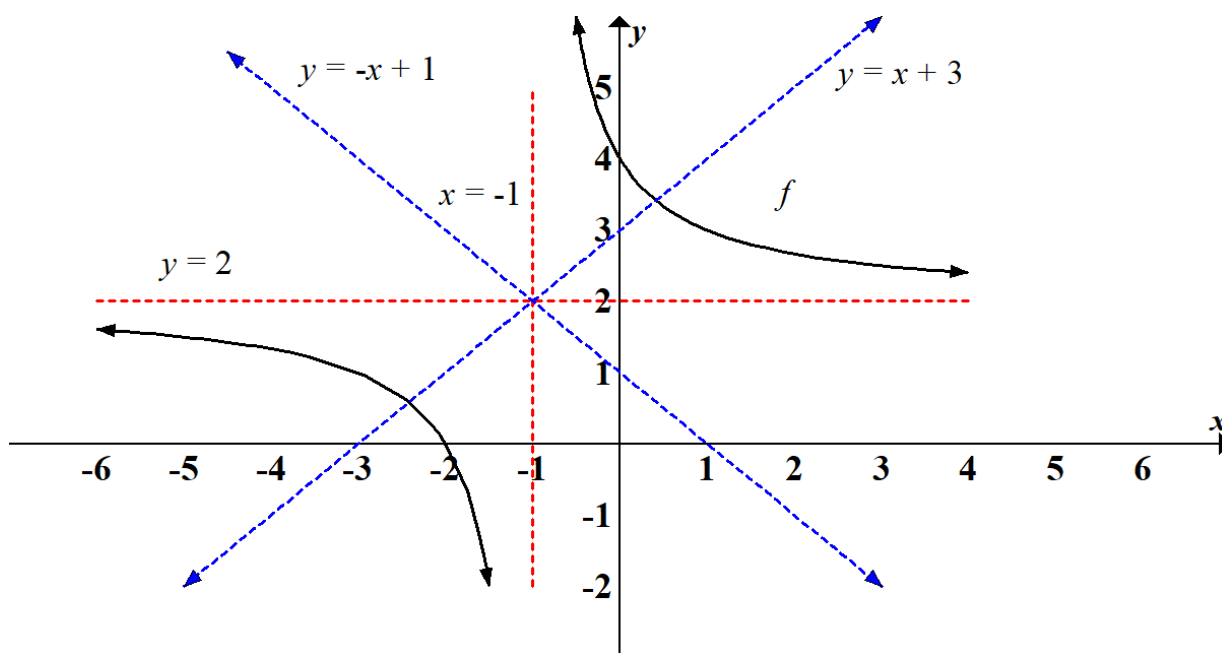


- 3.1. Write down the equations of asymptotes of f . (2)
- 3.2. Determine the value of p . (2)
- 3.3. Is $f(x)$ an increasing or a decreasing function? Motivate. (2)
- 3.4. Determine if the point $(1;5)$ lies on the axes of symmetry of f . (3)
- 3.4. Determine the value (s) of x where $f(x) \geq 7$. (2)

[11]

QUESTION 4

Given the graph of f , a hyperbola of the form $y = \frac{a}{x+p} + q$, answer the questions that follow.

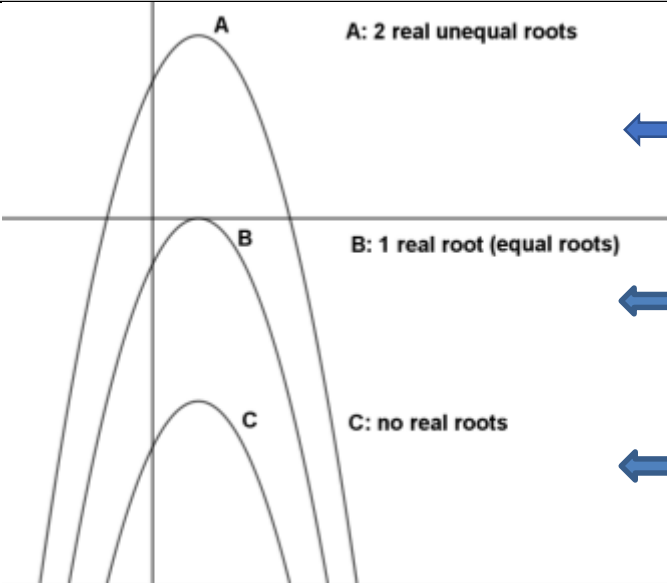


- 4.1. Write down the values of p and q . (2)
- 4.2. Determine the value of a , and write down the equation of f in the form of $y = \frac{a}{x+p} + q$. (3)
- 4.3. The axes of symmetry of f are $y = x + 3$ and $y = -x + 1$. The graph of f is transformed to g such that the axes of symmetry of g are given by $y = x - 3$ and $y = -x + 1$. Describe the transformation. Show all calculations to support your answer. (5)

[10]

SUMMARY ANG NOTES FOR ALL ALGEBRAIC FUNCTIONS

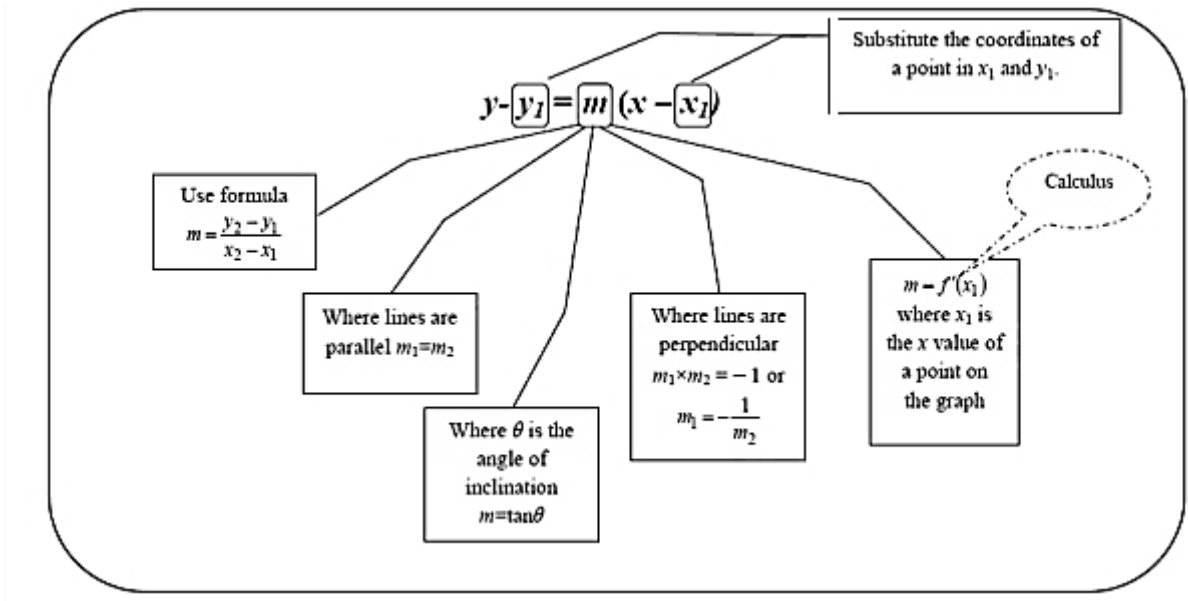
Function	$ax + q$	$\frac{a}{x+p} + q$	$ax^2 + bx + c$ $a(x+p)^2 + q$	$ab^{x+p} + q$
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R};$ $x \neq -p$	$x \in \mathbb{R}$	$x \in \mathbb{R}$
Range	$y \in \mathbb{R}$	$y \in \mathbb{R};$ $y \neq q$	$y \geq q$ if $a > 0$ or $y \leq q$ if $a < 0$.	$y > q$
Turning point			$\left(\frac{-b}{2a}; f\left(\frac{-b}{2a}\right)\right)$ or $\left(\frac{x_1 + x_2}{2}; f\left(\frac{x_1 + x_2}{2}\right)\right)$ OR $(-p; q)$	
Asymptotes		Vertical asymptote $x = -p$ Horizontal asymptote $y = q$		Horizontal Asymptote $y = q$
Axis of Symmetry		$y = \pm(x + p) + q$	$x = \frac{-b}{2a}$ OR $x = -p$	
x-intercept Let $y = 0$		y-intercept Let $x = 0$		

Nature of roots		Reflections
 A: 2 real unequal roots B: 1 real root (equal roots) C: no real roots	A: 2 x- intercepts B: 1 x-intercept C: no x- intercepts	Reflection about x-axis: $(x; y)$ becomes $(x; -y)$ Reflection about y-axis: $(x; y)$ becomes $(-x; y)$

Equations of algebraic graphs

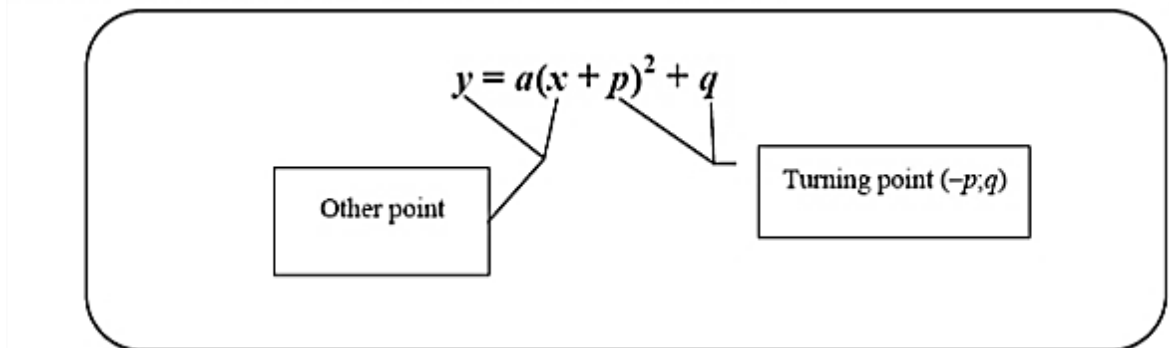
General rule: For every variable in a formula (other than x and y), you need 1 point to substitute. You could then solve simultaneous equations. There are shorter methods too:

The straight line $y = ax + q$ (Below, m is used for a)

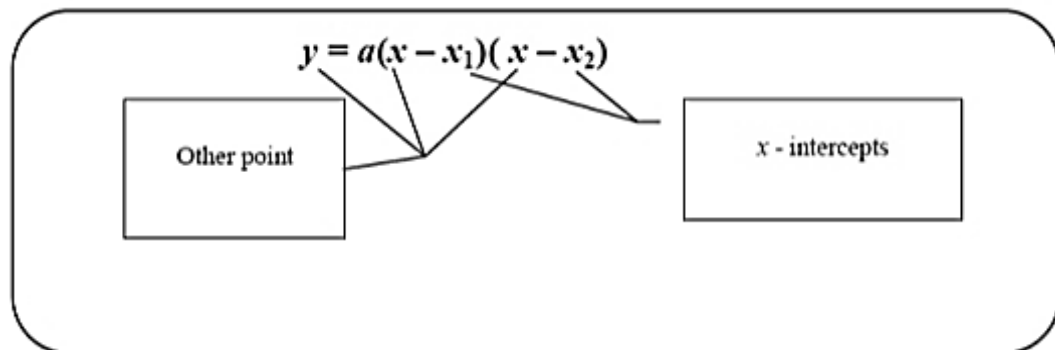


The quadratic function $y = ax^2 + bx + c$ (the simplified form of the 2 equations below)

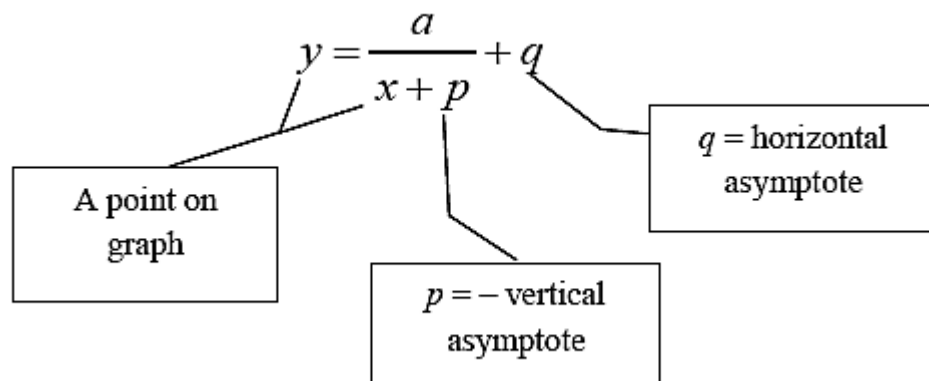
- When the turning point and another point is given



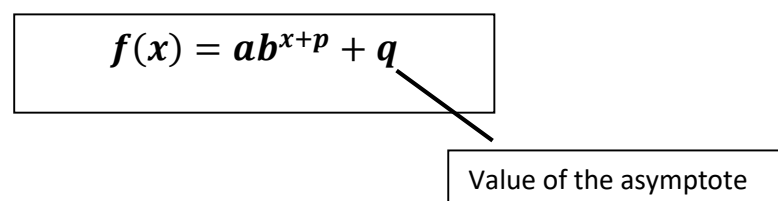
- When the intercepts and another point is given



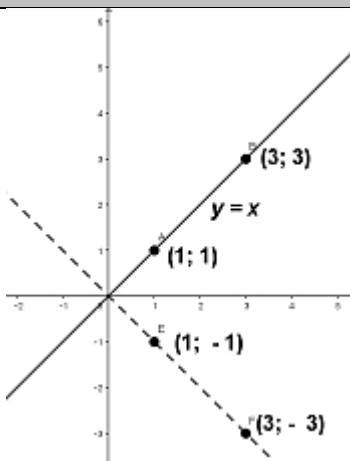
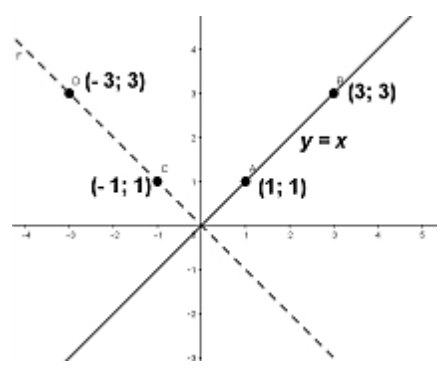
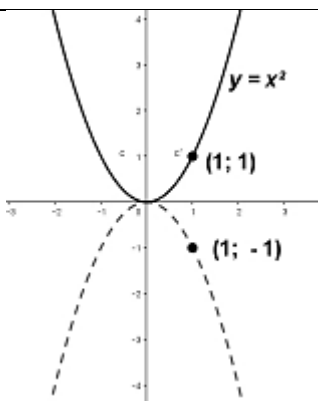
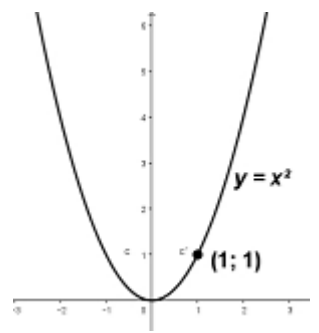
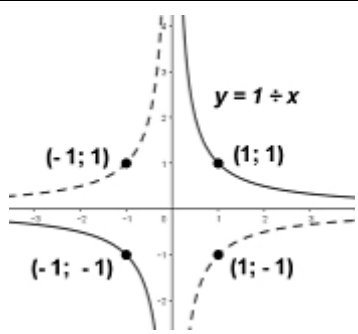
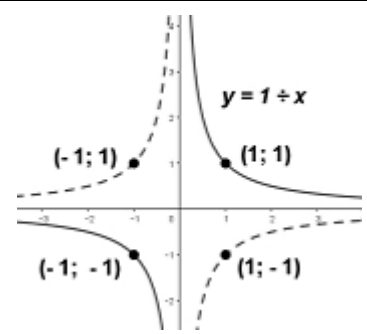
The hyperbola



The exponential function



Study these reflections of graphs about the x - and y - axis. Then answer the questions that follow.

Function	Reflected about the x -axis	Reflected about the y -axis
$f(x) = x$		
Reflected about the x -axis		
Reflected about the y -axis		
$f(x) = x^2$		
Reflected about the x -axis		
Reflected about the y -axis		
$f(x) = \frac{1}{x}$		
Reflected about the x -axis		
Reflected about the y -axis		

Sketching graphs i.e. not using a table of values

Be able to find the following from the equation of the graph.

- Its **shape** from the value of a .
- Its **y-intercept** or **vertical shift** from the value of q .
- Its horizontal shift from $-p$

Important points to plot are

- **x-intercepts** Let $f(x) = 0$ in the equation i.e. Let $y = 0$ these are the **roots**
- **y-intercept(s)** Determine y for $f(0) = 0$ i.e. Let $x = 0$
- **Turning point(s)** $(-p; q)$ for a quadratic function

If you need another point choose an x -value and substitute it into the equation of the graph.

- Lines that can be shown are **asymptotes** : $x = -p$ and $y = q$ for the hyperbola and exponential functions and **axes of symmetry** for the quadratic function and the hyperbola

If asked to reflect a graph about

- The x -axis the y -values change their sign
- The y -axis the x -values change their sign

TRIGONOMETRIC GRAPHS

Transformations of Trigonometric functions

- The effect of a : It changes the amplitude of the sine and cosine graphs and the y -value of the tangent graph at 90° and multiples thereof.
- The effect of q : It shifts the sine and cosine graphs vertically and the y -value of the tangent graph at 90° and multiples thereof, by q units.
- The effect of p : It causes a horizontal shift of $-p$ units.
- The effect of k : It causes a change in the period of the graphs. For the sine and cosine functions the period becomes $\frac{360^\circ}{k}$ and for the tangent graph the period becomes $\frac{180^\circ}{k}$. k is always considered to be positive.

EXAMPLE

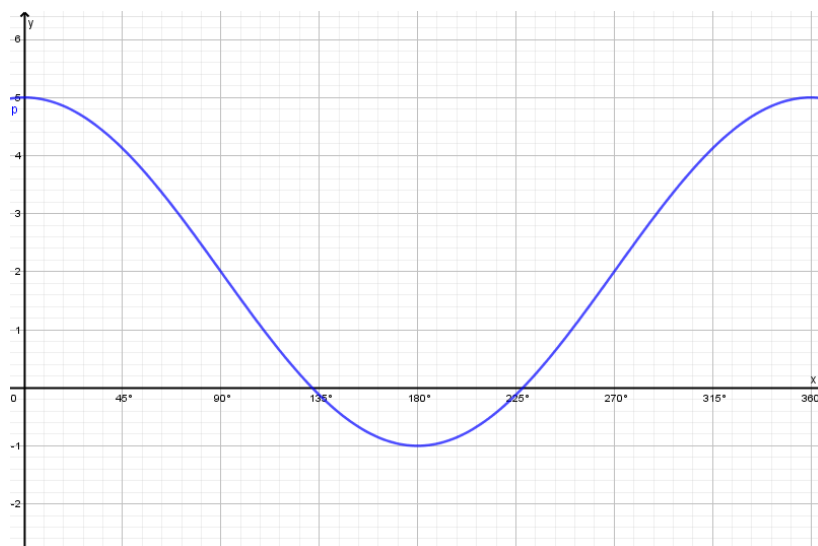
Sketch the given transformed trigonometric functions defined below. Discuss changes in the characteristic: domain; range; periodicity; amplitude; minima; maxima and asymptotes.

1. $y = 3 \cos x + 2$ if $x \in [0^\circ; 360^\circ]$

2. $y = \sin(3x + 90^\circ)$ if $x \in [0^\circ; 360^\circ]$

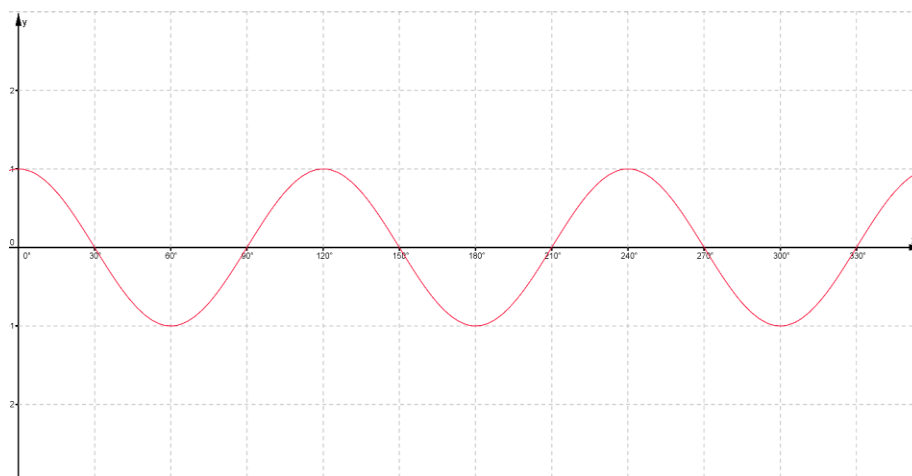
3. $y = \tan[2(x + 45^\circ)]$ if $x \in [0^\circ; 360^\circ]$

1. The sketch for $y = 3 \cos x + 2$ if $x \in [0^\circ; 360^\circ]$



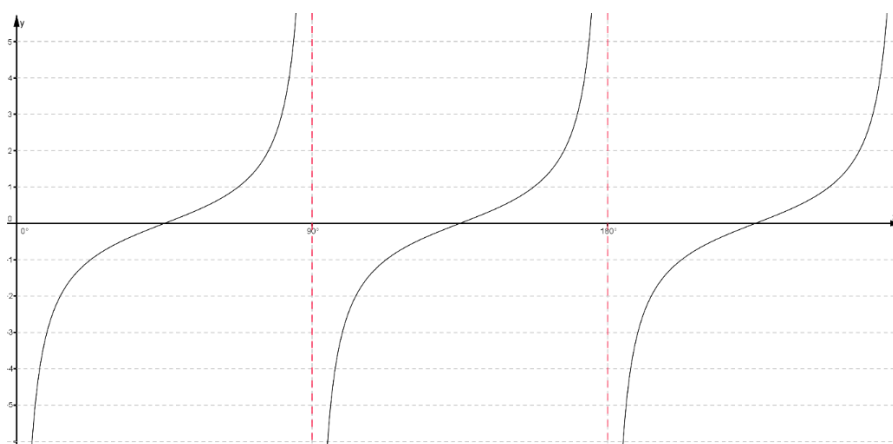
- this is a **vertical stretch** by a factor **3** and a **vertical shift by 2 units upwards**
- period is unchanged at **360°**
- domain $D = [0^\circ; 360^\circ]$
- minimum value = -1; maximum value = 5
- range $R = [-1; 5]$
- amplitude = $[5 - (-1)] \div 2 = 3$

2. The sketch for $y = \sin(3x + 90^\circ)$ if $x \in [0^\circ; 360^\circ]$



- ❖ this is a horizontal compression by factor 3 and a horizontal shift by 30° to the left
- ❖ period changes from 360° to 120°
- ❖ domain $D=[0^\circ; 360^\circ]$
- ❖ range $R=[-1; 1]$
- ❖ minimum value = -1; maximum value = 1
- ❖ amplitude is 1

3. The sketch for $y = \tan[2(x + 45^\circ)]$ if $x \in [0^\circ; 270^\circ]$

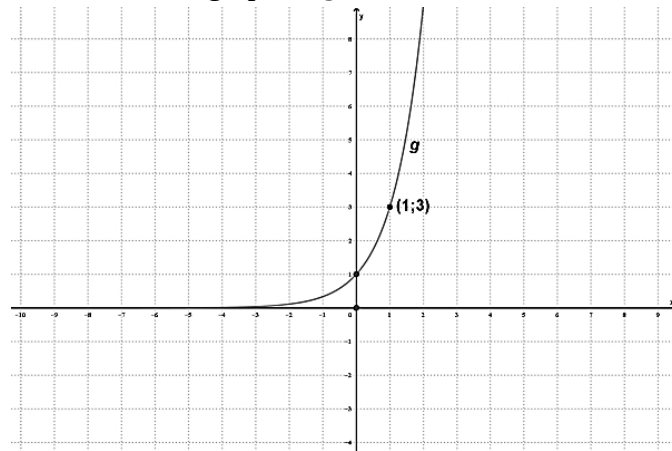


- ✦ this is a horizontal compression by factor 2 and a horizontal shift by 45° to the left
- ✦ period = 90° (changes from 180° to 90°)
- ✦ No amplitude because the maximum and the minimum values do not exist.
- ✦ asymptotes at $x = 90^\circ; 180^\circ; 270^\circ$ ($x = 90^\circ \cdot n; n \in \mathbb{Z}$)

EXERSICE 2

QUESTION 1

On the Cartesian plane below is the graph of $g(x) = 3^x$.

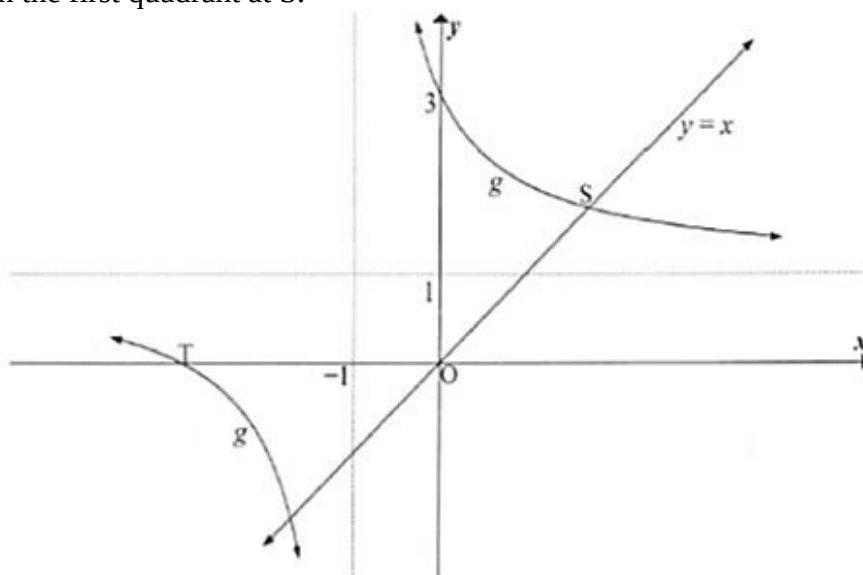


- 1.1 On the same Cartesian plane sketch the graph of $h(x) = \left(\frac{1}{3}\right)^x$
- 1.2 Describe the transformation from g to h .
- 1.3 Write the equation of h in terms of x in another way.
- 1.4 On the same Cartesian plane sketch the graph of m the reflection of g in the x -axis.
- 1.5 Write the equation of m in terms of $g(x)$ and in terms of x .
- 1.6 Draw the graph of $y = x$ on the same set of axes.

QUESTION 2

The diagram below shows hyperbola g defined by the equation $g(x) = \frac{2}{x+1} + 1$.

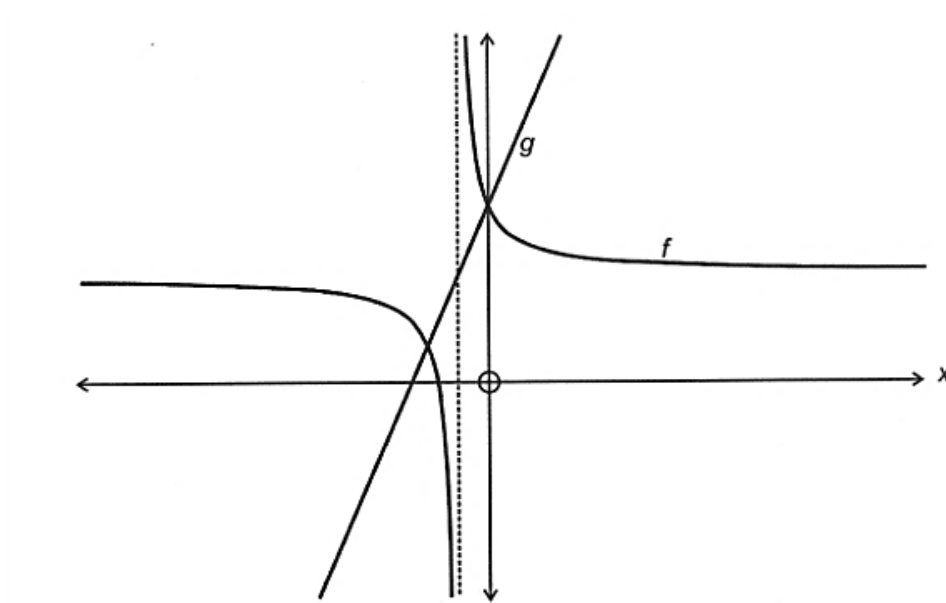
The graph of g intersects the x -axis at T and the y -axis at (0;3). The line $y = x$ intersects the hyperbola in the first quadrant at S.



- 2.1 Calculate the x -coordinate of T. (2)
- 2.2 Calculate the length of OS? (5)
- 2.3 For which values of k will the equation $g(x) = x + k$ have two real roots that are of opposite signs? (1)
- 2.4 For which value k will the equation $x^2 + x = k$ have no real roots (4)

QUESTION 3

In the diagram below, the graphs of $f(x) = \frac{a}{x+b} + c$ and $g(x) = 2x + 5$ are given.

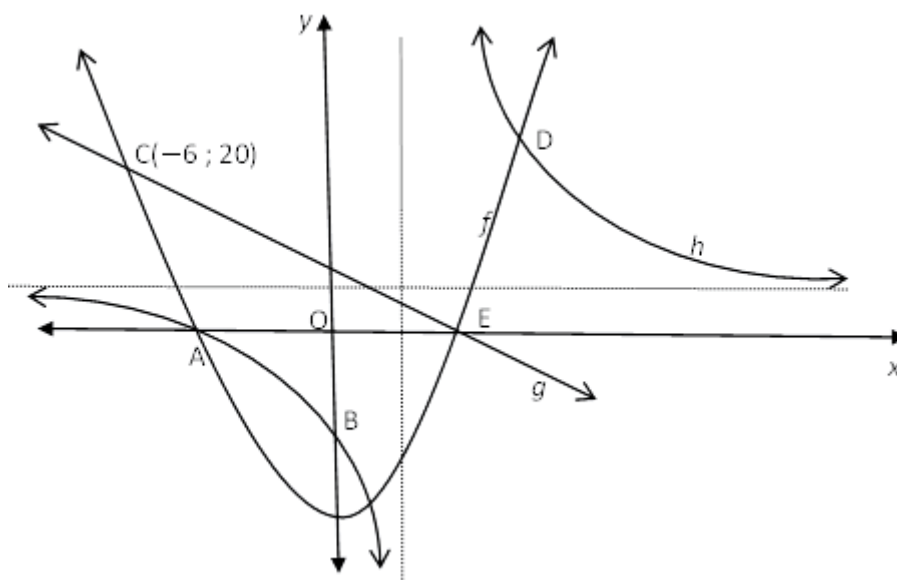


The graph of f has a vertical asymptote at $x = -1$, both graphs intersect the y -axis and the graph of g intersects the horizontal asymptote of f at the point $(-1; y)$.

- 3.1 Determine a , b and c . Show all working. (6)
- 3.2 If $f(x) = \frac{2}{x+1} + 3$ and $g(x) = 2x + 5$:
- 3.2.1 Determine the x -intercepts of f and g . (3)
- 3.2.2 Hence, or otherwise, solve for x if $f(x) \cdot g(x) \leq 0$. (3)

QUESTION 4

Sketched below are the graphs of: $g(x) = -2x + 8$; $f(x) = x^2 + k$ and $h(x) = \frac{6}{x-2} + 1$. A and B are the x- and y - intercepts of h respectively, C (-6 ; 20) and E are the points of intersection of f and g .



- 4.1 Calculate the coordinates of A, B and E. (6)
- 4.2 Show that the value of $k = -16$ (2)
- 4.3 Give the domain of f (1)
- 4.4 Give the range of f (1)
- 4.5 Write down the values of x for which $g(x) - f(x) \geq 0$ (2)
- 4.6 Determine the equation of the axis of symmetry of h if its gradient is negative. (3)
- 4.7 Write down the range of s , if $s(x) = f(x) + 2$. (3)
- 4.8 Write down the range of t , if $t(x) = h(x) + 2$. (3)

WORKSHEET ON SKETCHING OF TRIGONOMETRIC GRAPHS

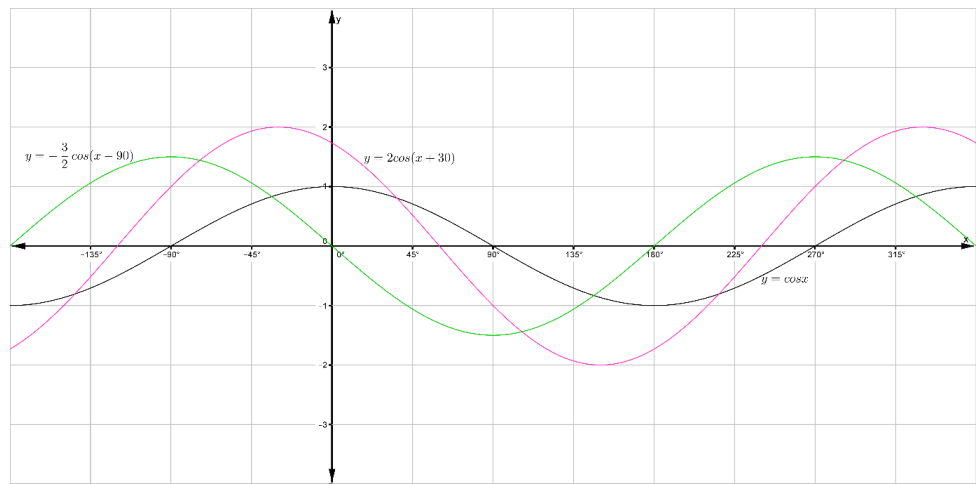
Sketch the graphs of the functions in each of the following sets on the same set of axes:

- 1 $y = \cos x, y = -4 \cos x, y = -\frac{5}{2} \cos x, y = -\cos x, y = \frac{5}{2} \cos x$ and $y = 5 \cos x$ when $x \in [-180^\circ; 360^\circ]$
- 2 $y = \tan x, y = -3 \tan x, y = -\frac{1}{3} \tan x, y = \frac{1}{2} \tan x$ and $y = 2 \tan x, x \in [-360^\circ; 360^\circ]$
- 3 $y = \sin x, y = \sin x + 1, y = \sin x + 3, y = \sin x - 1, y = \sin x - 2$ when $-360^\circ \leq x \leq 180^\circ$
- 4 $y = \cos x, y = \cos x + 2, y = \cos x - 1, y = \cos x - 2, y = \cos x + 4, x \in [-180^\circ; 360^\circ]$
- 5 $y = \sin x, y = 2 \sin x, y = \frac{1}{2} \sin x, y = -2 \sin x, y = -\frac{1}{3} \sin x, y = \frac{1}{3} \sin x, y = -\sin x$ and $y = 3 \sin x, x \in [-360^\circ; 360^\circ]$
- 6 $y = \cos x, y = \cos(x - 60^\circ)$ and $y = \cos(x + 30^\circ)$ when $-360^\circ \leq x \leq 360^\circ$
- 7 $y = \sin x, y = \sin(x + 45^\circ)$ and $y = \sin(x - 90^\circ)$ for $x \in [-360^\circ; 360^\circ]$
- 8 $y = \tan x$ and $y = \tan(x + 30^\circ)$ for $x \in [-360^\circ; 360^\circ]$
- 9 $y = \tan x$ and $y = \tan(x - 45^\circ)$ when $-180^\circ \leq x \leq 360^\circ$
- 10 $y = \sin x, y = \sin 2x, y = \sin 3x$ and $y = \sin \frac{1}{2}x$ for $x \in [-360^\circ; 360^\circ]$
- 11 $y = \tan x$ and $y = \tan 2x$ when $-360^\circ \leq x \leq 360^\circ$
- 12 $y = \tan x$ and $y = \tan \frac{1}{2}x$ for $x \in [-360^\circ; 360^\circ]$
- 13 $y = \tan x$ and $y = \tan \frac{3}{2}x, x \in [-360^\circ; 360^\circ]$
- 14 $y = \tan x$ and $y = \tan 3x, x \in [-360^\circ; 360^\circ]$
- 15 **WA15.** $y = \cos x, y = \cos \frac{1}{3}x$ and $y = \cos 4x, x \in [360^\circ; 360^\circ]$
- 16 **WA16.** $y = \tan x, y = -\frac{1}{3} \tan x - 2$ and $y = 3 \tan x + 1$, when $-360^\circ \leq x \leq 360^\circ$
- 17 **WA17.** $y = \sin x, y = -\sin(x - 45^\circ)$ and $y = -3 \tan(x + 90^\circ)$ for $x \in [-360^\circ; 180^\circ]$

WORKSHEET ON INTERPRETATION OF GRAPHS

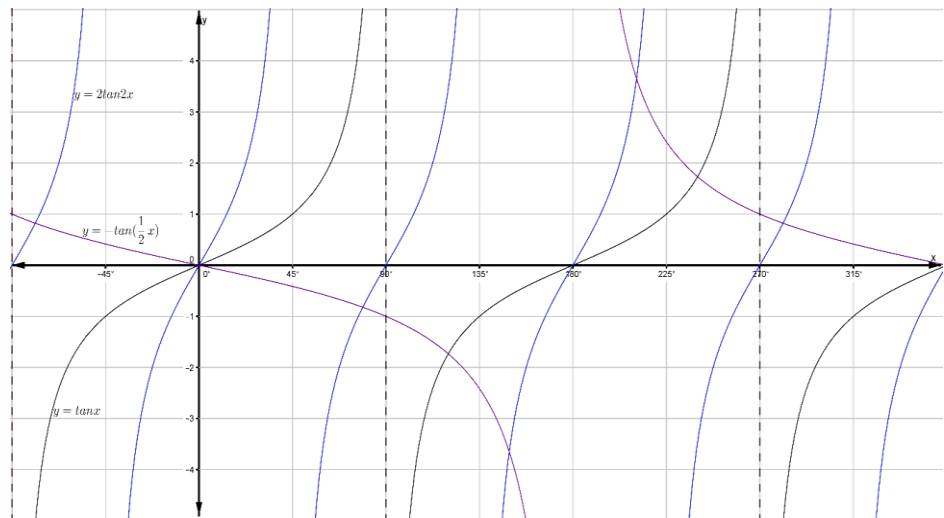
Study the following sets of graphs below and complete the table for each case.

1



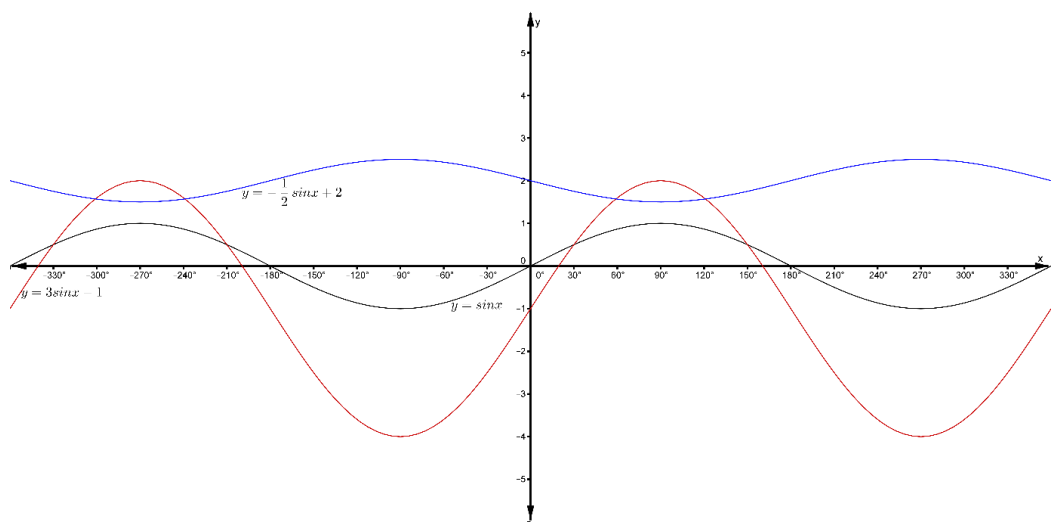
Function	Domain	Range	Amplitude	Period

2



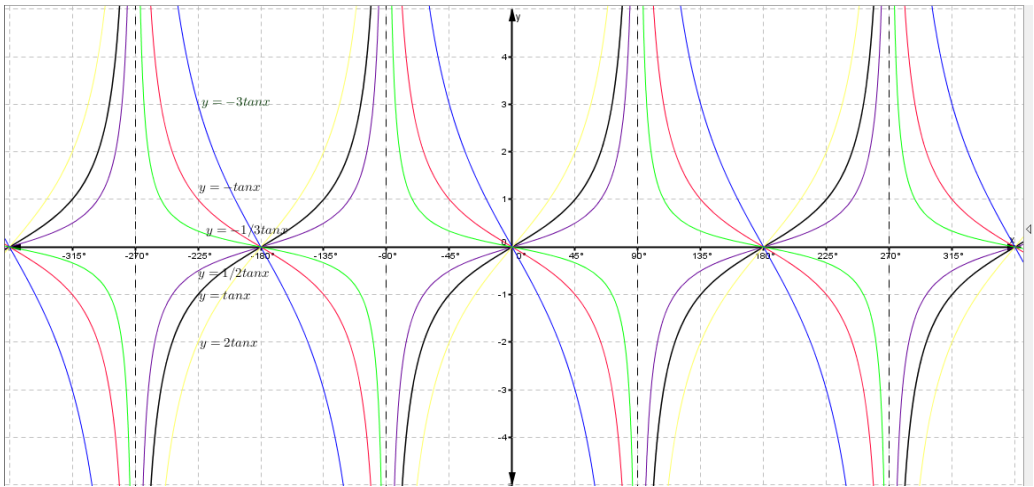
Function	domain	range	amplitude	period

3



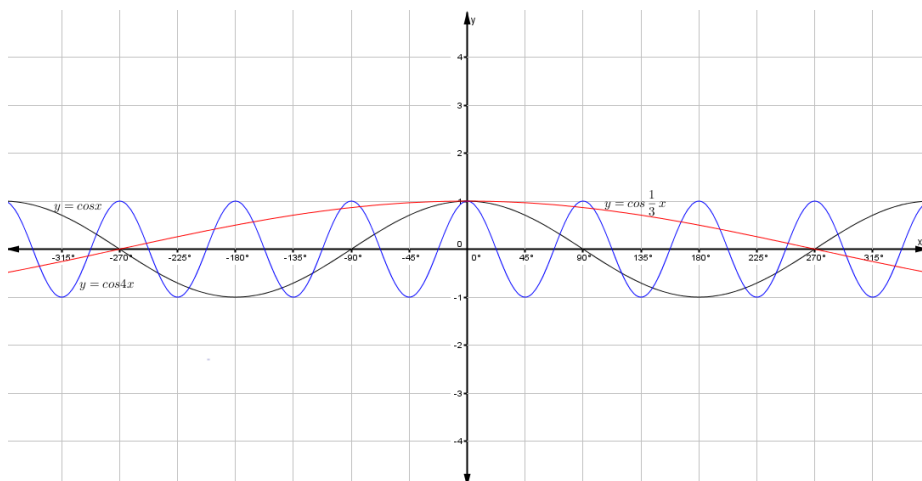
Function	domain	range	amplitude	period

4.



Function	domain	range	amplitude	period

5.



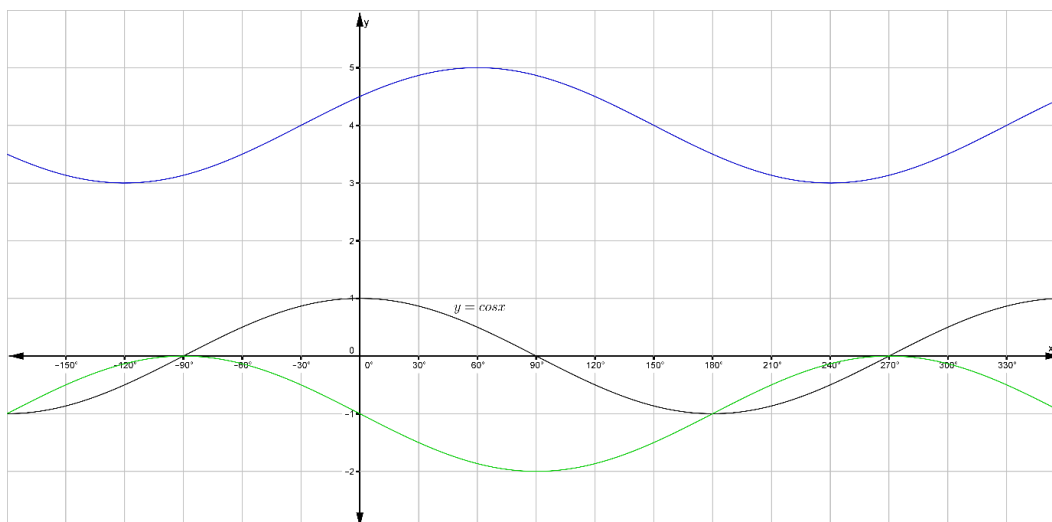
Function	domain	range	amplitude	period

WORKSHEET C

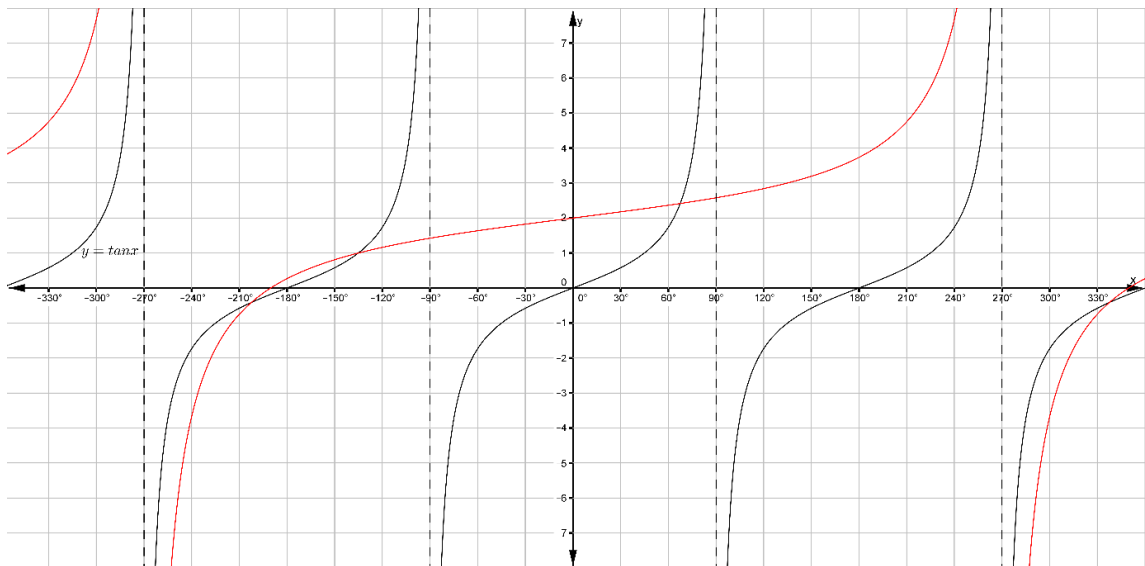
EQUATIONS OF DRAWN GRAPHS

Study the following graphs and write down the equations of each of the coloured ones.

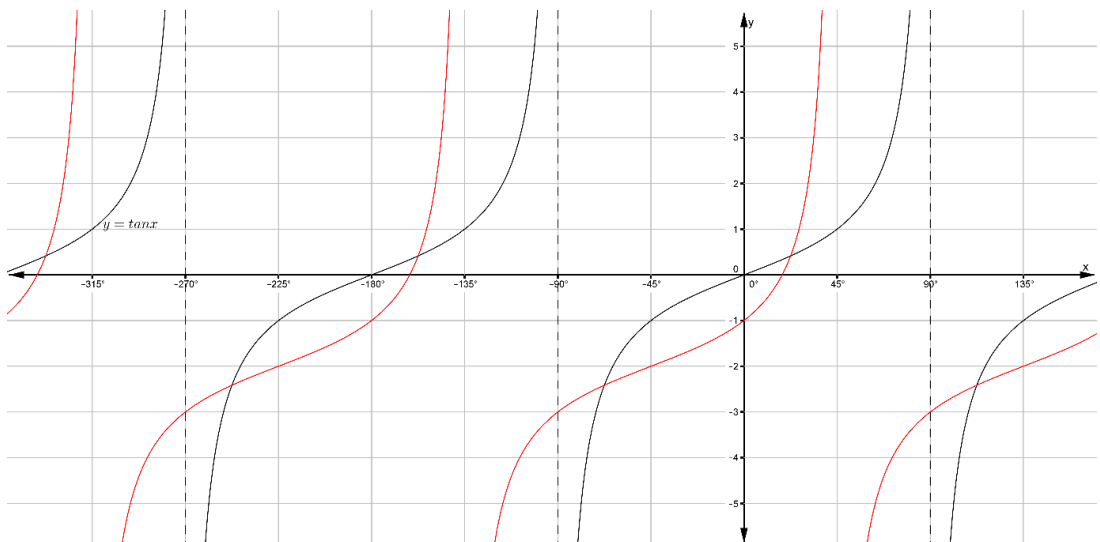
1



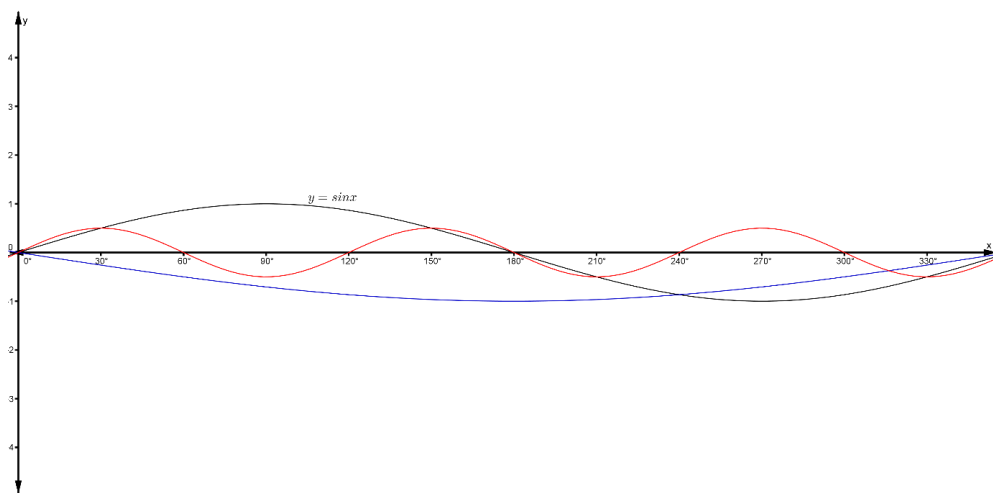
2.



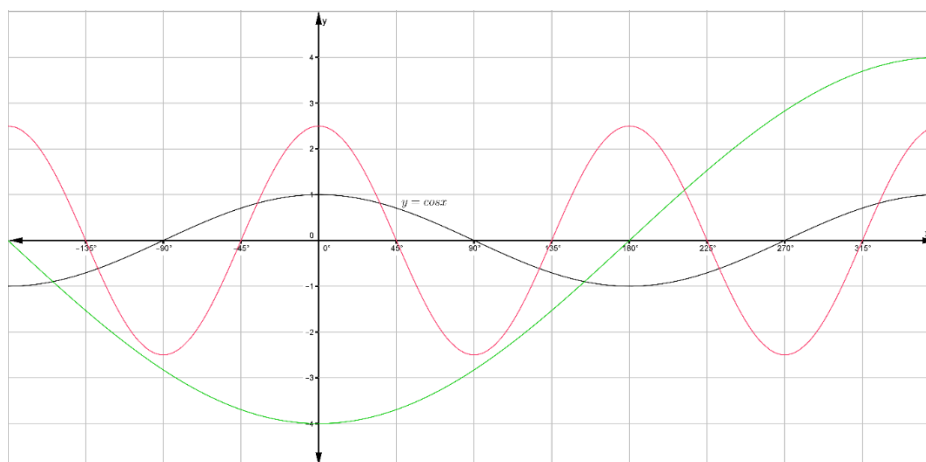
3.



4



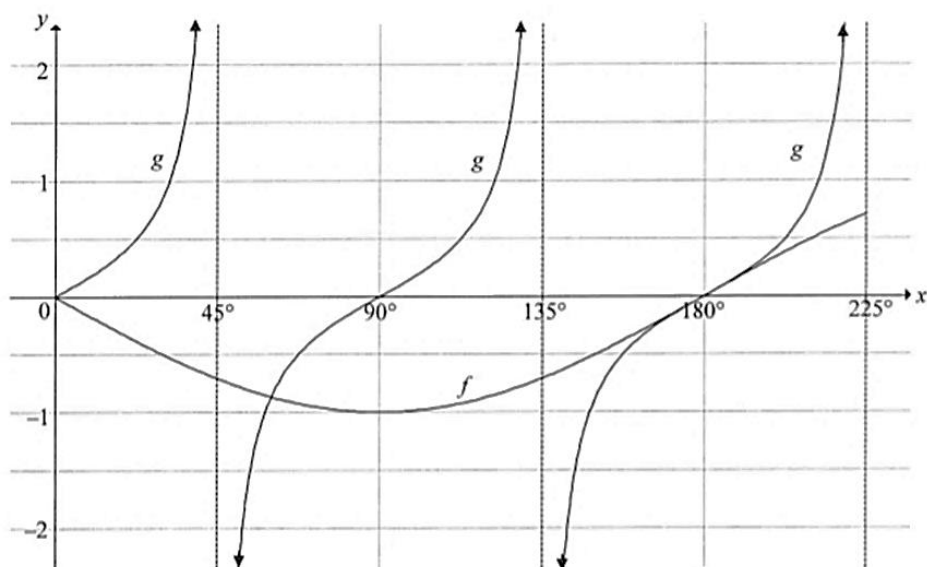
5.



EXAMINATION TYPE QUESTIONS

QUESTION 1

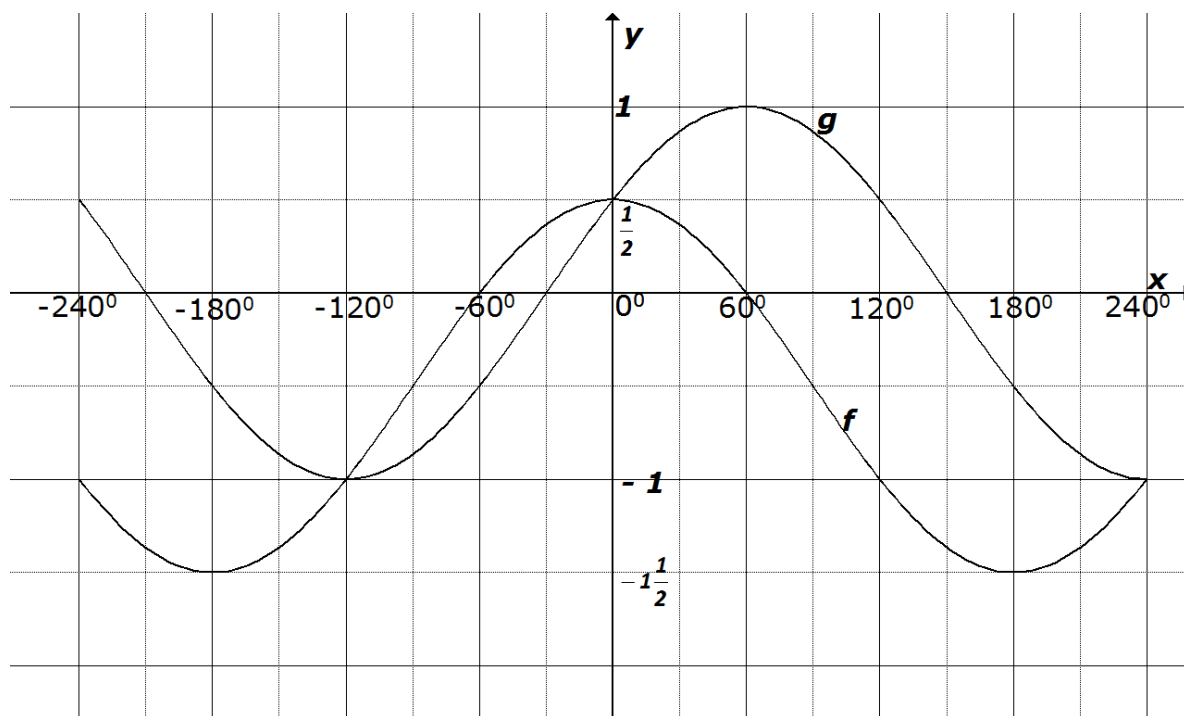
In the diagram, the graphs of the functions $f(x) = a \sin x$ and $g(x) = \tan bx$ are drawn on the same system of axes for $0^\circ \leq x \leq 225^\circ$.



- 1.1 Write down the values of a and b . (2)
- 1.2 Write down the period of $f(x)$. (2)
- 1.3 Determine values of x in the interval $90^\circ \leq x \leq 225^\circ$ for which $f(x) < g(x)$. (2)

QUESTION 2

In the diagram below, the graphs of $f(x) = \cos x + q$ and $g(x) = \sin(x + p)$ are drawn on the same system of axes for $-240^\circ \leq x \leq 240^\circ$. The graphs intersect at $\left(0^\circ; \frac{1}{2}\right)$, $(-120^\circ; -1)$ and $(240^\circ; -1)$.



- 2.1 Determine the values of p and q .
- 2.2 Determine the values of x in the interval $-240^\circ \leq x \leq 240^\circ$ for which $f(x) > g(x)$.
- 2.3 Describe a transformation that the graph of g has to undergo to form the graph of h , where $h(x) = -\cos x$.
- 2.4 Using the graphs to determine the solutions to $f(x) = g(x)$ for $-240^\circ \leq x \leq 240^\circ$.

E.TERM 3

WEEK	TOPIC	CONTENT
WEEK 1	Trigonometry Sin/Cos/Area rules	1. Prove and apply the sine, cosine and area rules.
WEEK 2		2. Solve problems in two dimensions using the sine, cosine and area rules.
WEEK 3		2. Solve problems in two dimensions using the sine, cosine and area rules.
WEEK 4	Measurements	Revise the volume and surface areas of right-prisms and cylinders. TRIMMED
WEEK 5		- Study the effect on volume and surface areas when multiplying any dimension by a constant factor k. - Calculate volume and surface areas of spheres, right prisms, right cones and combination of those objects (figures).
WEEK 6	Statistics	1. Revise measures of central tendency in ungrouped data. 2. Measures of central tendency in grouped data: calculation of mean estimate of grouped and ungrouped data and identification of modal interval and interval in which the median lies. 3. Revision of range as a measure of dispersion and extension to include percentiles, quartiles, inter-quartile and semi-inter-quartile range.
WEEK 7		4. Five number summary (maximum, minimum and quartiles) and box and whisker diagram. 5. Use the statistical summaries (measures of central tendency and dispersion), and graphs to analyse and make meaningful comments on the context associated with the given data. 6. Histograms
WEEK 8		7. Frequency polygons 8. Ogives (cumulative frequency curves) 9. Variance and standard deviation of ungrouped data 10. Symmetric and skewed data 11. Identification of outliers.
WEEK 9	Probability	1. The use of probability models to compare the relative frequency of events with the theoretical probability. 2. The use of Venn diagrams to solve probability problems, deriving and applying the following for any two events in a sample space S: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ A and B are Mutually exclusive if $P(A \text{ and } B) = 0$; A and B are complementary if they are mutually exclusive; and $P(A) + P(B) = 1$. Then $P(B) = P(\text{not}(A)) = 1 - P(A)$ 3. Revised the addition rule for mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$ The complementary rule: $P(\text{not } A) = 1 - P(A)$ and the identity $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
WEEK 10		4. Identify dependents and independent events and the product rule for independent events: $P(A \text{ and } B) = P(A) \times P(B)$ 5. The use of Venn diagrams to solve probability problems, Deriving and applying formulae for any three events A, B and C in a sample space S. 6. Use tree diagrams for the probability of consecutive or simultaneous events which are not necessarily independent

WEEK 1 – 3: TRIGONOMETRY

CONTENT

. Prove and apply the sine, cosine and area rules.
Solve problems in two dimensions using the sine, cosine and area rules.

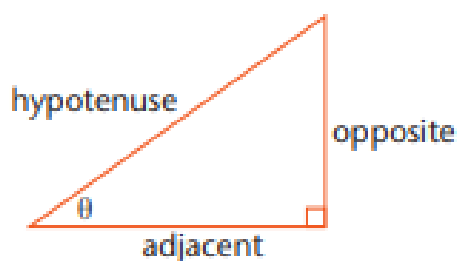
NOTES & EXAM TIPS

Solving right – angled triangles – Revision of Grade 10

For a right-angled triangle use the three trigonometric definitions:

KEY WORDS

solve a triangle – to calculate the lengths and sizes of the unknown sides and angles



$$\sin\theta = \frac{o}{h} \quad \cos\theta = \frac{a}{h} \quad \tan\theta = \frac{o}{a}$$

Remember:

- to find a side:

$$\frac{\text{unknown side}}{\text{known side}} = \sin, \cos, \text{ or } \tan \text{ of the known angle.}$$

$$\frac{\text{known side}}{\text{unknown side}} = \sin, \cos, \text{ or } \tan \text{ of the known angle.}$$

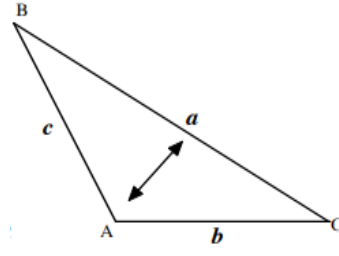
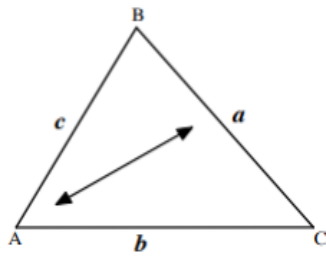
- To find an angle

$$\frac{\text{known side}}{\text{known side}} = \sin, \cos \text{ or } \tan \text{ of the unknown angle.}$$

Solving triangles with no right angles.

For triangles that have no right angles you cannot use the trigonometric definitions. You therefore need to know two new rules called the sine and cosine rules.

It is important to see how triangles are labelled. For the respective sides we use lower case letters but more specifically, the lower case letter of the angle opposite the side.



THE AREA RULE:

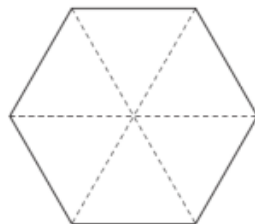
You have learnt that the area of a triangle is $A = \frac{1}{2} \text{base} \times \text{height}$. If you are not given or cannot calculate the height you need to know and use the area rule:

$$\text{Area} \Delta ABC = \frac{1}{2} bc \sin A \quad \text{or} \quad \text{area} \Delta ABC = \frac{1}{2} ac \cos B \quad \text{or} \quad \text{area} \Delta ABC = \frac{1}{2} ab \cos C$$

Application of Area Rule

The top of a table is in the shape of a rectangular hexagon

The three diagonal of the hexagon which are shown as dotted line in the diagram below each have length of 40 cm.



Calculate the area of the TOP.

Each triangle in the hexagon has centre angle $= \frac{360}{6} = 60^\circ$

Each triangle is therefore equilateral with sides 20 cm

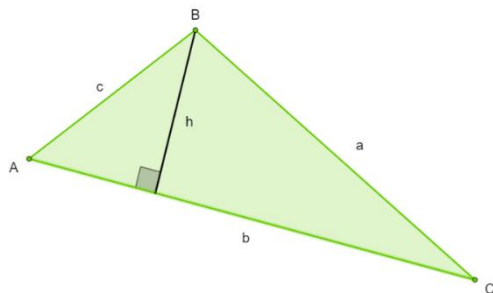
From the Formulae List: Area of a triangle $= \frac{1}{2} \times ab \sin C$

Area of each triangle $= \frac{1}{2} \times 20 \times 20 \times \sin 60 = 173.2 \text{ cm}^2$

Area of hexagon $= 6 \times 173.2 = 1039.3 \text{ cm}^2$

SINE RULE

Use the sine rule if you know a side – angle pair, (that is an angle and the side opposite it) and one other given side or angle.



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\text{unknown side}}{\sin \text{ of opp } \angle} = \frac{\text{known side}}{\sin \text{ of opp } \angle}$$

$$\begin{aligned} \sin A &= \frac{h}{c} & \text{and} & & \sin C &= \frac{h}{a} \\ h &= c \sin A & \text{and} & & h &= a \sin C \\ c \sin A &= a \sin C \\ \frac{\sin A}{a} &= \frac{\sin C}{c} \end{aligned}$$

COSINE RULE

For triangles that do not have a known side- angle pair you cannot use the sine rule. To use the cosine rule you need to have

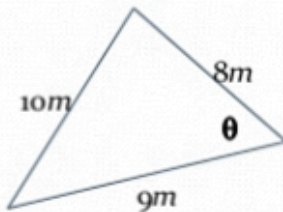
- Two side and an included angle (SAS)
- Three sides. (SSS)

$$c^2 = a^2 + b^2 - 2ab\cos C$$

- First, check to see if you can use the Sine Rule. It's easier!

You can use the Cosine Rule when

- You are told **ALL THREE SIDES** and asked to **FIND ANY ANGLE**

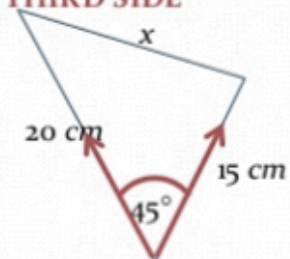


Here, we use

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

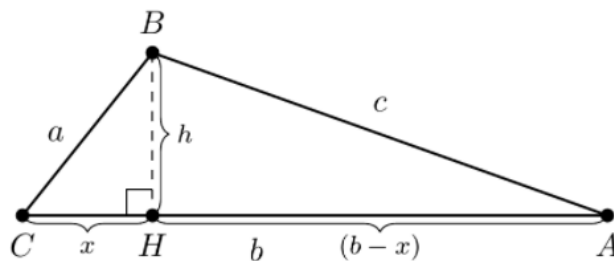
OR

- You are told **TWO SIDES** and **THEIR INCLUDED ANGLE** (i.e. the angle between those two sides) and asked to **FIND THE THIRD SIDE**



Here, we use

$$c^2 = a^2 + b^2 - 2ab \cos C$$



(Cosine rule) We begin by using Pythagoras' Theorem for the triangles CHB and HAB :

$$a^2 - x^2 = h^2 = c^2 - (b - x)^2$$

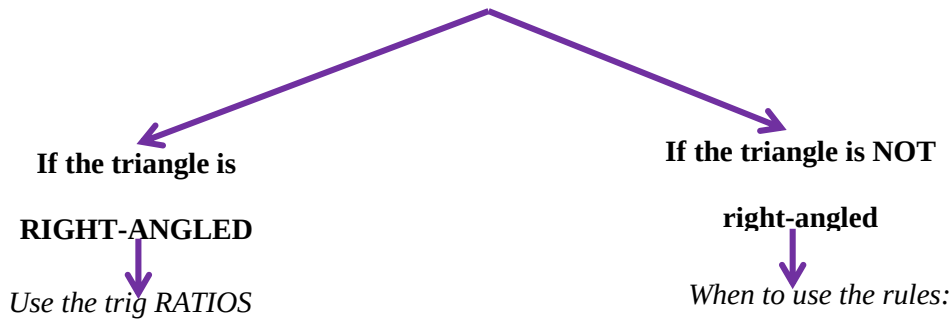
$$a^2 - x^2 = c^2 - b^2 + 2bx - x^2$$

$$a^2 + b^2 = c^2 + 2bx$$

From the definition of cosine for the triangle CHB , we have $x = a \cos C$, which leads to

$$a^2 + b^2 = c^2 - 2ab \cos C.$$

When to use what?



Remember:

3 properties of a triangle must be given in a triangle in order to work in that triangle
(**NOT** angle, angle, angle)

COSINE RULE if

- 3 sides of the triangle are given
- 2 sides and an included angle of the triangle is given

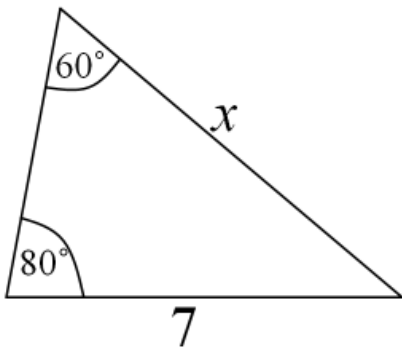
SINE RULE if

- Any condition that does NOT satisfy the cosine rule

EXAMPLES

No 1.

1. The triangle is not right-angled.
2. We do know a side and its opposite angle.
3. Therefore we use the Sine Rule.



Step 1: Remember that each fraction in the Sine Rule formula should contain a side and its opposite angle. Use variable (unknown side or angle) the numerator of the fraction.

$$\frac{x}{\sin 80^\circ} = \frac{7}{\sin 60^\circ}$$

Step 2 Solve the resulting equation to find the unknown side/angle.

$$x = \frac{7 \sin 80^\circ}{\sin 60^\circ}$$

$$x = 7,96$$

No 2

Step 1. Fill in the values you know into the formula

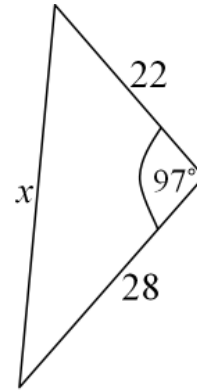
$$\begin{aligned}(\text{unknown side})^2 &= (\text{known side 1})^2 + (\text{known side 2})^2 \\ &- 2(\text{known side 1})(\text{known side 2})\cos \\ &< \text{between known sides}\end{aligned}$$

$$x^2 = 22^2 + 28^2 - 2 \times 22 \times 28 \cos 97^\circ$$

Step 2 Evaluate the right hand side and the square -root to find the length.

$$x^2 = 1418,143$$

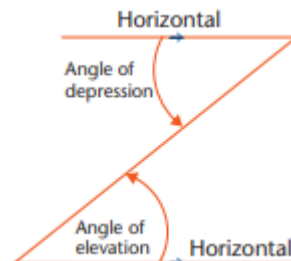
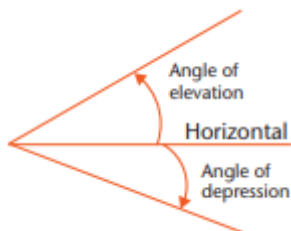
$$x = 37,7$$



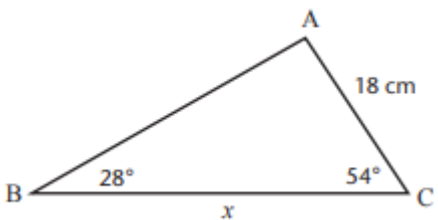
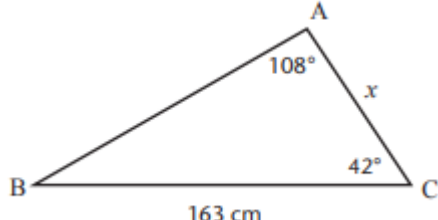
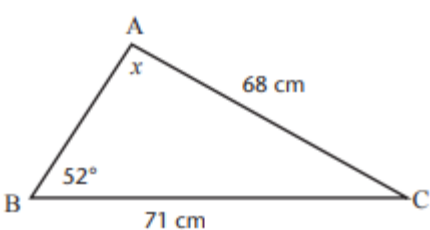
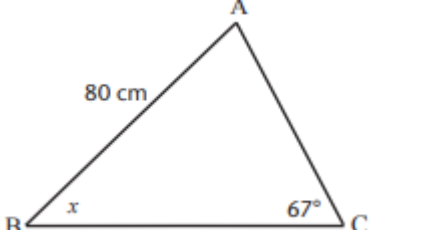
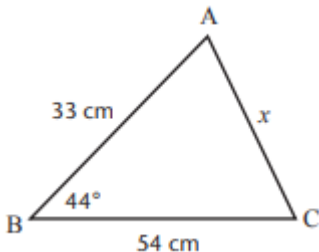
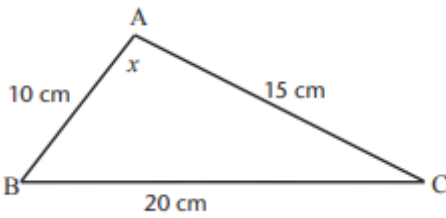
Two dimensional problems using the sine, cosine and area rules.

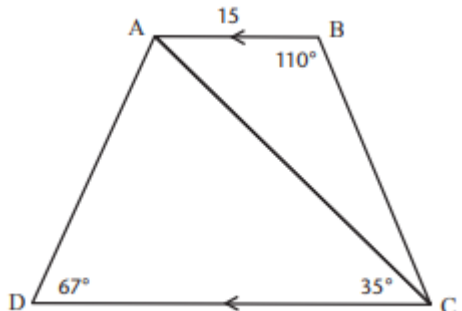
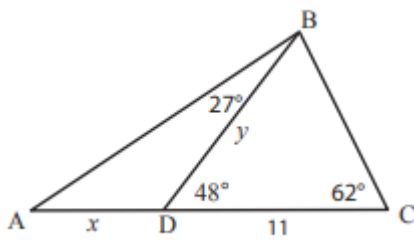
Remember:

The angle of elevation and depression are the angles between the line of sight and the horizontal.

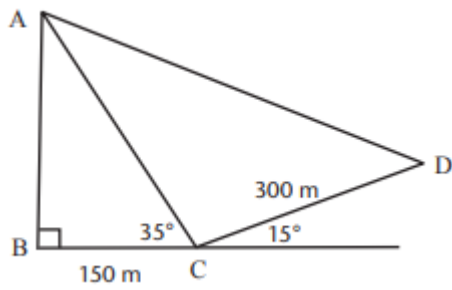


1. Use geometry to fill in as many of the angles as you can.
2. Start with the triangle which has the given length.
3. Find a side which links the two triangles.
4. Use this side to solve sides or angles in the other

Activity 1 Solve for x	Crawling
1. 	2. 
3. 	4. 
5. 	6. 

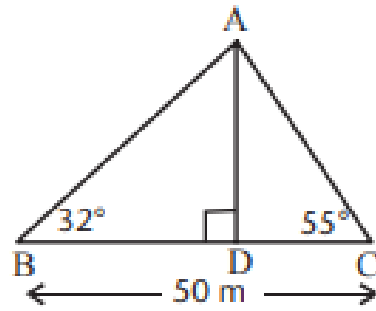
Activity 2	Walking
1. Given ABCD is a trapezium  Calculate the length of DC.	2.  Determine the values of x and y

3.



Calculate the length of AD

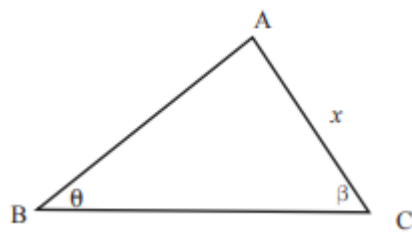
4.



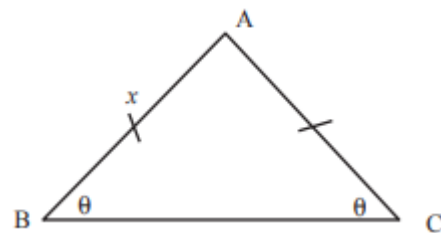
Calculate the length of AD

5. In each case find the length of BC in terms of the given letter and symbols.

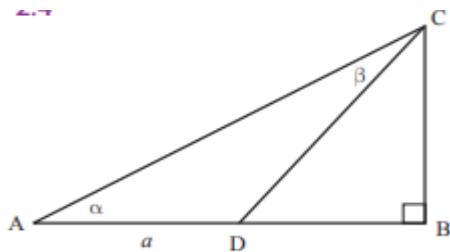
5.1



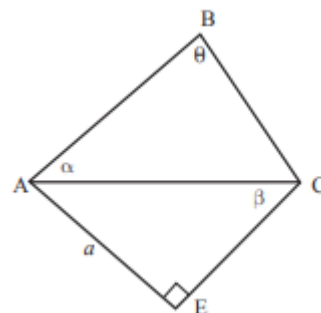
5.2



5.3



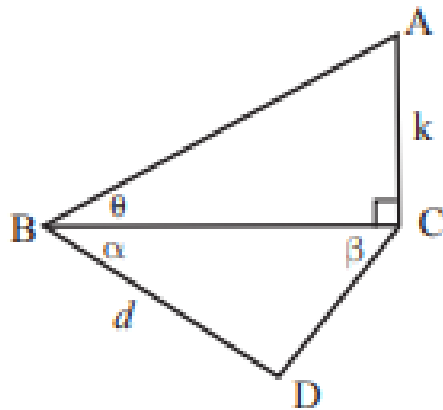
5.4



6. Use the given information in the diagram to show that:

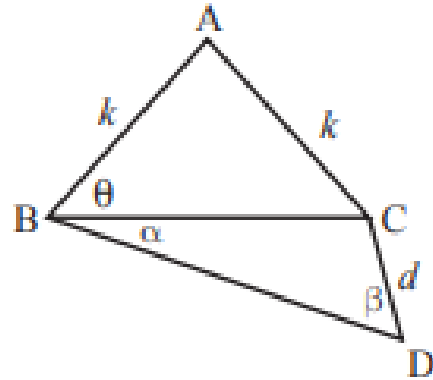
Jogging

6.1



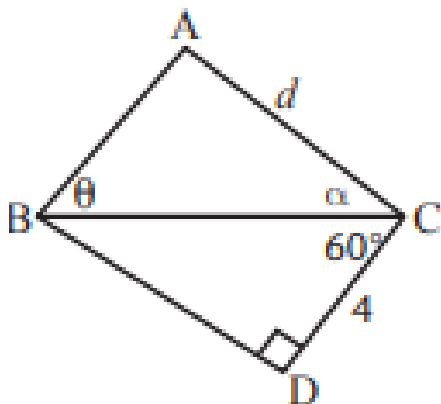
$$d = \frac{k \sin \beta}{\sin(\alpha + \beta) \cdot \tan \theta}$$

6.2



$$d = \frac{k \sin 2\theta \sin \alpha}{\sin \theta \sin \beta}$$

6.3

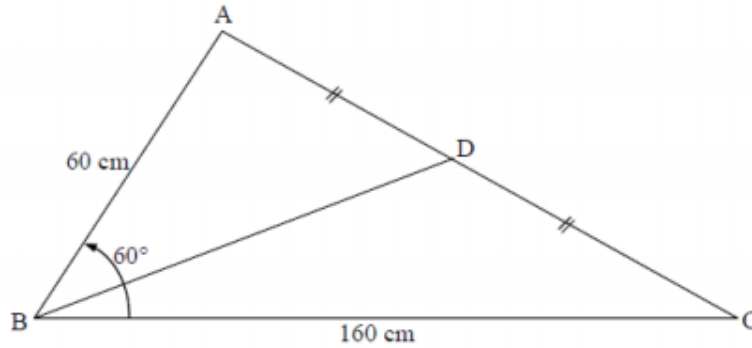


$$d = \frac{8 \sin \theta}{\sin(\alpha + \theta)}$$

Activity 3 Exam type Questions

Question 1

In $\triangle ABC$, $AB = 60 \text{ cm}$, $BC = 160 \text{ cm}$ and $\hat{ABC} = 60^\circ$. BD is the bisector of AC with D a point on AC.



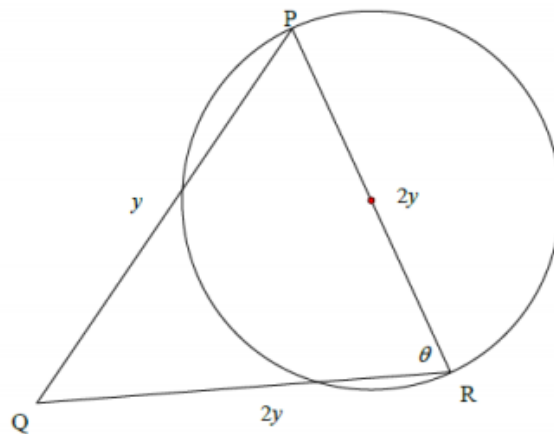
1.1 Calculate the length of AC. (3)

1.2 Determine the value of $\sin A$. Leave the answer in its simplest form. (3)

1.3 Calculate the area of $\triangle ABD$. Give your answer correct to ONE decimal. (3)

Question 2

In the diagram PR is the diameter of the circle. Triangle PQR is drawn with vertex Q outside the circle. $\hat{R} = \theta$, $PR = QR = 2y$ and $PQ = y$.



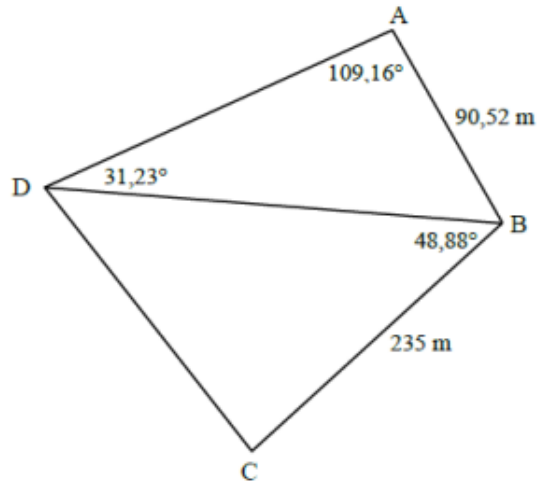
2.1 Determine the value of $\cos \theta$. (4)

2.2 If QR cuts the circumference of the circle at T. Determine PT in terms of y and θ . (3)

Question 3

Quadrilateral ABCD is drawn with $BC = 235\text{m}$ $AB = 90,52\text{m}$ It is given that

$$\hat{ADB} = 31,23^\circ \quad \hat{DAB} = 109,16^\circ \text{ and } \hat{CBD} = 48,88^\circ$$

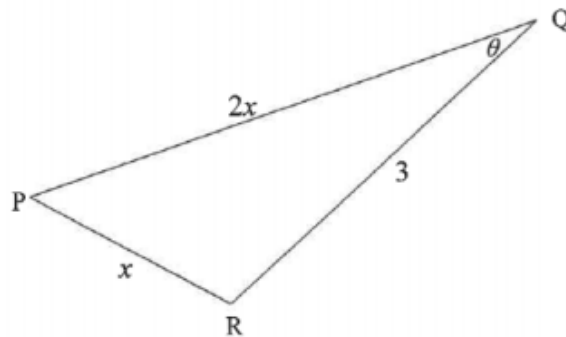


3.1 Determine the length of BD (3)

3.2 Determine the length of CD (3)

Question 4

In $\triangle PQR$, $QR = 3$ units $PR = x$ units $PQ = 2x$ units and $\hat{PQR} = \theta$



4.1 Show that $\cos\theta = \frac{x^2+3}{4x}$ (3)

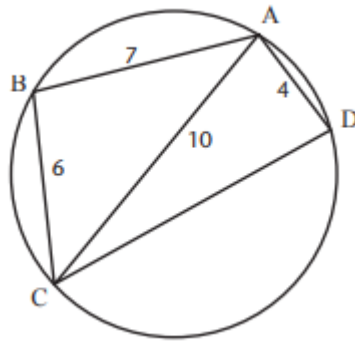
4.2 If $x = 2,4$ units

4.2.1 Calculate θ (3)

4.2.2 Calculate the area of $\triangle PQR$ (2)

Question 5

A, B, C and D lie on a circle. $AB = 7\text{ cm}$, $BC = 6\text{ cm}$, $AC = 10\text{ cm}$ and $AD = 4\text{ cm}$.



Calculate

5.1 the size of $\hat{B}$ (3)

5.2 the size of $\hat{ACD}$ (3)

WEEK 4 – 5 :TOPIC : MEASUREMENT

CONTENT:

- Study the effect on volume and surface areas when multiplying any dimension by a constant factor k.

Calculate volume and surface areas of spheres, right prisms, right cones and combination of those objects (figures).

NOTES & EXAM TIPS

Previous Knowledge:

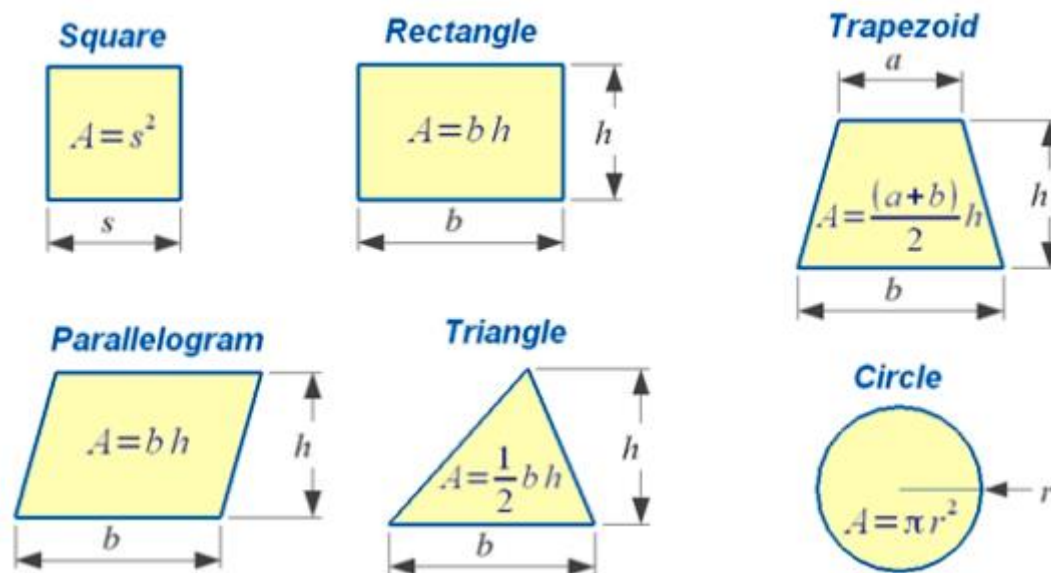
What are 2D shapes?

The two-dimensional shapes (2D shapes) are also known as flat shapes, are the shapes having two dimensions only. It has length and breadth. It does not have thickness. The two different measures used for measuring the flat shapes are area and the perimeter. Two-dimensional shapes are the shapes that can be drawn on the piece of paper. Some of the examples of 2D shapes are square, rectangle, circle, triangle and so on.

Area of 2D Shapes Formula

In general, the area of shapes can be defined as the amount of paint required to cover the surface with a single coat. Following are the ways to calculate area based on the number of sides that exist in the shape, area is the two dimensional space inside the boundary of a flat object. It is measured in square units.

Area formulae:

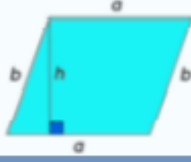
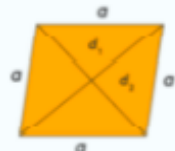
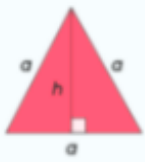


Let us write the formulas for different types of shapes in a tabular form.

Shape	Area	Terms	
Circle	$\pi \times r^2$	r = radius of the circle	
Triangle	$\frac{1}{2} \times b \times h$	b = base h = height	
Square	a^2	a = length of side	
Rectangle	$l \times w$	l = length w = width	
Parallelogram	$b \times h$	b=base h=vertical height	
Trapezium	$\frac{1}{2} (a + b) \times h$	a and b are the length of parallel sides h = height	

The perimeter formula can be defined as the sum of the length of all the sides of any geometric shape. Various shapes in geometry have a perimeter formula depending on the shape and size.

Perimeter formulae:

Name	Figure	Perimeter	Name	Figure	Perimeter
Rectangle		$2(a + b)$	Isosceles Right Triangle		$2a + d$
Square		$4a$	Parallelogram		$2(a + b)$
Triangle		$a + b + c$	Rhombus		$4a$
Right Triangle		$b + h + d$	Trapezium		$a + b + c + d$
Equilateral Triangle		$3a$	Circle		$2\pi r$

1• Revise the volume and surface areas of right-prisms and cylinders:

A right prism: Is a polyhedron with two identical and parallel faces. These are the bases of the prism. The other faces between the bases are rectangles, and are perpendicular to the bases.

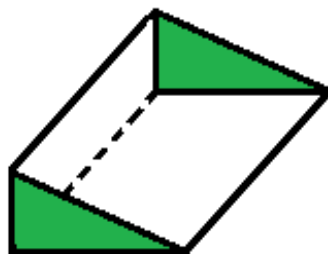
A right cylinder: Is a 3D object with two circular bases. The bases are parallel to each other and there is a curved surface between them.

Right Prism and right cylinder:

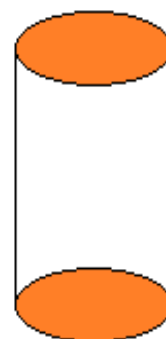
- Always have a pair of congruent bases which are parallel to each other.
- The bases are perpendicular to the lateral surfaces.
- The base of a right prism a polygon and the base of a right cylinder is a circle.



rectangular prism



triangular prism



right cylinder

In the figures: the shaded surfaces are bases.

The dimensions of the right prism are its length, breadth and height.

The dimensions of the right cylinder are its radius and height.

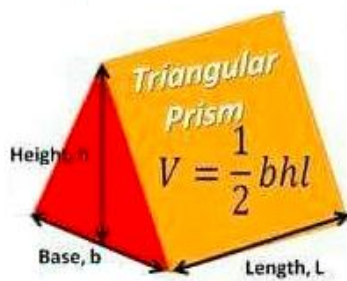
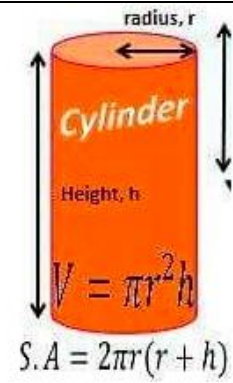
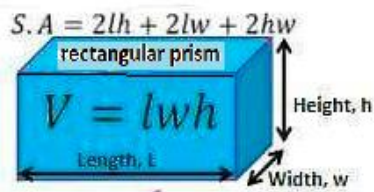
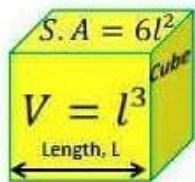
Surface area and volume of a right prism and a cylinder:

The **surface area** is given by the following formula:

$$S = 2(\text{area of base}) + \text{perimeter of base} \times \text{height of prism (cylinder)}.$$

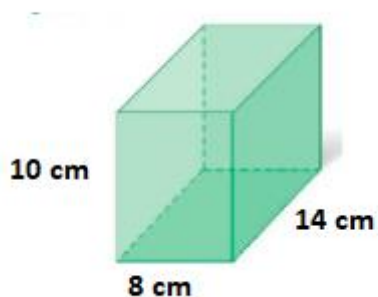
The **volume** of a right prism and a cylinder is given by the formula:

$$V = \text{area of base} \times \text{height of prism (cylinder)}$$



Worked Example: 1

Find the area and volume of the prism:



Answers:

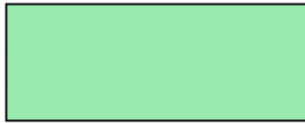
Area of prism:

$S = 2(\text{area of base}) + \text{perimeter of base} \times \text{height}$

$S = 2B + Ph$ use the formula

Determine perimeter and area of the base:

8 cm



14 cm

$P = 2(8) + 2(14) = 44$; perimeter of base.

$B = (8)(14) = 112$; area of base.

Substitute P, B and $h = 10$ into the surface area formula:

$$S = 2(112) + (44)(10)$$

$$S = 224 + 440$$

$$S = 664$$

The surface area is 664 cm^2

Volume of prism:

$V = \text{area of base} \times \text{height of prism}$

$$V = (8)(14)(10)$$

$$V = 1120$$

The volume is 1120 cm^3 .

2-The effect on volume and surface areas when multiplying any dimension by a constant factor k.

Prism:

- When we multiply any one dimension of a prism by a factor, k, the volume of the prism will change by the same factor.
- When we multiply any two dimensions of a prism by a factor, k, the volume of the prism will change by a factor of k^2 .
- When we multiply three dimension of a prism by a factor, k, the surface area of the prism changes by a factor of k^2 and the volume of the prism change by a factor k^3 .

The effects of changes to dimensions of a right prism

Right prism	Multiply by factor k		
	1 dimension	2 dimensions	3 dimensions
Change in volume(v)	$V \times k$	$V \times k^2$	$V \times k^3$
Change in surface area(s)	—	—	$S \times k^2$

Cylinder:

- The surface area of a cylinder changes by a factor of k^2 , when the radius and the height of cylinder changes are both multiplied by factor k .
- The volume of a cylinder changes by a factor of k when we multiply the height of cylinder by factor k .
- The volume of a cylinder changes by factor k^2 , when we multiply the radius (or diameter) by factor k .
- The volume of the cylinder changes by a factor of k^3 , when we multiply both the height and radius (or diameter) by factor k .

The effects of changes to dimensions of a right cylinder

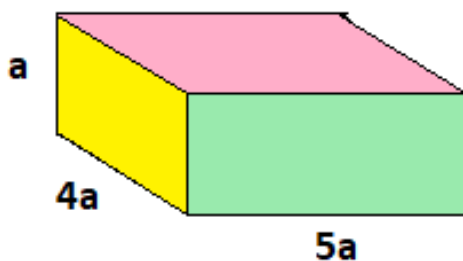
Right cylinder	Multiply by factor k		
	Height	Radius	Height
Change in volume(v)	$V \times k$	$V \times k^2$	$V \times k^3$
Change in surface area(s)	–	–	$S \times k^2$

Worked Example: 2

2- Given the right prism:

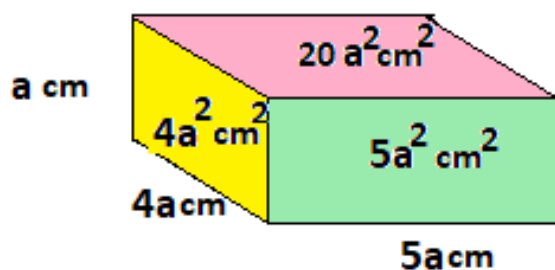
2.1 Determine the surface area.

2.2 Determine the volume.



Answers:

2.1



$$\text{Surface area} = 2(4a^2) + 2(5a^2) + 2(20a^2)$$

$$\text{Surface area} = 8a^2 + 10a^2 + 40a^2$$

$$\text{Surface area} = 58a^2 \text{ cm}^2$$

2.2

$V = \text{area of base} \times \text{height of prism}$

$$V = (4a)(5a)(a)$$

$$V = 20a^3 \text{ cm}^3$$

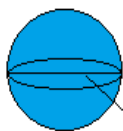
3- Calculate volume and surface areas of spheres, right prisms, right cones and combination of those objects (figures).

3.1 Spheres and hemispheres:

Key words:

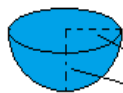
hemisphere – half a sphere

Sphere

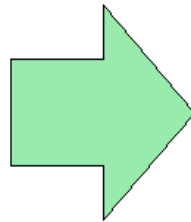


diameter = $2r$

hemisphere



(r) radius = $\frac{\text{diameter}}{2}$



- Volume of sphere = $\frac{4}{3} \pi r^3$

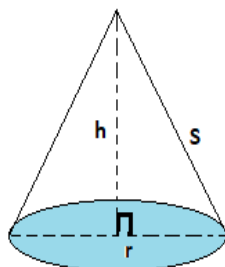
- Total surface area of sphere = $4\pi r^2$

- Volume of hemisphere = $\frac{2}{3} \pi r^3$

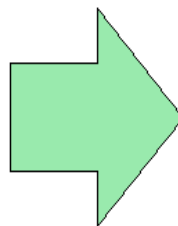
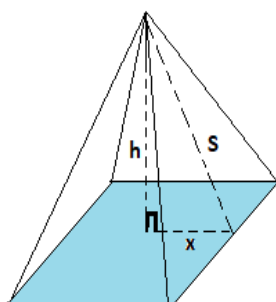
3.2 Right pyramids and right cones:

- Only one base.
- The perpendicular height measures the distance between the vertex of the pyramid or cone and the center of its.
- The slant height runs from the side of the base to the top of the pyramid or cone and is used to determine surface area.
- The base of right pyramid is a polygon; the base of right cone is a circle.

right cone

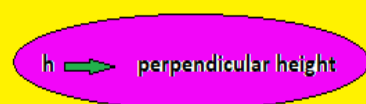


right pyramid



Volume of right cone = $\frac{1}{3} \pi r^2 h$

Area of right cone = $\pi r^2 + \pi r s$



Area of pyramid = area of base + area of lateral surface

Volume of pyramid = $\frac{1}{3}$ area of base X h

Pythagoras' Theorem is used in right cones and right pyramids:

Right Pyramid:

$$s^2 = x^2 + h^2$$

Right Cylinder:

$$s^2 = r^2 + h^2$$

3- Combined objects:

Combined of objects are objects that consist of a combination of two or more simple objects.

The volume of the object is only the sum or differences of the volume of the different 3D objects.

The surface area of the combined object, however is not that straightforward; the areas of the figures where the objects meet must either be neglected and/ or subtracted from the areas of the individual objects.

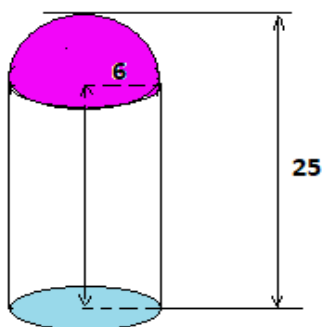
Worked Example: 3

Consider the cylindrical flask in the figure with its hemispherical top. The radius of the cylinder is 6 cm and the height

of the flask is 25 cm.

3.1 Determine the volume of the flask in cm^3 . Give answers in to two decimal places.

3.2 Determine the total surface area of the flask.



Answers:

3.1

The height of the cylinder without the hemispherical top is 19 cm.

$$V = \pi r^2 h + \frac{2}{3} \pi r^3$$

$$V = \pi (6)^2 (19) + \frac{2}{3} \pi (6)^3$$

$$V = 694 \pi + 144 \pi$$

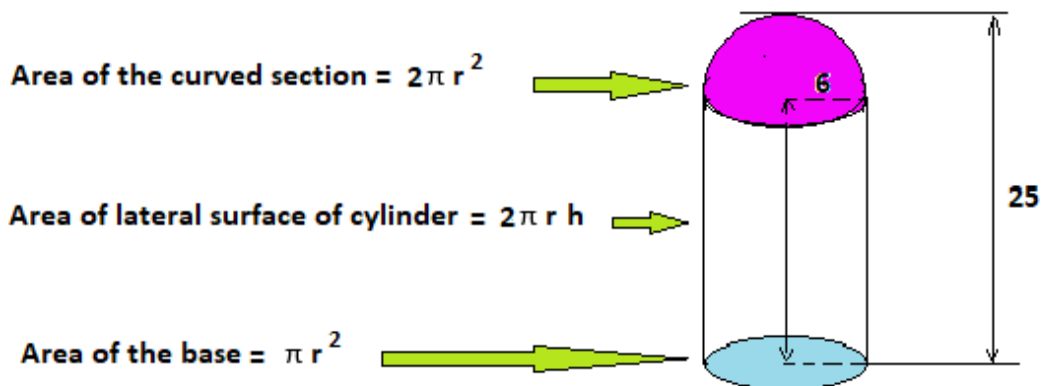
$$1 \text{ cm}^3 = 0,001 \text{ litres}$$

$$V = 2601,24 \text{ cm}^3 = 2,60 \text{ L}$$

The volume of the flask is $2601,24 \text{ cm}^3 = 2,60 \text{ L}$

3.2

total surface area: TSA



$$\text{TSA} = \pi r^2 + 2\pi r h + 2\pi r^2$$

$$\text{TSA} = 2\pi r h + 3\pi r^2$$

$$\text{TSA} = (3)\pi (36) + 2\pi (6) (19)$$

$$\text{TSA} = \pi r^2 + 2\pi r h + 2\pi r^2$$

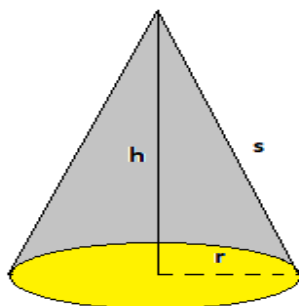
$$\text{TSA} = 336 \pi$$

$$\text{TSA} = 1055,58 \text{ cm}^2$$

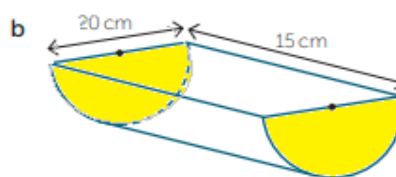
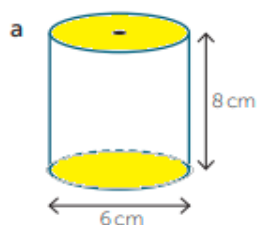
The total surface area of the flask is $1055,58 \text{ cm}^2$

LEARNERS' ACTIVITY

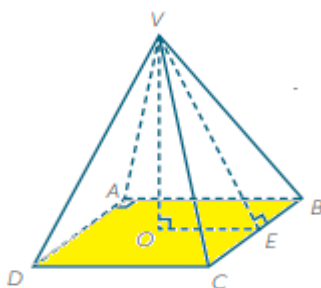
1- Calculate the volume and the total surface area of a cone of radius 6.6cm and height of 12.5cm.



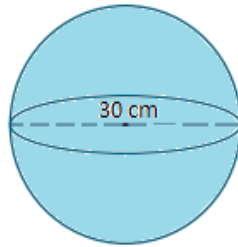
2- Calculate the surface area of each solid, correct to 2 decimal places



3- VABCD is a square-based pyramid with vertex V and A B E D C O V base ABCD, with V vertically above the center of the square base. The height of the pyramid is 4 cm and the side length of the base is 6 cm, find the surface area of the pyramid.



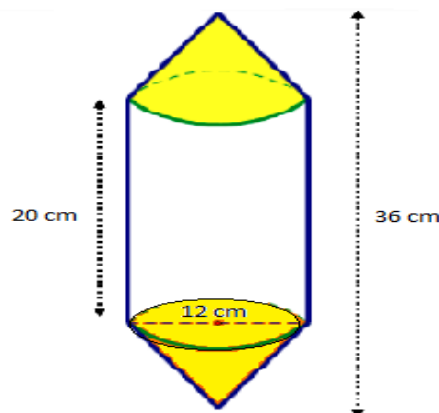
4- Calculate the volume of a sphere with a diameter of 30m.



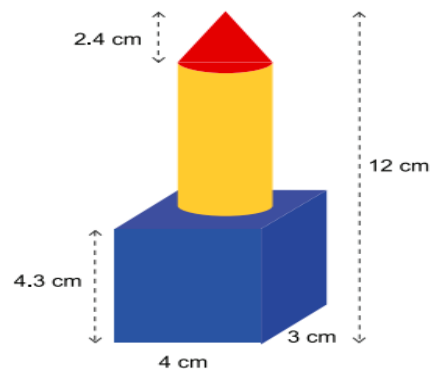
5- A sphere has volume 2800 cm^3 . Find the radius of the sphere, correct to the nearest millimetre.

6- A solid metal object is formed by placing two identical cones at each end of a cylinder. The total length of the object is 36 cm, the cylinder is 20 cm long and has a diameter of 12 cm.

6.1 Calculate the volume of the solid.



7- A Child's wooden toy is made from a cuboid, cylinder and cone. The cone fits perfectly on top of the cylinder. The radius of the cone is 1,1 cm. Calculate the volume of wood needed to make the toy. Give your answers correct to 2 decimal places.



WEEK 6 - 8: TOPIC:STATISTICS

CONTENT::
Measure of central

NOTES & EXAM TIPS

MEASURE OF CENTRAL TENDENCY

UGROUPED DATA

MEAN

$$\text{Mean} = \frac{\text{sum of all values}}{\text{total number of values}}$$

$$\bar{x} = \frac{\sum x_i}{n}$$

$$\bar{x} = \text{mean}$$

$$\sum x_i = \text{sum of all values}$$

$$n = \text{number of values}$$

MODE

The value that appears most frequently in a set of data points

Bimodal: a data set with two modes

Trimodal: a data set with three modes

MEDIAN

The median is the middle number in a set of data set .

$$\text{Position of Median} = \frac{1}{2}(n + 1)$$

$$n = \text{number of values}$$

If n =odd number , median is part of the data set

If n =even number, the median will be the average between the two middle numbers.

MEASURE OF DISPERSION

Range

Range = max value – min value

NB: Range is greatly influenced by outliers

Interquartile Range(IQR)

$$\text{IQR} = Q_3 - Q_1$$

Semi Interquartile range

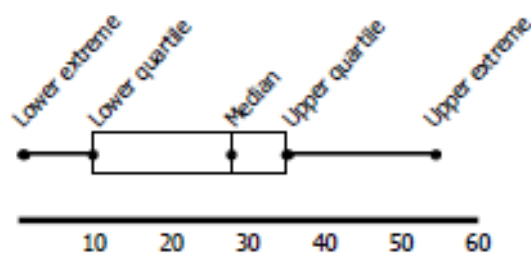
$$\text{Semi - IQR} = \frac{1}{2} (Q_3 - Q_1)$$

FIVE NUMBER SUMMARY

1. Minimum Value
2. Lower Quartile Q_1
3. Median (Q_2)
4. Upper Quartile Q_3
5. Maximum Value

BOX AND WHISKER PLOT

A box and whisker plot is a visual representation of a five number summary



REPRESENTING DATA

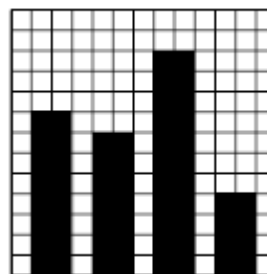
FREQUENCY TABLE

Mark	Tally	Frequency
4		2
5		2
6		4
7		5
8		4
9		2
10		1

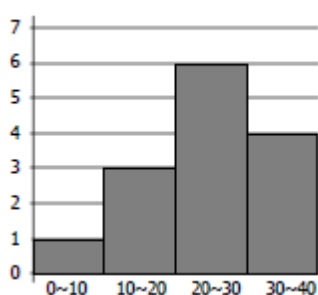
STEM AND LEAF PLOTS

Stem	Leaf
0	1, 1, 2, 2, 3, 4
1	0, 0, 0, 1, 1, 1
2	5, 5, 7, 7, 8, 8
3	0, 1, 1, 1, 2, 2
4	0, 4, 8, 9
5	2, 6, 7, 7, 8
6	3, 6

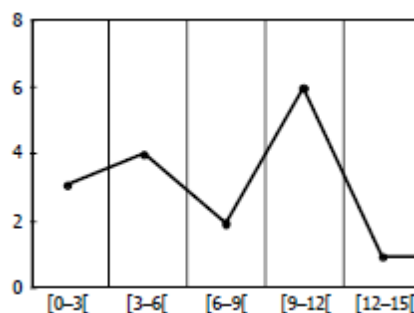
BAR GRAPH



HISTOGRAM



FREQUENCY POLYGON



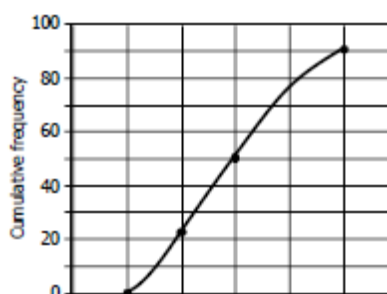
OUTLIERS

Any data that is more than 1,5 IQR to the left of Q_1 or the right of Q_3

Outlier $< Q_1 - (1,5 \times IQR)$ or

Outlier $> Q_3 + (1,5 \times IQR)$

OGIVES



TIPS AND FOCUS POINTS TO CONSIDAR

1. When presented with ungrouped data it is advisable that the data must be organised by arranging the set of data in ascending order before attempting to answer the questions.
2. Data can be provided in many different forms such as a frequency table or a bar graph. When making use of a bar graph it is important that both axis must be labelled with the frequency on the vertical axis. Each bar represents a group of data and can then be compared to each other.
3. Further be aware of the difference between a Bar Graph and a Histogram. A bar graph has gaps between the different bars where a histogram the different bars touch each other. Histograms are used to represent grouped and continuous data. Continuous data is data measured in term of real numbers.
4. Grouped data can be represented on a histogram or a frequency polygon.
5. Mean = $\frac{\sum fx}{n}$
Median is the middle score of an ordered set of data.
Mode is the most frequent score.
6. Range = largest value of data set – smallest value of the data set.
7. The data is divided into four parts. Firstly, the data is divided into 2 halves to get the median (Q_2). The lower halve and upper halve now get divided into to get the lower quartile (Q_1) and the upper quartile (Q_3).
8. The Interquartile range = $Q_3 - Q_1$.
9. The box and whisker plot show whether a data set is symmetrical, positively skewed or negatively skewed.
10. When we plot a frequency polygon we use the midpoints of each interval and join the midpoints with straight lines. Then we add an interval at the beginning and end of the data, with both having a frequency of 0.
11. A cumulative frequency graph or also known as a ogive always start on the x-axis and forms an S-shape curve. The end of each interval is plotted against the

Revision Exercises: (Grade 10)

QUESTION 1.

The marks obtained for a Mathematics Test for the learners in class 11A are captured in the table.

55	34	28	55	72	30	3	68	44	29
37	63	71	43	87	55	26	53	59	51

- 1.1 Find the mean of the set of data in the table above.
- 1.2 Calculate the median of the set of data in the table above.
- 1.3 Find the mode of the set of data in the table above.
- 1.4 Determine the lower quartile (Q1) of the above data.
- 1.5 Determine the upper quartile (Q3) of the above data.
- 1.6 Determine the interquartile range.
- 1.7 Draw a box and whisker graph to represent the data in the table above.
- 1.8 What observation can you make about the spread of the data using the box and whisker graph in question 1.7
- 1.9 Can you possibly identify an outlier and provide a reason for your answer?
- 1.10 Calculate the standard deviation of this data set.

QUESTION 2.

In the table below the time to complete a challenge in seconds were recorded for 100 participants. The data was then grouped in the intervals as indicated. The number of participants able to complete within that interval was counted and recorded.

Time(t) in seconds(s)	Count (Frequency)
$60 < t \leq 65$	12
$65 < t \leq 70$	13
$70 < t \leq 75$	17
$75 < t \leq 80$	20
$80 < t \leq 85$	29
$85 < t \leq 90$	7
$90 < t \leq 95$	2

- 2.1 Find the estimated mean of this grouped data.
- 2.2 Determine the modal group.
- 2.3 Determine the median group.

Exercises: (Grade 11)

3. The following 16 scores were recorded by a player A when playing darts during a regional competition. The scores are recorded in the table below.

19	27	22	17	13	30	26	18
22	10	21	32	23	20	13	22

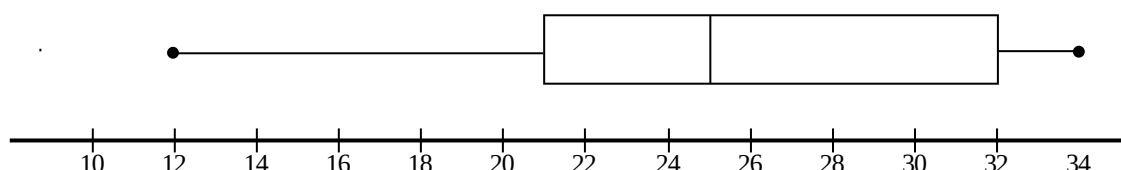
3.1 What is the median for the above-mentioned data.

3.2 Write down the upper and lower quartiles. (

3.3 Determine the standard deviation of the scores.

3.4 Draw a box and whisker diagram of the scores recorded in the table above for player A.

- 3.5 Another player B, playing in the same competition had his 15 scores recorded. The five number summary of his scores is **(12 ; 21 ; 25 ; 32 ; 34)**. This box and whisker diagram is showed below. How does player A and player B compare regarding their performance? Give reasons for your decision. (Player B is better, has a higher average score.)



4. Fifteen learners participated in a Mathematics Olympiad. The results, as a percentage, for each learner are shown below:

27	28	30	32	34	38	41	42	43	43	44	46	53	56	62
----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

4.1 Determine the median of the given data.

4.2 Determine the interquartile range for the data.

4.3 Draw a box and whisker diagram to represent the data.

4.3.1 By referring to the box and whisker diagram, comment on the skewness of the learners in this Olympiad. (Skewed to the left or negatively skewed)

5. Masibambane School learners were surveyed to find out how many times they went to the movies during the last year. The data was collected and recorded in the following frequency table:

Number of visits to the movies in a year	Frequency	Cumulative Frequency
0 - 5	8	8
6 - 10	10	18
11- 15	6	24
16 - 20	6	B
A	4	34
26 -30	4	38
31 - 35	2	C
36 - 40	0	40

- 5.1 Complete the table above by giving the values for A, B and C.
- 5.2 Draw the ogive of the data given in the table.
- 5.3 Estimate the number of visits to the movies made by the top 50% of the learners last year.
6. A group of Grade 11 learners were interviewed about the number of whatsapp messages(M) received and sent per day. The total number of messages received and sent was counted from 00h00 to 24h00. This information was captured in the table below.

Interval	Frequency	Cumulative Frequency
$0 < M \leq 20$	5	
$20 < M \leq 40$	15	
$40 < M \leq 60$	35	
$60 < M \leq 80$	50	
$80 < M \leq 100$	35	
$100 < M \leq 120$	10	

6.1 Complete the cumulative frequency table above.

6.2 Draw the Histogram of the info

above with the Interval on the horizontal axis and the Frequency on the vertical axis.

- 6.3 Draw an ogive (cumulative frequency curve) to represent the data.
- 6.4 Estimate the percentage of the learners who sent more than 50 messages in one day. (
- 6.5 In which direction is the data skewed?



SUB-TOPIC:

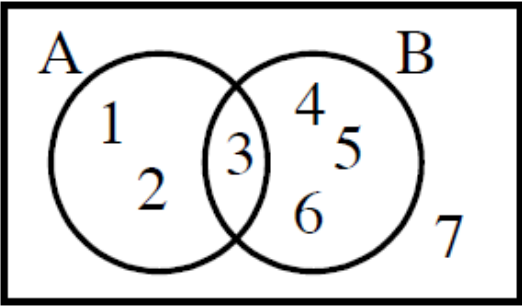
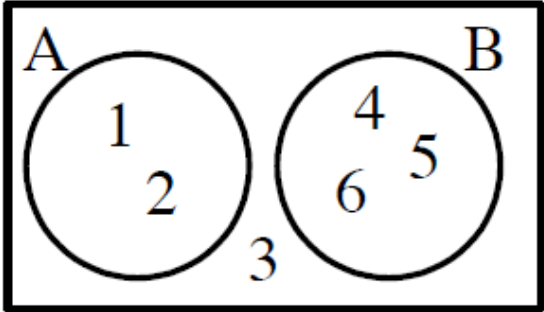
- Identities
- Reduction Formula
- Determine for which values of a variable an identity holds.

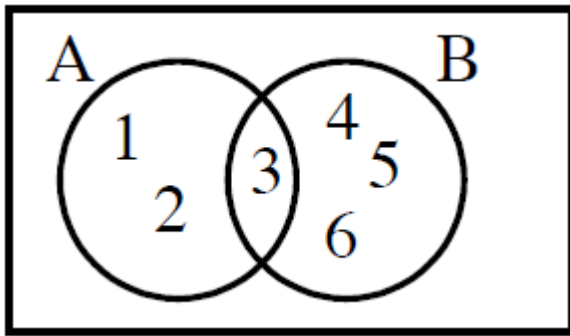
NOTES & EXAM TIPS

Terminology	Explanation	Formulae
1. Identity	Mutually Inclusive events A and B	• $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ • $P(A \text{ and } B) \neq 0$
2. Product Rule	Independent events A and B	• $P(A \text{ and } B) = P(A) \times P(B)$ • $P(A \text{ or } B) = P(A) + P(B) - P(A) \times P(B)$
3. Mutually exclusive [Addition/Sum Rule]	No intersection	• $P(A \text{ or } B) = P(A) + P(B)$ • $P(A \text{ and } B) = 0$
4. Exhaustive event	Mutual inclusive and exclusive events can be exhaustive, they contain all elements of the sample space.	• $P(A \text{ or } B) = 1$
5. Complementary events	Mutually exclusive, exhaustive events are complementary events.	• $P(A) + P(B) = 1$ • $P(A) + P(\text{Not } A) = 1$ • $P(A) = 1 - P(A')$

VENN DIAGRAMS

NOTES:

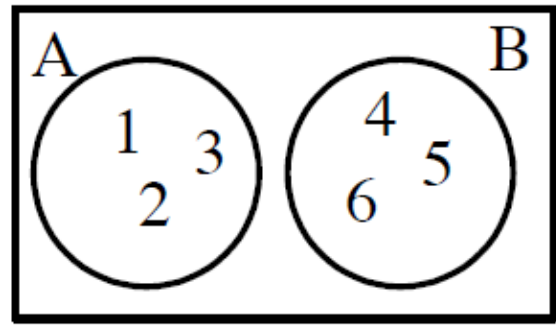
GIVEN: SAMPLE SPACE = {1 ; 2 ; 3 ; 4 ; 5 ; 6 ; 7}	
1. ADDITION RULE: INCLUSIVE EVENTS A and B $A = \{1 ; 2 ; 3 \}$ $B = \{3 ; 4 ; 5 ; 6 \}$ {3 } is common in BOTH sets A and B, this can be written mathematically as: $n(A \text{ and } B) = 3$ while {7 } is not in either sets. $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ 	2. MUTUALLY EXCLUSIVE EVENTS $A = \{1 ; 2 \}$ $B = \{4 ; 5 ; 6 \}$ $P(A \text{ and } B) = 0$ there is no common element in BOTH sets $n(A \text{ or } B)$ is the union/combination of the TWO events $n(A \text{ or } B) = n(A) + n(B) = 5$ $P(A \text{ or } B) = P(A) + P(B)$ 
3. INDEPENDENT EVENTS: PRODUCT RULE $P(A \text{ and } B) = P(A) \times P(B)$	
4. EXHAUSTIVE EVENTS $A = \{1 ; 2 ; 3 \}$ $B = \{3 ; 4 ; 5 ; 6 \}$ *$P(A \text{ or } B) = 1$ *Mutually exclusive events can be exhaustive	*Inclusive events can also be Exhaustive



$$P(A) = \frac{3}{6}; P(B) = \frac{4}{6}; P(A \text{ and } B) = \frac{1}{6};$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= \frac{3}{6} + \frac{4}{6} - \frac{1}{6} = 1$$



$$P(A) = \frac{3}{6}; P(B) = \frac{3}{6}; P(A \text{ and } B) = \frac{0}{6};$$

$$; P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

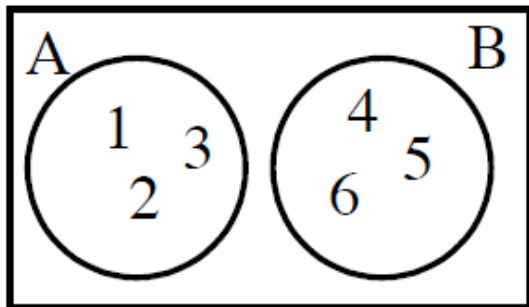
$$= \frac{3}{6} + \frac{3}{6} - \frac{0}{6} = 1$$

5. COMPLEMENTARY EVENTS:

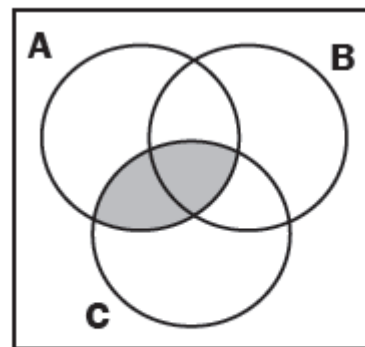
(Are Mutually exclusive, exhaustive events are Complementary)

$$*P(A) + P(B) = 1$$

$$*P(A') = 1 - P(A)$$

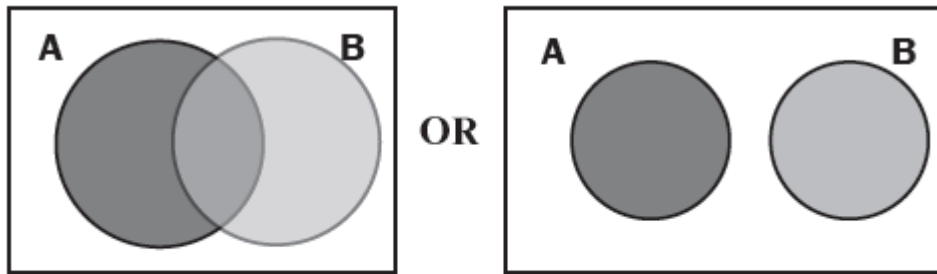


6.



The shaded area represents:
(A and C) or $(A \cap C)$

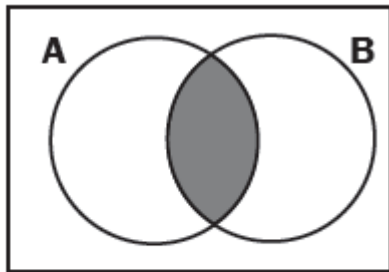
7.



The shaded areas represent:

(A or B) or $(A \cup B)$

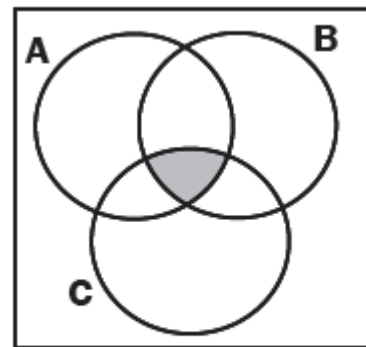
8.



The shaded area represents:

(A and B) or $(A \cap B)$

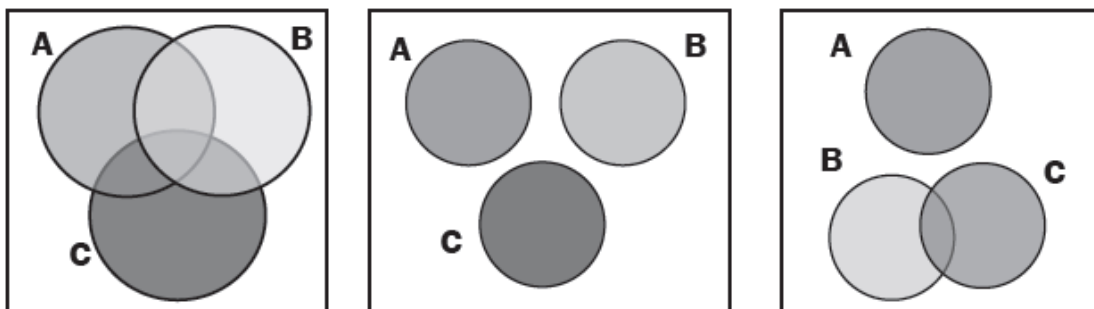
9.



The shaded area represents:

(A and B and C) or $(A \cap B \cap C)$

10.



The shaded areas represent:

(A or B or C) or $(A \cup B \cup C)$

TREE DIAGRAMS

Tree diagrams are used to illustrate combinations of events. Each branch shows a possible outcome and each branch shows the probability of the event. The sum of probabilities of all outcomes is always equal to 1.

EXAMPLES			
1.	A bag contains 3 green beads and 4 white beads. Two beads are taken from the bag at random, one after the other, replacing the first bead before the second draw.		
1.1	Draw a tree diagram to illustrate all the possible outcomes.		(4)
1.2	Determine probability that:		
	1.2.1 both are green.		(2)
	1.2.2 one is green and other white.		(2)
1.1	1st draw - 2nd draw - Possible outcomes - Probabilities (4)		
1.2.1	$P(\text{G and G}) = \frac{9}{49}$		(2)
1.2.2	$P(\text{G and W}) = \frac{12}{49} + \frac{12}{49} = \frac{24}{49}$		(2)

TWO WAY CONTINGENCY TABLE

Another way of illustrating combinations of TWO events is in terms of rows and columns.

EXAMPLES				
1.	In a survey amongst 300 Grade 11 learners, the following results were obtained to determine how many like fruit juice.			
		Like fruit juice	Do not like fruit juice	Totals
	Boys	100	A	190
	Girls	80	30	C
	Totals	B	D	300
	Determine:			
	1.1	The values of A, B, C and D.		(4)
	1.2	Whether the events Liking fruit juice and being a girl are independent events.		(4)
SOLUTIONS				
1.1	A = 90 B = 180 C = 110 D = 120			(4)
1.2	P(Liking fruit and being a girl) = $\frac{80}{300} = \frac{4}{15} = 0,2666666 \dots$ P(Liking fruit) x P(being a girl) = $\frac{180}{300} \times \frac{110}{300} = \frac{11}{50} = 0,22$ P(Liking fruit and being a girl) $\neq$ P(Liking fruit) x P(being a girl) $\therefore$ The events are not independent			(4)

ACTIVITIES

1. Given: $P(A) = 0,25$; $P(B) = 0,5$ and $P(A \text{ or } B) = 0,625$

1.1 Calculate $P(A \text{ and } B)$ (2)

1.2 Determine, giving reasons, if events A and B are

1.2.1 mutually exclusive (1)

1.2.2 inclusive (1)

1.2.3 complementary (2)

1.2.4 independent (3)

ACTIVITIES

1. $P(A) = 0,2$

$$P(B) = 0,5$$

$$P(A \text{ or } B) = 0,6$$

1.1 Calculate $P(A \text{ and } B)$. (2)

1.2 Are the events A and B independent? Show your calculations. (3)

2. For any two events A and B, its is given that:

$$P(A \text{ and } B) = 0,12$$

$$P(A \text{ or } B) = 0,83 \text{ and}$$

$$P(B) = 4P(A)$$

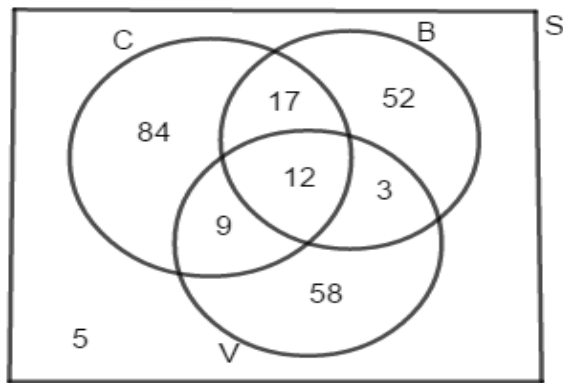
2.1 Are events A and B mutually exclusive ?Justify your answer. (2)

2.2 Calculate $P(B)$ (4)

- 2.3 Calculate $P(\text{not } A)$ (2)
3. Given: A and B are mutually exclusive events. $P(A) = 0,35$ and $P(B) = 0,52$
- Determine:
- 3.1 $P(A')$ (2)
- 3.2 $P(A \text{ and } B)$ (2)
- 3.3 $P(A \text{ or } B)$ (2)
4. Given: $P(A) = 0,6$; $P(B) = 0,3$ and $P(A \text{ or } B) = 0,8$ where A and B are two different events.
- Are the events A and B mutually exclusive? Justify with appropriate calculations and/or a diagram?
- (4)
5. Given: $P(A) = 0,12$; $P(B) = 0,35$ and $P(A \text{ and } B) = 0,428$.
- Determine whether events A and B are independent or not. Show ALL relevant calculations.
- (4)
6. Given: $P(W) = 0,4$
- $P(T) = 0,35$
- $P(T \text{ and } W) = 0,14$
- 6.1 Are the events W and T mutually exclusive? Give reasons for your answer. (2)
- 6.2 Are the events W and T independent? Give reasons for your answer. (2)

ACTIVITIES

1. A survey was carried out with 240 customers who bought food from a fastfood outlet on a particular day. The outlet sells cheese burgers (C), bacon (B) and vegetarian burgers (V). The Venn diagram below shows the number of customers who bought different types of burgers on the day.



- 1.1 How many customers did NOT buy burgers on the day? (1)
- 1.2 Are events B and C mutually exclusive? Give a reason for your answer. (2)
- 1.3 If a customer from this group is selected at random, determine the probability that he/she:
 - 1.3.1 Bought only a vegetable burger (1)
 - 1.3.2 Bought a cheese burger and a bacon burger (1)
 - 1.3.3 Did not buy a cheese burger (3)
 - 1.3.4 Bought a bacon or a vegetable burger (4)
2. A survey was conducted amongst 100 learners at a school to establish their involvement in three codes of sport, soccer, netball and volleyball. The results are shown below.
 - 55 learners play soccer (S)
 - 21 learners play netball (N)
 - 7 learners play volleyball (V)
 - 3 learners play netball only

- 2 learners play soccer and volleyball
- 1 learner plays ALL 3 sports

2.1 Draw a Venn diagram to represent the above information. (5)

2.2 What is the probability that one of the learners chosen at random from the group

2.2.1 plays netball or volleyball? (2)

2.2.2 Plays atleast two sporting codes. (2)

2.2.3 Plays only two sporting codes. (2)

3.	A survey was conducted among 200 learners about their preferences for chicken, mutton and beef. Below are the findings: • 90 prefer chicken • 64 prefer mutton • 77 prefer beef • 8 prefer all three types • 18 prefer mutton and beef • 27 prefer chicken and beef • 26 do not eat any meat at all • x number of learners prefer chicken and mutton but not beef		
	3.1	Draw a Venn diagram for this survey.	(6)
	3.2	How many students like only chicken?	(4)
	3.3	What is the probability that, if a student is randomly selected, he/she would like only two types of meat?	(3)

ACTIVITIES

1. A group of 33 learners was surveyed at a school. The following information from the survey is given:

- 2 learners play tennis, hockey and netball
- 5 learners play hockey and netball
- 7 learners play hockey and tennis
- 6 learners play tennis and netball
- A total of 18 learners play hockey
- A total of 12 learners play tennis
- 4 learners play netball ONLY

1.1 Draw the Venn diagram to represent the above information (5)

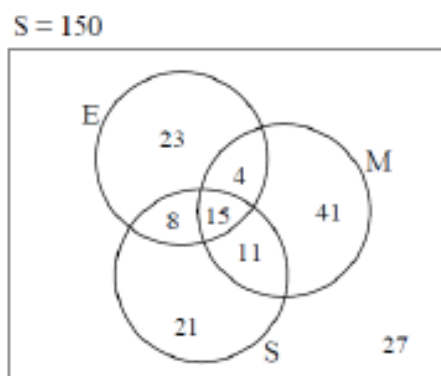
1.2 Determine the probability that a learner selected at random from this sample

1.2.1 plays netball (1)

1.2.2 plays hockey or netball (1)

1.2.3 does not play hockey (1)

2. A group of 150 students decided to go to a shopping mall on a Saturday. They had to decide whether they were going to Eat Out (E), go to the Movies (M) or just go Shopping (S). The information gathered is shown on the Venn-diagram below.



- 2.1 Calculate the probability that a student that is chosen randomly will not Eat Out nor go to the Movies. (2)
- 2.2 Determine if the events, Eat Out and go to the Movies are independent. Show all your calculation to support your answer. (4)
3. A survey of 80 students at a local library indicated the reading preferences below:
- 44 read the English magazines
 - 33 read the Afrikaans magazines
 - 39 read Sesotho magazines
 - 23 read both English and Sesotho magazines
 - 19 read both Afrikaans and Sesotho magazines
 - 9 read all three magazines
 - 69 read at least one magazine
- 3.1 How many students did not read any magazines (1)
- 3.2 Let the number of students who read English and Afrikaans but not Sesotho be represented by x . Draw a Venn diagram to represent reading preferences. (5)
- 3.3 Hence, show that $x = 5$. (3)
- 3.4 What is the probability, correct to THREE decimal places, that a student selected at random will read at least two of the three magazines? (3)

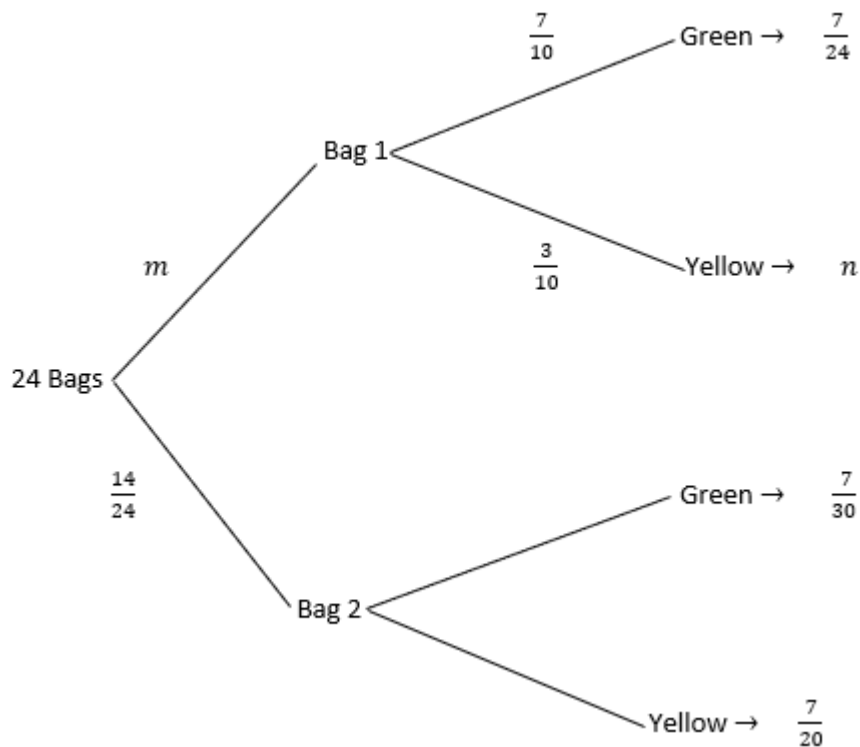
ACTIVITIES

1. A bag contains 3 green beads and 4 white beads. Two beads are taken from the bag at random, one after the other, **without replacing** the first bead before the second draw. (4)
- Draw a tree diagram to show all the possible outcomes.
2. A bag contains 3 red and 2 white balls. A second bag contains 5 red and 4 white balls. A bag is chosen at random and a ball is taken from the bag. (4)
- Draw the tree diagram to show all possible outcomes.

3. The probability of rain in Port Louis, the capital of Mauritius, is 95% in February. The probability of a festival in dry weather is 32% during February, and one-and-half times more likely when it is wet.
 - 3.1 Draw a tree diagram to illustrate all possible events and their possibilities. (4)
 - 3.2 What is the probability that:
 - 3.2.1 there will be a festival? (2)
 - 3.2.2 there will not be a festival in wet weather? (2)

ACTIVITIES

1. A car manufacturer offers two types of transmission in its entry-level model, automatic (AT) or manual (MT). Buyers can choose between petrol (P), diesel (D) or hybrid (H) engines, and the car comes in red (R), silver (S) or black (B).
 - 1.1 Draw a tree diagram to show all the possible options.
 - 1.2 What is the probability that a car drawn at random is:
 - 1.2.1 black.
 - 1.2.2 an automatic with a hybrid engine.
 - 1.2.3 a manual with a diesel engine and red
 - 1.2.4 a petrol engine with silver paint
2. There are 24 bags of marbles for sale in a shop. In ten of the bags there are 7 green marbles and three yellow marbles. The other bags each have x green marbles and 9 yellow marbles. A bag is chosen at random and a marble is then chosen at random from the bag. The tree diagram below illustrates the process and the outcomes.



- 2.1 Determine the values of m and n . (2)
- 2.2 Determine the value of x . (3)
- 2.3 What is the probability that a marble chosen is green (2)
3. Tshidi has a bag containing 80 marbles that are either green, yellow and red in colour. $\frac{3}{5}$ of the marbles are green and 10% of the marbles are yellow. Tshidi picks TWO marbles out of the bag, one at a time, **replacing** the first one
 - 3.1 How many red marbles are in the bag? (2)
 - 3.2 Draw a tree diagram to represent the above situation. (3)
 - 3.3 What is the probability that Tshidi will choose a GREEN and a YELLOW marble. (3)
4. A bag contains 6 red balls, 8 green balls and an unknown of yellow balls. The probability of randomly choosing a green ball from the bag is 25%
 - 4.1 Show that there are 32 balls in the bag.

- 4.2 A ball is drawn from the bag, the colour is recorded and it is not returned to the bag. Thereafter another ball is drawn from the bag, the colour is recorded and it is also not returned to the bag.
5. The probability that the first answer in a maths quiz competition will be correct is 0,4. If the first answer is correct, the probability of getting the next answer correct rises to 0,5. However, if the first answer is wrong, the probability of getting the next answer correct is only 0,3.
- 5.1 Represent the information on a tree diagram. Show the probabilities associated with each branch as well as the possible outcomes. (3)
- 5.2 Calculate the probability of getting the second answer correct. (3)

ACTIVITY

1. A group of elderly people were asked if they utilise the disabled parking bays at a shopping mall. Of the 400 respondents, 35 were disabled and 365 not disabled. The results of the questionnaire are tabled below:

	Disabled	Not disabled	Total
Used disabled parking bays	12	305	317
Did not use disabled parking bays	23	60	83
Total	35	365	400

Calculate the probability that a person drawn at random from the sample (round off your answers to TWO decimal places):

- 1.1 Is disabled (2)
- 1.2 Is a disabled person who uses the disabled parking bays (2)
- 1.3 Is not disabled and does not use the disabled parking bays (2)
- 1.4 Uses the disabled parking bays and is not disabled (2)

ACTIVITIES

1. A survey was conducted on 4 895 people to determine whether drinking more than five cups of coffee per day was dependent on gender. Their responses are given on the table below.

	More than 5 cups	Less than 5 cups	Total
Female	A	1 565	B
Male	1 485	C	2 658
Total	2 157	D	4 895

- 1.1 Calculate the missing values. (4)
- 1.2 What is the probability that:
 - 1.2.1 a female drinks more than five cups of coffee per day? (2)
 - 1.2.2 a person will be a female? (2)
- 1.3 Determine whether drinking more than five cups of coffee per day is dependent on gender. Substantiate your answer with relevant calculations, rounded off to TWO decimal places. (3)
2. 400 drivers were interviewed about the number of accidents they were involved in during a calendar year. The results were summarised in the table below:

Age of the driver	Number of accidents		Total
	2 or fewer	More than 2	
40 years and younger	200	C	300
Older than 40 years	A	50	E
Total	B	D	F

2.1 Calculate the values of A, B, C, D, E, and F. (3)

2.2 Is the number of accidents independent of the driver's age? Support your answer with the appropriate calculations. (4)

3. A survey on their preference of exercises was conducted among 140 people in two age groups. The information is summarised below.

AGE	TENNIS	RUNNING	GYM	TOTAL
35 years and younger	a	28	c	80
Older than 35 years	b	21	d	60
TOTAL	21	49	70	140

3.1 If it is given that preferring to play tennis and age are independent of each other, Determine the value of a . (3)

3.2 If it is given that $a = 12$, determine the probability that a random selected person prefers going to the gym or is in the age 35 years and younger (5)

F.TERM 4

WEEK	TOPIC	CONTENT
WEEK 2	Financial Mathematics	1. Use the simple and compound growth formulae to solve problems, including interest, hire purchase, inflation, population growth and other real-life problems.
WEEK 3		2. Understand the implication of fluctuating foreign exchange rates (e.g. on the petrol price, imports, exports, overseas travel). 3. Use simple and compound decay formulae: $A = (1 - in)$ and $A = (1 - i)^n$ to solve problems (including straight line depreciation and depreciation on a reducing balance).
WEEK 4		4. The effect of different periods of compound growth and decay, including nominal and effective interest rates

WEEK 2 – 4: FINANCIAL MATHEMATICS

1. Use the simple and compound growth formulae to solve problems, including interest, hire purchase, inflation, population growth and other real-life problems.
2. Understand the implication of fluctuating foreign exchange rates (e.g. on the petrol price, imports, exports, overseas travel).
3. Use simple and compound decay formulae:

$$A = (1 - in)$$
 and

$$A = (1 - i)^n$$
to solve problems (including straight line depreciation and depreciation on a reducing balance).
4. The effect of different periods of compound growth and decay, including nominal and effective interest rates

NOTES & EXAM TIPS

TERMINOLOGY

Inflation	The rate at which prices increase over time
Consumer Price Index (CPI)	the average prices of a basket of goods
Exchange rates	The value of one currency for the purpose of conversion to another
Population Growth	Change of population size over time
Hire purchase	Short term loan, deposit payable. Calculated using simple interest.
Reducing Balance	Interest is paid on the reducing balance, the lower the balance, the less you have to pay.
Nominal Interest Rate	Quoted period and compounded period is different eg 15% per annum compounded monthly.
Effective interest Rate	Quoted period and compound period is equal eg 0,75% per month compounded monthly.

The Compound Interest Formula	The Simple Interest Formula
$A = P(1 + i)^n$ A – Accumulated amount. This is the total amount paid after a period of n years. It includes the original amount P plus interest. P – Principal amount. This is the original amount borrowed or invested. n – the period of investment in years. i – interest rate per annum	$A = P(1 + in)$ A – Accumulated amount. This is the total amount paid after a period of n years. It includes the original amount P plus interest. P – Principal amount. This is the original amount borrowed or invested. n – the period of investment in years. i – interest rate per annum

EFFECTIVE AND NOMINAL INTEREST RATES	
$1 + i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m$	• Use the equation to convert from the nominal to effective rate of interest • This equation is not on the information pamphlet. • WHEN TO USE: if the rate quoted is per year, and the payments are in a different periods, then you must find the effective interest rate FIRST, before doing the sum. • WATCH OUT: the rate is 9% per year and payments are made at the end of every quarter. • If it says: the rate is 9% p.a. compounded quarterly and payments are made quarterly then you have nothing to worry about!!!

TIME LINES

- A timeline is a useful way to summarise and represent information in problems concerning investments and loans.
- Useful in financial mathematics when there are several deposits and withdrawals, as well as possible changes in interest rates
- Time line provides a visual summary of all the information in an investment fund

Worked Example

Kamohelo opens a savings account with a deposit of R9 400 and two years later a further R5 800 is added to the savings account. Five years after the savings was opened, another R12 600 is deposited into the account. The interest paid on the savings is 9% p.a. compounded monthly. Calculate the total amount saved at the end of eight years.

Solution:

T₀

T₁

T₂

T₅

T₈

9400

5800

12600

←

$i = \frac{0,09}{12}$

→

156

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- On the time line 1.1 represents the end of the second year, ***** represents the end of the fifth year and ***** represents the end of the eighth year. ***** is the beginning of the first year.
- The timeline gives a visual representation of all the information in the problem.

$$A = 9\,400 \left(1 + \frac{0.09}{12}\right)^{8 \times 12} + 5\,800 \left(1 + \frac{0.09}{12}\right)^{6 \times 12} + 12\,600 \left(1 + \frac{0.09}{12}\right)^{3 \times 12}$$

$$= R45\,681,60$$

NB: Discuss other methods that can be used to arrive at the above solution.

DEPRECIATION

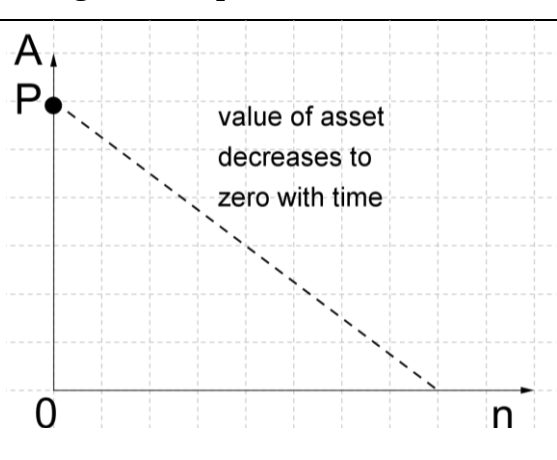
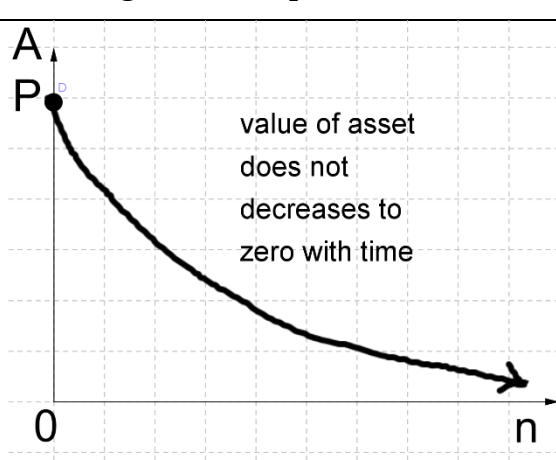
- Depreciation is the loss of value of an asset through age. Vehicles machinery and equipment reduce in value over a period of time and eventually need to be replaced
- There are two types of depreciation. **Straight line depreciation** and **reducing balance depreciation**.

Straight line depreciation:

- Depreciation is the same every year. It is a percentage of the original value of the asset.
- The value of the asset is reduced to zero over a period of time.

Reducing balance depreciation

- Depreciation is based on the previous year's value.
- Every year the depreciation will be a percentage of the reduced value of asset.
- The loss in value becomes less each year as the asset diminishes in value.
- The value of the asset does not reduce to zero over a period of time.

Straight line depreciation	Reducing balance depreciation
 A graph with a vertical axis labeled 'A' and a horizontal axis labeled 'n'. A point 'P' is marked on the vertical axis. A dashed line starts at 'P' and slopes downwards to the horizontal axis, representing a linear decrease. Text inside the graph area says 'value of asset decreases to zero with time'.	 A graph with a vertical axis labeled 'A' and a horizontal axis labeled 'n'. A point 'P' is marked on the vertical axis. A solid curve starts at 'P' and slopes downwards, approaching the horizontal axis asymptotically, representing exponential decay. Text inside the graph area says 'value of asset does not decrease to zero with time'.
Formula: $A = P(1 - in)$	$A = P(1 - i)^n$

EXERCISE

- 1.1 Calculate the effective rate if a sum of money is invested at a nominal rate of 10, 25% p.a. compounded quarterly.
- 1.2 Thato wants to go on holiday in 3 years' time. Her father gives her R25 000 to open a Savings Account. She invests this amount in an account which pays an interest rate of 7,5% p.a. compounded monthly for $1\frac{1}{2}$ years. At the end of the $1\frac{1}{2}$ years she deposits an additional R15 000 into the account. The interest rate for the remainder of the period is 9, 7% p.a. compounded half yearly.
- How much money will Thato have available when she goes on her holiday?
- 1.3 Jojo, a taxi owner, buys a new taxi for R280 000.
- 1.3.1 Calculate the value of the taxi after 4 years if the rate of depreciation is 18% p.a. based on reducing balance
- 1.3.2 Calculate the estimated cost of a taxi in 4 years' time if the expected inflation rate is 6,9% p.a.
- 1.3.3 How much additional money will Jojo need after 4 years if they sell the taxi at book value and use this money towards the purchase of a new taxi.
- 2 Sheena receives R1 500 as a gift. She invests her money in a savings account, earning interest at 15% per annum compounded semi-annually
- 2.1 How much money does Sheena have in her investment account at the end of 5 years?
- 2.2 Disa also receives R1 500, but she invests her money in an account which earns interest annually. If Sheena and Disa have the same amount of money at the end of 5 years, what annual interest rate is Disa earning?
- 3 A company buys new machinery at the beginning of 2013 for R23 000. The machinery depreciates on the reducing balance method at a rate of 13, 5% per annum.
- 3.1 Determine the book value of the machinery at the end of 2017.
- 3.2 Determine the expected cost of purchasing new machinery at the beginning of 2018 if the purchase price at the beginning of 2013 increases at 6,6 % compounded annually.
- 3.3 How much would the company have had to invest as a lump sum at the beginning of 2013, if they wanted to pay cash for the new machinery at the beginning of 2018 and the money is invested in a bank account earning interest of 4,7% p.a., compounded monthly?

4.1 A tractor bought for R120 000 depreciates to R11 090, 41 after 12 years by using the reducing balance method. Calculate the rate of depreciation per annum. (The rate was fixed over the 12 years.)

4.2 Calculate the effective interest rate if interest is 9, 8% p.a., compounded monthly.

4.3 Mrs Pillay invested R80 000 in an account which offers the following:

- 7,5 % p.a., compounded quarterly, for the first 4 years and thereafter
- 9,2% p.a., compounded monthly, for the next 3 years

Calculate the total amount of money that will be in the account at the end of 7 years if no further transactions happen on the account.

4.4 Exactly 8 years ago Tashil invested R30 000 in an account earning 6, 5% per annum, compounded monthly.

4.4.1 How much will he receive if he withdrew his money today? 3

4.4.2 Tashil withdrew R10 000 three years after making the initial deposit and re-invested R10 000 five years after making the initial deposit.

Calculate the difference between the final amount Tashil will now receive after eight years and the amount he would have received had there not been any transactions on the account after the initial deposit.

5.1 A school buys tablets at a total cost of R140 000. If the average rate of inflation is 6, 1% per annum over the next 4 years, determine the cost of replacing these tablets in 4 years' time.

5.2 An investment earns interest at a rate of 7 % per annum compounded semi-annually. Calculate the effective annual interest rate on this investment.

5.3 A savings account was opened with an initial deposit of R24 000. 18 months later R7 000 was withdrawn from the account. Calculate how much money will be in the savings account at the end of 4 years if the interest rate was 10,5% p.a. compounded monthly.

5.4 A car, costing R198 000, has a book value of R102 755, 34 after 3 years. If the value of the car depreciates at r % p.a. on a reducing balance, calculate r .

- 6.1 The cost price of a car is R635 000. The value of the car depreciates according to the reducing balance method at a rate of 15% p.a. Calculate the value of the car exactly 5 years after it was bought.
- 6.2 A loan of R50 000 is to be repaid in 48 equal monthly payments. Payments start one month after the loan has been granted. The interest rate on the loan is 16, 75% p.a. compounded monthly.
- 6.2.1 Calculate the value of the equal monthly payments.
- 6.2.2 Calculate the outstanding balance immediately after the 30th payment has been made. Give your answer to the nearest Rand.
- 6.2.3 How many years will it take for an investment to double its value if the Compound interest rate is 14, 75% p.a? Leave your answer correct to one decimal digit.
- 7 A business installs a server for R500 000. The value of the server depreciates at 20% per annum according to the diminishing-balance method.
- 7.1 Calculate the scrap value of the server at the end of 6 years.
- 7.2 The server needs to be replaced after 6 years.
- Calculate the cost of the new server if the inflation rate is at 7% per annum.
- The older server will be traded in.