



Western Cape
Government

Western Cape Education Department
Directorate: Curriculum FET

MATHEMATICS

REVISION BOOKLET
2025 TERM 2

Grade 11

This revision program is designed to assist you in revising the critical content and skills envisaged/ planned to be covered during the 2nd term. The purpose is to prepare you to understand the key concepts and to provide you with an opportunity to establish the required standard and the application of the knowledge necessary to succeed in the NCS examination.

If you wish to master Mathematics you need to remember:

- ✓ The final answer is by no means the most important in Mathematics. Systematic, detailed and logical layout of every step of your working is the most important.
- ✓ Do not accept the fact that you are careless. Carelessness can be overcome by checking your work. It is important to check the correctness and the validity of every step of your calculations. In this way carelessness is overcome.
- ✓ Never take short cuts in Mathematics by leaving out steps in your working.
- ✓ Despair in Mathematics can destroy your Mathematics. Never give up: try again and again and ... until you get it right. Continually say to yourself: I CAN!!!!
- ✓ The more you practice the better you will become!

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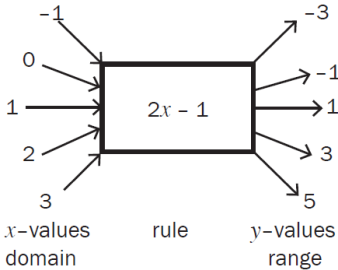
TOPIC: FUNCTIONS

1. CAPS EXTRACTION INDICATING PROGRESSION FROM GRADES 10-12

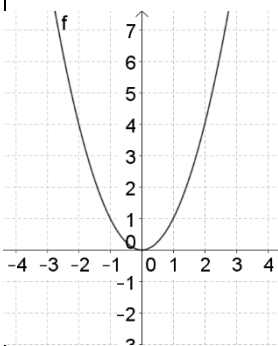
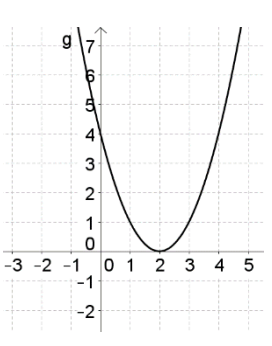
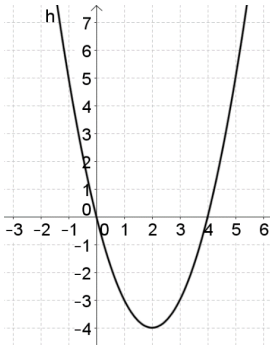
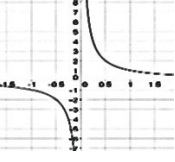
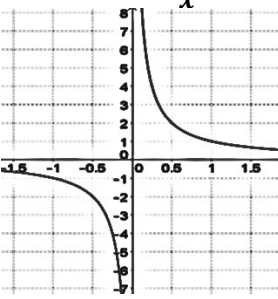
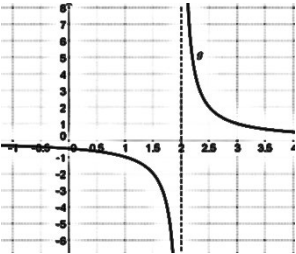
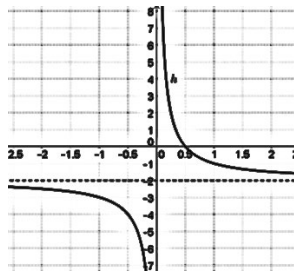
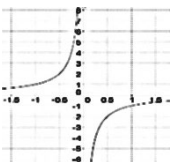
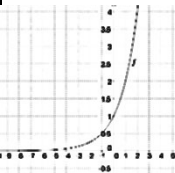
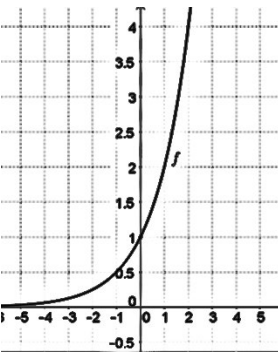
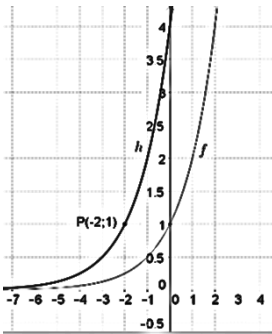
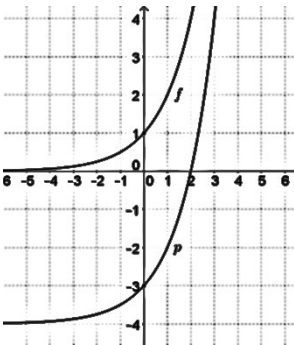
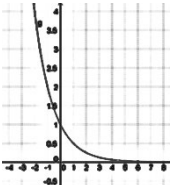
Grade 10	Grade 11
1. The concept of a function, where a certain quantity (output value) uniquely depends on another quantity (input value). Work with relationships between variables using tables, graphs, words and formulae. Convert flexibly between these representations. Note: that the graph defined by $y = x$ should be known from Grade 9. 2. Point by point plotting of basic graphs defined by $y = x^2$, $y = \frac{1}{x}$ and $y = b^x$; $b > 0$ and $b \neq 1$ to discover shape, domain (input values), range (output values), asymptotes, axes of symmetry, turning points and intercepts on the axes (where applicable). 3. Investigate the effect of a and q on the graphs defined by $y = a \cdot f(x) + q$, where $f(x) = x$, $f(x) = x^2$, $f(x) = \frac{1}{x}$ and $f(x) = b^x$, $b > 0, b \neq 1$.	1. Revise the effect of the parameters a and q and investigate the effect of p on the graphs of the functions defined by: 1.1. $y = f(x) = a(x + p)^2 + q$ 1.2. $y = f(x) = \frac{a}{x + p} + q$ 1.3. $y = f(x) = ab^{x+p} + q$ where $b > 0, b \neq 1$ 2. Investigate numerically the average gradient between two points on a curve and develop an intuitive understanding of the concept of the gradient of a curve at a point.

2. FUNCTIONS: TERMINOLOGY

- A **function** is a relation in which each input value (x – *value*) has only one output value (y – *value*).
- Functional notation:** The $f(x)$ **notation** is another way of representing the y –value in a **function**, $y = f(x)$.
Example: $f(x) = 2x - 1$.
If $x = -3$ we can find $f(-3) = 2(-3) - 1 = -7$
- The **domain** is the set of all possible x -values which will make the **function** "work" and will output real y –values.
- The **range** of a function is the possible values of y .



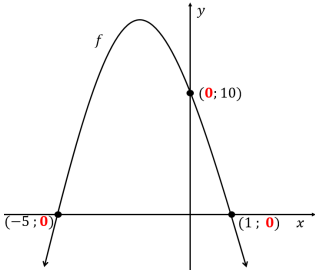
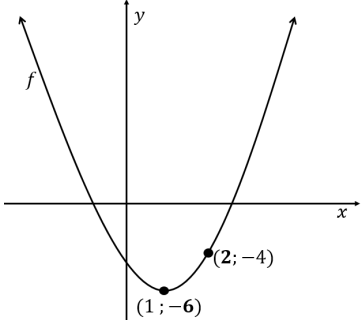
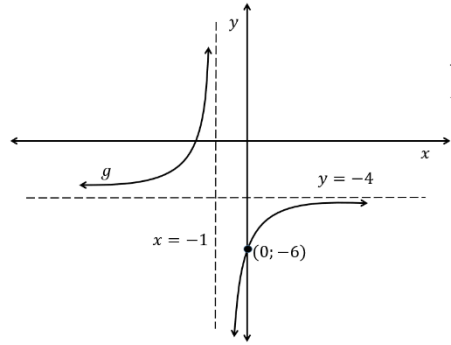
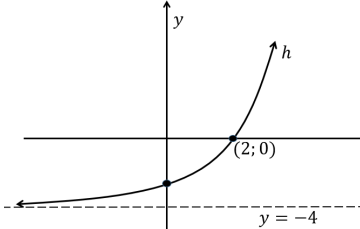
3. MOTHER FUNCTION AND INFLUENCE OF PARAMETERS

	Influence of	Mother function	Influence of p and q	
PARABOLA	$a > 0$ 	$f(x) = x^2$ 	$g(x) = (x - 2)^2$ 	$h(x) = (x - 2)^2 - 4$ 
	$a < 0$ 		Moved to 2 units to the right ($p < 0$)	Moved 4 units down ($q < 0$)
HYPERBOLA	$a > 0$ 1 st and 3 rd quadrant 	$f(x) = \frac{1}{x}$ 	$g(x) = \frac{1}{x - 2}$ 	$h(x) = \frac{1}{x} - 2$ 
	$a < 0$ 2 nd and 4 th quadrant 		Moved to 2 units to the right ($p < 0$)	Moved 2 units down ($q < 0$)
EXPONENTIAL GRAPH	$b > 1$ increasing 	$f(x) = 2^x$ 	$h(x) = 2^{x+2}$ 	$p(x) = 2^x - 4$ 
	$0 < b < 1$ decreasing 		Moved to 2 units to the left ($p > 0$)	Moved 4 units down ($q < 0$)

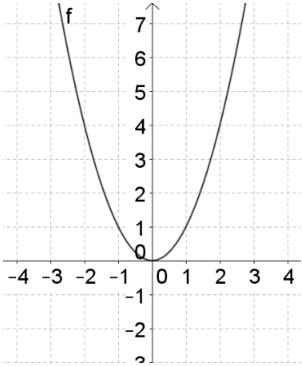
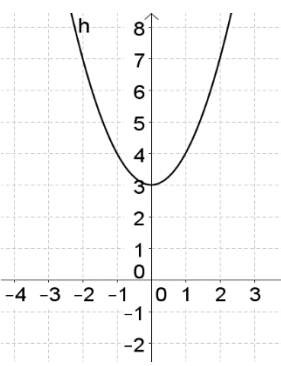
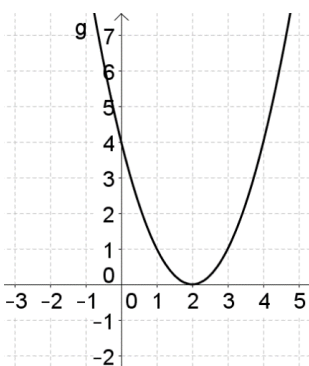
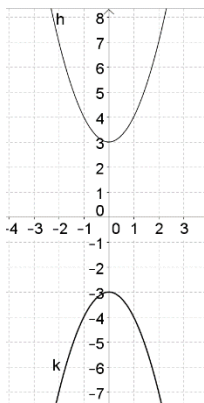
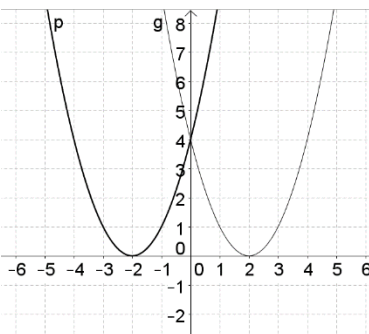
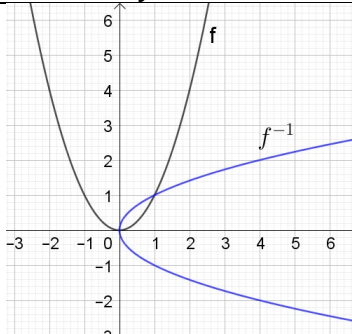
4. SKETCH OF GRAPHS

	Parabola	Hyperbola	Exponential Graph
Standard form	$y = ax^2 + bx + c$ $y = a(x + p)^2 + q$ $y = a(x - x_1)(x - x_2)$	$y = \frac{a}{x + p} + q$	$y = a \cdot b^{x+p} + q$
To sketch the graphs, you need: y - intercept ($x = 0$) x - intercept ($y = 0$) Turning point $\left(-\frac{b}{2a}; -\frac{\Delta}{4a}\right)$ OR $(-p; q)$ Asymptote(s) $x = -p$ $y = q$	Example 1 Sketch $f(x) = -2x^2 - 8x + 10$ OR $f(x) = -2(x + 2)^2 + 18$	Example 2 Sketch $g(x) = \frac{-2}{x+1} - 4$	Example 3 Sketch $h(x) = 2^x - 4$
Domain	$x \in R$	$x \in R; x \neq -1$	$x \in R$
Range	$y \in R; y \leq 18$	$y \in R; y \neq -4$	$y \in R; y > -4$
Symmetry axis	$x = -\frac{b}{2a}$ $x = -2$	$y = x + c$ Substitute $(-p; q) \rightarrow (-1; -4)$ $\therefore -4 = -1 + c$ $c = -3$ $y = x - 3$ OR $y = -x + c$ Substitute $(-p; q) \rightarrow (-1; -4)$ $\therefore -4 = -(-1) + c$ $c = -5$ $y = -x - 5$	

5. TO DETERMINE THE EQUATION OF A GRAPH

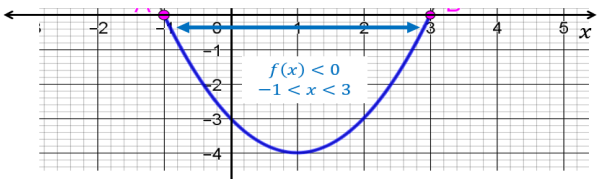
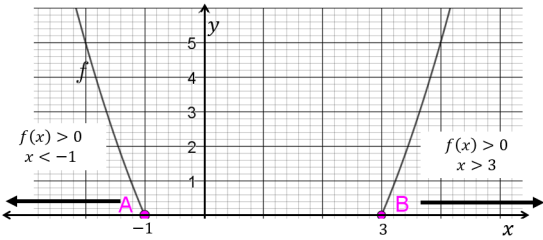
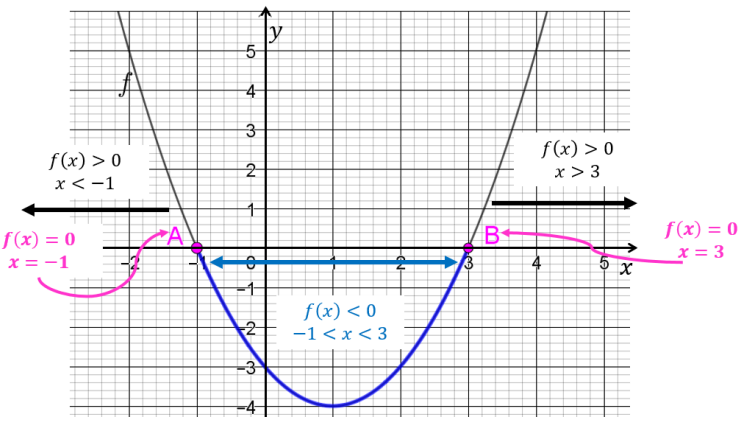
Parabola		Hyperbola	Exponential Graph
$y = ax^2 + bx + c$ $y = a(x + p)^2 + q$ $y = a(x - x_1)(x - x_2)$		$y = \frac{a}{x + p} + q$	$y = a \cdot b^{x+p} + q$
x –intercepts given: $y = a(x - x_1)(x - x_2)$ For a : Substitute another point given on the graph into the equation.	Turning point given: $y = a(x + p)^2 + q$ For a : Substitute a point given on the graph into the equation.	For p and q: Start by substituting the asymptotes into the equation. For a: Substitute a point given on the graph into the equation.	For q: Start by substituting the asymptote into the equation. For any other: Substitute a point given on the graph into the equation.
 $y = a(x + 5)(x - 1)$ Subst. (0; 10) $10 = a(0 + 5)(0 - 1)$ $10 = -5a$ $a = -2$ $y = -2(x + 5)(x - 1)$ $y = -2x^2 - 8x + 10$	 $y = a(x - 1)^2 - 6$ Subst. (2; -4): $-4 = a(2 - 1)^2 - 6$ $2 = a(1)^2$ $a = 2$ $y = 2(x - 1)^2 - 6$ OR $y = 2x^2 - 4x - 4$	 $y = \frac{a}{x + 1} - 4$ subst. (0; -6) $-6 = \frac{a}{0 + 1} - 4$ $-2 = a$ $y = \frac{-2}{x + 1} - 4$	 $y = a^x - 4$ subst. (2; 0): $0 = a^2 - 4$ $a^2 = 4$ $a = 2$ $y = 2^x - 4$

6. TRANSFORMATION

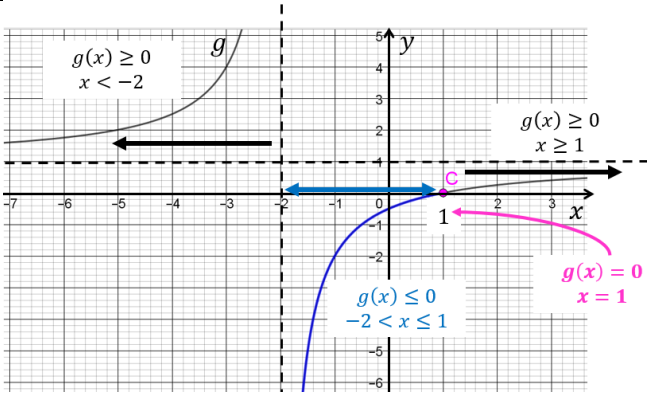
6.1 TRANSLATION			
$p > 0$ horizontal shift left $p < 0$ horizontal shift right $q > 0$ vertical shift up $q < 0$ vertical shift down			
EXAMPLE: PARABOLA		Translate 3 units up.	Translate 2 units to the right.
		Notation: $f(x) + 3$	Notation: $f(x - 2)$
	$f(x) = x^2$	$h(x) = x^2 + 3$	$g(x) = (x - 2)^2$
			
6.2 REFLECTION:			
- REFLECTION in the $(x - axis)$ OR in the line $y = 0$ Notation: $f(x) \rightarrow -f(x)$ - REFLECTION in the $(y - axis)$ OR in the line $x = 0$ Notation: $f(x) \rightarrow f(-x)$ - REFLECTION in the line $y = x$ Notation: $f(x) \rightarrow f^{-1}(x)$			
EXAMPLE: PARABOLA	$h(x) = x^2 + 3$ $\rightarrow k(x) = -x^2 - 3$	$g(x) = (x - 2)^2$ $\rightarrow p(x) = (-x - 2)^2$	$f: y = x^2$ $f^{-1}: x = y^2$ $\therefore y = \pm\sqrt{x}$
			
	EXPONENTIAL FUNCTION	$f(x) = 2^x + 3$ $\rightarrow -f(x) = -(2^x) - 3$	$f(x) = 2^x - 2$ $\rightarrow f(-x) = 2^{-x} - 2$
			$f(x) = 2^x$ $\rightarrow f^{-1}(x) = \log_2 x$

7. GRAPHS AND INEQUALITIES: EXAMPLES

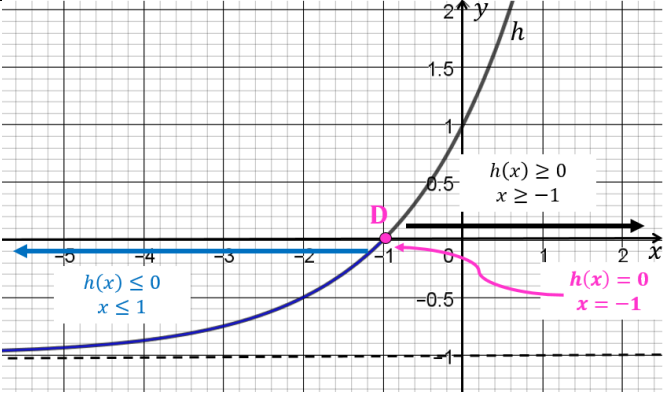
PARABOLA



HYPERBOLA



EXPONENTIAL FUNCTION



SECTION A (Routine questions)

NOTE:

- Questions in this revision were compiled from past exam National papers for GR 11 and GR 12.
- Some questions have been altered to fit the content covered in the FET lessons for GR 11.

QUESTION 1 (GR11 DBE NOV 2017)

Given $f(x) = \frac{-3}{x+2} + 1$ and $g(x) = 2^{-x} - 4$

- 1.1 Determine $f(-3)$.
- 1.2 Determine x if $g(x) = 4$.
- 1.3 Write down the asymptotes of f .
- 1.4 Write down the range of g .
- 1.5 Determine the coordinates of the x – and y – intercepts of f .
- 1.6 Determine the equation of the axis of symmetry of f which has a negative gradient.
Leave your answer in the form $y = mx + c$.
- 1.7 Sketch the graphs of f and g on the same axes. Clearly show ALL the intercepts with the axes and the asymptotes.

QUESTION 2 (GR 11 DBE November 2016)

Given $f(x) = -2x^2 + x + 6$

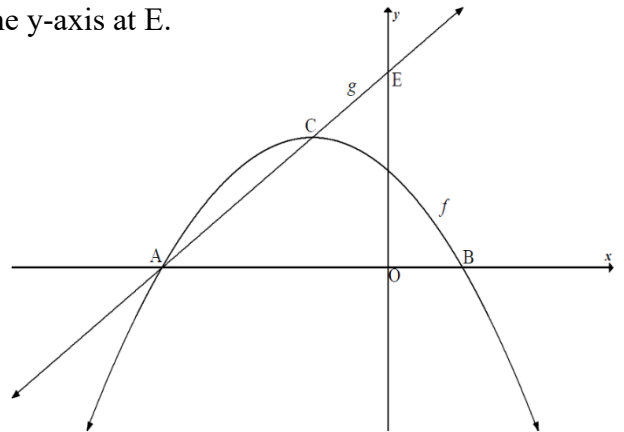
- 2.1 Calculate the coordinates of the turning point of f .
- 2.2 Determine the y – intercept of f .
- 2.3 Determine the x – intercepts of f .
- 2.4 Sketch the graph of f showing clearly all intercepts with the axes and turning point.
- 2.5 Determine the values of k such that $f(x) = k$ has equal roots.
- 2.6 If the graph f is shifted TWO units to the right and ONE unit upwards to form h , determine the equation h in the form $y = a(x + p)^2 + q$.

QUESTION 3 (NSC DBE 2016)

The sketch below shows graphs of $f(x) = -x^2 - 2x + 3$ and $g(x) = mx + q$.

Graph f has x -intercepts at A and B(1; 0) and a turning point at C.

The straight line g passing through A and C, cuts the y -axis at E.

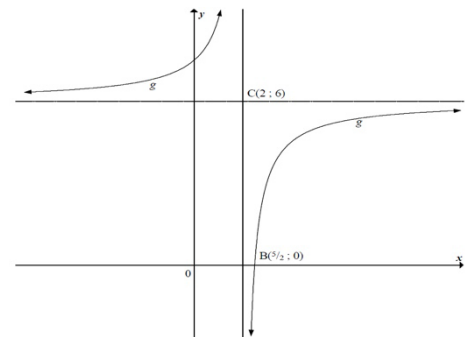


- 3.1 Write down the coordinates of the y -intercept of f .
- 3.2 Show that the coordinates of C are $(-1; 4)$.
- 3.3 Write down the coordinates of A.
- 3.4 Determine the values for m and q .

QUESTION 4 (GR11 DBE NOV 2015)

Sketched below is the graph of $g(x) = \frac{a}{x-p} + q$.

$C(2; 6)$ is the point of intersection of the asymptotes of g . $B\left(\frac{5}{2}; 0\right)$ is the x -intercept of g .



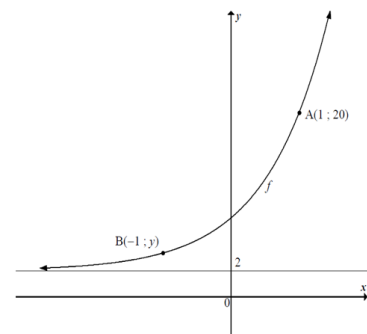
- 4.1 Determine the equation of g in the form $g(x) = \frac{a}{x-p} + q$.
- 4.2 Is g an increasing or decreasing function?
- 4.3 Give the domain and the range of g .
- 4.4 Use the graphs to solve for x if: $g(x) \leq 0$

QUESTION 5 (GR11 DBE NOV 2016)

The sketch below represents the graph of $f(x) = 2 \cdot b^{x+1} + q$

The graph of f passes through the points A $(1; 20)$ and B $(-1; y)$.

The line $y = 2$ is an asymptote of f .



- 5.1 Show that the equation of f is $f(x) = 2 \cdot (3)^{x+1} + 2$
- 5.2 Calculate the y -coordinate of point B.
- 5.3 Write down the range of f .
- 5.4 Calculate the coordinates of the y -intercept of f .
- 5.5 Is this graph an increasing or decreasing function?

SECTION B (Complex questions)

QUESTION 1 (GR11 DBE NOV 2015)

The sketch below represents the graphs of two parabolas, f and g .

$$f(x) = \frac{1}{2}x^2 - 8$$

The turning point of g is $C(2; 9)$ and the y –intercept of g is $A(0; 5)$.

B and D are the x –intercepts of f and g respectively.

1.1 Show that $g(x) = -x^2 + 4x + 5$.

1.2 Calculate the length of BD .

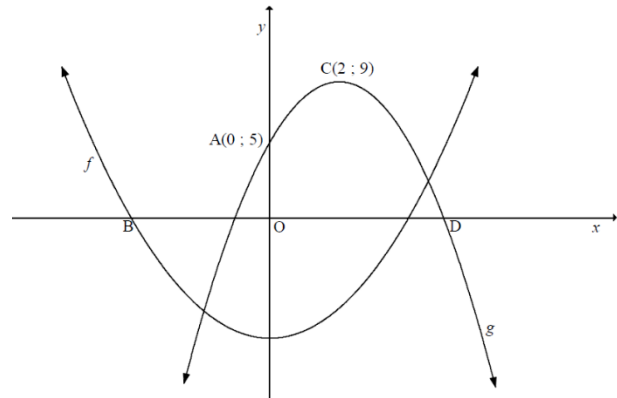
1.3 Give the equation of the axis of symmetry of f .

1.4 Give the range of $g(x) + 1$

1.5 Use the graphs to solve for x , if:

1.5.1 $f(x) \geq 0$

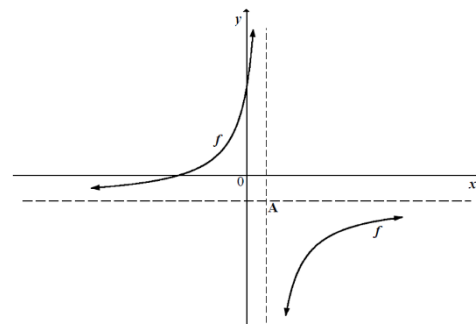
1.5.2 f and g are both strictly increasing.



QUESTION 2 (GR11 DBE NOV 2015)

The sketch below represents the graph of $f(x) = \frac{-9}{x-1} - 2$

A is the point of intersection of the asymptotes of f .



2.1 Write down the coordinates of A .

2.2 Give the domain of f .

2.3 Give the equation of the horizontal asymptote for $f(x) - 1$.

2.4 Determine the coordinates of the x – and y –intercepts of f .

2.5 Write down the equation of the axis of symmetry of f that has a negative gradient.

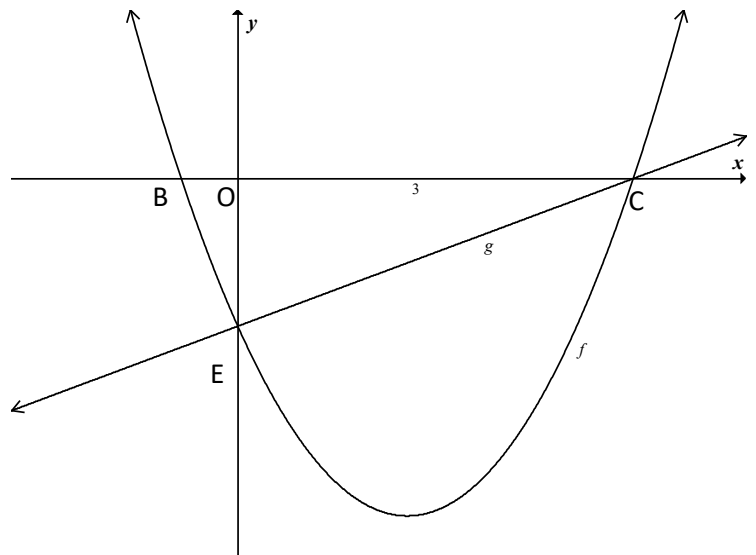
QUESTION 3 (GR11 DBE NOV 2015)

Given $h(x) = 4 \cdot (2^{-x}) + 1$

- 3.1 Is h an increasing or decreasing function?
- 3.2 Determine the coordinates of the y – intercept of h .
- 3.3 Explain why h does not have an x –intercept.
- 3.4 Give the equation of the asymptote of f .
- 3.5 Sketch the graphs of h , clearly showing all asymptotes, intercepts with the axes and at least one other point on h .
- 3.6 Describe the transformation from h to g if $g(x) = 4(2^{-x} + 2)$

QUESTION 4 (NSC March 2011)

A parabola f intersects the x -axis at B and C and the y -axis at E. The axis of symmetry of the parabola has equation $x = 3$. The line through E and C has equation $g(x) = \frac{x}{2} - \frac{7}{2}$.



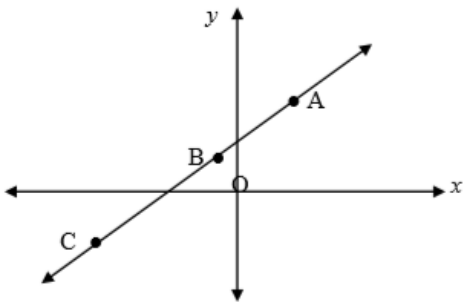
- 4.1 Show that the coordinates of C are $(7 ; 0)$.
- 4.2 Calculate the x -coordinate of B.
- 4.3 Determine the equation of f in the form $y = a(x - p)^2 + q$.
- 4.4 Write down the equation of the graph of h , the reflection of f in the x -axis.
- 4.5 Write down the maximum value of $t(x)$ if $t(x) = 1 - f(x)$.

TOPIC: 11 Analytical Geometry

Formulae	Questions linked to Formulae
1. Distance: $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	- Find the distance between two points. - Prove that a Δ is isosceles, equilateral, or right-angled. - Prove that a quadrilateral is a parallelogram. - Determine the perimeter.
2. Gradient: $m = \frac{y_2 - y_1}{x_2 - x_1}$	- Parallel lines: $m_1 = m_2$ - Perpendicular lines: $m_1 \times m_2 = -1$ - Show that an angle is a right angle - Angle of inclination: $\tan \theta = m$ - Determine whether a quadrilateral is a parallelogram. - Determine whether 3 points are collinear (i.e. form a straight line) - If $\tan \theta > 0$, angle is acute, If $\tan \theta < 0$, angle is obtuse.
3. Midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$	- Find the midpoint of a line segment. - Find the length of a line, given one point and the midpoint.
4. Angle of inclination: This is the angle formed between a line and the positive x -axis $\tan \theta = m$	- Find the inclination of a line. - Determine the angle formed by two lines. - Determine the angles of a Δ.
5. Equation of a straight line: $y = mx + c$ Or $y - y_1 = m(x - x_1)$	- Find the equation of a line, given - the gradient and one other point. - Find the equation of a line given- 2 points on the line

Collinear points

Co-linear points are points that lie in the same straight line. Hence the gradient of a line joining two points on the line must be the same.



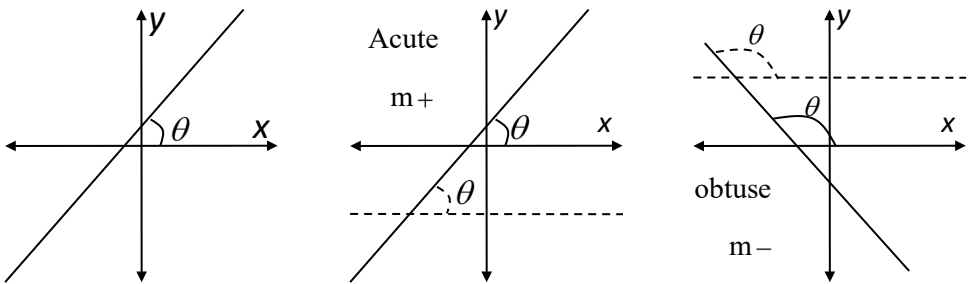
A, B and C are collinear, $\therefore M_{AB} = M_{BC} = M_{AC}$

There are 2 ways to prove points co-linear:

- 1. Prove 3 gradients equal
- 2. Prove 2 gradients equal, and state a common point

Angle of Inclination:

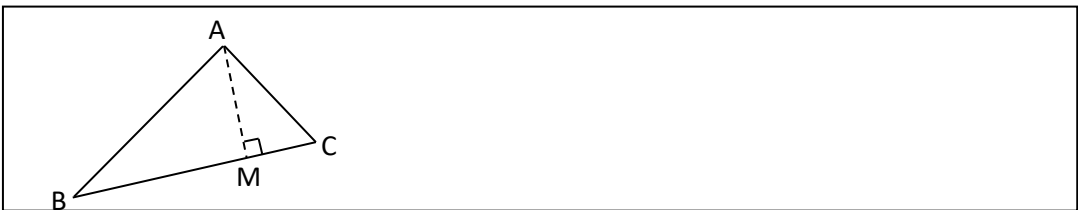
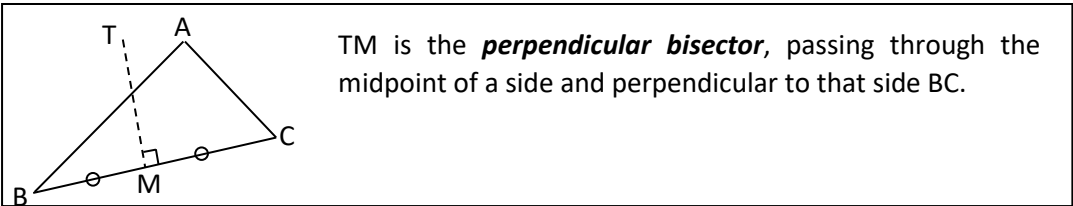
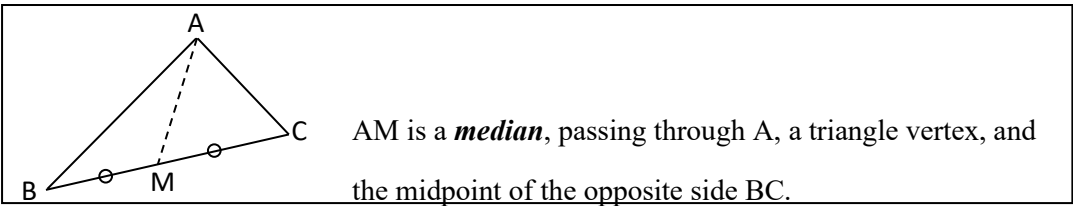
The angle that a line forms with the positive $x - axis$ is called the angle of inclination.



The inclination is an angle in the interval $[0^\circ; 180^\circ]$

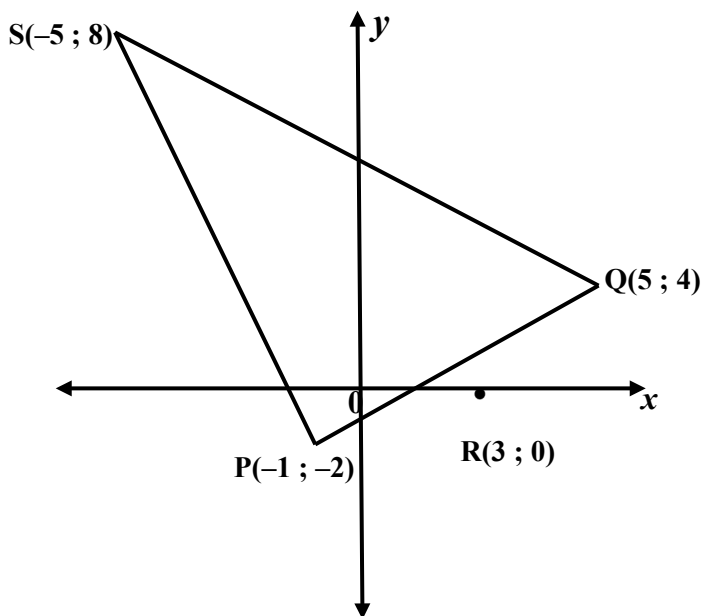
If θ is the inclination, then $\tan\theta = m$

Medians, Perpendicular bisectors and Altitude



Section A**QUESTION 1**

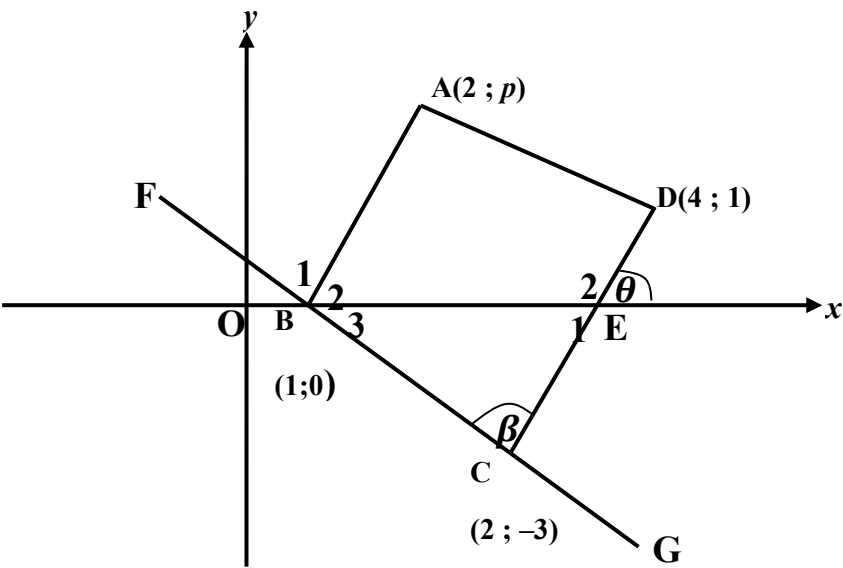
In the diagram below, $S(-5 ; 8)$, $P(-1 ; -2)$, $Q(5 ; 4)$ and $R(3;0)$ are points on a Cartesian plane.



- 1.1 Calculate the length of PR. (2)
 - 1.2 Calculate the length of SP. (2)
 - 1.3 If it is also given that $QR = \sqrt{20}$ and $SQ = \sqrt{116}$, what type of quadrilateral is SPRQ? Give a reason for your answer. (2)
 - 1.4 Determine the coordinates of M, the midpoint of PQ. (2)
 - 1.5 Hence, or otherwise, determine whether the points S, M and R are collinear. (3)
- [11]**

QUESTION 2

In the diagram below is quadrilateral ABCD, with vertices $A(2 ; p)$, $B(1 ; 0)$, $C(2 ; -3)$ and $D(4 ; 1)$. FBCG is a straight line.



- 2.1

The equation of AD is $y = -\frac{1}{2}x + 3$ and A is the point $(2 ; p)$.
Calculate the value of p .

(2)
- 2.2

Prove that $AD \perp AB$.

(3)
- 2.3

Determine the coordinates of E.

(4)
- 2.4

Determine the value of β .

(6)
- 2.5

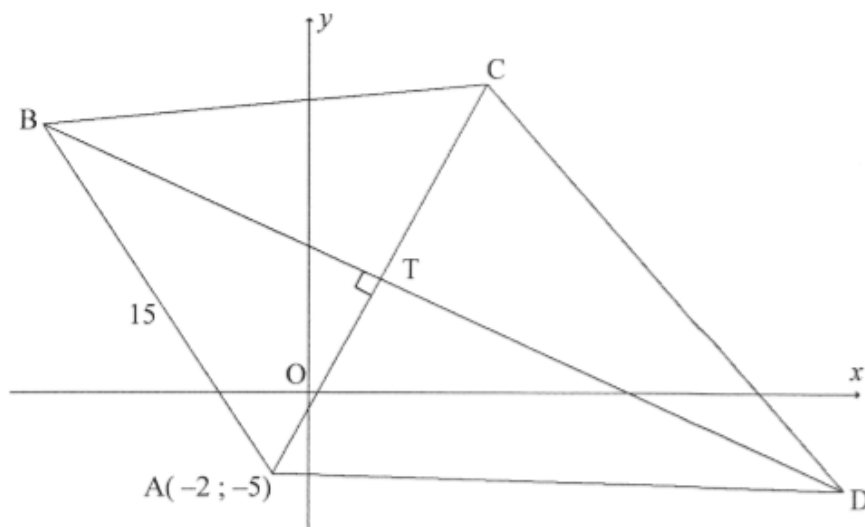
What type of quadrilateral is ABCD? Give a reason for your answer.

(2)
- [17]

Question 3

$A(-2; -5)$, B , C and D are the vertices of Quadrilateral $ABCD$ such that diagonal AC is perpendicular to diagonal BD at T .

The equation of BD is given by $2y + x = 18$ and $AB = 15$ units



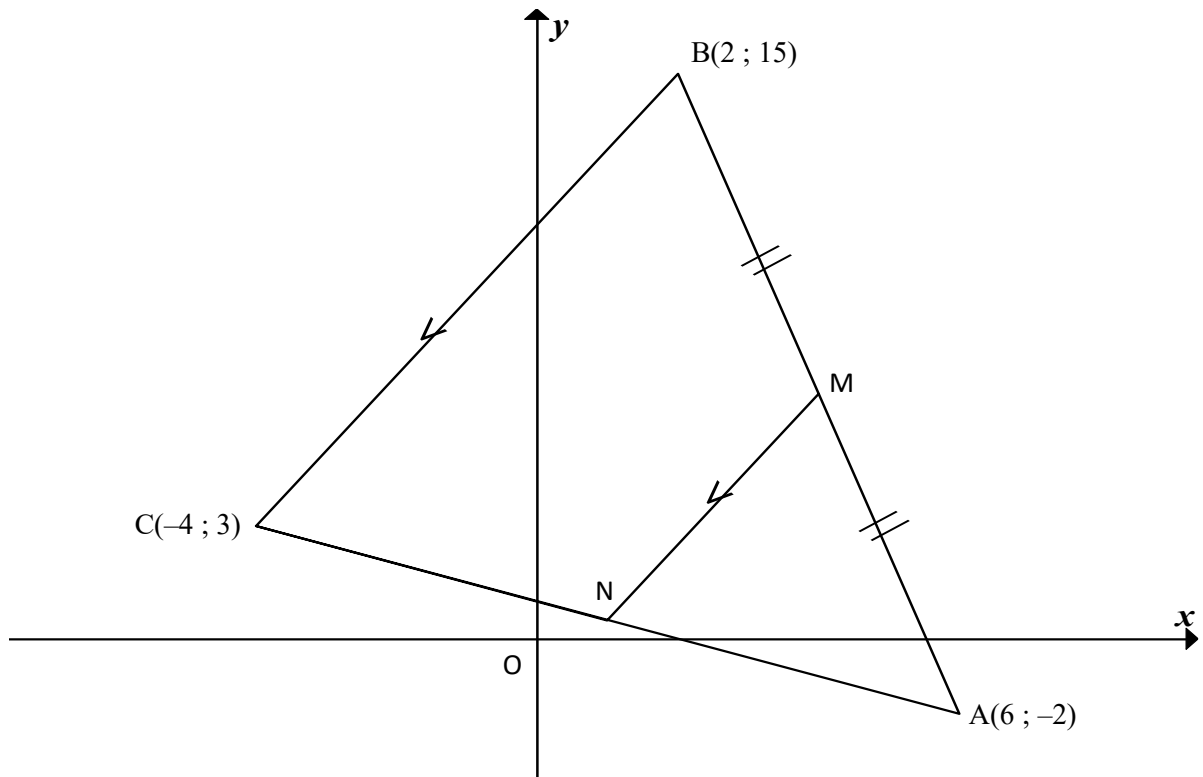
- 3.1 Determine the gradient of line AC (2)
- 3.2 Determine the equation of AC in the form $y = mx + c$ (2)
- 3.3 If the equation of AC is $y = 2x - 1$, calculate the coordinates of T (3)
- 3.4 If $ABCD$ is a kite with $AB = BC$
 - 3.4.1 Determine the coordinates of C (2)
 - 3.4.2 Calculate the length of BT (4)

[13]

QUESTION 4

In the diagram, $A(6 ; -2)$, $B(2 ; 15)$ and $C(-4 ; 3)$ are the vertices of $\triangle ABC$.

M is the midpoint of AB . N is a point on CA such that $MN \parallel BC$.



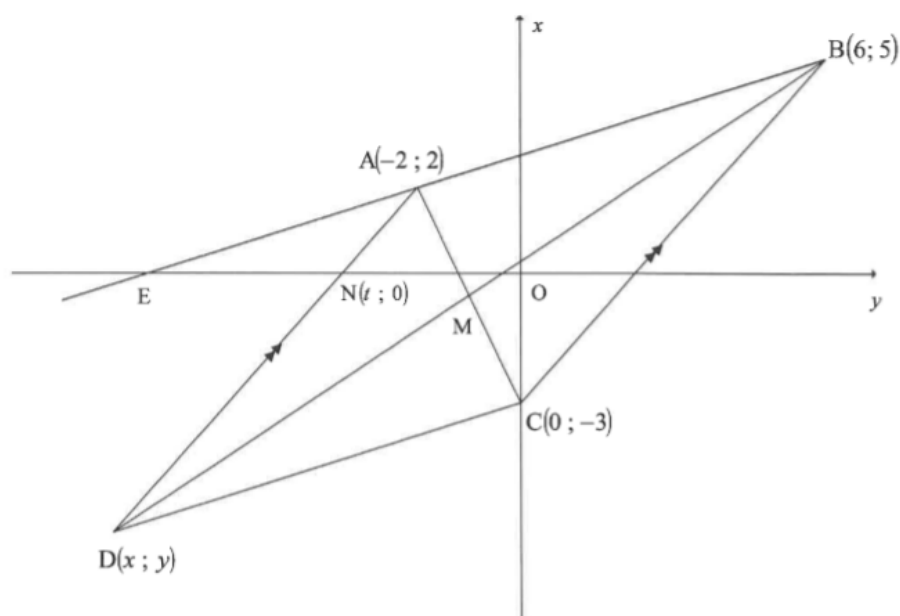
- 4.1 Determine the coordinates of M , the midpoint of AB . (2)
- 4.2 Determine the gradient of line MN . (3)
- 4.3 Hence, or otherwise, determine the equation of line MN , in the form $y = mx + c$. (2)
- 4.4 Calculate, with reasons, the coordinates of point N . (4)
- 4.5 If $ABCD$ (in that order) is a parallelogram, determine the coordinates of point D . (4)

[15]

Question 5

In the diagram $A(-2; 2)$, $B(6; 5)$, $C(0; -3)$ and $D(x; y)$ are the vertices of a quadrilateral having $AD \parallel BC$. BA produced has an x -intercept at E . BD and AC intersect at M .

$N(t; 0)$ is a point on AD .



- 5.1 Calculate the gradient of BC (2)
- 5.2 Determine the equation of AD (3)
- 5.3 Determine the value of t (2)
- 5.4 Calculate the length of AN (2)
- 5.5 If DC is defined by $y = \frac{3}{8}x - 3$, determine the coordinates of D (4)
- 5.6 Prove that $ABCD$ is a parallelogram (3)
- 5.7 Calculate the coordinates of M (3)

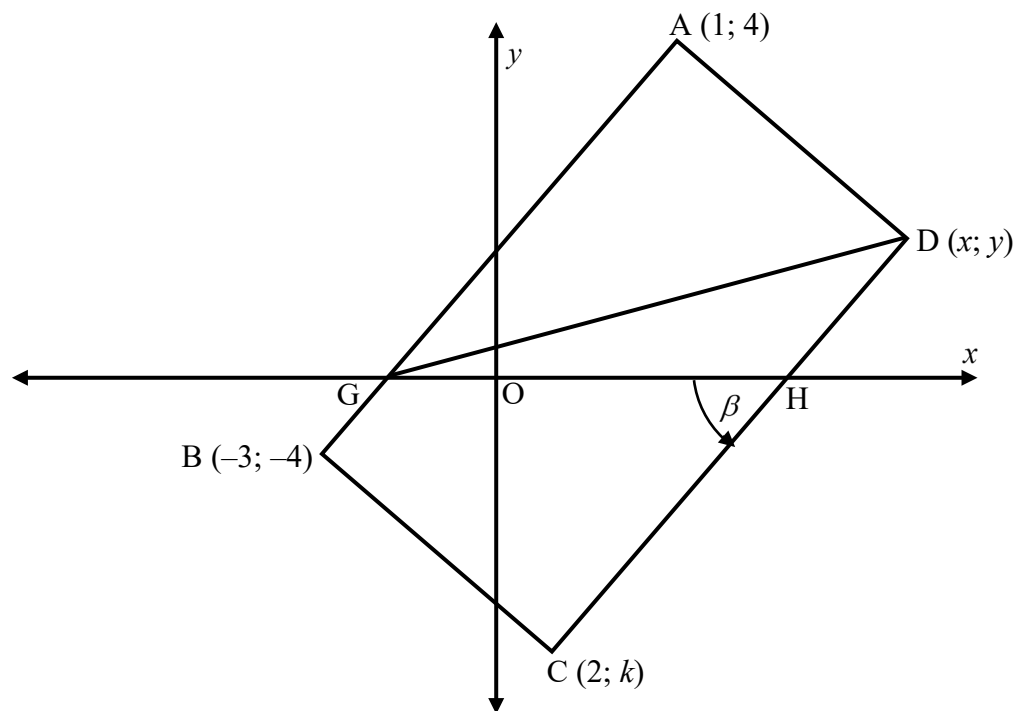
[19]

SECTION B

QUESTION 1

In the diagram below, A (1 ; 4), B (−3 ; −4) , C (2 ; k) and D (x ; y) are the vertices of a rectangle.

AB and DC cuts the x – axis at G and H respectively. GD is drawn. $\widehat{GHC} = \beta$.



- 1.1

Calculate the gradient of BG.

(2)
- 1.2

Determine the equation of AB in the form $y = mx + c$.

(2)
- 1.3

Calculate the:
- 1.3.1

Value of k (y-coordinate of C).

(3)
- 1.3.2

Coordinates of D.

(3)
- 1.3.3

Size of β .

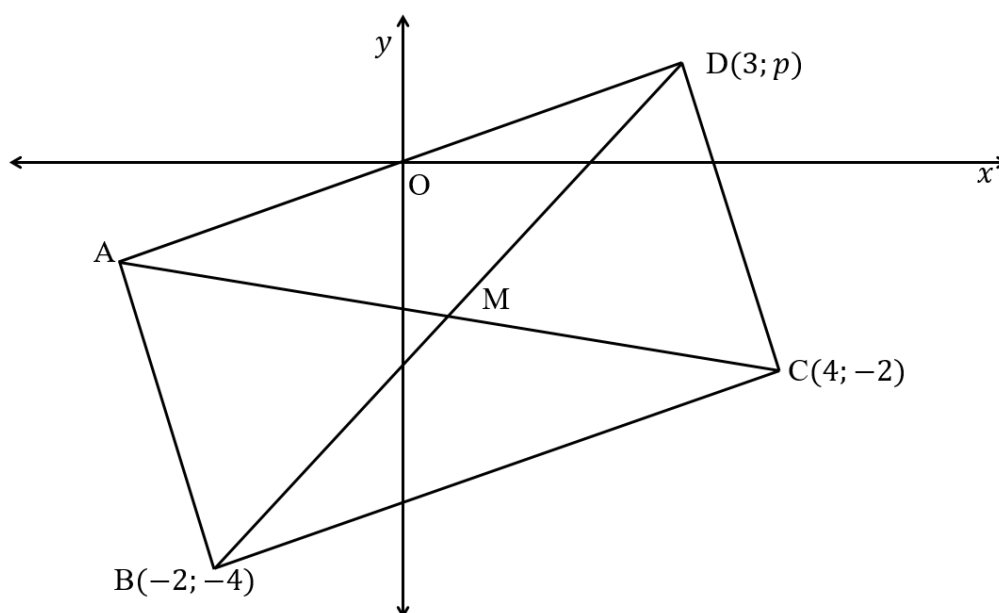
(3)
- 1.3.4

Area of ΔDHG

(7)
- [21]**

QUESTION 2

In the diagram A, B(-2; -4), C(4; -2) and D(3; p) are the vertices of a rectangle. The diagonals AC and BD intersect at M.

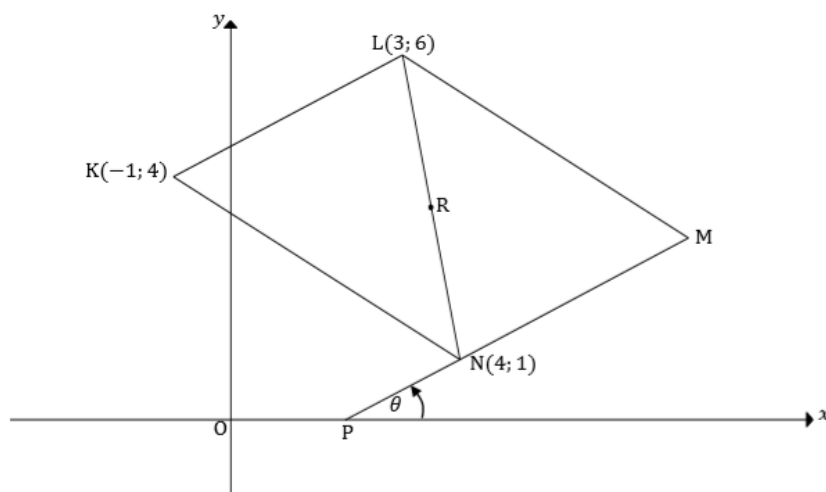


- 2.1 If the length of AC is $\sqrt{50}$ units, show that $p = 1$. (4)
- 2.2 Determine the coordinates of M. (2)
- 2.3 Calculate the gradient of CD. (2)
- 2.4 Determine the equation of line AB in the form $y = mx + c$. (2)
- 2.5 Determine the coordinates of A (2)

[12]

Question 3

In the diagram, $K(-1; 4)$; $L(3; 6)$; M and $N(4; 1)$ are vertices of a parallelogram. R is the midpoint of LN . P is the x -intercept of the line MN produced.



3.1 Calculate the following:

3.1.1 Gradient of KL (2)

3.1.2 The coordinates of R (3)

3.1.3 The coordinates of M (4)

3.2 Determine the equation of NM in the form $y = mx + c$ (3)

3.3 Calculate the:

3.3.1 The coordinates of P . (2)

3.3.2 The size of θ , the inclination of PM . (2)

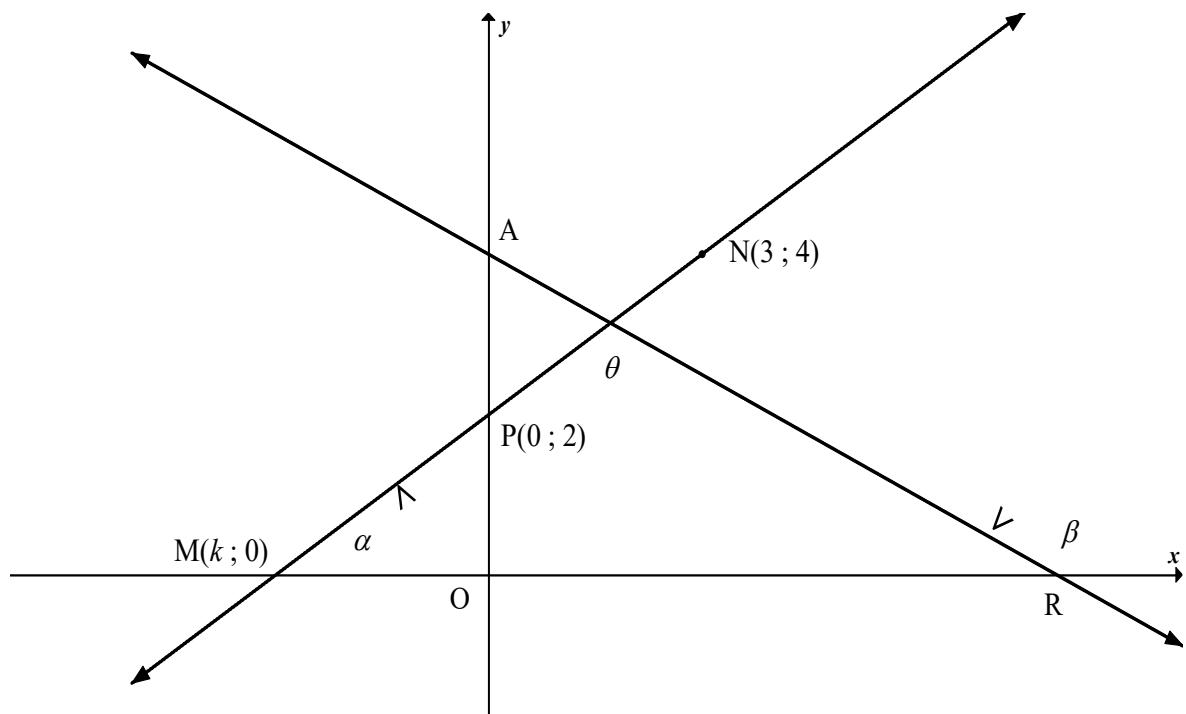
3.3.3 The size of $\angle KPN$ (4)

[20]

QUESTION 4

In the diagram, R and A are the x - and y -intercepts respectively of the straight line AR. The equation of AR is $y = -\frac{1}{2}x + 4$. Another straight line cuts the y -axis at $P(0; 2)$ and passes through the points $M(k; 0)$ and $N(3; 4)$.

α and β are the angles of inclination of the lines MN and AR respectively.



- 4.1 Given that M, P and N are collinear points, calculate the value of k . (3)
- 4.2 Determine the size of θ , the obtuse angle between the two lines. (4)
- 4.3 Calculate the length of MR. (3)
- 4.4 Calculate the area of $\triangle MNR$. (3)

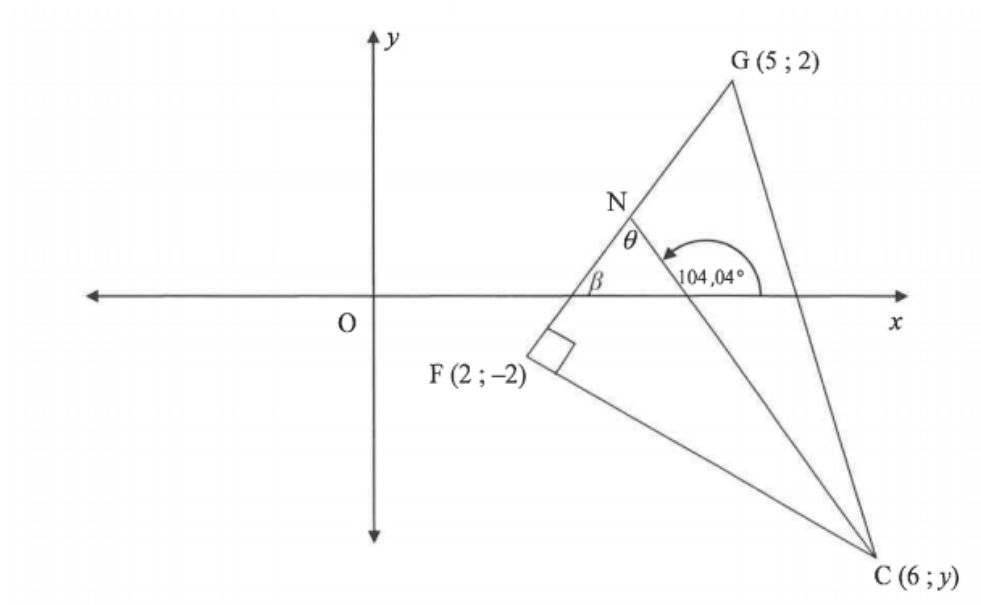
[13]

Question 5

In the diagram, $F(2; -2)$, $G(5; 2)$ and $C(6; y)$ are the vertices of $\triangle FGC$, $FG \perp FC$.

N is a point on FG such that the inclination of NC is $104,04^\circ$.

The angle of inclination of FG is β and $\angle FNC = \theta$



5.1 Calculate the gradient FG (2)

5.2 Calculate the value of y (3)

5.3 Calculate the size of θ (3)

5.4 Calculate the length of NC (4)

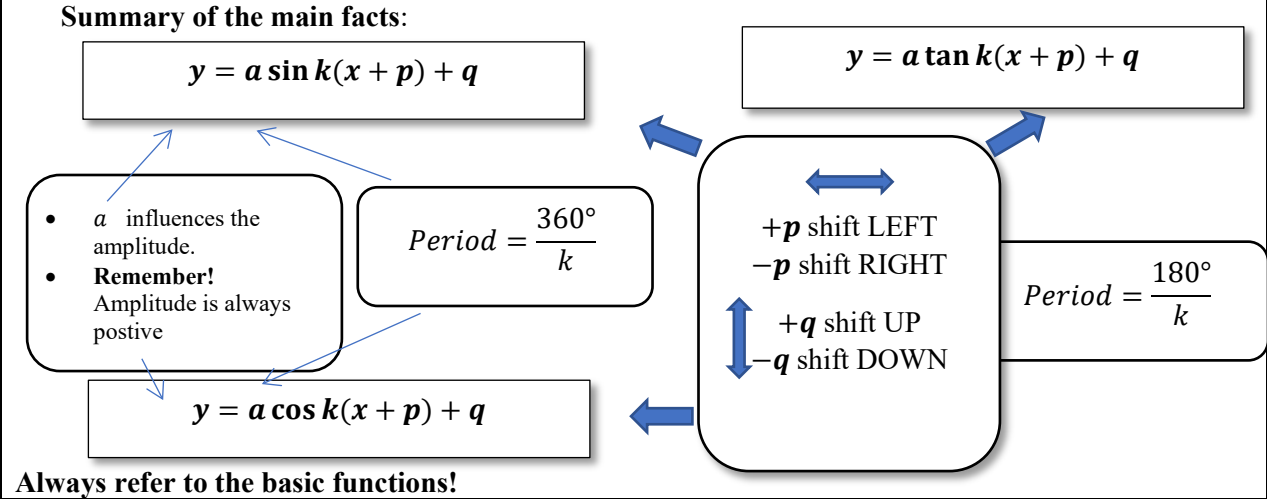
[12]

TOPIC: Trigonometry Graphs Revision

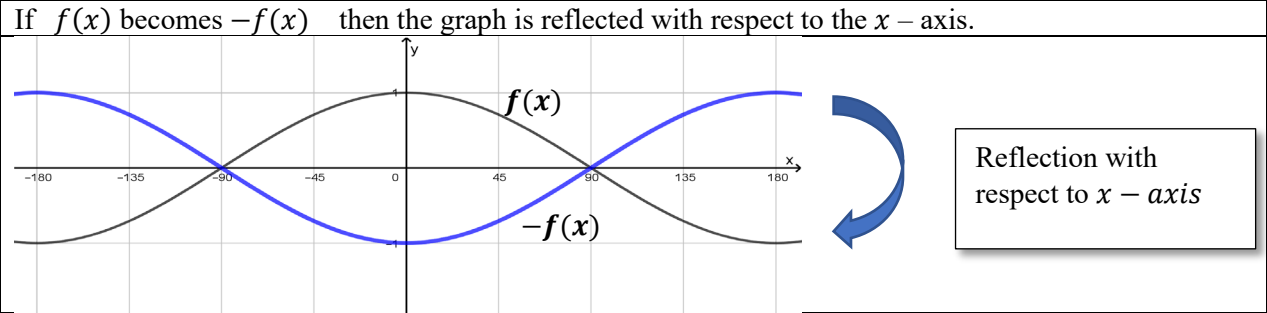
A. Basic Graphs

Sine Graph	
	The graph: $y = \sin x$ Amplitude: 1 Period: 360° Key features: Graph starts at 0. $(0^\circ; 0)$
Cosine Graph	
	The graph: $y = \cos x$ Amplitude: 1 Period: 360° Key features: Graph starts at 1. $(0^\circ; 1)$
Tangent Graph	
	The graph $y = \tan x$ Period: 180° Key features: Asymptotes at 90° and 270° . Equation of asymptotes: $x = 90^\circ$ and $x = 270^\circ$ Zero points at $0^\circ; 180^\circ$ and 360°

B. Parameters and Transformations



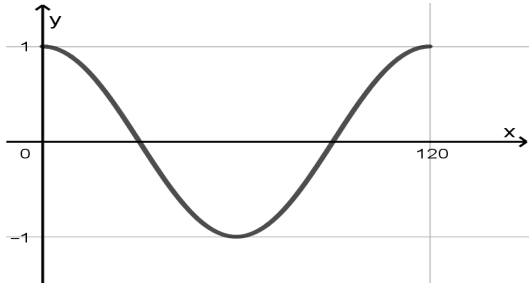
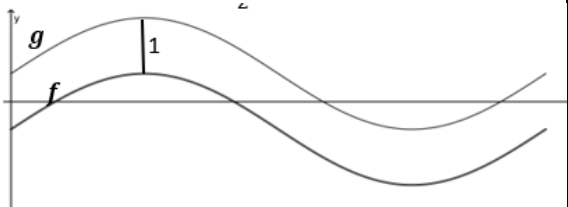
C. Reflections



SECTION A

Question 1

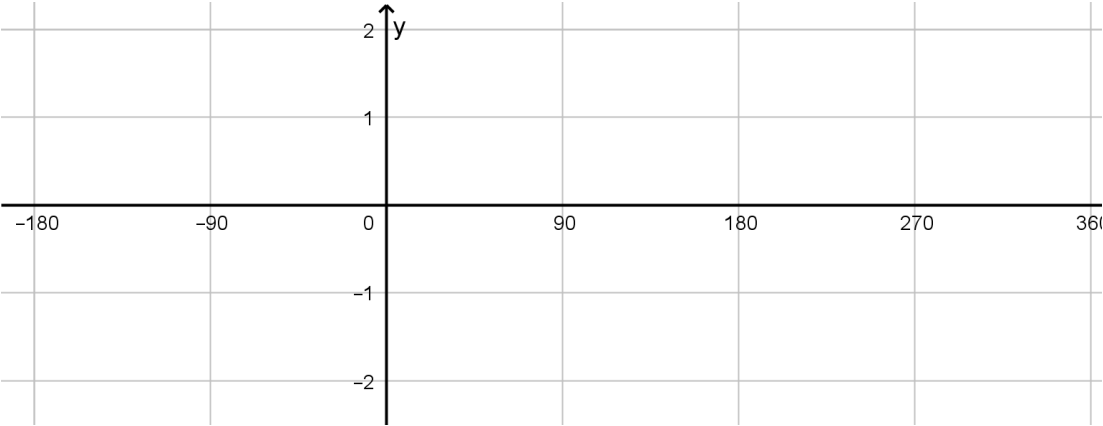
Answer the following multiple-choice questions. Choose the correct answer.

1. The graph of $y = \tan \theta$, $\theta \in [0^\circ; 360^\circ]$ is not defined for $\theta = \dots\dots$ a) 0° and 180° b) 180° and 360° c) 0° and 90° d) 90° and 270° e) 0° and 270°	5. The graph of $y = \sin 2(x + 30^\circ)$ has the same period as: a) $y = 3 \tan x$ b) $y = 2 \cos(x + 60^\circ)$ c) $y = 2 \sin(x + 30^\circ)$ d) $y = \cos(x + 60^\circ)$ e) $y = \sin 3(x + 30^\circ)$
2. The period of $y = \sin \frac{1}{2}x$ is: a) $\frac{1}{2}$ b) 360° c) 720° d) 180° e) 2	6. The range of $y = 1 + \sin x$ is: a) $x \in [-360^\circ; 360^\circ]$ b) $y \in [-1; 1]$ c) $0 \leq y \leq 2$ d) $-2 \leq y \leq 2$ e) $-2 \leq y \leq 0$
3. The period of $y = 2 \cos 3\theta$ is: a) 360° b) $1 \frac{1}{2}$ c) 6 d) 2 e) 120°	7. The maximum value of $\sqrt{\cos x}$ is: a) 1 b) -1 c) $\frac{1}{2}$ d) -2 e) 2
4. Which graph is represented below?  a) $y = 3 \cos \theta$ b) $y = \cos 3\theta$ c) $y = 3 \sin \theta$ d) $y = \sin 3\theta$ e) None of the above	8. If $f(x) = \sin x - \frac{1}{2}$ then $g(x) = \dots$  a) $g(x) = 2 \sin x$ b) $g(x) = 2 \cos x$ c) $g(x) = \frac{1}{2} - \sin x$ d) $g(x) = \sin x + \frac{1}{2}$ e) $g(x) = -\frac{1}{2} + \sin x$

Question 2

Consider: $f(x) = 2 \sin x$

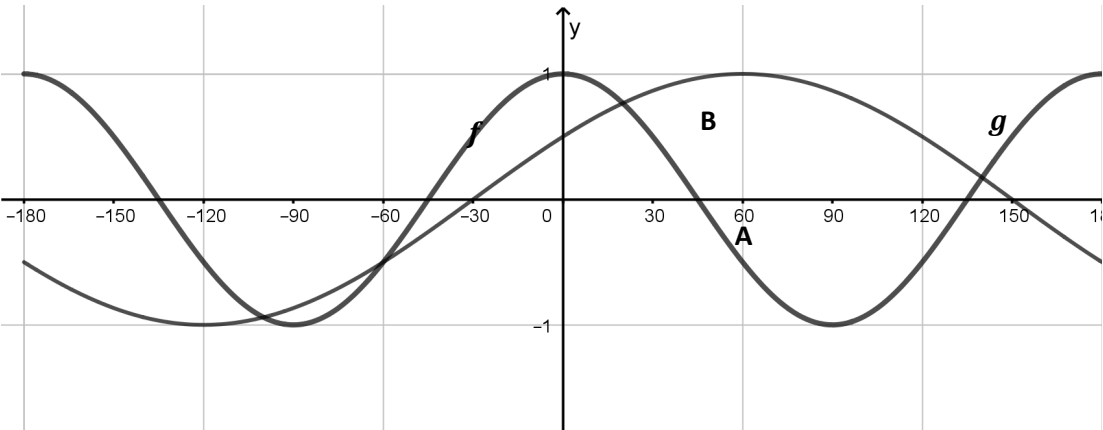
- 2.1 Draw a sketch graph of f , for $x \in [-180^\circ < x < 360^\circ]$.
(5)



- 2.2 Write down the range of g if $g(x) = 2 f(x)$ (2)
2.3 Write down the period of f . (1)
2.4 Give the value of θ if $f(x + \theta) = 2 \cos \theta$ (2)
[10]

Question 3

The graphs below represent the functions $f(x) = \cos 2x$ and $g(x) = \sin(x + 30^\circ)$.

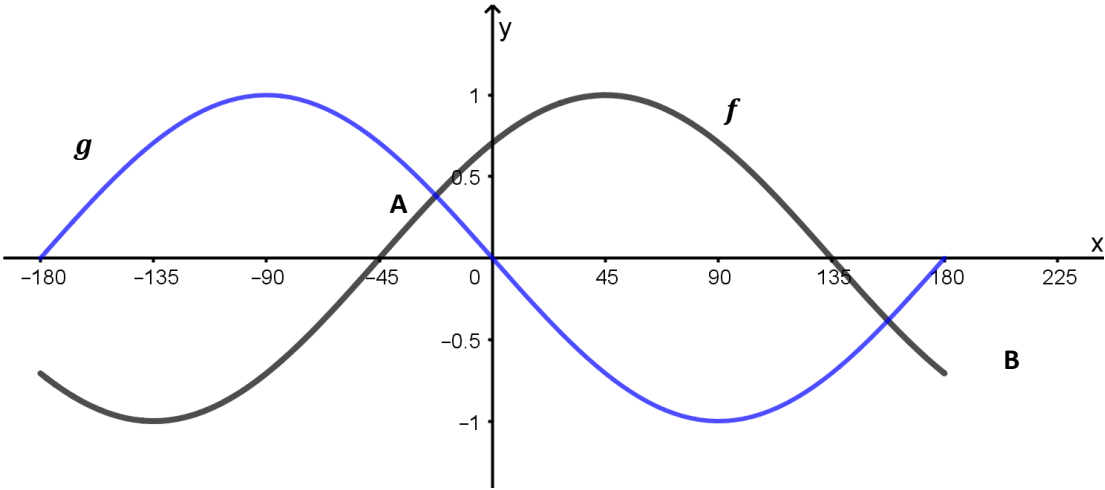


- 3.1 Give the period of f . (1)
3.2 Give the coordinates of A. (1)
3.3 For which values of x are $f(x)$ and $g(x)$ both increasing? (2)
3.4 3.4.1 Using a calculator, find $g(20^\circ)$ (1)
3.4.2 Using a calculator, find $f(20^\circ)$ (1)
3.5 Write down the coordinates van B. (2)
3.6 For which values of x is $g(x) < f(x)$ for $x \in [0^\circ ; 180^\circ]$ (2)
3.7 If g is translated 30° to the left to make h , give the equation of h . (2)

[12]

Question 4

4.1 In the diagram below, the graphs of $f(x) = \cos(x + p)$ and $g(x) = q \sin x$ are shown for the interval $-180^\circ \leq x \leq 180^\circ$.

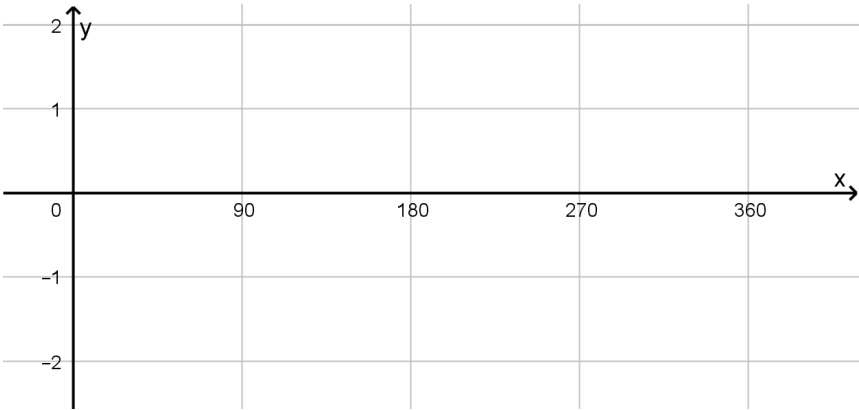


- 4.1 Determine the values of p and q (2)
- 4.2 The graphs intersect at $A(-22,5^\circ ; 0,38)$ and B. Determine the coordinates of B. (2)
- 4.3 Determine the value(s) of x in the interval $-180^\circ \leq x \leq 0^\circ$ for which $f(x) < g(x)$ (3)

[7]

Question 5

- 5.1 On the same set of axes below, draw sketch graphs of each of $f(x) = -2\sin x$ and $g(x) = \cos 2x$ for the domain $x \in [0^\circ; 360^\circ]$. (6)
- You must show clearly any intercepts with the axes as well as the coordinates of the maximum and minimum values for each graph.



- 5.2 Without doing any working, write down the period of the graph $k(x) = \sin 3x$ (1)

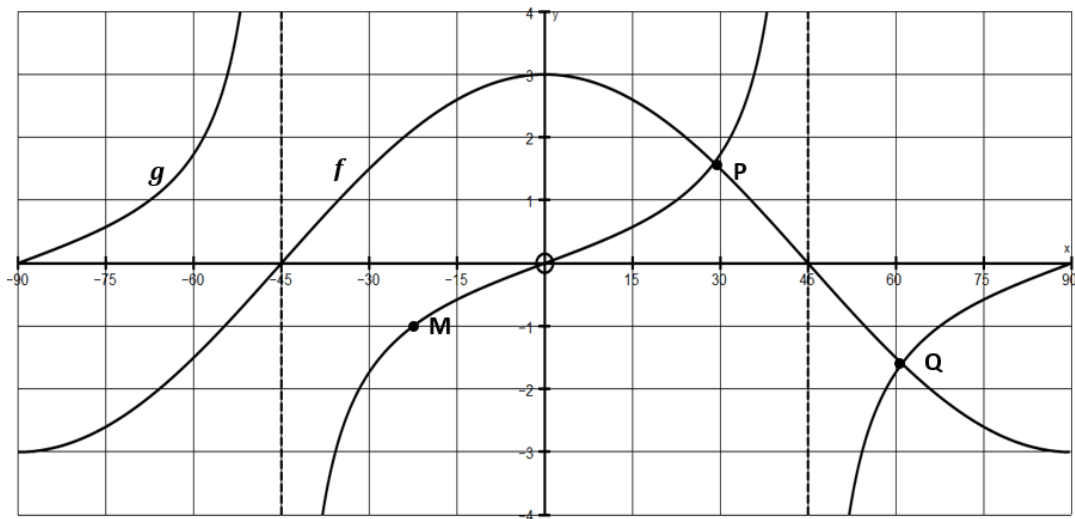
- 5.3 State the amplitude of the graph $f(x) + 1$ (1)
- 5.4 What is the range of $g(x) = \cos 2x$? (1)
- 5.5 If $f(x)$ has been shifted 30° to the right, write down the equation of the new graph. (1)
- [10]

SECTION B



Question 1

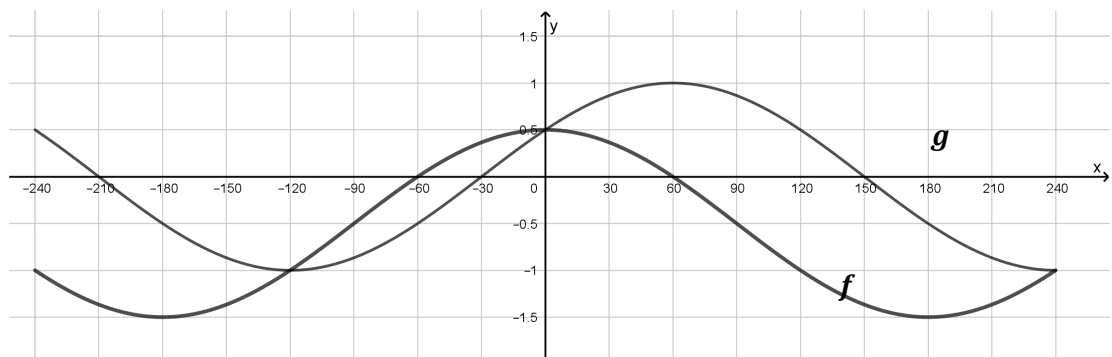
- 1 The graphs of the functions $f(x) = a \cos(bx)$ and $g(x) = c \tan 2x$ for $x \in [-90^\circ; 90^\circ]$ are sketched below. The graphs intersect at $P(29^\circ; 1,6^\circ)$ and Q:



- 1.1 Write down the range of f . (2)
- 1.2 If $M(-22,5^\circ; -1)$ lies on g , determine the value of c . (1)
- 1.3 Write down the values of a and b . (2)
- 1.4 Without calculations, determine accurately the coordinates of Q. (2)
- 1.5 If the system of axes is shifted 45° to the right and the graphs remain fixed, write down the equation that is now represented by the graph f . (2)
- 1.6 Determine the values of x for which $f(x) \cdot (g(x))^2 > 0$. (3)
- [12]

Question 2

2. In the diagram below, the graphs of $f(x) = \cos x + q$ and $g(x) = \sin(x + p)$ are drawn on the same system of axes for $-240^\circ \leq x \leq 240^\circ$. The graphs intersect at $(0^\circ; \frac{1}{2})$, $(-120^\circ; -1)$ and $(240^\circ; -1)$.



- 2.1

Determine the values of p and q .

(2)
- 2.2

Determine the values of x in the given interval for which $f(x) > g(x)$

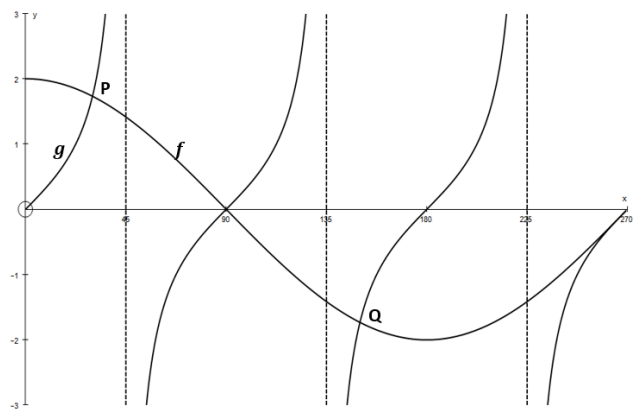
(2)
- 2.3

Determine the equation of $h(x)$ if h is the graph of f reflected with respect to the x -axis.

(1)
- [5]

Question 3

In the figure, the graphs of $f(x) = a \cos x$ and $g(x) = \tan ax$ are shown for the domain $[0^\circ; 270^\circ]$. These graphs intersect at $P(30^\circ; \sqrt{3})$ and at Q as shown.



- 3.1

What is the value of a ?

(1)
- 3.2

Determine the periods of f and g .

(2)
- 3.3

Write down the co-ordinates of Q .

(2)
- 3.4

For what value(s) of x in the given domain is:
- 3.4.1

$f(x) > 0$?

(1)
- 3.4.2

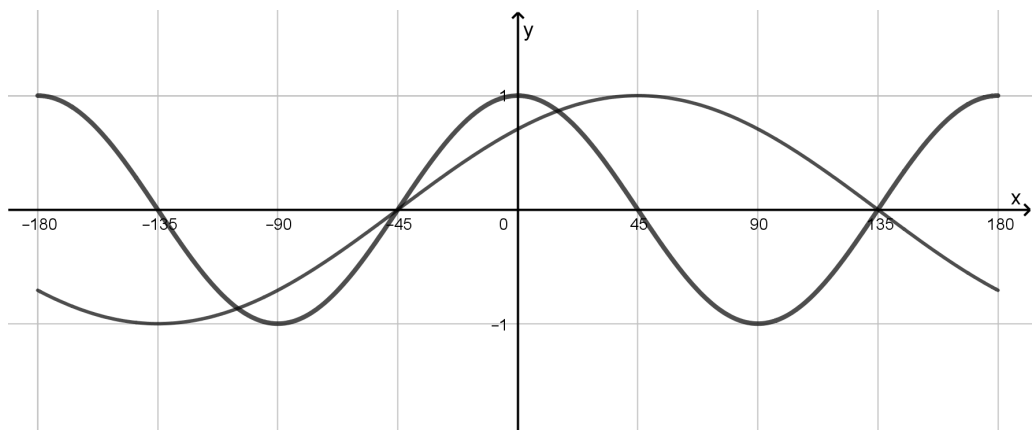
$f(x)$ and $g(x)$ increasing as x increases in value?

(2)

Question 4

4.

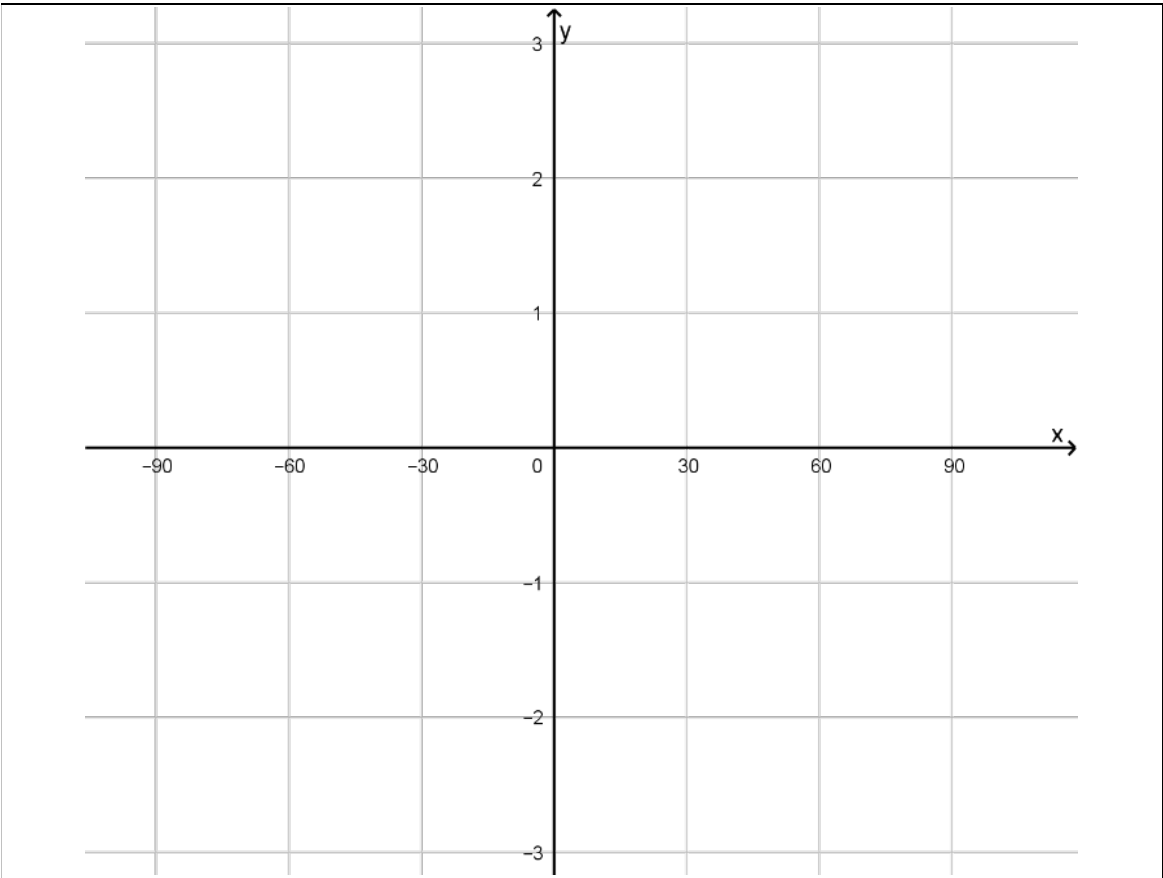
In the diagram below the graphs of $f(x) = a \cos bx$ and $g(x) = \sin(x + p)$ are drawn for, $x \in [-180^\circ; 180^\circ]$.



- 4.1 Write down the values of a, b and p . (3)
- 4.2 For which values of x in the given interval does the graph of f increase as the graph of g increases? (2)
- 4.4 Determine the minimum value of h if $h(x) = f(x) - 1$ (1)
- 4.5 Describe how the graph g must be transformed to form the graph k , where $k(x) = -\cos x$. (2)

[8]

Question 5	
Given: $f(x) = \tan(x - 45^\circ)$	
5.1	Sketch the graph of $f(x)$ for $-90^\circ \leq x \leq 90^\circ$ on the grid below: You need to show the intercepts as well as the asymptotes clearly. Remember to show at least two important points on your graph.

		(6)
5.2	What is the period of f ?	(1)
5.3	Write down the asymptote of f ?	(1)
5.4	Describe the shift of f to g if $g(x) = -f(x)$	(1)
		[9]

TOPIC TRIGONOMETRY

✓ Definitions

For any size θ :

$$\sin \theta = \frac{y}{r} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{x}{r} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{y}{x} = \frac{\text{opposite}}{\text{adjacent}}$$

✓ Identities

$\tan \theta = \frac{\sin \theta}{\cos \theta}$	$\frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$
$\sin^2 \theta + \cos^2 \theta = 1$	$\sin^2 \theta = 1 - \cos^2 \theta$ $\cos^2 \theta = 1 - \sin^2 \theta$

✓ 180° , 360° and $(-\theta)$ rule

REDUCTION FORMULAE			
Quadrant 2	Quadrant 3	Quadrant 4	Negative angles
$\sin(180^\circ - \theta) = \sin \theta$	$\sin(180^\circ + \theta) = -\sin \theta$	$\sin(360^\circ - \theta) = -\sin \theta$	$\sin(-\theta) = -\sin \theta$
$\cos(180^\circ - \theta) = -\cos \theta$	$\cos(180^\circ + \theta) = -\cos \theta$	$\cos(360^\circ - \theta) = \cos \theta$	$\cos(-\theta) = \cos \theta$
$\tan(180^\circ - \theta) = -\tan \theta$	$\tan(180^\circ + \theta) = \tan \theta$	$\tan(360^\circ - \theta) = -\tan \theta$	$\tan(-\theta) = -\tan \theta$

✓ 90° rule

CO-FUNCTIONS	
Quadrant 1	Quadrant 2
$\sin(90^\circ - \theta) = \cos \theta$	$\sin(90^\circ + \theta) = \cos \theta$
$\cos(90^\circ - \theta) = \sin \theta$	$\cos(90^\circ + \theta) = -\sin \theta$

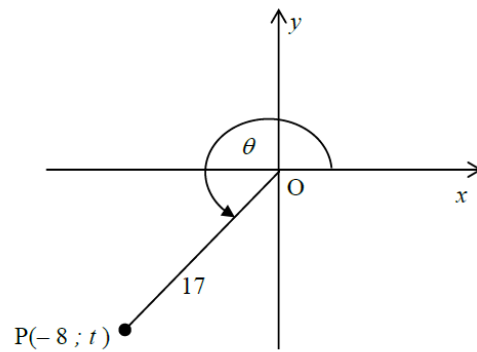
✓ Special angles

Know the $45^\circ/30^\circ/60^\circ$ ratios without a calculator.

$\sin 45^\circ = \frac{1}{\sqrt{2}}$	$\sin 60^\circ = \frac{\sqrt{3}}{2}$	$\sin 30^\circ = \frac{1}{2}$
$\cos 45^\circ = \frac{1}{\sqrt{2}}$	$\cos 60^\circ = \frac{1}{2}$	$\cos 30^\circ = \frac{\sqrt{3}}{2}$
$\tan 45^\circ = 1$	$\tan 60^\circ = \sqrt{3}$	$\tan 30^\circ = \frac{1}{\sqrt{3}}$

QUESTION 1(NOV 2016)

- 1.1 In the diagram below, $P(-8 ; t)$ is a point in the Cartesian plane such that $OP = 17$ units and reflex $\angle XOP = \theta$



- 1.1.1 Calculate the value of t . (2)
- 1.1.2 Determine the value of each of the following WITHOUT using a calculator:
- (a) $\cos(-\theta)$ (2)
- (b) $1 - \sin \theta$ (2)

If $\sin 17^\circ = a$, WITHOUT using a calculator, express the following in terms of a :

- 1.2
- 1.2.1 $\tan 17^\circ$ (3)
- 1.2.2 $\sin 107^\circ$ (2)
- 1.2.3 $\cos^2 253^\circ + \sin^2 557^\circ$ (4)
- 1.3 Simplify fully, WITHOUT the use of a calculator: (6)

$$\frac{\cos(-225^\circ) \cdot \sin 135^\circ + \sin 330^\circ}{\tan 225^\circ}$$

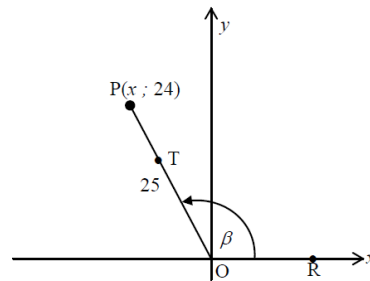
- 1.4 Prove the identity:

$$\frac{1}{(\cos x + 1)(\cos x - 1)} = \frac{-1}{\tan^2 x \cdot \cos^2 x} \quad (4)$$

- 1.5 Determine the general solution for $2 \sin x \cdot \cos x = \cos x$ (6)

QUESTION 2 (NOV 2015)

In the diagram below, $P(x; 24)$ is a point such that $OP = 25$ and $\angle ROP = \beta$, where β is an obtuse angle.



2.1 2.1.1 Calculate the value of x . (2)

2.1.2 Determine the value of each of the following WITHOUT using a calculator: (1)

(a) $\sin \beta$ (2)

(b) $\cos(180^\circ - \beta)$ (2)

(c) $\tan(-\beta)$ (2)

2.1.3 T is a point on OP such that $OT = 15$. Determine the coordinates of T, WITHOUT using a calculator. (4)

2.2 Determine the value of the following expression: (4)

$$\frac{2 \sin x \cdot \cos x (1 + \tan^2 x)}{\tan x}$$

2.3 Consider: $\frac{1 - \cos^2 A}{4 \cos(90^\circ + A)}$ (3)

2.3.1 Simplify the expression to a single trigonometric term. (6)

2.3.2 Hence, determine the general solution of [24]

$$\frac{1 - \cos^2 2x}{4 \cos(90^\circ + 2x)} = 0,21$$

QUESTION 3 (NOV 2017)

3.1 Simplify fully: $\sin(90^\circ - x) \cdot \cos(180^\circ + x) + \tan x \cos x \sin(x - 180^\circ)$ (6)

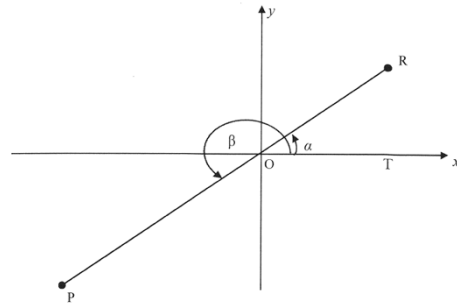
3.2 Prove WITHOUT using a calculator, that

$$\frac{\sin 315^\circ \cdot \tan 210^\circ \cdot \sin 190^\circ}{\cos 100^\circ \cdot \sin 120^\circ} = \frac{-\sqrt{2}}{3} \quad (6)$$

3.3 In the diagram below, R is a point in the first quadrant such that $\angle TOR = \alpha$.

RO is extended to P such that $OP = 2 RO$ and $\angle TOP = \beta$.

It is given that $\sin \alpha = \frac{3}{5}$.



WITHOUT using a calculator, determine:

3.3.1 The value of $\tan \alpha$ (3)

3.3.2 The value of $\sin \beta$ (3)

3.3.3 The coordinates of P. (4)

3.4 Prove the identity:

$$\frac{\sin \theta - \tan \theta \cdot \cos^2 \theta}{\cos \theta - 1 + \sin^2 \theta} = \tan \theta \quad (4)$$

QUESTION 4 (NOV 2014)

- 4.1 If $\cos 23^\circ = p$, express, **without the use of a calculator**, the following in terms of p :

4.1.1 $\cos 203^\circ$ (2)

4.1.2 $\sin 293^\circ$ (3)

- 4.2 Simplify the following expression to a single trigonometric term:

$$\frac{\sin(360^\circ - x) \cdot \tan(-x)}{\sin(180^\circ) \cdot (\sin^2 A + \cos^2 A)} \quad (6)$$

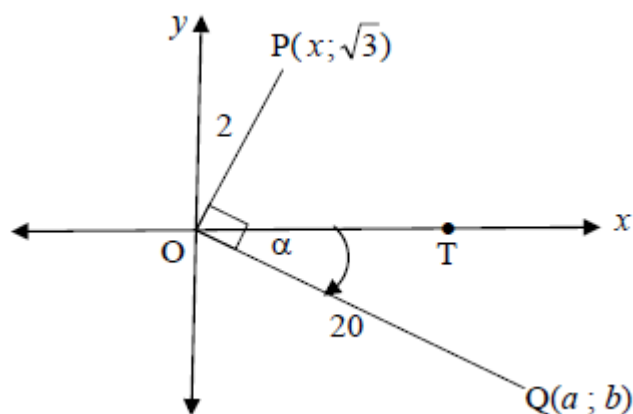
- 4.3

4.3.1 Prove the identity: $\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = \frac{2}{\cos x}$ (5)

- 4.3.2 For which values of x in the interval $0^\circ \leq x \leq 360^\circ$ will the identity in QUESTION 4.3.1 be undefined? (2)

- 4.4 Determine the general solution of: $\sin 2x = 4 \cos 2x$ (5)

- 4.5 In the diagram below $P(x; 3)$ is a point on the Cartesian plane such that $OP = 2$. $Q(a; b)$ is a point such that $\angle TOQ = \alpha$ and $OQ = 20$. $\angle POQ = 90^\circ$.



- 4.5.1 Calculate the value of x . (2)

- 4.5.2 Hence, calculate the size of α . (3)

(5)
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QUESTION 5

DO NOT USE A CALCULATOR FOR THIS QUESTION.

Given $\cos \theta = -\frac{1}{3}$ and $0^\circ \leq \theta \leq 180^\circ$

5.1

Determine, with the aid of a diagram, the value of the following

5.1.1 $\tan (180^\circ + \theta)$ (3)

5.1.2 $3\sin (\theta - 90^\circ)$ (2)

5.2

Given: $\frac{\sin(-210^\circ)}{\cos(300^\circ)} + \frac{\cos(x+90^\circ)}{\sin(360^\circ+x)}$

Simplify the following expression:

5.2.1 $\frac{\sin(-210^\circ)}{\cos(300^\circ)} + \frac{\cos(x+90^\circ)}{\sin(360^\circ+x)}$ (5)

5.2.2 For which values of x is the expression in 5.2.1 undefined for $-360^\circ \leq x \leq 360^\circ$? (2)

5.3 Prove that $\tan \theta \sqrt{\frac{1}{\sin^2 \theta} - 1} = 1$ (4)

5.4 Determine the general solution of the following equation:

$2\sin^2 \theta = 1 + \sin \theta$ (6)

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