



Western Cape
Government

Western Cape Education Department
Directorate: Curriculum FET

MATHEMATICS

REVISION BOOKLET 2025 TERM 3

Grade 11

This revision program is designed to assist you in revising the critical content and skills envisaged/ planned to be covered during the 3rd term. The purpose is to prepare you to understand the key concepts and to provide you with an opportunity to establish the required standard and the application of the knowledge necessary to succeed in the Grade 11 examination.

If you wish to master Mathematics you need to remember:

- ✓ The final answer is by no means the most important in Mathematics. Systematic, detailed and logical layout of every step of your working is the most important.
- ✓ Do not accept the fact that you are careless. Carelessness can be overcome by checking your work. It is important to check the correctness and the validity of every step of your calculations. In this way carelessness is overcome.
- ✓ Never take short cuts in Mathematics by leaving out steps in your working.
- ✓ Despair in Mathematics can destroy your Mathematics. Never give up: try again and again and ... until you get it right. Continually say to yourself: I CAN!!!!
- ✓ The more you practice the better you will become!

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1. CAPS- REQUIREMENTS: GRADE 11	
Trigonometry	
Grade 11-CAPS bl. 15	Grade 11-CAPS bl. 37
(d) Establish the sine, cosine and area rules. Solve problems in 2-dimensions.	1. Prove and apply the sine, cosine and area rules. 2. Solve problems in two dimensions using the sine, cosine and area rules. Comment: • The proofs of the sine, cosine and area rules are examinable .

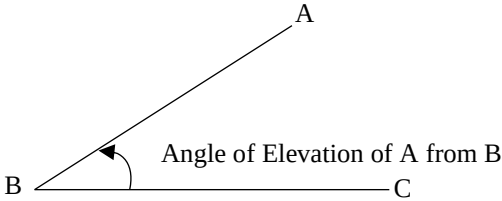
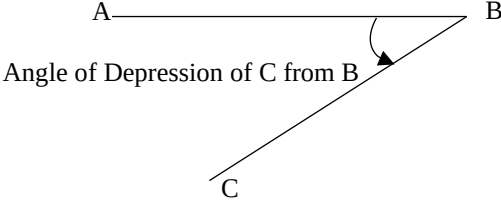
Naming sides in triangles

Note:

- We use **small caps** to name the side **opposite** an angle (vertex)
- In a Δ we get the **shortest side** opposite the **smallest angle** and vice versa
- There can only be **1 obtuse angle** in a Δ (opposite the longest side)

2. FORMULAE

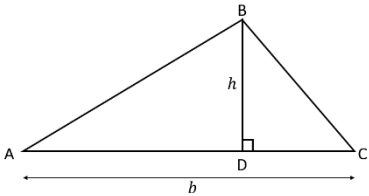
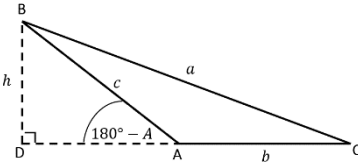
Area rule :	$Area\ van\ \Delta ABC = \frac{1}{2}ab\ sin\ C = \frac{1}{2}ac\ sin\ B = \frac{1}{2}bc\ sin\ A$
• In the Area rule: Area of $\Delta ABC = \frac{1}{2}bc\ sin\ A$, the sides b (AC) and c (AB) are adjacent to the angle A; we say $\hat{A}$ is the included angle to the 2 sides • So, to calculate the area, we need 2 sides and the included angle • Area is measured in (units)²	
Sine rule:	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
The sine-rule is usually used when: • 2 sides and an angle, opposite 1 of the 2 sides, are given • 2 angles and a side are given	
cos rule:	$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\ cos\ A & \therefore & A = \frac{b^2 + c^2 - a^2}{2bc} \\ b^2 &= a^2 + c^2 - 2ac\ cos\ B & \therefore & B = \frac{a^2 + c^2 - b^2}{2ac} \\ c^2 &= a^2 + b^2 - 2ab\ cos\ C & \therefore & C = \frac{a^2 + b^2 - c^2}{2ab} \end{aligned}$
Note: The cos rule is used when: • 3 sides are given (and you have to calculate an angle) • 2 sides and the included angle given (and you have to calculate the 3rd side)	

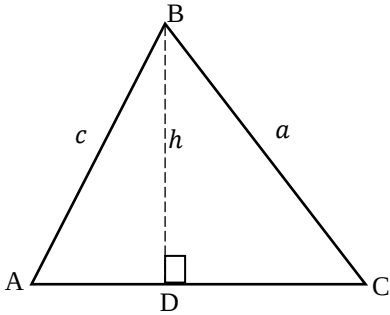
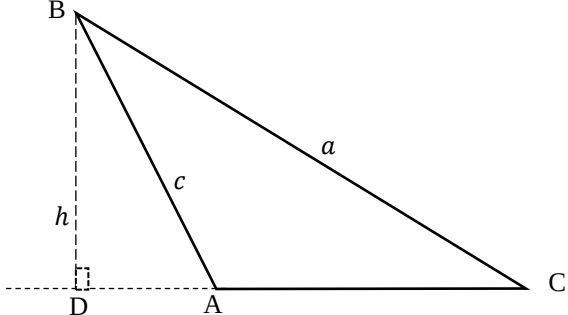
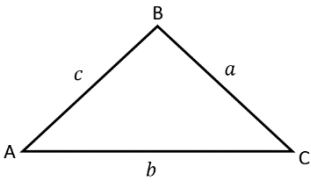
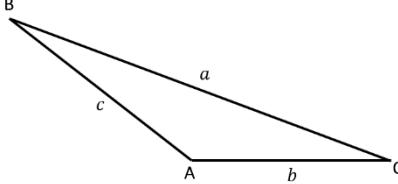
Angles of Elevation/ Depression	
Angle of Elevation (from horizontal upwards) 	Angle of Depression (from horizontal downwards) 

Note: Calculations/ Proofs when working with problems in 2D

- Usually, 2 triangles with a common/ connecting side will be given; one that is right-angled and the other one that is scalene – identify them.
- In the right-angled triangle, we use the 3 trig ratios ($\sin \theta$; $\cos \theta$; $\tan \theta$) to calculate sides/ angles or write sides in terms of another.
- In the scalene triangle, we use the sin/ cos-rule to calculate sides/ angles.
- Also note that Euclidean Geometry may sometimes be needed to calculate the size of angles.
- Start with the triangle that has the most given information and calculate the common side depending on the type of triangle, otherwise start with what was asked.
- Usually this common side will provide a link to the follow-on question/ required answer.
- Only use Area rule if asked to calculate Area.

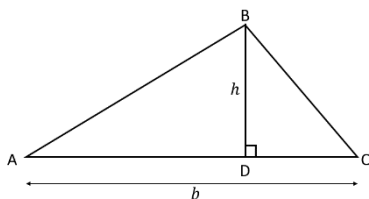
3. PROOFS

Area rule : $A = \frac{1}{2}ab \sin C$	
If $\hat{A}$ is an acute angle  Proof: Area of $\triangle ABC = \frac{1}{2}bh$... (1) But $\sin A = \frac{h}{c} \therefore h = c \sin A$ Subst. in (1) Area of $\triangle ABC = \frac{1}{2}bc \sin A$ Similarly, it can be shown that: Area of $\triangle ABC = \frac{1}{2}ab \sin C$ and Area of $\triangle ABC = \frac{1}{2}ac \sin B$	If $\hat{A}$ is an obtuse angle  Proof: Area of $\triangle ABC = \frac{1}{2}bh$... (1) But $\sin(180^\circ - A) = \frac{h}{c} \therefore h = c \sin A$ Subst. in (1) Area of $\triangle ABC = \frac{1}{2}bc \sin A$ Similarly, it can be shown that: Area of $\triangle ABC = \frac{1}{2}ab \sin C$ and Area of $\triangle ABC = \frac{1}{2}ac \sin B$

Sine rule: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	
Fig.1 	Fig. 2 
Proof: $\sin A = \frac{h}{c}$ [in $\triangle ABD$] and $\sin C = \frac{h}{a}$ [in $\triangle CBD$] [$\because \sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$] $\therefore h = c \sin A$ and $h = a \sin C$ $\therefore c \sin A = a \sin C \Rightarrow \therefore \frac{c \sin A}{ac} = \frac{a \sin C}{ac}$ $\therefore \frac{\sin A}{a} = \frac{\sin C}{c}$ Similarly, through construction of a $\perp$ height from C onto AB, we can proof that: $\frac{\sin A}{a} = \frac{\sin B}{b}$ $\therefore \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	
OR If $\hat{A}$ is an acute angle  If $\hat{A}$ is an obtuse angle  Use the Area rule for $\triangle ABC$: $\frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B$ Dividing each term by $\frac{1}{2}abc$ gives: $\frac{\sin A}{a} = \frac{\sin C}{c} = \frac{\sin B}{b}$	
The sine-rule is usually used when: • 2 sides and an angle, opposite 1 of the 2 sides, are given • 2 angles and a side are given	

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

If $\hat{A}$ is an acute angle



$$\begin{aligned}\text{In } \triangle BDC: a^2 &= BD^2 + CD^2 \text{ (Theorem of Pythagoras)} \\ &= BD^2 + (b - AD)^2 \\ &= BD^2 + b^2 - 2bAD + AD^2\end{aligned}$$

$$\text{But } BD^2 + AD^2 = c^2 \quad (\text{Pythagoras})$$

$$\text{Hence } a^2 = b^2 + c^2 - 2bAD \quad \dots \dots (1)$$

$$\text{In } \triangle ABD: \cos A = \frac{AD}{c} \quad \therefore AD = c \cos A \quad \dots \dots (2)$$

Subst. (2) in (1)

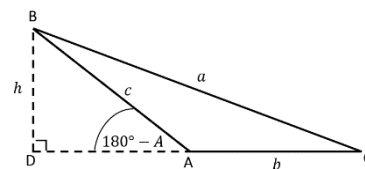
$$\therefore a^2 = b^2 + c^2 - 2bc \cos A$$

Similarly, it can be shown that:

$$b^2 = a^2 + c^2 - 2ac \cos B \quad \text{and}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

If $\hat{A}$ is an obtuse angle



$$\begin{aligned}\text{In } \triangle BDC: a^2 &= BD^2 + CD^2 \text{ (Theorem of Pythagoras)} \\ &= BD^2 + (b + AD)^2 \\ &= BD^2 + b^2 + 2bAD + AD^2\end{aligned}$$

$$\text{But } BD^2 + AD^2 = c^2 \quad (\text{Pythagoras})$$

$$\text{Hence } a^2 = b^2 + c^2 + 2bAD \quad \dots \dots (1)$$

$$\begin{aligned}\text{In } \triangle ABD: \cos(180^\circ - A) &= \frac{AD}{c} \\ \therefore AD &= c \cos A \quad \dots \dots (2)\end{aligned}$$

Subst. (2) in (1)

$$\therefore a^2 = b^2 + c^2 - 2bc \cos A$$

Similarly, it can be shown that:

$$b^2 = a^2 + c^2 - 2ac \cos B \quad \text{and}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

3. REVISION EXERCISES

Note:

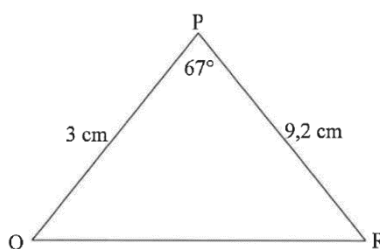
- Questions in this revision are compiled from national examination papers for Gr 11 and 12.
- Some questions have been changed to suit the content covered in the FET lessons for Gr 11.

SECTION A (Routine questions)

QUESTION 1 (Nov 2018/ 2016)

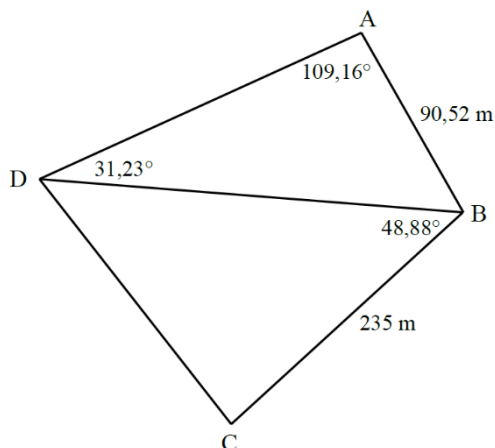
- 1.1 In the diagram is $\hat{P} = 67^\circ$, $PQ = 3$ cm and $PR = 9,2$ cm.
Determine the length of QR.

(3)



GR 11 REVISION: TRIGONOMETRY FORMULAE (Solving triangles)

- 1.2 Quadrilateral ABCD is drawn with $BC = 235$ m and $AB = 90,52$ m. It is also given that $\hat{ADB} = 31,23^\circ$; $\hat{DAB} = 109,16^\circ$ and $\hat{CBD} = 48,88^\circ$.

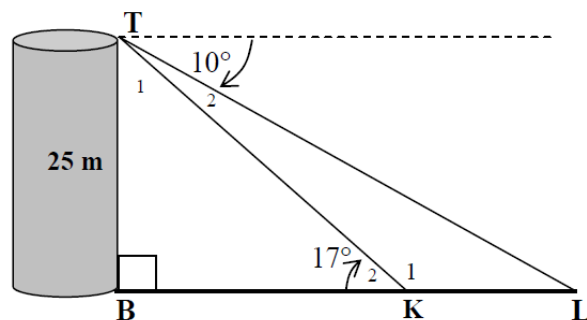


Determine the length of :

- 1.2.1 BD (3)
1.2.2 CD (3)

QUESTION 2 (Model 2007)

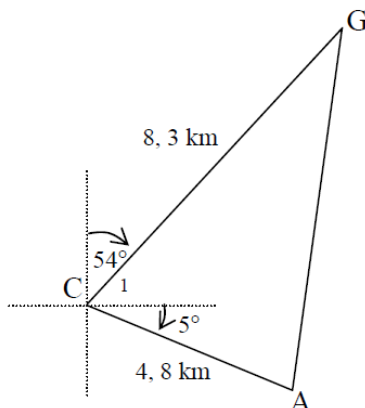
- 2.1 The diagram below is a representation of a 25 m vertical observation tower TB and two cars K and L on a road. The angle of depression from T to car L is 10° . The angle of elevation from car K to the top of the tower is 17° . B, K and L lie in a straight line and lie on the same horizontal plane as the base of tower TB.



- 2.1.1 Calculate the size of $\hat{L}$. (1)
2.1.2 Calculate the length of KT. (3)
2.1.3 Hence, calculate the distance between the two cars. (4)

GR 11 REVISION: TRIGONOMETRY FORMULAE (Solving triangles)

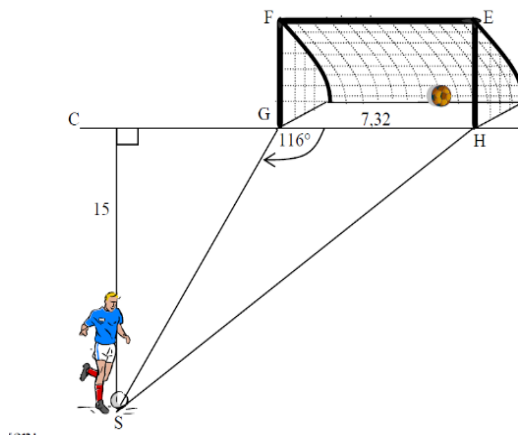
- 2.2 A game ranger G is 8,3 km from control centre, C, at a bearing of 54° east when he receives a call that there is an injured antelope, A, that needs attention. The antelope is located 4,8 km at a bearing 5° south of east from the control centre. The diagram below is a representation of the above-mentioned situation.



- 2.2.1 Calculate how far the game ranger is from the injured antelope. (4)
- 2.2.2 Calculate the area of $\triangle GCA$. (3)

QUESTION 3 (Nov 2007)

A soccer player aims towards the goal which is 15 metres from the back line CH on a soccer field. The angle from the left goal post, FG to the soccer player, S is 116° . The goal posts are 7,32 m wide. The diagram represents the above situation. Calculate:

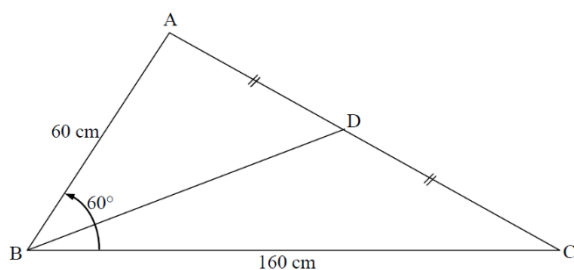


- 3.1 The size of $\widehat{CGS}$. (1)
- 3.2 How far the soccer player is from the left goal post FG (calculate the distance GS). (3)
- 3.3 How far the soccer player is from the right goal post EH. (3)
- 3.4 The approximate size of $\widehat{GSH}$, the angle within which the soccer player could possibly score a goal. (4)

QUESTION 4 (Nov 2013)

4.1 Prove that in any acute angled $\triangle ABC$ is $c^2 = a^2 + b^2 - 2ab \cos C$. (6)

4.2 In $\triangle ABC$, $AB = 60$ cm, $BC = 160$ cm and the angle of elevation of A from B is 60° .
D is the bisector of AC with D a point on AC.

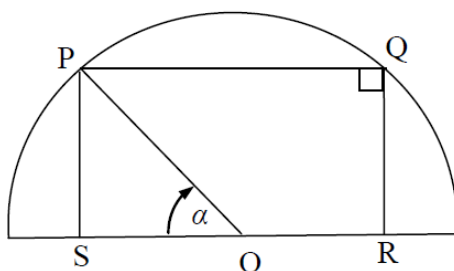


4.2.1 Calculate the length of AC. (3)

4.2.2 Determine the value of $\sin A$. Leave the answer in its simplest surd form. (3)

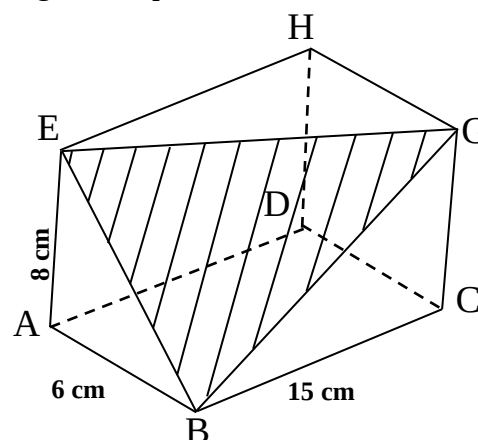
4.2.3 Calculate the area of $\triangle ABD$. Give your answer correct to ONE decimal place. (3)

4.3 In the diagram, O is the centre of a semi circle. PQRS is a rectangle inscribed in the semi circle such that O lies on RS. $\widehat{SOP} = \alpha$
Calculate the size of α for which PQRS will be a square. (3)



QUESTION 5 (June 2017)

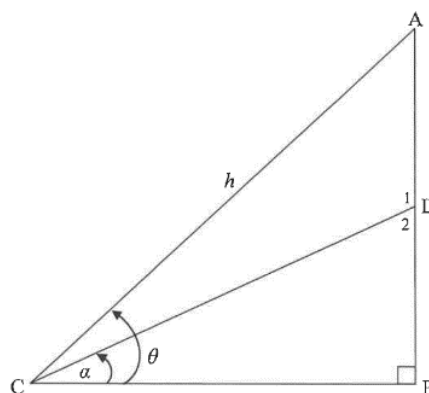
A rectangular block of wood ABCDEFGH with $AB = 6$ cm, $EA = 8$ cm and $BC = 15$ cm is given. A cut is made through E, G and B to show a triangular shape EBG as shown in the diagram. Corner F is thus removed.



- 5.1 Show through calculation, that the length of $EG = \sqrt{261}$ cm (2)
- 5.2 Hence, calculate the size of $\hat{EBG}$. (5)

QUESTION 6 (Nov 2018)

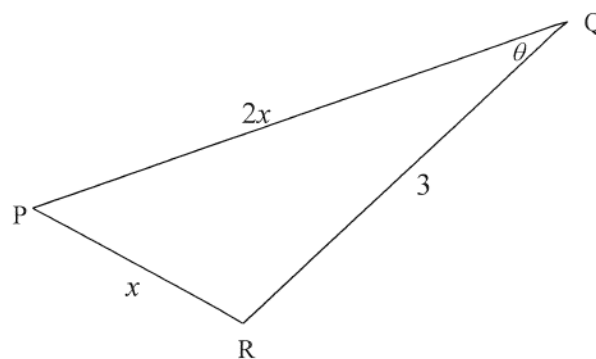
In the diagram below, $\hat{DCB} = \alpha$, $AC = h$ units and $\hat{ACB} = \theta$.



- 6.1 Determine the size of $\hat{ACD}$ in terms of θ and α . (1)
- 6.2 Prove that $AD = \frac{h \sin(\theta - \alpha)}{\cos \alpha}$ (4)
- 6.3 Determine the length of AD if $h = 17$ units, $\theta = 58^\circ$ and $\alpha = 23^\circ$. (2)
- 6.4 Calculate the area of $\triangle ADC$. (3)

SECTION B (Mixed)**QUESTION 7 (Nov 2017)**

In $\triangle PQR$, $QR = 3$ units, $PR = x$ units, $PQ = 2x$ units and $\widehat{PQR} = \theta$.



7.1 Show that: $\cos \theta = \frac{x^2 + 3}{4x}$ (3)

7.2 If $x = 2,4$ units:

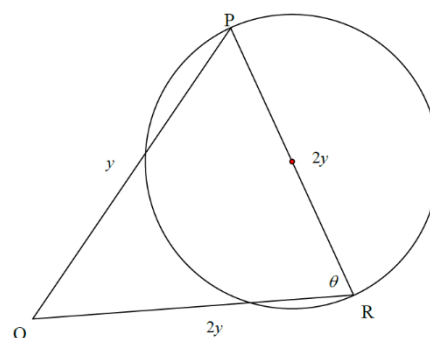
7.2.1 Calculate the size of θ (3)

7.2.2 Determine the area of $\triangle PQR$. (2)

7.3 Calculate the values of x for which the triangle exists. (4)

QUESTION 8 (Nov 2015/ 2016)

8.1 In the diagram, PR is the diameter of the circle. Triangle PQR is drawn with vertex Q outside the circle. $\widehat{R} = \theta$, $PR = QR = 2y$ and $PQ = y$.



8.1.1 Determine the value of $\cos \theta$. (4)

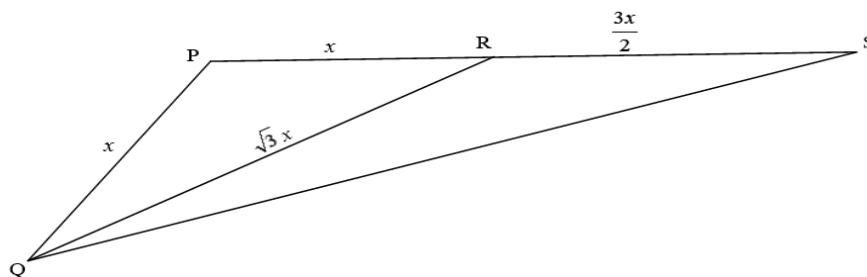
8.1.2 If QR cuts the circumference of the circle at T , determine PT in terms of y and θ . (3)

8.2 If $c^2 = a^2 + b^2 - 2ab \cos C$, deduce that: $1 + \cos C = \frac{(a+b+c)(a+b-c)}{2ab}$ (4)

QUESTION 9 (June 2015)

Triangle PQS forms a certain area of a park. R is a point on PS and QR divides the area of the park into two triangular parts, as shown below, for a festive event.

$PQ = PR = x$ units, $RS = \frac{3x}{2}$ units and $RQ = \sqrt{3}x$ units.

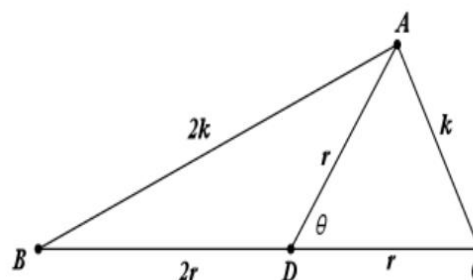


9.1 Calculate the size of $\hat{P}$. (4)

9.2 Hence, calculate the area of triangle QRS in terms of x in its simplest form. (5)

QUESTION 10 (CAPS p 42)

In $\triangle ABC$, $\hat{ADC} = \theta$, $DA = DC = r$, $AC = k$ and $AB = 2k$



Prove that $\cos \theta = \frac{1}{4}$

GRADE 11 STATISTICS CONSOLIDATION

1. Summary of types of data:

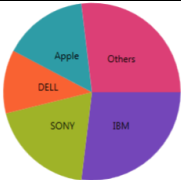
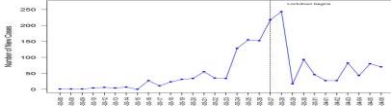
The table shows the difference between **univariate** and **bivariate data**

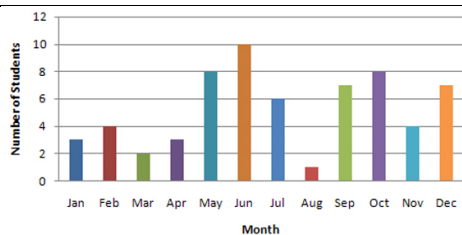
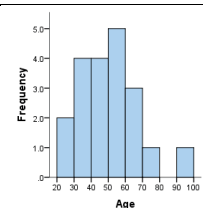
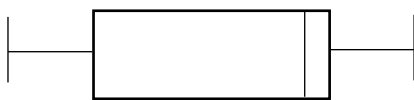
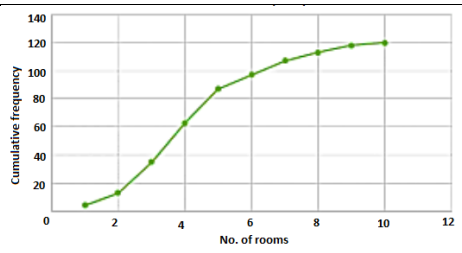
Univariate Data	Bivariate Data
Data that consists of a single variable Example: Mathematics test scores	Data that consists of two variables Example: time spent studying mathematics and the corresponding mathematics test marks.
Data is analysed by: - Determining the measures of central tendency- mean, mode, median. - Determining measures of dispersion- range, interquartile range, variance and standard deviation. - Drawing different graphs.	Data is analysed by: - Identifying the dependent and independent variables. - Determining if a relationship or correlation exist between the variables. - Determining the strength of the relationship or correlation.
Interpretation questions like: how many learners have achieved a pass mark?	Interpretation questions like: Describe the relationship/ correlation between the time spent studying and test scores.

Discrete data: Exact values that are countable whole numbers, for example the number of people that have recovered from the COVID-19 virus.

Continuous data: It can be values anywhere within a range of real number values, for example height, mass and time.

2. Summary of types of graphs

Type	Example	Explanatory notes
Pie chart Proportional relationships at a specific point in time		To convert data into degrees $\frac{\text{frequency}}{\text{total outcomes}} = 360^\circ$
Line graph Trends/ changes over time.		Multiple trends can be compared

Bar graph Comparison of discrete non-numerical data. The heights of each bar represents the frequency.		Cannot be used to illustrate continuous data. Bar graphs are separated by distinct gaps between the bars												
Histogram Trends in numerical continuous data. The area of the bars represents frequency.		Used only for continuous data with no gaps between bars. The bars can be of different widths, in keeping with interval size.												
Box and whisker diagram Created by using a five number summary. Illustrates the spread of data. Can be used to illustrate and discuss skewness		Scale used must be accurate to extract an accurate five-point summary. 1. Minimum value 2. Q_1, 3. Q_2 4. Q_3 5. maximum value.												
Stem and leaf plot Used to summarise grouped data and simultaneously gives a <i>picture</i> of the data	Stem and Leaf Plot <table><tr><td>4</td><td>1</td></tr><tr><td>5</td><td>2 7 8</td></tr><tr><td>6</td><td>5 6</td></tr><tr><td>7</td><td>0 5 8 8 8</td></tr><tr><td>8</td><td>0 0</td></tr><tr><td>9</td><td>5</td></tr></table>	4	1	5	2 7 8	6	5 6	7	0 5 8 8 8	8	0 0	9	5	The stems are the digits in the LHS column and the leaves are the digits in the RHS column.
4	1													
5	2 7 8													
6	5 6													
7	0 5 8 8 8													
8	0 0													
9	5													
Cumulative frequency curve / Ogive It is a graph that shows the information in a cumulative frequency table. You can draw an Ogive of ungrouped and grouped discrete data or grouped continuous data		The total of scores are called the cumulative frequency. It is calculated by adding the frequencies of all the previous scores. A smooth graph of cumulative frequencies.												

3. Important symbols used in Statistics:

Σ	The Greek letter <i>sigma</i> , which means <i>sum of</i> , used to show that u must add all the values together
n	The number of scores (data items) in a data set.
$\bar{x}$	The mean/ average of all the scores in the data set.
σ	The standard deviation

4. Measuring Data

Central tendency:

The measures of central tendency are the three different averages i.e. the mean, median and mode.

Dispersion:

The measures of dispersion, i.e. the range, interquartile range, semi-IQR, variance and standard deviation are used to measure the spread and variability of the data.

Concept	How to determine:	Relevant Formulae
Mean / Average	$\text{mean} = \frac{\text{sum of all the scores}}{\text{number of scores}}$	Ungrouped data: $\bar{x} = \frac{\sum x}{n} \text{ or } \bar{x} = \frac{\sum fx}{n}$ Grouped data $\bar{x} = \frac{\sum(f \times \text{interval midpoint})}{n}$
Median: Middlemost scores	Arrange the scores in ascending order, if the number of the scores is: Odd , then the median is the score exactly in the middle. Even , add the two middle scores together and divide the result by 2	$\text{median}\left(\frac{n+1}{2}\right)$ use this formula for large samples. For grouped data the median can be estimated using an Ogive.
Mode	The score that occurs most often. More than one mode can exist. For grouped data use the modal interval	
Percentile	A percentile is a measure that tells us what % of the total frequency scored at, or below the measure. It divides data into 100 equal parts.	To find the position of the k^{th} percentile which is a particular percentile: $P_k = \frac{k(n+1)}{100}$
Quartiles	Divide data into 4 equal parts. The 1 st quartile is the 25 th percentile The 2 nd quartile is the 50 th percentile The 3 rd quartile is the 75 th percentile.	For large samples $Q_1 = \left(\frac{n+1}{4}\right) \text{th score}$ $Q_2 = \left(\frac{n+1}{2}\right) \text{th score}$ $Q_3 = \left(\frac{3(n+1)}{4}\right) \text{th score}$
Range	The difference between the highest and lowest scores in a given data set.	Range is equal to highest score – lowest score
Interquartile Range (IQR)	The difference between the upper quartile (Q_3) and the lower quartile (Q_1) in a given set	$\text{IQR} = Q_3 - Q_1$
Semi - IQR	Half the difference between the upper and lower quartile in any given set	$\text{Semi-IQR} = \frac{Q_3 - Q_1}{2}$

5. Variance and Standard Deviation:

Measures of dispersion, taking into account all of the data, which is linked to the mean. The **Variance** is the mean of the sum of the squares of the deviations from the mean.

We find the variance by:

- Finding the mean: $\bar{x} = \frac{\Sigma x}{n}$
- Finding the deviation from the mean of each item of the data set:
 - **Deviation = data item (x) – mean ($\bar{x}$)** ($x - \bar{x}$)
- Squaring each deviation:
 - **(deviation)² = (x – $\bar{x}$)²**
- Finding the sum of the squares of the deviation:
 - **$\Sigma(\text{deviation})^2 = \Sigma(x - \bar{x})^2$**
- Finding the mean of the squares of the deviations by dividing by the number of terms in the data set:
 - **Variance = $\frac{\Sigma(\text{deviation})^2}{\text{number of data items}} = \frac{\Sigma(x - \bar{x})^2}{n}$**

The **Standard Deviation** is the square root of the variance:

$$\sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}$$

- When data elements are closely/ tightly clustered together, the standard deviation and variance will be small, when they are spread apart, the standard deviation and the variance will be relatively large.
- A data set with more data items near to the mean will have less spread and a smaller standard deviation
- A data set with more data items far from the mean will have a greater spread and a larger standard deviation.

Example 1

- a) Calculate the variance and standard deviation of the following two sets of data representing the number of runs scored by two cricketers over 5 matches.

Batsman A	40	45	51	52	62
Batsman B	15	28	44	78	85

- b) Use the two standard deviations to compare the distribution of data in the two sets.

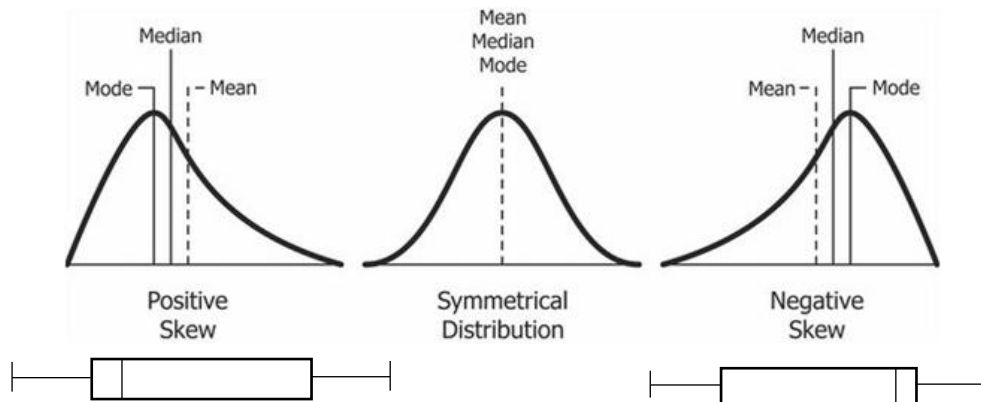
Solution 1

1. Find the mean of each set.
Mean of Batsman A = 50
Mean of Batsman B = 50
2. Find the deviation from the mean of each item in the data set.
3. Square each deviation.
4. Find the variance.
5. Find the standard deviation

Batsman A			Batsman B		
Data Item	Deviation from mean	$(Deviation)^2$	Data Item	Deviation from mean	$(Deviation)^2$
40	$40 - 50 = -10$	100	15	$15 - 50 = -35$	1225
45	$45 - 50 = -5$	25	28	$28 - 50 = -22$	484
51	$51 - 50 = 1$	1	44	$44 - 50 = -6$	36
52	$52 - 50 = 2$	4	78	$78 - 50 = 28$	784
62	$62 - 50 = 12$	144	85	$85 - 50 = 35$	1225
		274			3754
Variance = $\frac{\Sigma(deviation)^2}{number\ of\ data\ items}$ $\frac{274}{5} = 54.8$			Variance = $\frac{\Sigma(deviation)^2}{number\ of\ data\ items}$ $\frac{3754}{5} = 750.8$		
Standard Deviation = $\sqrt{variance}$ $= \sqrt{54.8}$ $\approx 7.$			Standard Deviation = $\sqrt{variance}$ $= \sqrt{750.8}$ ≈ 27.4		
The larger standard deviation with Batsman B indicates that the data items are generally much further from the mean than the data items with Batsman A . This implies that the data items of Batsman B are more spread out than the data items of Batsman A . In terms of cricket the difference in the standard deviations shows that Batsman A is possibly more predictable.					

6. The distribution/ spread of Data

The distribution is the nature and shape of the spread of data. It can be either symmetric or skewed.



Key features of skewed data:

Skewness is the tendency for the values to be more frequently around the high or low ends of the x – axis.

- With a **positively skewed distribution**, the tail on the right side is longer than the left side. (median < mean)
Most of the values tend to cluster toward the left side of the x -axis (i.e. the smaller values) with increasingly fewer values on the right side of the x -axis (i.e. the larger values)
- With a **negatively skewed distribution**, the tail on the left side is longer than the right side. (median > mean)
Most of the values tend to cluster toward the right side of the x -axis (i.e. the larger values) with increasingly fewer values on the left side of the x -axis (i.e. the smaller values).

7. Outliers

- An outlier is a data entry that is far removed from the other entries in a data set e.g. a data entry that is much smaller or larger than the rest of the data values.
- An outlier has an influence on the mean and the range of the data set, but has no influence on the mean or lower and upper quartiles.
- Any data item that is less than $Q_1 - 1,5 \times IQR$ or more than $Q_3 + 1,5 \times IQR$ is an outlier.

Example: Are any of the entries in the data set outliers?

1, 8, 12, 14, 14, 15, 17, 17, 19, 26, 32

First find the IQR

$$IQR = Q_3 - Q_1 = 19 - 12 = 7$$

Lower quartile (Q_1) < $Q_1 - 1,5 \times IQR$

$$< 12 - 1,5 \times 7$$

$$< 1,5$$

1 is an outlier

Upper quartile (Q_3) > $Q_3 + 1,5 \times IQR$

$$> 19 + 1,5 \times 7$$

$$> 29,5$$

32 is also an outlier

STATISTICS REVISION EXERCISES

Section A

QUESTION 1

Mr Ngwane is the sales manager for a furniture shop. Every month his 15 staff members report on the number of customers who visited during the previous month.

The results were given as follows:

12 15 15 19 22 23 26 26 32 33 33 33 33 35 35

- 1.1 Determine the:

1.1.1 Median of the data (1)

1.1.2 Interquartile range (3)

1.1.3 Mean of the data (2)

1.1.4 Standard deviation of the data (2)
- 1.2 Determine the percentage of customers who visited the furniture shop that are outside one standard deviation of the mean (3)

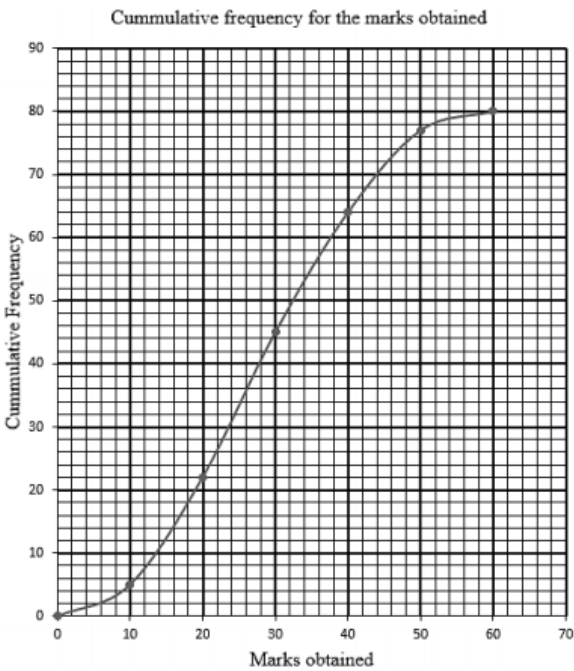
[11]

QUESTION 2

A group of learners wrote a standardised English test that was scored out of 60.

The results were represented in a cumulative frequency graph below.

- 2.1 How many learners wrote the test? (1)
- 2.2 How many learners scored at least 20 out of 60? (2)
- 2.3 Using the graph, estimate the median test score. (2)



2.4 Complete the frequency table below (5)

Marks obtained	Frequency
$0 < x \leq 10$	
$10 < x \leq 20$	
$20 < x \leq 30$	
$30 < x \leq 40$	
$40 < x \leq 50$	
$50 < x \leq 60$	

2.5 Write down the modal group. (1)
[11]

QUESTION 3

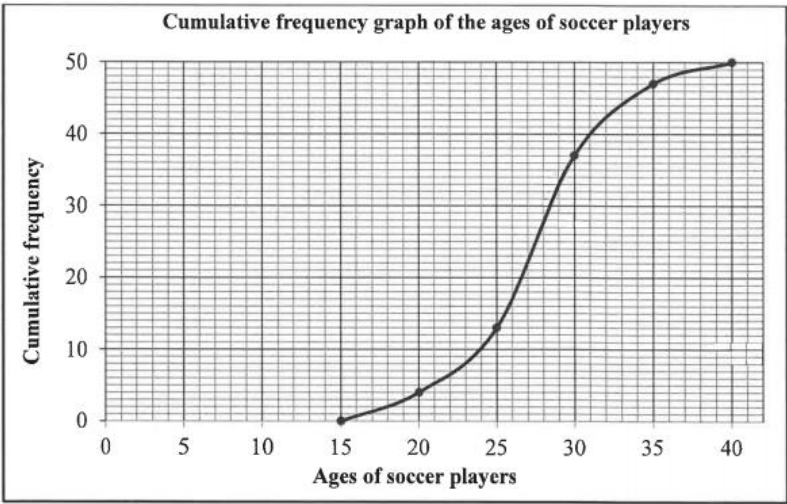
A school held a sports day. One of the items on the program was an obstacle race. Teams of 10 parents and learners participated in the race. The table below shows the time taken, in minutes, by each member of a particular team to complete the race.

4	12	13	16	17	18	20	22	22	25
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- 3.1 How long, in minutes, did it take for the fastest member of this team to complete the race? (1)
- 3.2 Determine the mean time taken by this team. (2)
- 3.3 Calculate the standard deviation for the data. (1)
- 3.4 How many members of the team completed the obstacle race outside of two standard deviations of the mean? (3)
- 3.5 It took another team a total time of $x + 5$ minutes to complete the race. Calculate the value of x if the overall mean of the two teams combined was 18 minutes. (3)
[10]

QUESTION 4

4.1 A survey was conducted of the ages of players at a soccer tournament. The results are shown in the cumulative frequency graph (ogive) below.



- 4.1.1 How many learners took part in the soccer tournament? (1)
- 4.1.2 Determine the number of players between the ages of 24 and 31 years old. (2)
- 4.1.3 Complete the frequency column of the table below in the answer book. (3)

CLASS INTERVAL	FREQUENCY	CUMULATIVE FREQUENCY
$15 \leq x < 20$		4
$20 \leq x < 25$		13
$25 \leq x < 30$		37
$30 \leq x < 35$		47
$35 \leq x < 40$		50

- 4.1 4 Draw a frequency polygon for the data(4)
- 4.2 Two grade 11 Mathematics classes have the same number of learners. The five-number summaries of the marks obtained by these classes for a test are shown below.

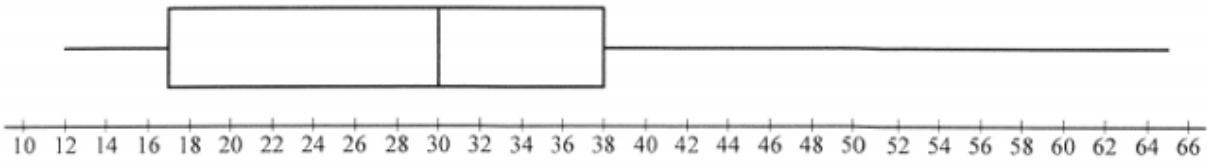
CLASS A (30; 48; 65; 82; 90)
CLASS B (50; 58; 65; 75; 90)

The parents of learners in CLASS A and CLASS B observe that both classes have the same median and the same maximum mark and therefore claim that there is no difference in the performance between these classes. Do you agree with this claim?
Use at least **two** different arguments to justify your answer. (3)

[13]

QUESTION 5

5.1 Mr Brown conducted a survey on the amount of airtime (in rand) EACH student had on his or her cellphone. He summarised the data in the box and whisker diagram below:



- 5.1.1 Write down the five-number summary of the data.(2)
- 5.1.2 Determine the interquartile range.(1)
- 5.1.3 Comment on the skewness of the data.(1)
- 5.2 A group of 13 students indicated how long it took (in hours) before their cellphone batteries required recharging. The information is given below.

5	8	10	17	20	29	32	48	50	50	63	<i>y</i>	107
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- 5.2.1 Calculate the value of *y* if the mean for this data set is 41. (2)
- 5.2.2 If *y* = 94, calculate the standard deviation of the data. (2)
- 5.2.3 The mean time before another group of 6 students needed to recharge the batteries of their cellphones was 18 hours. Combine these groups and calculate the overall mean time needed for these two groups to recharge the batteries of their cellphones. (3)

[11]

QUESTION 6

A student conducted a survey among his friends and relatives to determine the relationship between the age of a person and the number of marketing phone calls he or she received within one month. The information is given in the table below.

AGE OF PERSON IN SURVEY	FREQUENCY	CUMULATIVE FREQUENCY
$20 < x \leq 30$	7	7
$30 < x \leq 40$		27
$40 < x \leq 50$	25	
$50 < x \leq 60$		64
$60 < x \leq 70$		72
$70 < x \leq 80$	4	
$80 < x \leq 90$		80

- 6.1 Complete the frequency and cumulative frequency columns in the given table. (4)
- 6.2 How many people participated in this survey? (1)
- 6.3 Write down the modal class. (1)
- 6.4 Draw an ogive (cumulative frequency graph) to represent the data in the table. (3)
- 6.5 Determine the percentage of marketing calls received by people older than 54 years. (3)

[12]

Section B

QUESTION 1

Each child in a group of four-year-old children was given the same puzzle to complete. The time taken (in minutes) by each child to complete the puzzle is shown in the table below.

TIME TAKEN (t) (IN MINUTES)	NUMBER OF CHILDREN
$2 < t \leq 6$	2
$6 < t \leq 10$	10
$10 < t \leq 14$	9
$14 < t \leq 18$	7
$18 < t \leq 22$	8
$22 < t \leq 26$	7
$26 < t \leq 30$	2

- 1.1 How many children completed the puzzle? (1)
- 1.2 Calculate the estimated mean time taken to complete the puzzle. (2)
- 1.3 Complete the cumulative frequency column in the table (2)
- 1.4 Draw a cumulative frequency graph (ogive) to represent the data (3)
- 1.5 Use the graph to determine the median time taken to complete the puzzle. (2)

[10]

QUESTION 2

A survey was conducted among 100 people about the amount that they paid on a monthly basis for their cellphone contracts. The person carrying out the survey calculated the estimated mean to be R309 per month. Unfortunately, he lost some of the data thereafter. The partial results of the survey are shown in the frequency table below:

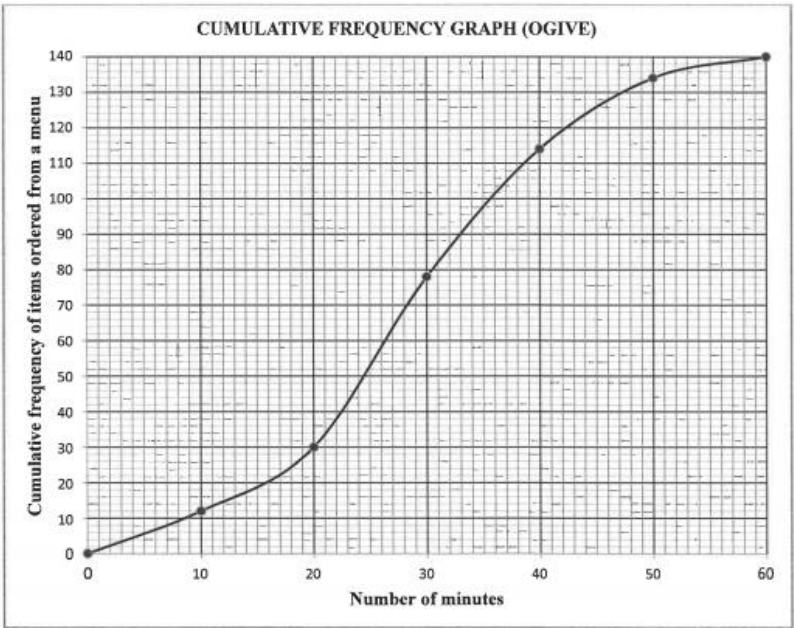
AMOUNT PAID (IN RANDS)	FREQUENCY
$0 < x \leq 100$	7
$100 < x \leq 200$	12
$200 < x \leq 300$	<i>a</i>
$300 < x \leq 400$	35
$400 < x \leq 500$	<i>b</i>
$500 < x \leq 600$	6

- 2.1 How many people paid R200 or less on their monthly cell phone contracts? (1)
- 2.2 Use the information above to show that $a = 24$ and $b = 16$ (5)
- 2.3 Write down the modal class for the data. (1)
- 2.4 Draw an ogive (cumulative frequency graph) to represent the data. (4)
- 2.5 Determine how many people paid more than R420 per month for their cell phone contracts. (2)

[13]

QUESTION 3

The cumulative frequency graph (ogive) drawn below shows the total number of food items ordered from the menu over a period of 1 hour.



- 3.1 Write down the total number of food items ordered from the menu during this hour.(1)
 - 3.2 Write down the modal class of the data. (1)
 - 3.3 How long did it take to order the first 30 food items? (1)
 - 3.4 How many food items were ordered in the last 15 minutes? (2)
 - 3.5 Determine the 75th percentile for the data. (2)
 - 3.6 Calculate the interquartile range of the data. (2)
- [9]

QUESTION 4

Reggie works part-time as a waiter at a local restaurant. The amount of money (in rand) he made in tips over a 15-day period is given below.

35	70	75	80	80
90	100	100	105	105
110	1110	115	120	125

- 4.1.1 Calculate
- a) The mean of the data

(2)
- b) The standard deviation of the mean.

(2)
- 4.1.2. Mary also works part-time as a waitress at the same restaurant. Over the same 15day period, Mary collected the same amount in tips as Reggie, but her standard deviation was R14.
- Using the available information, comment on the:
- a) Total amount in tips that they each collected over the 15day period.

(1)
- b) Variation that each of them received in daily tips over this period.

(1)

[6]

QUESTION 5

The monthly profit (in thousands of rands) made by a company in a year is given in the table below.

110	112	156	164	167	169
171	176	192	228	278	360

- 5.1 Calculate the:
- 5.1.1 Mean profit for the year.

(3)
- 5.1.2. Median profit for the year.

(1)
- 5.2 Draw a box and whisker diagram to represent the data
- (2)
- 5.3 Hence,determine the interquartile range of the data.
- (1)
- 5.4 Comment on the skewness in the distribution of the data.
- (1)
- 5.5 For the given data:
- 5.5.1 Calculate the standard deviation

(1)
- 5.5.2 Determine the number of months in which the profit was less than one

(2)

[11]

QUESTION 6

An organisation decided that it would set up blood donor clinics at various colleges. Students would donate blood over a period of 10 days. The number of units of blood donated per day by students of college X is shown in the table below.

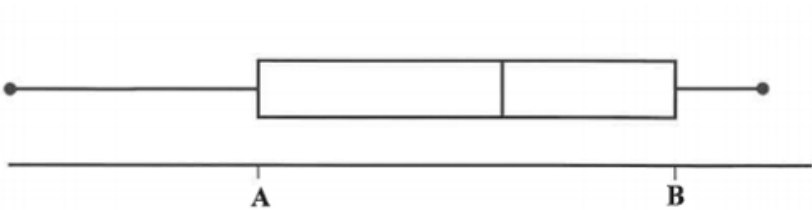
Days	1	2	3	4	5	6	7	8	9	10
Units of blood	45	59	65	73	79	82	91	99	101	106

- 6.1 Calculate:
- 6.1.1 The mean of the units of blood donated per day over a period of 10 days

(2)
- 6.1.2 The standard deviation.

(2)
- 6.1.3 How many days is the number of units of blood donated at College X outside one standard deviation from the mean?

(3)
- 6.2 The number of units of blood donated by students of College X is represented in the box and whisker diagram below:



- 6.2.1 Describe the skewness of the data
- (1)
- 6.2.2 Write down the values of A and B, the lower quartile and the upper quartile respectively, of the data set.
- (2)

[10]