



MATHEMATICS

REVISION BOOKLET MEMORANDUM 2025 TERM 2

Grade 11

This revision program is designed to assist you in revising the critical content and skills envisaged/ planned to be covered during the 2nd term. The purpose is to prepare you to understand the key concepts and to provide you with an opportunity to establish the required standard and the application of the knowledge necessary to succeed in the NCS examination.

The revision program covers the following topics:

- Functions and Graphs
- Analytical Geometry
- Trigonometric Functions
- Trigonometry

If you wish to master Mathematics you need to remember:

- ✓ The final answer is by no means the most important in Mathematics. Systematic, detailed and logical layout of every step of your working is the most important.
- ✓ Do not accept the fact that you are careless. Carelessness can be overcome by checking your work. It is important to check the correctness and the validity of every step of your calculations. In this way carelessness is overcome.
- ✓ Never take short cuts in Mathematics by leaving out steps in your working.
- ✓ Despair in Mathematics can destroy your Mathematics. Never give up: try again and again and ... until you get it right. Continually say to yourself: I CAN!!!!
- ✓ The more you practice the better you will become!

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TOPIC: FUNCTIONS MEMORANDUM
ONDERWERP: FUNKSIES MEMORANDUM
SECTION A / AFDELING A
QUESTION 1 / VRAAG 1 (GR11 DBE NOV 2017)

$$f(x) = \frac{-3}{x+2} + 1 \text{ and / en } g(x) = 2^{-x} - 4$$

1.1

$$\begin{aligned} f(-3) &= \frac{-3}{-3+2} + 1 \\ f(-3) &= \frac{-3}{-1} + 1 \\ f(-3) &= 3 + 1 \\ f(-3) &= 4 \end{aligned}$$

1.2

$$\begin{aligned} g(x) &= 4 \\ 4 &= 2^{-x} - 4 \\ 4 &= 2^{-x} - 4 \\ 8 &= 2^{-x} \\ 2^3 &= 2^{-x} \\ 3 &= -x \\ -3 &= x \end{aligned}$$

1.3

$$x = -2; y = 1$$

1.4

$$y > -4 \quad \textbf{OR/OF} \quad y \in (-4; \infty)$$

1.5

y-intercept/afsnit:

$$\begin{aligned} f(x) &= \frac{-3}{0+2} + 1 \\ y &= -\frac{1}{2} \end{aligned}$$

$$\therefore \text{y-intercept/afsnit: } \left(0; -\frac{1}{2}\right)$$

x-intercept/afsnit:

$$\begin{aligned} 0 &= \frac{-3}{x+2} + 1 \\ -1 &= \frac{-3}{x+2} \\ -x-2 &= -3 \\ -x &= -1 \\ x &= 1 \end{aligned}$$

$$\text{x-intercept/afsnit: } (1; 0)$$

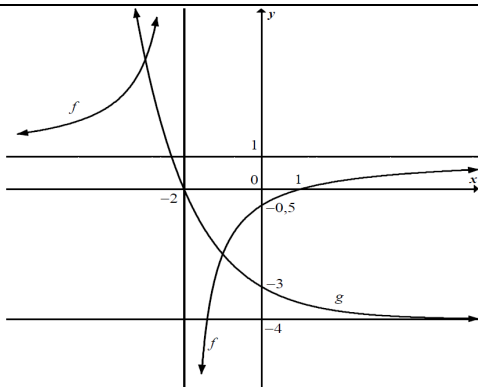
1.6

$$\begin{aligned} y &= -x + c \\ \text{subst. } (-2; 1) \\ 1 &= -(-2) + c \\ -1 &= c \\ \therefore y &= -x - 1 \end{aligned}$$

OR/OF

$$\begin{aligned} y-1 &= -(x-(-2)) \\ y-1 &= -x-2+1 \\ y &= -x-1 \end{aligned}$$

1.7

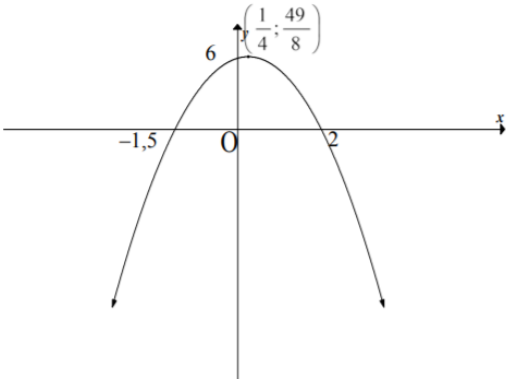


y-intercept/afsnit:

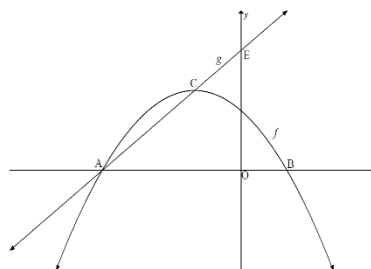
$$g(x) = 2^{-0} - 4 = -3$$

x-intercept/afsnit:

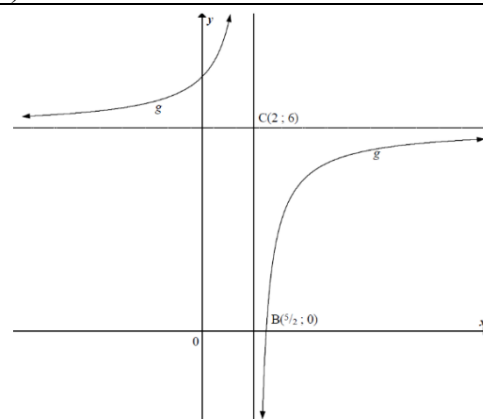
$$\begin{aligned} 0 &= 2^{-x} - 4 \\ 4 &= 2^{-x} \\ 2^2 &= 2^{-x} \\ \therefore x &= -2 \end{aligned}$$

QUESTION 2 / VRAAG 2 (GR 11 DBE November 2016)			
Given $f(x) = -2x^2 + x + 6$			
2.1	$x = -\frac{b}{2a}$ $x = -\frac{1}{2(-2)}$ $x = \frac{1}{4}$ $y = -2\left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right) + 6$ $y = \frac{49}{8}$ $\therefore \left(\frac{1}{4}; \frac{49}{8}\right)$	2.2	$y\text{-intercept/afsnit:}$ $f(x) = -2(0)^2 + (0) + 6 = 6$ $\therefore (0; 6)$
		2.3	$x\text{-intercept/afsnit:}$ $0 = -2x^2 + x + 6$ $0 = 2x^2 - x - 6$ $0 = (2x + 3)(x - 2)$ $x = -\frac{3}{2}; x = 2$ $\therefore \left(-\frac{3}{2}; 0\right) \text{ and / en } (2; 0)$
2.4			
2.5	$k = \frac{49}{8}$		
2.6	New turning point / nuwe draaipunt $\left(\frac{9}{4}; \frac{57}{8}\right)$ $y = -2\left(x - \frac{9}{4}\right)^2 + \frac{57}{8}$		

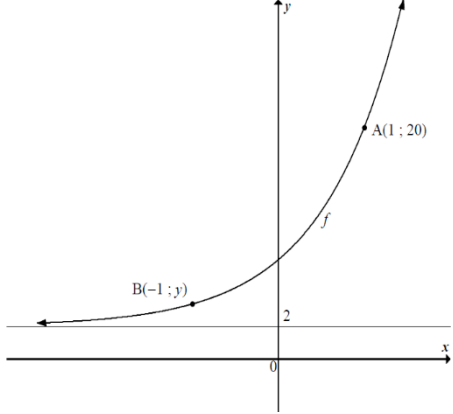
QUESTION 3 / VRAAG 3 (NSC DBE 2016)			
3.1	(0 ; 3)		
3.2	$x = -\frac{b}{2a}$ $x = -\frac{(-2)}{2(-1)}$ $x = -1$ $y = -(-1)^2 + (-1) + 6$ $y = 4$ $\therefore C(-1 ; 4)$	3.3	B(1 ; 0) By symmetry / <i>Deur simmetrie</i> A(-3; 0) OR / OF $0 = -x^2 - 2x + 3$ $0 = x^2 + 2x - 3$ $0 = (x + 3)(x - 1)$ $x = -3 ; x = 1$ $\therefore A(-3; 0)$
3.4	A(-3; 0) and C(-1 ; 4) $m = \frac{y_1 - y_2}{x_1 - x_2}$ $m = \frac{4 - 0}{-1 - (-3)}$ $m = 2$ $\therefore y = 2x + q$ <i>subst</i> (-3; 0) $0 = 2(-3) + q$ $6 = q$ $\therefore y = 2x + 6$		



QUESTION 4 / VRAAG 4 (GR11 DBE NOV 2015)	
4.1	$g(x) = \frac{a}{x-2} + 6$ <i>subst</i> $\left(\frac{5}{2}; 0\right)$ $0 = \frac{a}{\frac{5}{2}-2} + 6$ $-6 = \frac{a}{\frac{1}{2}}$ $-3 = a$ $\therefore g(x) = \frac{-3}{x-2} + 6$
4.2	increasing function. / <i>stygende funksie</i>
4.3	Domain / Definisieversanemling: $x \in R ; x \neq 2$ Range / waardeversameling: $y \in R ; y \neq 6$
4.4	$2 < x \leq \frac{5}{2}$



QUESTION 5 / VRAAG 5 (GR11 DBE NOV 2016)

5.1	$f(x) = 2 \cdot b^{x+1} + q$ $f(x) = 2 \cdot b^{x+1} + 2$ $subst. (1 ; 20)$ $20 = 2 \cdot b^{1+1} + 2$ $18 = 2 \cdot b^2$ $9 = b^2$ $\therefore b = 3$ $f(x) = 2 \cdot (3)^{x+1} + 2$			
5.2	$f(x) = 2 \cdot (3)^{x+1} + 2$ Subst. $(-1 ; y)$ $y = 2 \cdot (3)^{-1+1} + 2$ $y = 4$	5.4	$f(0) = 2 \cdot (3)^{0+1} + 2$ $y = 2 \cdot 3 + 2 = 8$ $(0 ; 8)$	
5.3	$y \in R ; y > 2$	5.5	increasing function / <i>stygende funksie</i> .	
5.4	$f(0) = 2 \cdot (3)^{0+1} + 2$ $y = 2 \cdot 3 + 2 = 8$ $(0 ; 8)$			
5.5	increasing function / <i>stygende funksie</i> .			

SECTION B / AFDELING B

QUESTION 1 / VRAAG 1 (GR11 DBE NOV 2015)

1.1

$$y = a(x - 2)^2 + 9$$

$$\text{Subst. } (0; 5)$$

$$5 = a(0 - 2)^2 + 9$$

$$-4 = a(4)$$

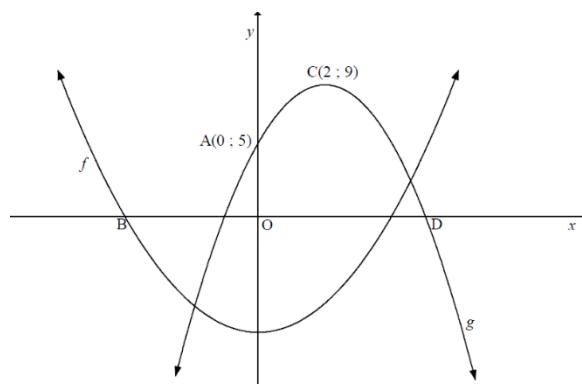
$$-1 = a$$

$$y = -1(x - 2)^2 + 9$$

$$y = -1(x^2 - 4x + 4) + 9$$

$$y = -x^2 + 4x - 4 + 9$$

$$\therefore g(x) = -x^2 + 4x + 5$$



1.2

$$g(x) = -x^2 + 4x + 5$$

$$0 = -x^2 + 4x + 5$$

$$0 = x^2 - 4x - 5$$

$$0 = (x - 5)(x + 1)$$

$$x = 5; x = -1$$

$$\therefore D(5; 0)$$

$$f(x) = \frac{1}{2}x^2 - 8$$

$$0 = \frac{1}{2}x^2 - 8$$

$$0 = x^2 - 16$$

$$0 = (x - 4)(x + 4)$$

$$x = 4; x = -4$$

$$\therefore B(-4; 0)$$

$$BD = 4 + 5 = 9 \text{ unit / eenhede}$$

1.3

$$x = 0$$

1.4

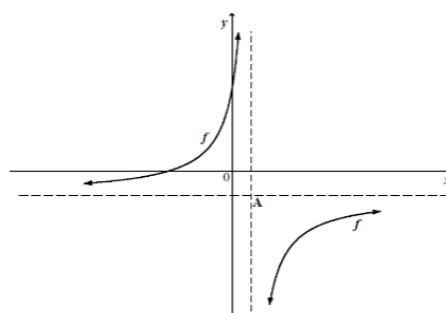
$$y \in R; y \leq 10$$

1.5.1

$$x \leq -4 \text{ or / of } x \geq 4$$

1.5.2

$$0 < x < 2$$



QUESTION 2 / VRAAG 2 (GR11 DBE NOV 2015)

$$f(x) = \frac{-9}{x-1} - 2$$

2.1

$$A(1; -2)$$

2.2

$$x \in R; x \neq 1$$

2.3

$$y = -3$$

2.4

y-intercept/afsnit:

$$y = \frac{-9}{0-1} - 2$$

$$y = 9 - 2 = 7$$

$$\therefore (0; 7)$$

x-intercept/afsnit:

$$0 = \frac{-9}{x-1} - 2$$

$$2 = \frac{-9}{x-1}$$

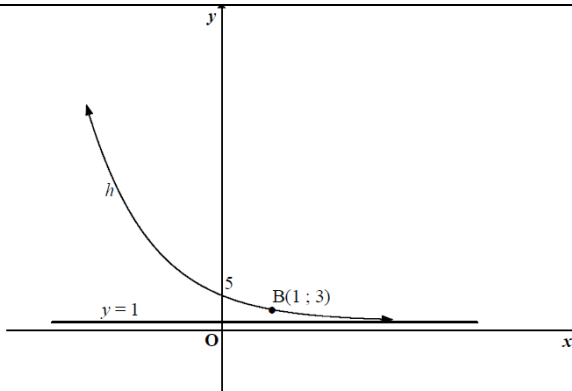
$$2x - 2 = -9$$

$$2x = 7$$

$$x = \frac{7}{2}$$

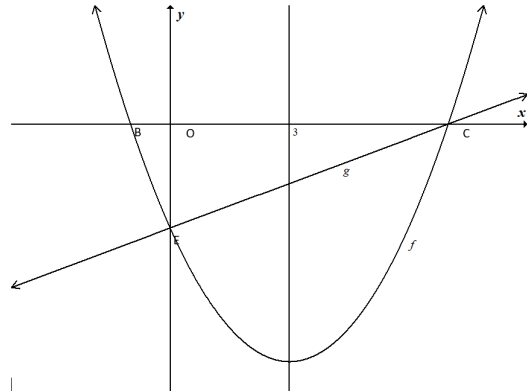
$$\therefore \left(\frac{7}{2}; 0\right)$$

2.5	$y = -x + c$ <i>subst.</i> (1 ; -2) $-2 = -(1) + c$ $-1 = c$ $\therefore y = -x - 1$
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QUESTION 3 / VRAAG 3 (GR11 DBE NOV 2015)	
Given $h(x) = 4 \cdot (2^{-x}) + 1$	
3.1	decreasing function / afnemende funksie.
3.2	y –intercept/<i>afsnit</i>: $h(x) = 4 \cdot (2^{-x}) + 1$ $y = 4 \cdot (2^{-0}) + 1$ $y = 5$ $\therefore (0 ; 5)$
3.3	x –intercept/<i>afsnit</i>: $h(x) = 4 \cdot (2^{-x}) + 1$ $0 = 4 \cdot (2^{-x}) + 1$ $-1 = 4 \cdot (2^{-x})$ $\frac{-1}{4} = (2^{-x})$ which is impossible, since $2^{-x} > 0$ for all $x \in R$ <i>wat onmoontlik is, want $2^{-x} > 0$ vir alle $x \in R$</i> Therefore/<i>Dus</i>: no solution/<i>geen oplossing</i>, which means there will be no x-intercept/<i>wat beteken daar</i> <i>sal geen x-afsnit wees nie.</i> OR/OF The graph lies above its asymptote $y = 1$ because the coefficient of 2^{-x} is 4 <i>Die grafiek lê bokant sy asimptoot $y = 1$ want die koëffisiënt van 2^{-x} is 4.</i>
3.4	$y = 1$
3.5	
3.6	$g(x) = 4(2^{-x} + 2)$ $g(x) = 4(2^{-x}) + 8$ The graph of h is translated 7 units upwards to form g <i>Die grafiek van h word 7 eenhede na bo getransleer om g te vorm.</i>

QUESTION 4 / VRAAG 4 (NSC March 2011)

4.1	$g(x) = \frac{x}{2} - \frac{7}{2}$ $x\text{-intercept/afsnit:}$ $0 = \frac{x}{2} - \frac{7}{2}$ $x = 7$ $\therefore C(7; 0)$
4.2	$C(7; 0)$ By symmetry / <i>Deur simmetrie</i> $B(-1; 0)$



4.3	$y = a(x - 7)(x + 1)$ $\text{subst. } E(0; -\frac{7}{2})$ $-\frac{7}{2} = a(0 - 7)(0 + 1)$ $-\frac{7}{2} = -7a$ $a = \frac{1}{2}$ $y = \frac{1}{2}(x^2 - 6x - 7)$ $y = \frac{1}{2}x^2 - 3x - \frac{7}{2}$ $\text{Subst } x = 3$ $y = \frac{1}{2}(3)^2 - 3(3) - \frac{7}{2}$ $y = -8$ TP / DP (3 ; -8) $\therefore y = \frac{1}{2}(x - 3)^2 - 8$	OR / OF $f(x) = \frac{1}{2}(x - 3)^2 + q$ $0 = \frac{1}{2}(7 - 3)^2 + q$ $q = -8$ $y = \frac{1}{2}(x - 3)^2 - 8$ OR / OF $f(x) = \frac{1}{2}(x + 1)(x - 7)$ $= \frac{1}{2}(x^2 - 6x - 7)$ $= \frac{1}{2}[(x - 3)^2 - 16]$ $= \frac{1}{2}(x - 3)^2 - 8$
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4.4	$h(x) = -f(x)$ $h(x) = -\frac{1}{2}x^2 + 3x + \frac{7}{2}$ OR / OF $\therefore h(x) = -\frac{1}{2}(x - 3)^2 + 8$
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4.5	Maximum value is / <i>maksimumwaarde is</i> : 9
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TOPIC: ANALYTICAL GEOMETRY MEMORANDUM
ONDERWERP: ANALITIESE MEETKUNDE MEMORANDUM

SECTION A / AFDELING A		
QUESTION 1 / VRAAG 1		
SUGGESTED ANSWER/ VOORGESTELDE ANTWOORD		Marks/ Punte
1.1	$PR = \sqrt{(3+1)^2 + (0+2)^2}$ $= \sqrt{20} \text{ or } 2\sqrt{5}$	(2)
1.2	$SP = \sqrt{(-5+1)^2 + (8+2)^2}$ $= \sqrt{116} \text{ or } 10,77$	(2)
1.3	Kite – adjacent sides are equal Vlieër- aanliggende sye is gelyk	(2)
1.4	$M(x; y) = \left(\frac{-1+5}{2}; \frac{-2+4}{2} \right)$ $= (2; 1)$	(2)
1.5	$m_{SM} = \frac{8-1}{-5-2}$ $= -1$ $m_{MR} = \frac{1-0}{2-3}$ $= -1$ $\therefore$ S, M and R are collinear (same gradient and common point) Kolinier (dieselfde gradient and gemene punt)	(3)
		[11]

QUESTION 2 / VRAAG 2		
	SUGGESTED ANSWER/ VOORGESTELDE ANTWOORD	Marks/ Punte
2.1	$p = -\frac{1}{2}(2) + 3 = 2$	(2)
2.2	$m_{AD} = \frac{1-2}{4-2} = -\frac{1}{2}$ $m_{AB} = \frac{0-2}{1-2} = 2$ $m_{AD} \times m_{AB} = -\frac{1}{2} \times 2$ $= -1$ $\therefore AD \perp AB$	(3)
2.3	$m_{CD} = \frac{1+3}{4-2} = 2$ $y = 2x + c$ $1 = 2(4) + c$ $c = -7$ $y = 2x - 7$ $2x - 7 = 0$ $x = 3\frac{1}{2}$ $E\left(3\frac{1}{2}; 0\right)$	(4)

2.4	$\tan \theta = 2$ $\theta = 63,43^\circ$ $\therefore \hat{E} = 63,43^\circ \text{ (vertically opposite } \angle s \text{)}$ $m_{BC} = \frac{0+3}{1-2} = -3$ $\tan \hat{FBE} = -3$ $\hat{FBE} = 108,43^\circ$ $\therefore \beta = 45^\circ$	(6)
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2.5	Trapezium – one pair of opposite sides parallel Tapesium – een paar teenoorstaande sye parallel.	(2)
		[17]

QUESTION 3 / VRAAG 3		
3.1	$BD = -\frac{1}{2}x + 9$ $\therefore m_{BD} = -\frac{1}{2}$ $\therefore m_{AC} = 2$	(2)
3.2	$y - y_1 = m(x - x_1)$ $y - (-5) = 2(x - (-2))$ $y = 2x - 1$	(2)
3.3	$2x - 1 = -\frac{1}{2}x + 9$ $\frac{5}{2}x = 10$ $x = 4$ $y = 2(4) - 1$ $y = 7$ $T = (4; 7)$	(3)
3.4.1	$4 = \frac{-2+x}{2}$ $8 = -2 + x$ $x = 10$ $7 = \frac{-5+y}{2}$ $14 = -5 + y$ $y = 19$ $C(10; 19)$	(2)
3.4.2	$AT = \sqrt{(4 - (-2))^2 + (7 - (-5))^2}$ $= \sqrt{180}$ $= 6\sqrt{5}$ $BT^2 + AT^2 = AB^2$ (Pythagoras) $BT = \sqrt{15^2 + (\sqrt{180})^2} = 3\sqrt{5}$	(4)
3.4.3	BC is the diameter/ middellyn [subt.right $\angle$ /ondersp.reg $\angle$] or / of [conv. $\angle$ in semi-circle / omgk. $\angle$ in halfsirkel] Radius = $\frac{15}{2} = 7,5$ units / eenhede	(2)
		15
QUESTION 4 / VRAAG 4		
4.1	$m = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$ $m = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$ $m = \left(4; \frac{13}{2} \right)$	(2)

4.2	$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{15 - 3}{2 - (-4)}$ $= 2$ $m_{MN} = m_{BC} = 2 \quad [BC \parallel MN]$	(2)
4.3	$y - y_1 = m(x - x_1) \quad \text{or / of} \quad y = mx + c$ $y = \frac{13}{2} = 2(x - 4) \quad \frac{13}{2} = 2(4) + c$ $y = 2x - \frac{3}{2} \quad -\frac{3}{2} = c$ $y = 2x - \frac{3}{2}$	(2)
4.4	N is a midpoint of AC N is die middelpunt van AC [Line through midpoint of one side parallel to second side] [Lyn deur middelpunt van een sy parallel aan die tweede sy] $N\left(\frac{-4+6}{2}; \frac{3+(-2)}{2}\right)$ $N\left(1; \frac{1}{2}\right)$ or / of $m_{AC} = \frac{3 - (-2)}{(-4) - 6}$ $= -\frac{1}{2}$ Equation of AC $y - y_1 = m(x - x_1)$ $y - 3 = -\frac{1}{2}(x - (-4))$ $y = -\frac{1}{2}x + 1$ $-\frac{1}{2}x + 1 = 2x - \frac{3}{2}$ $-x + 2 = 4x - 3$ $x = 1$ $y = 2(1) - \frac{3}{2} = \frac{1}{2}$ $N = \left(1; \frac{1}{2}\right)$	(4)
4.5	N is the midpoint of BD and the midpoint of AC [diagonals of parm bisect] N is die midpnt van BD en midpt van AC [hoeklyne van parm halveer] $\left(\frac{2+x}{2}; \frac{y+15}{2}\right) = \left(1; \frac{1}{2}\right)$ $\frac{2+x}{2} = 1 \quad \frac{y+15}{2} = \frac{1}{2}$ $x = 0 \quad y = -14$ $D = (0; -14)$	(4)
Question 5		
5.1	B (6; 5) C (0; -3) $m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-3 - 5}{0 - 6} = \frac{-8}{-6} = \frac{4}{3}$	(2)

5.2	$m_{AD} = m_{BC} = \frac{4}{3} (AD \parallel BC)$ $y = \frac{4}{3}x + c$ $2 = \frac{4}{3}(-2) + c$ $\frac{14}{3} = c$ $\therefore y = \frac{4}{3}x + \frac{14}{3}$	(3)
5.3	$y = \frac{4}{3}x + \frac{14}{3}$ $0 = \frac{4}{3}t + \frac{14}{3}$ $-\frac{14}{3} = \frac{4}{3}t$ $t = -\frac{14}{4} = -\frac{7}{2}$	(2)
5.4	$AN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{((-2) - (-\frac{7}{2}))^2 + (2 - 0)^2}$ $= \sqrt{\frac{25}{4}}$ $= \frac{5}{2}$	(2)
5.5	$\frac{3}{8}x - 3 = \frac{4}{3}x + \frac{14}{3}$ $\frac{23}{24}x = -\frac{23}{3}$ $x = -8$ $y = \frac{4}{3}(-8) + \frac{14}{3}$ $y = -6$ $D(-8; -6)$	(4)
5.6	$m_{AB} = \frac{5 - 2}{6 - (-2)} = \frac{3}{8}$ $m_{AB} = m_{DC}$ $\therefore AB \parallel DC$ But / maar $AD \parallel BC$ $\therefore ABCD$ is a parallelogram [opp sides are $\parallel$ teenoorstaande sye is $\parallel$ or / of M is the midpoint of AC	

	M is die middelpunt $M\left(\frac{(-2) + 0}{2}; \frac{2 + (-3)}{2}\right)$ $M = \left(-1; \frac{1}{2}\right)$ M is the midpoint of BD / M is die middelpunt van BD $M\left(\frac{(-8) + 6}{2}; \frac{(-6) + 5}{2}\right)$ $M = \left(-1; \frac{1}{2}\right)$ $\therefore ABCD$ is a parallelogram [diagonals bisect each other] [Hoeklyne halveer mekaar]	(3)
5.7	M is the midpoint of AC [diagonals bisect] M is die middelpunt van AC [hoeklyne halveer mekaar] $M\left(\frac{(-2) + 0}{2}; \frac{(2) + (-3)}{2}\right)$ $M = \left(-1; -\frac{1}{2}\right)$	(3)
		[19]

SECTION B / AFDELING B		
QUESTION 1 / VRAAG 1		
SUGGESTED ANSWER/ VOORGESTELDE ANTWOORD		Marks
1.1	$m_{BG} = \frac{-4 - 4}{-3 - 1}$ $= 2$	(2)
1.2	$y = 2x + c$ $\text{sub } (-3; -4)$ $-4 = 2(-3) + c$ $\therefore c = 2$ $y = 2x + 2$ Of/ Or $y = 2x + c$ $\text{sub}(1; 4)$ $4 = 2(1) + c$ $\therefore c = 2$ $y = 2x + 2$ Of/ Or $y - y_1 = m(x - x_1)$ $y + 4 = 2(x - (-3))$ $y = 2x + 2$ Or /Of $y - y_1 = m(x - x_1)$ $y - 4 = 2(x - 1)$ $y = 2x + 2$	(2)
1.3.1	$m_{BC} = -\frac{1}{2}$ $-\frac{1}{2} = \frac{k - (-4)}{2 - (-3)}$ $\frac{1}{2} = \frac{k + 4}{5}$ $-5 = 2k + 8$ $2k = -13$ $k = -\frac{13}{2} = -6\frac{1}{2}$	(4)

1.3.2	By translation / Deur translasie $(x; y) \rightarrow \left(x + 5; y - 2\frac{1}{2}\right)$ $(1; 4) \rightarrow \left(6; 1\frac{1}{2}\right)$ $D = \left(6; \frac{3}{2}\right)$ Or / Of $\frac{y - \left(-\frac{13}{2}\right)}{x - 2} = \frac{2}{1}$ $y = 2x - \frac{21}{2}$ $\frac{y - 4}{x - 1} = -\frac{1}{2}$ $2y - 8 = -x + 1$ $2\left(2x - \frac{21}{2}\right) - 8 = -x + 1$ $4x - 21 - 8 = -x + 1$ $\therefore x = 6$ $y = 2(6) - \frac{21}{2}$ $\therefore y = \frac{3}{2}$ $D\left(6; \frac{3}{2}\right)$	(3)
1.3.3	$m_{CD} = 2$ $\tan \angle DHX = 2$ $\angle DHX = \tan^{-1}(2)$ $\angle DHX = 63,49^\circ$ $\beta = 63,49^\circ$ [vertically opposite angles / vertikaal teenoorgestelde hoeke]	(3)

1.3.4	$y = 2x + c$ $-\frac{13}{2} = 2(2) + c$ $-\frac{21}{2} = c$ $y = 2x - \frac{21}{2}$ $0 = 2x - \frac{21}{2}$ $\therefore x = 5,25$ $GH = 6,25$ $\text{Area/Oppvl} = \frac{1}{2} GH \cdot y_D$ $= \frac{1}{2} \times 6,25 \times 1,5 = 4,69 \text{ sq units / vierkante eenhede}$	(7)
		21

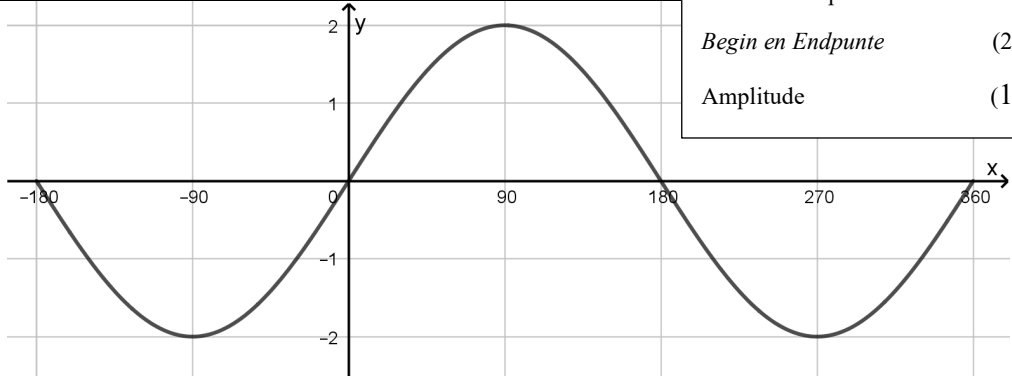
TOPIC: TRIGONOMETRY GRAPHS

ONDERWERP TRIGONOMETRIESE GRAFIEKE

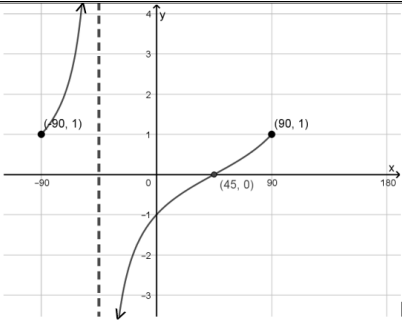
SECTION/ AFDELING A

Question/ Vraag 1

1.	d)	5.	a)
2.	c)	6.	c)
3.	e)	7.	a)
4.	b)	8.	d)

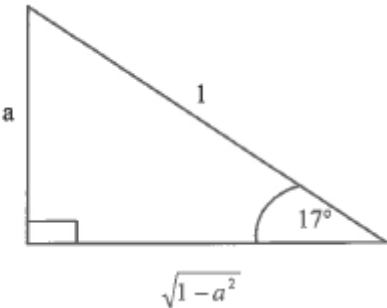
Question/ Vraag 2		Turning Points /Draaipunte (1)
2.1		Shape/ Vorm (1)
		Start and endpoints
		Begin en Endpunte (2)
		Amplitude (1)
2.2	$y \in [-4; 4]$	(2)
2.2	360°	(1)
2.3	$\theta = -90^\circ$	(2)
		[10]

Question/ Vraag 3		
3.1	180°	(1)
3.2	$A(45^\circ; 0)$	(1)
3.3	$x \in (-90^\circ; 0^\circ)$ or / of $-90^\circ < x < 0^\circ$	(2)
3.4	3.4.1 0,76	(1)
	3.4.2 0,76	(1)
3.5	$(20^\circ; 0,76)$	(2)
3.6	$B(-60^\circ; 20^\circ)$	(2)
3.7	$h(x) = \cos x$	(2)
		[12]
Question/ Vraag 4		
4.1	$p = 45^\circ ; q = -1$	(2)
4.2	$B(157,5^\circ ; -0,38)$	(2)
4.3	$x \in (-180^\circ; -22,5^\circ)$ or/of $-180 < x < -22,5^\circ$ for/vir $x \in \mathbb{R}$.	(3)
		[7]
Question/ Vraag 5		
5.1		(6)
5.2	120°	(1)
5.3	1	(1)
5.4	$y \in [-2; 2]$	(1)
5.5	$y = -2 \sin(x - 30^\circ)$	(1)
		[10]

SECTION/ AFDELING B		
Question/ Vraag 1		
1.1	$y \in [-3; 3]$	(2)
1.2	$c = 2$	(1)
1.3	$a = 3$ and $b = 2$	(2)
1.4	$Q(60^\circ; -1,6)$	(2)
1.5	$y = -3 \sin 2x$	(2)
1.6	$-45^\circ < x < 45^\circ$	(3)
		[12]
Question/ Vraag 2		
2.1	$p = 30^\circ$ and $q = -\frac{1}{2}$	(2)
2.2	$-120^\circ < x < 0^\circ$	(2)
2.3	$h(x) = -\cos x - \frac{1}{2}$	(1)
		[5]
Question/ Vraag 3		
3.1	$a = 2$	(1)
3.2	Period $f: 360^\circ$ $g: 90^\circ$	(2)
3.3	$Q(165^\circ; -\sqrt{3})$	(2)
3.3	3.3.1 $0^\circ < x < 90^\circ$ 3.3.2 $180^\circ < x < 235^\circ$	(1) (2)
Question/ Vraag 4		
4.1	$a = 1, b = 2$ and $p = -45^\circ$	(3)
4.3	$-90^\circ < x < 0^\circ$	(2)
4.4	-2	(1)
4.5	Reflected with respect to x - axis	(2)
		[8]
Question/ Vraag 5		
5.1		(6)
5.2	180°	(1)
5.3	$x = -45^\circ$	(1)
5.4	Reflection with respect to x - axis	(1)
		[9]

TOPIC: TRIGONOMETRY

ONDERWERP: TRIGONOMETRIE

QUESTION/ VRAAG 1.		
1.1.1	$x^2 + y^2 = r^2$ $(-8)^2 + (t)^2 = 17^2$ $t^2 = 225$ $t = -15$	✓ sub in pyth ✓ answer (2)
1.1.2 a	$\cos(-\theta)$ $= \cos \theta$ $= \frac{-8}{17}$	✓ $\cos \theta$ ✓ answer (2)
1.1.2 b	$1 - \sin \theta = 1 - \frac{-15}{17}$ $= \frac{17}{17} + \frac{15}{17}$ $= \frac{32}{17}$	✓ sub CA ✓ answer (2)
1.2.1	$\tan 17^\circ = \frac{a}{\sqrt{1-a^2}}$ 	✓ Sketch/Pythagoras ✓ Skets/ Pythagoras ✓ $\sqrt{1-a^2}$ ✓ CA answer/antw Answer only : Full marks/ Slegs antwoord: Vol punte (3)
1.2.2	$\sin 107^\circ$ $= \sin(90^\circ + 17^\circ)$ $= \cos 17^\circ$ $= \sqrt{1-a^2}$ OR/OF $\sin 107^\circ$ $= \sin(180^\circ - 73^\circ)$ $= \sin 73^\circ$ $= \sqrt{1-a^2}$	✓ $\cos 17^\circ$ ✓ CA $\sqrt{1-a^2}$ ✓ $\sin 73^\circ$ ✓ CA $\sqrt{1-a^2}$ (2)
1.2.3	$\cos^2 253^\circ + \sin^2 557^\circ$ $= (-\cos 73^\circ)^2 + (-\sin 17^\circ)^2$ $= (-a)^2 + (-a)^2$ $= 2a^2$	✓ $\cos^2 73^\circ$ ✓ $\sin^2 17^\circ$ ✓ subst of ratios/ subst van trig verhoudings ✓ answer/antw (4)

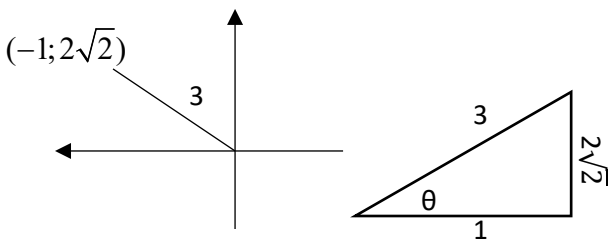
1.3	$\frac{\cos(180^\circ + 45^\circ)\sin(180^\circ - 45^\circ) + \sin(360^\circ - 30^\circ)}{\tan(180^\circ + 45^\circ)}$ $= \frac{(-\cos 45^\circ)(\sin 45^\circ) - \sin 30^\circ}{\tan 45^\circ}$ $= \frac{\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2}}{1}$ $= -1$	✓ $-\cos 45^\circ$ ✓ $\sin 45^\circ$ ✓ $-\sin 30^\circ$ ✓ $\tan 45^\circ$ ✓ Special angle ratios <i>Speciale hoeke</i> ✓ CA answer/antw (6)
1.4	$RHS = \frac{-1}{\tan^2 x \cos^2 x}$ $= \frac{-1}{\frac{\sin^2 x}{\cos^2 x} \times \cos^2 x}$ $= \frac{-1}{\sin^2 x}$ $= \frac{-1}{1 - \cos^2 x}$ $= \frac{1}{\cos^2 x - 1}$ $= \frac{1}{(\cos x + 1)(\cos x - 1)}$ $= LHS$	✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ simplification/vereen ✓ identity/ identiteit ✓ factors /faktore
1.5	$2\sin x \cos x - \cos x = 0$ $\cos x(2\sin x - 1) = 0$ $\cos x = 0 \quad \text{or} \quad \sin x = \frac{1}{2}$ $x = 90^\circ + 360^\circ k, k \in \mathbb{Z} \quad x = 30^\circ + 360^\circ k, k \in \mathbb{Z} \quad \text{or}$ $x = 270^\circ + 360^\circ k, k \in \mathbb{Z} \quad x = 150^\circ + 360^\circ k, k \in \mathbb{Z}$ or $x = 90^\circ + 180^\circ k, k \in \mathbb{Z}$ or $x = \pm 90^\circ + 360^\circ k, k \in \mathbb{Z}$	✓ factors / faktore ✓ both equations/ <i>beide verg.</i> ✓ BOTH general solutions for $\cos x = 0$ / <i>Altwee algemene</i> <i>oplossings vir</i> $\cos x = 0$ ✓✓ general solutions for $\sin x = \frac{1}{2}$ <i>algemene</i> <i>oplossings vir</i> $\sin x = \frac{1}{2}$ ✓ $k \in \mathbb{Z}$ (6) [22]

QUESTION/ VRAAG 2.		
2.1.1	$x^2 + y^2 = r^2$ $x^2 + (24)^2 = 25^2$ $x^2 = 49$ $x = -7$	✓ subst ✓ answer/antw (2)
2.1.2a	$\sin \beta = \frac{24}{25}$	✓ answer/antw
2.1.2b	$\cos(180^\circ - \beta)$ $= -\cos \beta$ $= -\left(-\frac{7}{25}\right)$ $= \frac{7}{25}$	✓ reduction/reduksie ✓ answer/antw (2)
2.1.2c	$\tan(-\beta)$ $= -\tan \beta$ $= -\left(\frac{24}{-7}\right)$ $= \frac{24}{7}$	✓ reduction/reduksie ✓ answer/antw (2)
2.1.3	$\sin \beta = \frac{24}{25} = \frac{y_T}{15}$ $\therefore y_T = 14\frac{2}{5}$ $\cos \beta = -\frac{7}{25} = \frac{x_T}{15}$ $\therefore x_T = -4\frac{1}{5}$ $\therefore T\left(-4\frac{1}{5} ; 14\frac{2}{5}\right)$	✓ equating ratios/ stel verh's gelyk ✓ y_T ✓ equating ratios/ stel verh's gelyk ✓ x_T (4)

2.2	$\frac{2 \sin x \cdot \cos x \cdot (1 + \frac{\sin^2 x}{\cos^2 x})}{\frac{\sin x}{\cos x}}$ $= 2 \sin x \cdot \cos x \cdot (\frac{\cos^2 x + \sin^2 x}{\cos^2 x}) \times \frac{\cos x}{\sin x}$ $= 2 \cos^2 x (\frac{1}{\cos^2 x})$ $= 2$ OR/OF $\frac{2 \sin x \cdot \cos x \cdot (1 + \frac{\sin^2 x}{\cos^2 x})}{\frac{\sin x}{\cos x}}$ $= 2 \sin x \cdot \cos x \cdot (1 + \frac{\sin^2 x}{\cos^2 x}) \times \frac{\cos x}{\sin x}$ $= 2 \cos^2 x (1 + \frac{\sin^2 x}{\cos^2 x})$ $= 2(\cos^2 x + \sin^2 x)$ $= 2$	✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ identity/identiteit ✓ simplify/vereenv ✓ answer/antw (4) ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ simplify/vereenv ✓ identity/identiteit ✓ answer/antw (4)
2.3.1	$\frac{1 - \cos^2 A}{4(-\sin A)}$ $= \frac{\sin^2 A}{-4\sin A}$ $= -\frac{1}{4}\sin A$	✓ $-\sin A$ ✓ identity/identiteit ✓ answer/antw (3)
2.3.2	$-\frac{1}{4}\sin 2x = 0,21$ $\sin 2x = -0,84$ ref/verw $\angle = 57,14^\circ$ $\therefore 2x = 237,14^\circ + k.360^\circ$ or $2x = 302,86^\circ + k.360^\circ$ $x = 118,57^\circ + k.180^\circ$ or $x = 151,43^\circ + k.180^\circ ; k \in \mathbb{Z}$	✓ correct equation/ korrekte vgl ✓ $-0,84$ ✓ ref/verw $\angle$ $= 57,14^\circ$ ✓ $237,14^\circ$ & $302,86^\circ$ ✓ $118,57^\circ$ & $151,43^\circ$ ✓ $k.180^\circ ; k \in \mathbb{Z}$ (6)
QUESTION/ VRAAG 3		
3.1	$\sin(90^\circ - x) \cdot \cos(180^\circ + x) + \tan x \cdot \cos x \cdot \sin(x - 180^\circ)$ $= \cos x \cdot (-\cos x) + \frac{\sin x}{\cos x} \cdot \cos x \cdot (-\sin x)$ $= -\cos^2 x - \sin^2 x$ $= -(\cos^2 x + \sin^2 x)$ $= -1$	✓ $\cos x$ ✓ $-\cos x$ ✓ $\frac{\sin x}{\cos x}$ ✓ $-\sin x$ ✓ common factor/gemene fakt. ✓ identity/identiteit (6)

3.2	$\text{LHS} = \frac{\sin 315^\circ \cdot \tan 210^\circ \cdot \sin 190^\circ}{\cos 100^\circ \cdot \sin 120^\circ}$ $= \frac{(-\sin 45^\circ)(\tan 30^\circ)(-\sin 10^\circ)}{(-\sin 10^\circ)(\sin 60^\circ)}$ $= \frac{-\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{3}}}{\frac{\sqrt{3}}{2}}$ $= -\frac{\sqrt{2}}{3}$	✓ $-\sin 45^\circ$ ✓ $\tan 30^\circ$ ✓ $-\sin 10^\circ$ ✓ $-\sin 10^\circ$ ✓ $\sin 60^\circ$ ✓ subst. of special angles/invert. van sp hoeke (6)
3.3.1	$x^2 + y^2 = r^2$ [Pythagoras] $(x)^2 + (3)^2 = 5^2$ $x^2 = 16$ $x = 4$ $\tan \alpha = \frac{3}{4}$	✓ subst in pyth ✓ $x = 4$ ✓ answer/antw (3)
3.3.2	$\sin \beta$ $= \sin(180^\circ + \alpha)$ $= -\sin \alpha$ $= \frac{-3}{5}$	✓ $\beta = 180^\circ + \alpha$ ✓ $-\sin \alpha$ ✓ answer/antw (3)
3.3.3	$\frac{y}{10k} = \frac{-3k}{5k}$ $y = -6k$ $\therefore x = -8k$ $P(-8k; -6k)$	✓ $\frac{y}{10}$ OR/OR ✓ $\frac{y}{10k}$ ✓ $\frac{-3}{5}$ ✓ $\frac{-3k}{5k}$ ✓ $y = -6$ ✓ $y = -6k$ ✓ $x = -8$ ✓ $x = -8k$ (4)

4.3.2	Undefined if/ongedefinieerd as: $\sin x = -1$ or $\cos x = 0$ $\therefore x = 90^\circ ; 270^\circ$	✓ 90° ✓ 270° (2)
4.4	$\sin 2x = 4 \cos 2x$ $\tan 2x = 4$ $2x = 75,96^\circ + k \cdot 180^\circ$ $x = 37,98^\circ + k \cdot 90^\circ ; k \in \mathbb{Z}$	✓ $\tan 2x = 4$ ✓ $75,96^\circ$ ✓ $37,98^\circ$ ✓ $k \cdot 90^\circ$ ✓ $k \in \mathbb{Z}$ (5)
4.5.1	$x^2 + y^2 = r^2$ $x^2 + (\sqrt{3})^2 = 2^2$ $x^2 = 1$ $x = \pm 1$ $x = 1$ (since P lies in the 1 st quadrant/aangesien P in die 1 ^{ste} kwadrant lê)	✓ subst ✓ $x = 1$ (2)
4.5.2	$\sin \hat{POT} = \frac{\sqrt{3}}{2}$ $\hat{POT} = 60^\circ$ $\hat{POT} + \alpha = 90^\circ$ $\alpha = 90^\circ - 60^\circ$ $= 30^\circ$	✓ correct ratio/ korrekte verh ✓ 60° ✓ answer/antw (3)

4.5.3	$\sin(-30^\circ) = \frac{b}{20}$ $b = 20 \sin(-30^\circ)$ $b = -10$ $\cos(-30^\circ) = \frac{a}{20}$ $a = 20 \cos(-30^\circ)$ $a = 10\sqrt{3} \quad \text{OR/OF} \quad 17,32$ $Q(10\sqrt{3}; -10) \quad \text{OR/OF} \quad Q(17,32; -10)$ OR/OF $OQ^2 = 400$ $a^2 + b^2 = 400$ $PQ^2 = 2^2 + 20^2$ $PQ^2 = 404$ $(a-1)^2 + (b-\sqrt{3})^2 = 404$ $a^2 - 2a + 1 + b^2 - 2\sqrt{3}b + 3 = 404$ $400 - 2a + 4 - 2\sqrt{3}b = 404$ $2a = -2\sqrt{3}b$ $a = -\sqrt{3}b$ $(-\sqrt{3}b)^2 + b^2 = 400$ $4b^2 = 400$ $b^2 = 100$ $b = -10 \quad (b < 0)$ $a = -\sqrt{3}(-10)$ $a = 10\sqrt{3}$ $\therefore Q(10\sqrt{3}; -10)$	✓ correct ratio/ korrekte verh ✓ $b = 20 \sin(-30^\circ)$ ✓ $b = -10$ ✓ correct ratio/ korrekte verh ✓ $a = 10\sqrt{3} \quad \text{OR} 17,32$ (5) ✓ subst into distance formula/subst in afstandformule ✓ subst into distance formula/subst in afstandformule ✓ $a = -\sqrt{3}b$ ✓ $b = -10$ ✓ $a = 10\sqrt{3} \quad \text{OR} 17,32$ (5) [33]
QUESTION/ VRAAG 5.		
5.1		✓ diagram / <i>diagram</i>
5.1.1	$\tan(180^\circ + \theta)$ $= \tan \theta$ $= -2\sqrt{2}$	✓ reduction / <i>reduksie</i> ✓ answer / <i>antwoord</i> (3)
5.1.2	$3 \sin(\theta - 90^\circ)$ $= 3(-\cos \theta)$ $= 3\left(-\frac{1}{3}\right)$ $= -1$	✓ reduction / <i>reduksie</i> ✓ answer / <i>antwoord</i> (2)

5.2.1	$\frac{\sin(-210^0)}{\cos(300^0)} + \frac{\cos(x+90^0)}{\sin(360^0+x)}$ $\frac{\sin 30^0}{\cos 60^0} + \frac{-\sin x}{\sin x}$ $\frac{1}{\frac{2}{1}} + -1$ $= 1 - 1$ $= 0$	✓✓ sin and cos reduction sin en cos reduksie ✓✓ cos and sin reduction cos en sin reduksie ✓ answer / antwoord (5)
5.2.2	$\sin(360^0 + x) \neq 0$ $360^0 + x \neq 0^0 + k.360^0$ or/of $360^0 + x \neq 180^0 + k.360^0$ $x \neq -360^0; -180^0; 0; 180^0; 360^0$	✓✓ answers / antwoorde (2)
5.3	$\text{LHS/LK} : \tan \theta \sqrt{\frac{1}{\sin^2 \theta} - 1}$ $= \frac{\sin \theta}{\cos \theta} \sqrt{\frac{1 - \sin^2 \theta}{\sin^2 \theta}}$ $= \frac{\sin \theta}{\cos \theta} \sqrt{\frac{\cos^2 \theta}{\sin^2 \theta}}$ $= \frac{\sin \theta}{\cos \theta} \times \frac{\cos \theta}{\sin \theta}$ $= 1$ $= \text{RHS/RK}$	✓ single fraction/enkel breuk ✓ changing tan/verander tan ✓ identity / identiteit ✓ taking square root / Vierkantswortel (4)
5.4	$2\sin^2 \theta = 1 + \sin \theta$ $2\sin^2 \theta - \sin \theta - 1 = 0$ $(2\sin \theta + 1)(\sin \theta - 1) = 0$ $\sin \theta = -\frac{1}{2}$ or/of $\sin \theta = 1$ $\therefore \theta = -30^0 + k.360^0$ or/of $\theta = 210^0 + k.360^0$ OR/OF $\theta = 330^0 + k.360^0$ or/of $\theta = 210^0 + k.360^0$ or/of $\theta = 90^0 + k.360^0; k \in \mathbb{Z}$	✓ standard form / standaardvorm ✓ factorising / faktorisering ✓ solving / los op ✓✓✓ solutions / oplossings Penalise 1 mark if no $k \in \mathbb{Z}$ Penaliseer 1 punt indien geen $k \in \mathbb{Z}$ (6) [22]