KWAZULU-NATAL PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

CURRICULUM GRADE 10 -12 DIRECTORATE

NCS (CAPS) SUPPORT

LAST PUSH

REVISION DOCUMENT

MATHEMATICS

GRADE 12

2026

Study copy distributed by EduResource ZA | eduresourcza.com

This document has been compiled by the FET Mathematics Subject Advisors together with Lead Teachers. It seeks to unpack the contents and to give guidance to teachers.

TABLE OF CONTENTS

TOPICS		PAGE NUMBERS
1.	Algebra and Equations	3 – 6
2.	Sequences and Series	7 – 11
3.	Functions	11 – 22
4.	Financial Mathematics	23 – 28
5.	Differential Calculus	29 – 37
6.	Probability	37 – 45
7.	Statistics	45 – 54
8.	Analytical Geometry	55 – 64
9.	Trigonometry	65 – 78
10.	Euclidean Geometry	79 – 85
C.	Answers	86 – 100

Study copy distributed by EduResource ZA | eduresourcza.com

TOPIC: 1 **ALGEBRA, EQUATIONS AND INEQUALITIES [25 ± 3 MARKS]**

GUIDELINES, SUMMARY NOTES, & STRATEGIES

CONCEPT	HOW TO LEARN IT	RELEVANT FORMULAE AND KEYWORDS
Quadratic equations	Solve quadratic equations (by factorisation, completing the square and using quadratic formula)	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
Inequalities	Remove brackets, standard form, factorise, critical values, method and solve	
Solve exponential equations	Apply the laws of exponents to expressions involving rational exponents	
Surd equations	Isolate the surd and square both sides. Remember to check solutions of a surd equation.	
Simultaneous equations	Solve equations with two unknowns, one of which is linear and the other quadratic, algebraically.	Involves making y or x subject of the formula in the linear equation and then substituting into the other equation.
Nature of roots	The nature of roots and the conditions for which the roots are real, non-real, equal, unequal, rational and irrational.	$\Delta=b^2-4ac$ $\Delta<0$ Non-real/imaginary $\Delta\geq 0$ Real $\Delta=0$ Real and equal (1 root) $\Delta>0$ Real and unequal (2 roots) $\Delta>0$ and a perfect square, e.g. $\Delta=16$ $\Delta>0$ and not a perfect square, e.g. $\Delta=20$

1.1 **Solve for** x

1.1.1	$x(2x+1)=0$	(2)	**L1**	1.1.8	$x^2-9=0$	(2)	**L1**
1.1.2	$-x^2+12=-x$	(3)	**L1**	1.1.9	$14x^2=7x$	(3)	**L1**
1.1.3	$(2x-1)(x+5)=0$	(2)	**L1**	1.1.10	$(2x-5)(x^2-2)=0$	(3)	**L1**
1.1.4	$x(5-x)=-6$	(3)	**L2**	1.1.11	$2x(3x-4)=0$	(2)	**L1**
1.1.5	$x^2+7x=0$	(3)	**L1**	1.1.12	$(x+2)(3x-7)=0$	(2)	**L1**
1.1.6	$-\frac{1}{2}x(x-6)=0$	(2)	**L1**	1.1.13	$-x(x^2-4)=0$	(3)	**L1**
1.1.7	$(x^2-4x)^2-8(x^2-4x)-48=0$	(5)	**L2**	1.1.14	$\frac{2x^2-3x}{3}-\frac{5}{2x^2-3x}=\frac{2}{3}$	(6)	**L2**

1.2 Solve for x **(Correct to TWO decimal places)**

1.2.1	$2x-6=-5x^2$	(3)	**L1**	1.2.5	$x(2x+3)-7=0$	(3)	**L1**
1.2.2	$4x^2-12x-6=0$	(3)	**L1**	1.2.6	$3x^2-2x=4$	(4)	**L1**
1.2.3	$2x^2-4x+1=0$	(3)	**L1**	1.2.7	$5x^2-2=11x$	(4)	**L1**
1.2.4	$5-10x-3x^2=0$	(3)	**L1**	1.2.8	$x(10x-2)=6$	(4)	**L1**

Study copy distributed by EduResource ZA | eduresourcza.com

1.3 INEQUALITIES

1.3.1 $2x^2 - 2 \geq 3x$ (4) **L2**

1.3.2 $-x^2 + 3x > -4$ (4) **L2**

1.3.3 $-3(x+7)(x-5) < 0$ (4) **L2**

1.3.4 $x(5x-3) + 4 \geq 6x$ (4) **L2**

1.3.5 $3^x\left(x^2 - 3x + 2\right) \leq 0$ (4) **L3**

1.3.6 $5x - 2(x^2 - 6) \leq 0$ (4) **L2**

1.3.7 $\dfrac{2x-2}{x-3} > 3$ (5) **L2**

1.3.8 $(3x+2)(x-3) > 14$ (4) **L2**

1.3.9 $\dfrac{-4}{2x^2 - 7x - 4} \geq 0$ (4) **L3**

1.3.10 $\left(\frac{1}{5}\right)^x (3-x) > 0$ (2) **L3**

1.4 Simplification of EXPONENTIAL and SURD expressions without using a calculator:

1.4.1 $\dfrac{2^{2015} + 2^{2013}}{4^{1006}}$ (3) **L2**

1.4.2 $\sqrt{3}.\sqrt{48} - \dfrac{4^{x+1}}{2^{2x}}$ (3) **L2**

1.2.3 $\left(3\sqrt{2} - 12\right)\left(2\sqrt{2} + 1\right)$ (3) **L2**

1.4.4 $\sqrt{\dfrac{7^{2020} - 7^{2018}}{12}}$ (3) **L2**

1.4.5 $\dfrac{5^{2007} + 5^{2010}}{5^{2008} + 5^{2009}}$ (3) **L2**

1.4.6 $\dfrac{\sqrt{75x} - \sqrt{27x}}{\sqrt{27x}}$ (3) **L2**

1.4.7 $\left(\dfrac{\sqrt{3^{2011}} - \sqrt{3^{2009}}}{\sqrt{3^{2008}}} + \sqrt{3}\right)^2$ (4) **L3**

1.4.8 $\sqrt{\dfrac{3^{x+1} - 3^x}{3^{x-1}} + 3}$ (3) **L2**

1.4.9 $\dfrac{1}{\sqrt{1}+\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} + \dfrac{1}{\sqrt{3}+\sqrt{4}} + \ldots + \dfrac{1}{\sqrt{99}+\sqrt{100}}$ (3) **L3**

1.4.10 Express $\dfrac{4+\sqrt{27}}{2+\sqrt{3}}$ in the form of $a + b\sqrt{3}$ if $a, b \in \mathbb{Z}$ (4) **L2**

1.5 Solve the following EXPONENTIAL equations:

1.5.1 $3^{1-2x} - 1 = 0$ (3) **L2**

1.5.2 $\dfrac{3^{x^2}}{27} = 9^x$ (3) **L2**

1.5.3 $\dfrac{5^{2x} - 1}{5^x + 1} = 4$ (3) **L2**

1.5.4 $3^x(3^x - 9) = 0$ (2) **L2**

1.5.5 $3^{x+1} + 3^{x-1} + 3^x = 39$ (4) **L2**

1.5.6 $2^{\frac{2x}{3}} + 2^{\frac{x}{3}} - 6 = 0$ (3) **L3**

1.5.7 $4^{x-2} - 16^{3x+4} = 0$ (3) **L2**

1.5.8 $2^{2x} - 6.2^x = 16$ (4) **L2**

1.5.9 $9.2^{x-1} = 2.3^x$ (3) **L2**

1.5.10 $3^x + 3^{-x+1}.5 = 8$ (4) **L3**

1.6 Solve the following SURD equations:

1.6.1 $\sqrt{5-x} = x + 1$ (5) **L2**

1.6.2 $\sqrt{x-3} - 4 = 5$ (4) **L2**

1.6.3 $6 + \sqrt{x+7} = x + 1$ (5) **L2**

1.6.4 $2 + \sqrt{x+2} = 6 - x$ (5) **L2**

1.6.5 $11 - 2x + \sqrt{2x^2 - 29x + 114} = 0$ (4) **L3**

1.6.6 $x = 2\sqrt{2x-1} + 2$ (5) **L2**

1.6.7 $2 = \sqrt{8-m} + m$ (3) **L2**

1.6.8 $\sqrt{7x+2} + 2x = 8$ (4) **L2**

1.6.9 $2\sqrt{x-3} = x - 3$ (5) **L2**

1.6.10 $\sqrt{2x+5} - \dfrac{3}{\sqrt{2x+5}} = 2$ (6) **L3**

1.6.11 $x - 7\sqrt{x} = -12$ (4) **L2**

1.6.12 $2\sqrt{x+6} + 2 = x$ (4) **L2**

1.7 Solve the following equations simultaneously:

1.7.1 $y + x = 2$ and $x^2 + 3xy + 8 = 0$ (6) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

1.7.2 $x+2=2y$ and $\frac{1}{x}+\frac{1}{y}=1$ (5) **L3**

1.7.3 $\frac{2^x.2^y}{4}=1$ and $-2x^2+3xy=y^2$ (6) **L3**

1.7.4 $x-3=2y$ and $x^2-y^2=45$ (6) **L2**

1.7.5 $x+4=2y$ and $y^2-xy+21=0$ (6) **L2**

1.7.6 $x=2y+1$ and $x^2-2y+3xy=6$ (6) **L2**

1.7.7 $2x-y=7$ and $x^2+xy+y^2=21$ (6) **L2**

1.7.8 $2y+8-8x=0$ and $2x^2+4x=y^2+6$ (6) **L2**

1.7.9 $3x+y=-1$ and $9x^2=y^2$ (5) **L2**

1.7.10 $2^{y+2}=8^x$ and $x^2+y^2=2$ (5) **L3**

1.7.11 $(3x-y)^2+(x-5)^2=0$ (4) **L4**

1.7.12 $6y=x^2-2x-6$ and $\sum_{n=3}^{4}x(n-2)^2=10y$ (6) **L4**

1.8 Mathematical modelling

1.8.1 Calculate the sum of the digits of $2^{2009}\times 5^{2000}$ (5) **L4**

1.8.2 Given: $(\sqrt{6p}+\sqrt[4]{128})(\sqrt{6p}-\sqrt[4]{128})=54+a\sqrt{2}$. Calculate $a-p$ (4) **L3**

1.8.3 Determine the range of the function $y=x+\frac{1}{x}$, $x\neq 0$ and x is real. (6) **L4**

1.8.4 Consider the product $1\times 2\times 3\times 4\times .....\times 30$
Determine the largest value of k such that 3^k is a factor of this product. (4) **L4**

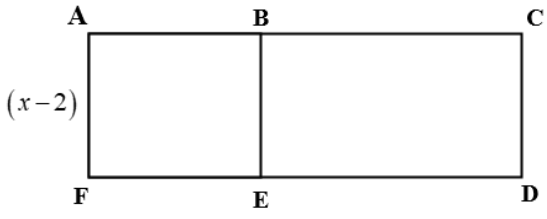
1.8.5 If n is the largest integer for which $n^{200}>5^{300}$, determine the value of n. (3) **L4**

1.8.6 Determine the sum of the digits of $5^{2009}\times 2^{2010}\times 24$ (4) **L4**

1.8.7 If it is given that $2^x\times 3^y=24^6$, determine the numerical value of $x-y$ (3) **L4**

1.8.8 Without the use of the calculator, prove that $(x+x^{-1})^2-(x-x^{-1})^2=4$ (4) **L3**

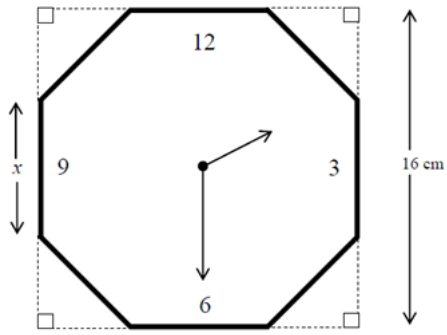
1.8.9 ACDF is a rectangle with an area of $(x^2+2x-8)\text{cm}^2$. B is on AC and E is a point on FD such that ABEF is a square with sides of length $(x-2)$cm each.



Calculate the length of ED. (5) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

1.8.10 Jabu manufactures wall clock faces in the shape of a regular octagon (eight-sided shape having sides of equal length). He cuts off identical isosceles triangles from the corners of the square sheet of glass having sides $16cm$ long.



If the length of a side of the clock face is x, calculate the value of x correct to TWO decimal places

(7) **L4**

1.9 Nature of roots

1.9.1 Discuss the nature of the roots of the equation $2(x-3)^2+2=0$ (2) **L2**

1.9.2 Determine the value(s) of p if $g(x)=-2x^2-px+2$ has a maximum value of $3\frac{1}{8}$. (4) **L3**

1.9.3 The roots of the equation $f(x)=0$ are $x=\frac{4\pm\sqrt{16-4m(-m+5)}}{2m}$,

Determine the value(s) of m for which the roots will be non-real (4) **L2**

1.9.4 Show that the maximum value of $\sqrt{-x^2+4x+12}$ is 4 (4) **L3**

1.9.5 Show that the roots of $3x^2+(k+2)x=1-k$ are real and rational for all value(s) of k. (4) **L3**

1.9.6 Given: $\frac{x^2-x-6}{3x-9}$

(a) For which value(s) of x will the expression be undefined? (2) **L2**

(b) Simplify the expression fully. (3) **L2**

1.9.7 The solution to the quadratic is given by $x=\frac{-2\pm\sqrt{9-r}}{3}$, where $r\in Q$.

Determine the value of r if the roots of the equation are non-real. (3) **L2**

1.9.8 Without solving the following equation, determine the nature of roots of:

$2-(x-1)^2=4$ (3) **L2**

1.9.9 Given : $\text{P}=\sqrt{\frac{3}{x+4}}-\frac{1}{x}$

(a) For which value(s) of x will P be undefined? (2) **L3**

(b) For which value(s) of x will P be non-real? (3) **L2**

1.9.10 Given: $2x^2-5x=3$

(a) Show that the roots of $2x^2-5x=3$ are real and rational. (2) **L2**

(b) Determine the value that must be added to the negative root so that the roots are equal. (3) **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

TOPIC:2 SEQUENCES AND SERIES [25 ± 3 MARKS]

FOR SUMMARY NOTES & STRATEGIES: Refer 2025 Last Push Document

EC OR TAMBO DISTRICT MAR 2026 (adapted)

2.1 Given the quadratic pattern: 5; 12; 21; 32; …

2.1.1 Determine the general term T_n of the pattern. (4) **L1**

2.1.2 Calculate the value of the 25th term. (2) **L2**

2.1.3 Which term of the sequence has the value of 1152? (2) **L2**

2.1.4 Determine which two consecutive terms have a first difference of 147. (4) **L3**

2.2 The first four terms of a quadratic number pattern are: -1 ; $-4x$; 11 ; $2x+12$;…

2.2.1 Calculate the value of x (4) **L2**

2.2.2 If $x=-2$, show that $T_n=-3n^2+18n-16$ (3) **L2**

2.2.3 Consider the first n terms of the quadratic number pattern. Calculate the least values of n for which the sum of the first differences will be less than -1380. (4) **L3**

KZN MAR 2026

2.3 Given: 26; 45 and 70 are the third, fourth and fifth terms of a quadratic number pattern.

2.3.1 Calculate the first term of the quadratic number pattern. (1) **L2**

2.3.2 Determine the expression for the nth term of the pattern. (4) **L2**

NSC MAY/JUN 2026

2.4 Given the arithmetic series: 6 + 10 + 14 + …

2.4.1 Which term in the series has the value of 602? (4) **L2**

2.4.2 Show that $S_n=2n^2+4n$ (2) **L2**

2.4.3 Calculate the value of $\sum_{n=100}^{150}(4n+2)$ (4) **L3**

MP SEP 2025

2.5 Given the arithmetic pattern: 181; ...; 9; 5; 1.

2.5.1 Determine the nth term of the sequence. (2) **L1**

2.5.2 Calculate the number of terms in the sequence. (2) **L1**

2.5.3 Calculate the sum of the series 181 + ... + 17 + 13. (3) **L2**

2.5.4 Write the series in 2.5.3 in sigma-notation. (2) **L2**

WC SEP 2025

2.6 The following arithmetic sequence is given: -7 ; -1 ; 5 ; ... ; 167

2.6.1 How many terms are there in this sequence? (2) **L1**

2.6.2 Calculate the value of n for which the sum of n terms will be 8208. (2) **L2**

2.7 Given the series: $(x)+\frac{(x)^2}{2}+\frac{(x)^3}{4}+...$

2.7.1 Determine the nth term in terms of x (2) **L2**

2.7.2 Determine the value(s) of x for which the series will converge. (2) **L1**

2.7.3 If the sum of the first two terms is equal to $\frac{5}{8}$ and $x>0$, determine S_∞ (6) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

LP SEP 2025 (adapted)

2.8 Given: -4 ; 0 ; 4 ; 8 ; ...

2.8.1 Write down the next three terms of the sequence. (1) **L1**

2.8.2 Determine the general term of the sequence. (2) **L2**

2.8.3 Determine the term of the sequence that is equal to 56. (2) **L2**

2.8.4 Calculate the value of the 12th term of the sequence. (1) **L2**

2.8.5 Show that all the terms of this sequence are even. (2) **L3**

2.9 Given: $3(2)^5 + 3(2)^4 + 3(2)^3 + ...$

2.9.1 Will the series converge? Give a reason for your answer. (2) **L2**

2.9.2 Calculate the sum of the first seven terms of the series. (2) **L2**

2.9.3 Calculate the sum to infinity of the series. (2) **L2**

2.9.4 Calculate: $\sum_{k=8}^{\infty} 3(2)^{6-k}$ (2) **L2**

EC SEP 2025

2.10 Evaluate: $\sum_{k=1}^{10} \frac{3}{2}(2)^k$ (4) **L2**

2.11 Given: $S_n = 2n^2 - 6n$

2.11.1 Calculate the sum of the first thirty terms. (2) **L2**

2.11.2 Determine the value of n if $T_n = 300$ (3) **L2**

KZN SEP 2025 (adapted)

2.12 Consider the arithmetic sequence: $x-3$; $2x+1$; $4x-1$; ...

2.12.1 Determine the value of x (3) **L2**

2.12.2 Calculate the numerical value of the 7th term. (3) **L2**

2.12.3 If $x=6$, determine which two consecutive terms have a product of 77259? (3) **L3**

2.13 Determine the number of terms in the following geometric sequence:

$\frac{1}{2}; \frac{\sqrt{3}}{2}; \frac{3}{2}; \frac{81\sqrt{3}}{2}$ (4) **L2**

2.14 Solve for p if $\sum_{k=0}^{\infty} 9p^k = \sum_{m=1}^{7} (-27)\left(-\frac{2}{3}\right)^m$ and $-1 < p < 1$ (6) **L3**

FS SEP 2025

2.15 Given the geometric sequence: $\frac{9}{2}$; 9 ; 18 ; ... ; 2304

2.15.1 Determine the value of r, the common ratio. (1) **L1**

2.15.2 Does the sequence converge? Motivate your answer. (2) **L2**

2.15.3 How many terms are there in the sequence? (3) **L2**

2.15.4 Determine the sum of the first 10 terms of the sequence. (2) **L2**

2.16 If the first term of the arithmetic series is 18, the common difference is 3 and the last term is 99.

2.16.1 Write down the next two terms of the series (2) **L1**

2.16.2 Calculate the sum of the series. (5) **L2**

2.16.3 If the even numbers are removed from the series, calculate the sum of the remaining terms in the series. (5) **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

DBE MAY/JUN 2026

2.17 Two clocks, A and B, were observed from 09: 00 to 10: 00. The following observations were made:

- The time on both clocks was exactly 09:00.
- Clock A measured time correctly from 09:00 to 10:00
- Clock B measured halftime that clock A measured in the first five minutes, clock B measured 50% of the time it measured in the previous 5 minutes.

2.17.1 What was the time on clock B when clock A reached 10:00? (5) **L3**

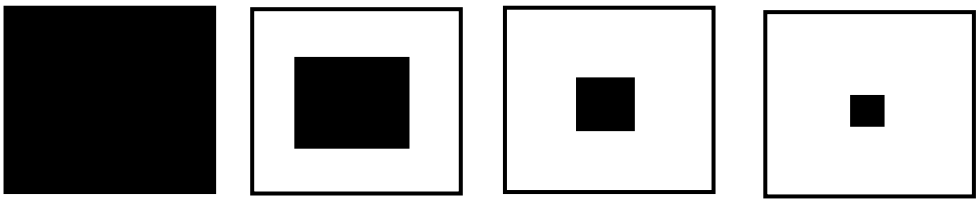
2.17.2 How long will it take clock B to reach the time of 09:06? justify your answer. (2) **L3**

IEB MAR 2017

2.18 The nth term of a quadratic sequence is given by $T_n = an^2 + bn + c$ .If $T_1 = -1, T_2 = 4$ and $T_3 = 11$, determine the values of a, b and c . (6) **L3**

MSI REVISION BOOK

2.19 A sequence of squares, each having side 1, is drawn as shown below. The first square is shaded, and the length of the side of each shaded square is half the length of the side of the shaded square in the previous diagram.



2.19.1 Determine the area of the unshaded region in DIAGRAM 3. (2) **L3**

2.19.2 What is the sum of the areas of the unshaded regions of the first seven squares? (5) **L3**

2.20 Given the geometric sequence: $3;6;12;24;\ ...$

2.20.1 Determine the formula for T_n . (2) **L2**

2.20.2 Which term in this sequence is equal to 768? (3) **L2**

HUDSON PARK HIGH SCHOOL JUN 2016

2.21 The 10th, 11th and 12th terms of an arithmetic sequence are: $2x+3$; $4x+10$; $10x-3$

2.21.1 Calculate the value of *x*, showing that it will be 5. (2) **L2**

2.21.2 Hence, determine T_{10} and T_{11} (2) **L2**

2.21.3 Now, calculate T_{500} (4) **L2**

2.22 Given: $\sum_{p=0}^{n}\left(\frac{1}{3}p+\frac{1}{6}\right)=20\frac{1}{6}$. Calculate the value of n (4) **L3**

KUTLWANONG

2.23 Consider the finite arithmetic sequence: $18;13;8;3;...;-72$

2.23.1 Write down the fifth term. (1) **L1**

2.23.2 Calculate the number of terms. (3) **L2**

2.23.3 Determine the sum of all the negative terms. (4) **L3**

2.23.4 How many terms are multiples of 5? (4) **L3**

2.24 Given the linear pattern: $5x-2$; $6x$; $7x+2$

2.24.1 Write down the next term. (1) **L1**

2.24.2 Determine the nth term (3) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

2.24.3 If the 15th term is 88, calculate x (3) **L2**

FS SEP 2025

2.25 The general term of a quadratic sequence is $T_n = an^2 - 5n + c$, If it is given that $T_4 = 18$ and $T_3 = T_2 + 5$. (5) **L3**

Calculate the value of a and c.

NC SEP 2025

2.26 Consider the geometric series: $726 + 243 + 81 + ...$

2.26.1 Calculate T_9 (3) **L1**

2.26.2 It is given that: $729 + 243 + 81 + ... T_m = 1093\frac{1}{3}$, calculate m. (3) **L2**

2.27 Consider the following infinite geometric series: $1 + r + r^2 + r^3 + ...$ (5) **L4**

The second and each consecutive term that follows, is twice the sum of all the terms in the series following that term. Determine the value of the 2nd and 3rd terms of the series.

GR 12 IEB SEQ & SER 2015-2021

2.28 Given the quadratic sequence: $\frac{2^3-1}{1}; \frac{3^3-1}{2}; \frac{4^3-1}{3}; \frac{5^3-1}{4} ...$, determine the nth term.

Simplify your answer as far as possible. (4) **L3**

2.29 The sum of the first n terms of the geometric sequence $9 + 6 + 4 + ...$ is greater than 25.

Calculate the smallest value of n. (6) **L4**

2.30 A geometric series is such that $S_4 = 7\frac{1}{2}$; $S_5 = 15\frac{1}{2}$; and $S_6 = 31\frac{1}{2}$ .Determine , in terms of n, the sum to n terms of the series. (7) **L3**

DGHS JAN TEST 2025

2.31 A convergent geometric series consisting of only positive terms has first term a, constant ratio r and n^{th} term, T_n, such that $\sum_{n=3}^{\infty} T_n = \frac{1}{4}$

2.31.1 If $T_1 + T_2 = 2$, show that $a = \frac{a}{1+r}$ (2) **L2**

2.31.2 Calculate the value(s) of a and r (6) **L3**

ORIGINAL

2.32 Given the sequence: $3; 3; 9; 6; 15; 9; 21...$

2.32.1 Calculate the difference between 37th and 36th terms. (4) **L3**

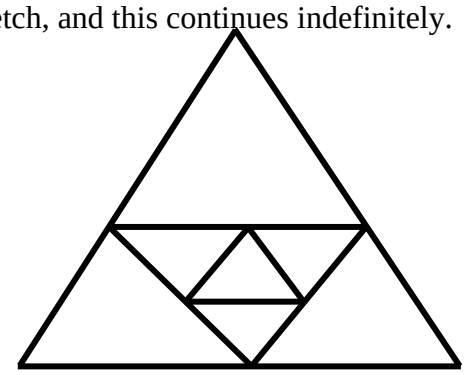
2.32.2 Calculate the sum to 19 terms (5) **L3**

2.32.3 Prove that all the terms of the infinite sequence are divisible by 3 (3) **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

GP JUN 2025

2.33 An equilateral triangle ΔRST with sides of length $12p$ units is drawn. A second triangle is drawn by joining the midpoints of the sides of the first triangle RST. Each triangle thereafter is drawn by joining the midpoints of the sides of the previous triangle as shown on the sketch, and this continues indefinitely.



2.33.1 Write down, in terms of p, the length of each side of the second triangle. (1) **L2**

2.33.2 Calculate, in terms of p, the perpendicular height of ΔRST. (2) **L2**

2.33.3 Show that the sum of the areas of all the triangles formed will not exceed $48\sqrt{3}p^2$ (5) **L4**

FOR MORE LEVEL 4 QUESTIONS REFER TO 2021 HYGHFLYERS DOCUMENT.

Study copy distributed by EduResource ZA | eduresourcza.com

TOPIC: 3 **FUNCTIONS AND INVERSES [35 ± 3 MARKS]**

REFER TO LAST PUSH 2025 DOCUMENT FOR NOTES

1. Refer to IEB MAY 2024 (2025 DOCUMENT PAGE 16 NO4)

- Changing $f(x)=\frac{x+a}{x+b}$ to $f(x)=\frac{a}{x+p}+q$.
- Determining the minimum length between the curves of a hyperbolic function.
- Determining the range after a function has been transformed.

2. Refer to DBE NOV 2024 (2025) DOCUMENT PAGE 18 NO 10)

- Turning point of a quadratic function.
- Equation of a line.
- Maximum length of a vertical line between two functions.
- Values of a variable for which a function can be a tangent to another function, (**unfamiliar question)**.

FS SEP 2023

3.1 Given the function: $h(x)=-\frac{2}{x-2}+2$.

3.1.1	Write down the equations of the asymptotes of h.	(2)	**L1**
3.1.2	Calculate the x-intercept of h.	(2)	**L2**
3.1.3	Draw the graph of h. Clearly show all asymptotes and intercepts with the axes.	(4)	**L2**
3.1.4	Determine the equation of the axis of symmetry of h, in the form $y=mx+c$ where $m<0$.	(3)	**L2**
3.1.5	Determine the range of $-h(x)+1$	(2)	**L3**
3.1.6	Determine the values of x where $h(x)\le 0$.	(2)	**L2**

FS SEP 2025

3.2 Given $f(x)=x^2-2x-3$ and $g(x)=x+3$

3.2.1	Write down the y-intercepts of both graphs.	(2)	**L1**
3.2.2	Sketch the graphs of f and g^{-1} on the same set of axes, indicating the turning point and all intercepts with the axes.	(5)	**L3**
3.2.3	Determine the values of for which $f(x)=k$ will have two unequal positive real roots.	(3)	**L3**
3.2.4	For which values of x will $f^{/}(x).f^{//}(x)\ge 0$?	(3)	**L4**

EC JUN 2017

3.3 3.3.1 Given a function, $f: y+4=(x-5)^2$

(a)	Write down the equation of the axis of symmetry of f.	(1)	**L1**
(b)	Determine the x-intercept of f.	(3)	**L2**
(c)	Sketch the graph of f, clearly showing the intercepts with the axes and the turning point.	(4)	**L2**
(d)	Write down the range of f.	(1)	**L1**
(e)	f is transformed to g, where the x-intercepts of g is the same as that of f and the turning point of g is (5; 4). Describe the transformation and write the equation of g.	(2)	**L3**

3.3.2 Given: $f(x)=x^2+3$ and $g(x)=kx-1$, determine the value(s) of k if g is the tangent to the graph of f. (5) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

IEB MAY 2026

3.4 A parabola has the equation $y = 2(x-p)^2 - q$.

3.4.1 If $p > 0$ and $q > 0$, draw a possible sketch of the parabola. (3) **L2**

3.4.2 Determine the positive distance between the x-intercepts in terms of p and q. (3) **L4**

DBE NOV 2015

3.5 The function defined as $y = \frac{a}{x+p} + q$ has the following properties:

- The domain is $x \in R, x \neq -2$.
- $y = x + 6$ is an axis of symmetry.
- The function is increasing for all $x \in R, x \neq -2$.

Draw a neat sketch of this function. Your sketch must include all the asymptotes. (4) **L4**

FS SEP 2018

3.6 3.6.1 Given: $f(x) = 3^{-x}$

(a) Write down the equation of f^{-1}, the inverse of f, in the form $f^{-1}(x) = \ldots$ (2) **L2**

(b) On the set axes, sketch the graphs of $f(x)$ and $f^{-1}(x)$. Clearly indicate the intercepts with the axes and one other point on the graphs and label the TWO graphs. (4) **L2**

3.6.2 A linear function satisfies the following conditions: $p(-3) = 10$ and $p'(x) = -2$. Determine the inverse of p in the form $p^{-1}(x) = \ldots$ (4) **L4**

WC SEP 2017

3.7 Given: $f(x) = \left(\frac{1}{5}\right)^x$

3.7.1 Determine the equation of f^{-1} in the form y = … (1) **L1**

3.7.2 Sketch the graphs of f and f^{-1} on the same system of axes on the diagram sheet. Clearly show all intercepts with the axes. (4) **L2**

3.7.3 Write down the domain of f^{-1}. (2) **L1**

3.7.4 For which values of x will $f(x).f^{-1}(x) \geq 0$? (2) **L3**

3.7.5 Write down the range of $g(x)$ if $g(x) = -f(x) - 3$ (2) **L3**

KZN JUN 2019

3.8 Given $t(x) = 8^x$

3.8.1 Write down the equation of t^{-1}, the inverse of t, in the form $y = \ldots$ (2) **L2**

3.8.2 Show that $t\left(x + \frac{1}{3}\right) = 2.t(x)$. (3) **L4**

3.8.3 Sketch t and t^{-1} in the same system of axes, showing the line of reflection and intercepts with the axes. (4) **L2**

IEB NOV 2025

3.9 If $g(x) = -a^{-x} + k$ then:

3.9.1 (a) Determine the coordinates of the x – intercepts of the graph of g in terms of a and k (3) **L2**

(b) Sketch the graph of $y = g(x)$ if $0 < a < 1$ and $k > 2$. (3) **L2**

3.9.2 Given that: $f = \{(-1; 2); (2; -1); (4; 7); (-3; 1); (5; 6)\}$

(a) Is f a function? Explain. (2) **L1**

(b) Give the domain of f^{-1}. (1) **L3**

(c) Determine p if $f[f^{-1}(2)] = p$. (2) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

FS SEP 2024

3.10 Given the functions $f(x) = -\frac{2}{x} - 1$ and $g(x) = k^x$. The point $(1\ ;\ 3)$ lies on g.

3.10.1 Determine the value of k. (2) **L1**

3.10.2 Write down the equations of the asymptotes of f. (2) **L1**

3.10.3 Write down the equation of g^{-1}, the inverse of g. (2) **L2**

3.10.4 Determine the x-intercept(s) of f. (2) **L2**

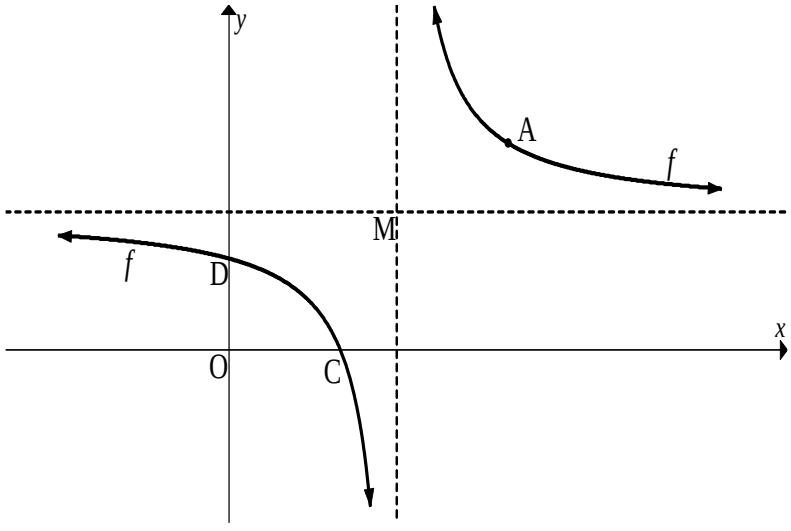
3.10.5 Draw a neat sketch of f and g^{-1} on the same set of axes, clearly indicate all asymptotes and intercepts with the axes. (5) **L2**

3.10.6 For which values of x will the axis of symmetry of f, which has a negative gradient, intersect with the graphs of f. (4) **L3**

3.10.7 Consider the graph of h where $h(x) = g^{-1}(x+2)$. Determine the value(s) of x for which $g^{-1}(x+2) \leq 1$. (2) **L3**

DBE MAY 2025

3.11 The graph of $f(x) = \frac{4}{x-3} + 4$ is drawn alongside. M is the point where the asymptotes of f intersect. C and D are the x- and y-intercepts respectively of f. A is the point on f that is closest to M.



3.11.1 Write down the coordinates of M. (2) **L1**

3.11.2 Calculate the coordinates of D. (2) **L1**

3.11.3 If $y = x + t$ is the equation of a line of symmetry of f, calculate the value of t. (2) **L2**

3.11.4 Determine the values of x for which $f(x) \leq 0$. (4) **L3**

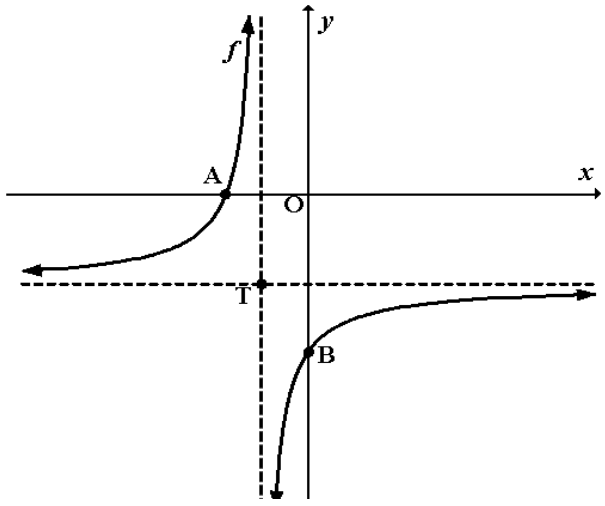
3.11.5 Calculate the coordinates of A. (3) **L3**

3.11.6 A single transformation is applied to f to obtain a new graph defined as $h(x) = \frac{-4}{x+3} + 4$. A$^{/}$ is the image of A under this transformation. Calculate the length of AA$^{/}$. (2) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

IEB MAY 2026

3.12 The function $f(x)=\frac{-3}{x+1}-2$ is given, and the graph is sketched alongside. A is the x-intercept of f, B is the y-intercept of f, and T is the point of intersection of the asymptotes.

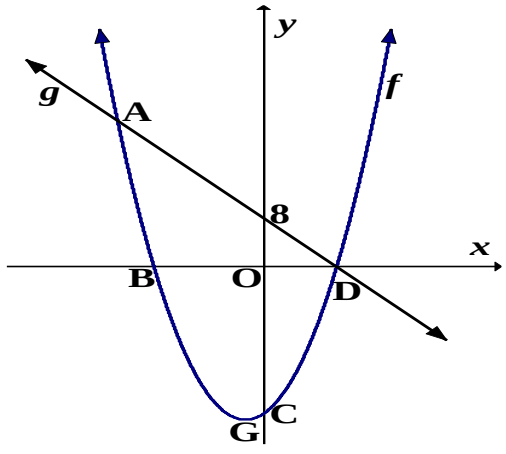


3.12.1 Show that A, T, and B do not lie on a straight line. (5) **L3**

3.12.2 If $y=\frac{k}{x+1}-2$, determine the value of k so that A, T, and B are in a straight line. (5) **L4**

WC SEP 2025

3.13 The graphs of $f(x)=(x-4)(x+6)$ and $g(x)=mx+c$ are shown alongside.
The x-intercepts of f are points B and D.
C is the y-intercept of f and G is the turning point of f.



3.13.1 Write down the coordinates of B and D. (2) **L1**

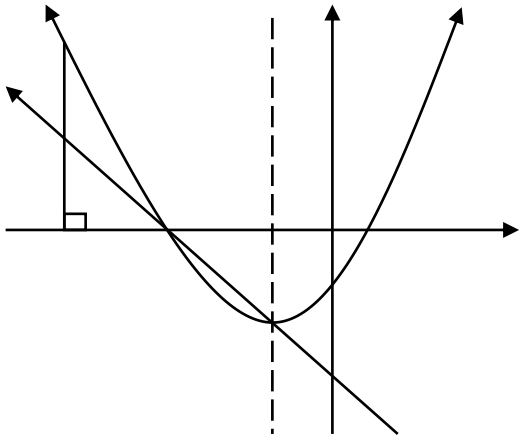
3.13.2 Determine the values of m and c. (3) **L2**

3.13.3 Determine the coordinates of $A^{/}$, the reflection of A in the line $y=x$ (5) **L3**

3.13.4 Determine the value of k for which $g(x)-k$ will be a tangent to f. (5) **L4**

NW JUN 2025

3.14 In the diagram alongside, the graphs of $f(x)=ax^2+bx+c$ and $g(x)=-4x-12$ are drawn. B and C are x-intercepts of f. C is the x-intercept of g. f and g intersect at A and C. A is the turning point of f. The line $x=-1$ is the axis of symmetry of f.



3.14.1 Determine the coordinates of A, B and C. (4) **L2**

3.14.2 Show that $f(x)=2x^2+4x-6$. (3) **L2**

3.14.3 Write down the coordinates of the turning point of $f(x-3)-2$ (2) **L2**

3.14.4 Write down the value of k if $f(x)=k$ has equal roots. (1) **L2**

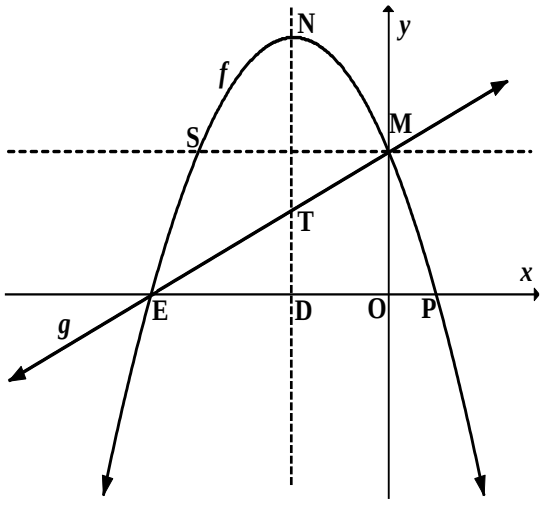
3.14.5 Write down the range of $-f(x)$ (2) **L2**

3.14.6 If $f(x)=2x^2+4x-6$, determine the length of OT if PRT is perpendicular to the x-axis with P a point on f, R is a point on g and $\text{PRT}=16$ units. (6) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

LP SEP 2025

3.15 The graphs of $f(x) = -x^2 - 4x + 5$ and $g(x) = mx + c$ are given alongside. Points E, M and P are the intercepts of the graphs with the axis. N is the turning point of *f* and NTD is the axis of symmetry of *f*. T is the point on the graph of *g*.



3.15.1 Determine the coordinates of E and P. (3) **L1**

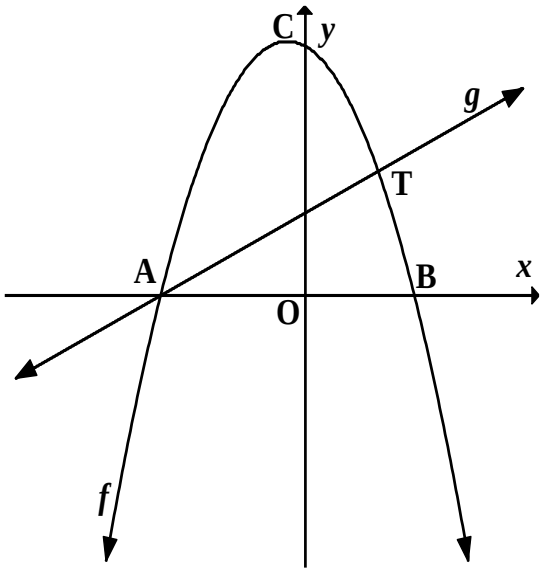
3.15.2 Determine the values of *m* and *c*. (2) **L2**

3.15.3 Calculate the length of NT. (6) **L3**

3.15.4 Determine the coordinates of point S, the mirror image of point M with respect of the axis of symmetry. (2) **L3**

FS SEP 2024

3.16 The sketch shows the graphs of the functions $f(x) = -x^2 - x + 12$ and $g(x) = x + 4$.
A and B are the *x*-intercepts of *f*, while C is the turning point of *f*. The functions intersect at A and T.



3.16.1 Determine the *x* coordinates of A and B. (3) **L1**

3.16.2 Determine the range of $f(x)$. (3) **L2**

3.16.3 Determine the maximum value of $k(x) = 3^{f(x)-12}$ (2) **L2**

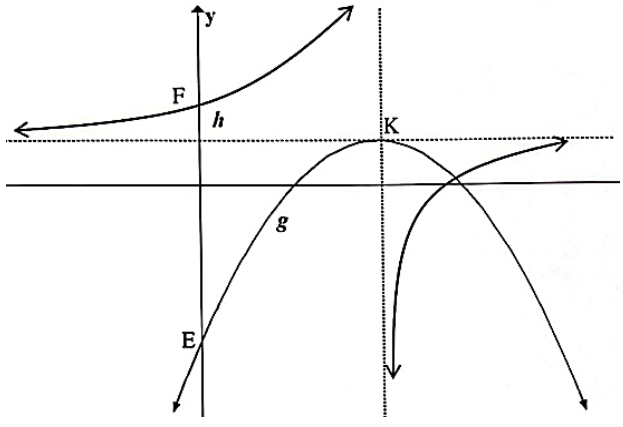
3.16.4 Determine for which value(s) of *x* will $x.f(x) > 0$. (2) **L2**

3.16.5 For which value(s) of *k* will $-x^2 - x + 12 = k$ have two negative unequal roots? (2) **L3**

3.16.6 Write down the equation of *h* in the form $h(x) = a(x+p)^2 + q$ if *h* is the reflection of *f* in the straight line $x = 1$ (4) **L3**

NC SEP 2025

3.17 Sketched alongside are the graphs of $g(x) = ax^2 + bx - 7$ and $h(x) = -\frac{2}{x-3} + 2$.
E and F are the *y*-intercepts of *g* and *h* respectively. The turning point K is also the point of intersection of the two asymptotes of *h*.



3.17.1 Write down the coordinates of K. (2) **L1**

3.17.2 Calculate the length of EF. (4) **L2**

3.17.3 Determine the values of *a* and *b* in $g(x) = ax^2 + bx - 7$. (4) **L2**

3.17.4 Write down the range of $h(x) + 3$. (2) **L2**

3.17.5 For which values of *p* will $g(x) = p$ have TWO unequal roots? (2) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

3.17.6 Determine the equation of the line of symmetry of h, $m < 0$, in the form $y = mx + c$. (5) **L2**

3.17.7 Describe the transformation from graph g to the graph of t, where $t(x) = (x-3)^2$. (2) **L3**

3.17.8 It is given that $g(x) = f'(x)$. Determine the x-value(s) where the gradient of a tangent to f is equal to 1. (3) **L3**

NW SEP 2025

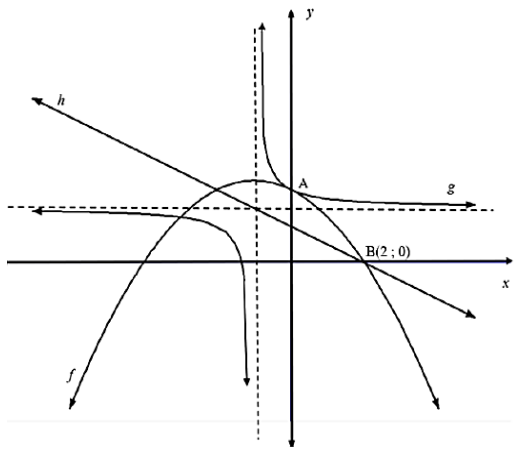
3.18 Sketched alongside are the graphs of

$f(x) = -\frac{1}{2}x^2 + bx + c$ and

$g(x) = \frac{a}{x+p} + q.$

The axis of symmetry of f, $x = -1$, is the vertical asymptote of g. The line h is an axis of symmetry of g. A is the y – intercept of f and g.

$B(2\ ;\ 0)$ is an x-intercept of f and h.



3.18.1 Show that the equation of f is $f(x) = -\frac{1}{2}x^2 - x + 4$. (2) **L2**

3.18.2 Write down the coordinates of the y-intercept of f. (2) **L1**

3.18.3 Determine the coordinates of the turning point of f. (2) **L1**

3.18.4 Determine the equation of h. (3) **L2**

3.18.5 Determine the equation of g. (4) **L3**

3.18.6 Determine the coordinates of the x-intercept of g. (3) **L1**

3.18.7 For which values of x will it be $g(x).f'(x) \leq 0$? (3) **L3**

3.18.8 Determine the equation of $k(x)$ if k is the reflection of g about the line $x = 4$. (3) **L3**

3.18.9 For which values of p will both x-intercepts of $f(x) + p$ be greater than – 2? (4) **L4**

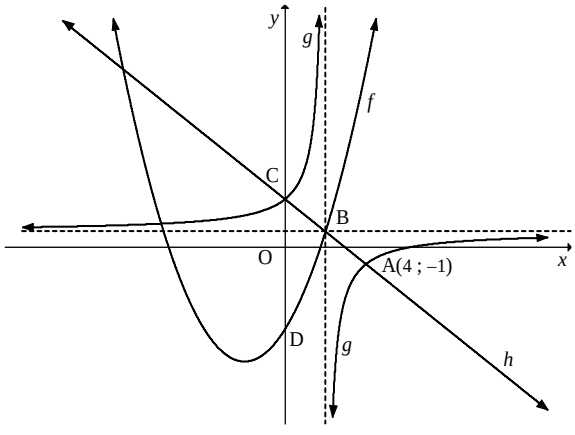
KZN SEP 2025

3.19 The graphs of $f(x) = \frac{1}{2}(x+p)^2 + q$ and

$g(x) = \frac{a}{x+r} + t$ are sketched alongside.

The line $h(x) = -x + 3$ is an axis of symmetry of g and h. Point C is the y-intercept of both g and h. $E(-2\ ;\ w)$ are the coordinates of the turning point of f.

B, a point on f, is the point of intersection of the asymptotes of g.



3.19.1 Write down the coordinates of C. (1) **L1**

3.19.2 Show that the coordinates of B are $(2\ ;\ 1)$ (4) **L3**

3.19.3 Determine the values of a, r and t. (4) **L2**

3.19.4 Determine the equations of the asymptotes of the graph of j if $j(x) = g(x+3) - 1$. (2) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

3.19.5 Show that the equation of *f* is given by $f(x)=\frac{1}{2}x^2+2x-5$. (3) **L2**

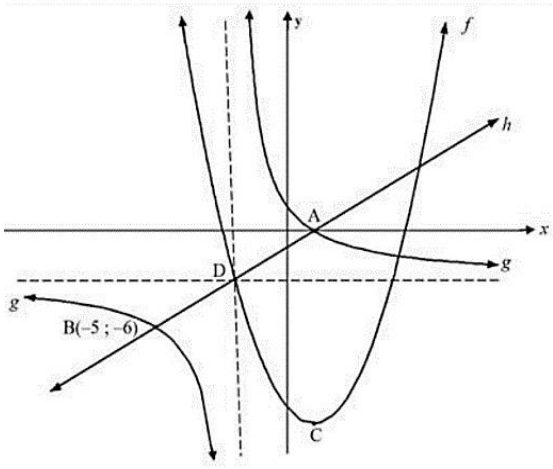
3.19.6 If $f(x)=k$, determine the values of k for $f(x)$ has TWO negative roots. (2) **L3**

3.19.7 Determine the values of d such that $\frac{1}{2}(x+d)^2+2(x+d)-5=-(x+d)+3$ will have one positive and one negative root. (5) **L4**

NW SEP 2024

3.20 The graphs of $g(x)=\frac{a}{x+r}+t$ and $f(x)=(x+p)^2+q$ are sketched below.

The line $h(x)=x-1$ is an axis of symmetry of *g*. Point A is the *x*-intercept of *g*. A and $B(-5;-6)$ are the points of intersection of *g* and *h*. The axis of symmetry of *f* intersects the *x* – axis at A. C is the turning point of *f*. D, a point on *f*, is the point of intersection of the axis of symmetry of *g*.



3.20.1 Determine the coordinates of A (2) **L1**

3.20.2 Show that the coordinates of D is given by: $D(-2\ ;\ -3)$ (2) **L2**

3.20.3 Determine the equation of *g*. (3) **L2**

3.20.4 Show that the equation of *f* is: $f(x)=x^2-2x-11$ (3) **L2**

3.20.5 Determine the *x*-intercepts of *f*. (3) **L3**

3.20.6 For which values of *x* will $f'(x).f''(x)<0$? (2) **L3**

3.20.7 Calculate the maximum value of $\frac{f''(x)}{f(x)+14}$. (3) **L4**

3.20.8 For which value(s) of *m* will $(x+m)^2-2(x+m)-11=x-1$ have TWO different negative roots? (7) **L4**

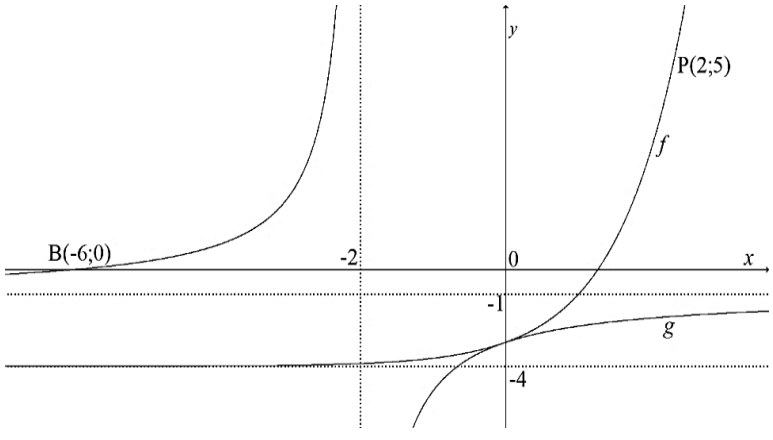
FS SEP 2025

3.21 The diagram below shows the graphs of the functions of $f(x)=b^x+c$ and $g(x)=\frac{a}{x+p}+q$.

$B(-6;0)$ is the *x*-intercept of *g*.

The graphs of f and g have a common *y*-intercept.

$P(2;5)$ is a point on *f*.



3.21.1 Write down the values of p and q. (2) **L2**

3.21.2 Determine the equation of g. (3) **L1**

3.21.3 Determine the equation of the inverse of the axis of symmetry of g for $m<0$. (4) **L3**

3.21.4 Write down the equation of the asymptote of *f*. (1) **L1**

3.21.5 Determine the equation of *f*. (3) **L2**

3.21.6 Determine the equation of h if $h(x)=f(x)+4$ (2) **L2**

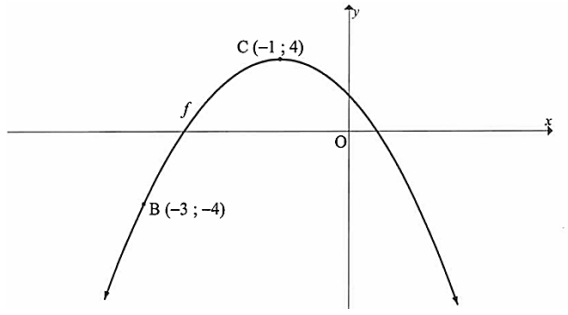
Study copy distributed by EduResource ZA | eduresourcza.com

3.21.7 Determine the equation of h^{-1}, the inverse of h, in the form $y = ...$ (2) **L1**

3.21.8 For which values of x is $f(x) \geq g(x)$? (2) **L2**

DBE MAY 2025

3.22 The graph of $f(x) = a(x+p)^2 + q$ is drawn below. C(– 1 ; 4) is the turning point of *f*.
B(– 3 ; – 4) is a point on *f*.



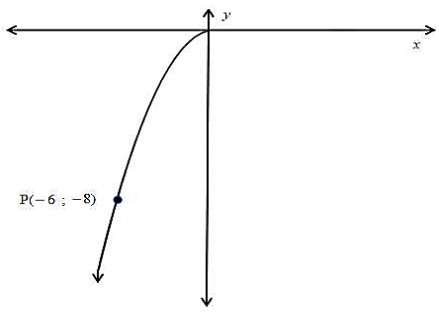
3.22.1 Show that $f(x) = -2x^2 - 4x + 2$. (3) **L2**

3.22.2 Determine the values of *k* for which $h(x) = f(x) + k$ will have no real roots. (2) **L2**

3.22.3 The graph of $y = g'(x)$ where g' is the derivative of *g*, is obtained when *f* is reflected over the line $y = 4$. Draw a sketch graph of g if $g(0) < 0$.
Clearly indicate any stationary points on your graph. (4) **L3**

GP SEP 2024

3.23



The graph of $f(x) = ax^2$, $x \leq 0$ is sketched alongside.

The point $P(-6\ ;\ -8)$ lies on the graph of f.

3.23.1 Calculate the value of a. (2) **L1**

3.23.2 Determine the equation of $f^{-1}(x)$, in the form $y = ...$ (2) **L2**

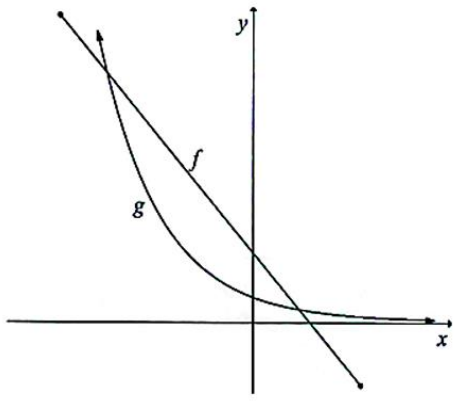
3.23.3 Write down the range of $f^{-1}(x)$. (1) **L1**

3.23.4 Sketch the graph of $f^{-1}(x)$. Indicate the coordinates of any point on the graph different to $(0;0)$. (1) **L2**

3.23.5 The graph of f is reflected across the line $y = x$, and thereafter it is reflected across the x - axis. Determine the equation of the new function in the form $y = ...$ (2) **L2**

KZN SEP 2024

3.24



The following graphs are drawn below:

- $f(x) = -2x + 3$
- $g(x) = 2^{-x}$

3.24.1 Calculate the *x* – intercept of *f*. (2) **L1**

3.24.2 For which values of *x* is $-2^{-x+1}.x + 6.2^{-x-1} < 0$? (4) **L4**

3.24.3 A new graph *p* is formed by reflecting *g* in the line $y = x$. Write down the equation of p in the form y = … (2) **L2**

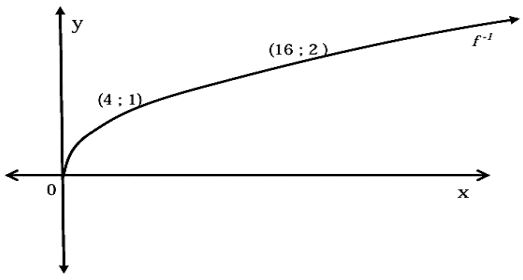
3.24.4 Write down the range of f^{-1}, the inverse of *f*. (2) **L1**

3.24.4 Determine the coordinates of the point of intersection between *f* and f^{-1}. (3) **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

UNKNOWN

3.25



The graph of the inverse of $f(x) = ax^2, x \geq 0,$ is drawn below. (4 ; 1) and (16 ; 2) are points on $f^{-1}(x)$.

Determine:

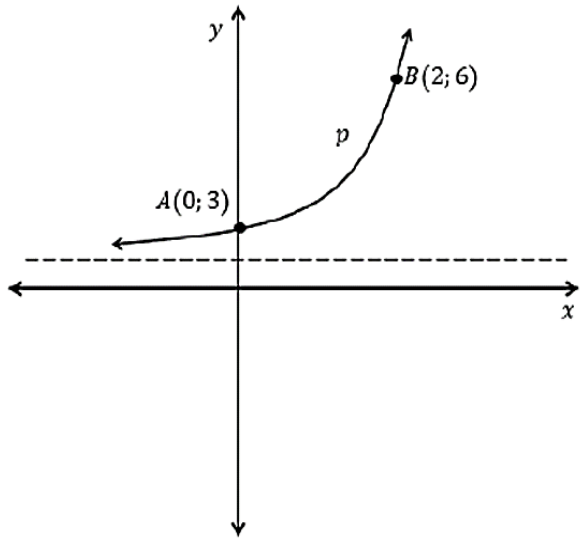
3.25.1 The value of a. (2) **L1**

3.25.2 The equation of f^{-1}. (2) **L2**

3.25.3 x, if $f(x+2) = 16$. (2) **L3**

WC SEP 2025

3.26 The graph of $p(x) = 2^x + q$ passes through A(0; 3) and B(2 ; 6).



3.26.1 Calculate the value of q. (2) **L1**

3.26.2 The graph of p is translated 2 units down to obtain $k(x)$. Determine the equation of $k^{-1}(x)$ in the form y = … (3) **L2**

3.26.3 For which values of x is:

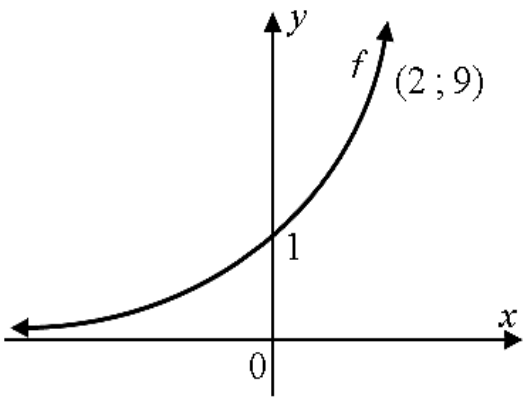
(a) $k^{-1}(x) \leq 0$ (2) **L2**

(b) $p(x+3).k^{-1}(x-5) > 0$ (2) **L4**

3.26.4 If p is reflected about the horizontal asymptote and translated 3 units to the left to form $h(x)$, determine the equation of $h(x)$. (2) **L3**

LP JUN 2025 AMENDED

3.27



The sketch below represents the graph of $f(x) = b^x + q$ which intercepts the y – axis at (0; 1) and passes through the point $(2\ ;\ 9)$.

3.27.1 Show that the equation of $f(x) = 3^x$. (3) **L2**

3.27.2 Determine the equation of $f^{-1}(x)$ (2) **L1**

3.27.3 Sketch the graph of $f^{-1}(x)$. Show all intercepts with the axes. (3) **L2**

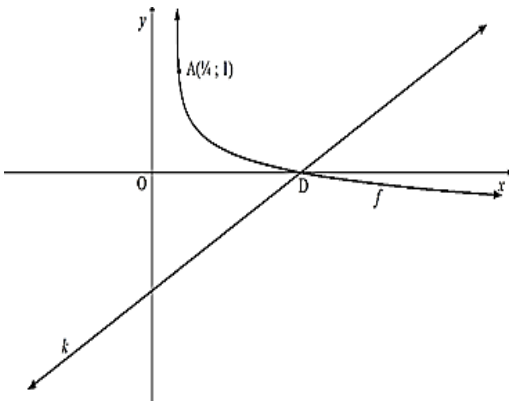
3.27.4 Given that $g(x) = 2x - 1$, calculate the value of P if:

$$P = \sum_{x=-2}^{15} f(x) - \sum_{x=1}^{20} g(x)$$ (5) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

KZN SEP 2025

3.28



The graphs of $f(x)=\log_b x$ and $k(x)=mx-3$ are drawn below. $A\left(\frac{1}{4};1\right)$ is the point on f and D is the x-intercept of both f and k.

3.28.1 Write down the coordinates of D. (1) **L2**

3.28.2 Determine the values of b. (2) **L2**

3.28.3 Determine the values of m. (2) **L1**

3.28.4 Write down the domain of f. (1) **L1**

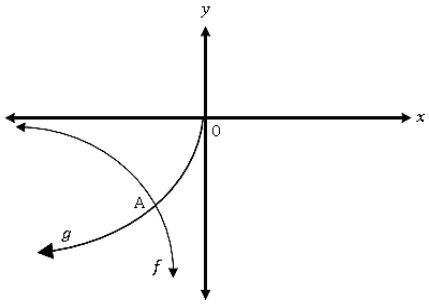
3.28.5 Determine the equation of f^{-1}, the inverse of f in the form of $y=....$ (2) **L1**

3.28.6 Determine the values of x for which $\frac{1}{4}\le f^{-1}(x)\le 16$. (3) **L3**

3.28.7 Sketch the graphs of f^{-1} and k^{-1} on the same system of axes. Clearly indicate the intercepts with the axes. (4) **L2**

GP JUNE 2016

3.29



The sketch below shows the graphs of $f(x)=\frac{1}{x}$ for $x<0$ and $g(x)=-\sqrt{-x}$ for $x\le 0$.

3.29.1 Write down the equation of the asymptote of the graph of f. (2) **L2**

3.29.2 The graphs of f and g intersect at the point $A(p;\ -1)$. Calculate the value of p. (2) **L2**

3.29.3 Determine the equation of $g^{-1}(x)$ the inverse of g, in the form $y=.....$ (3) **L2**

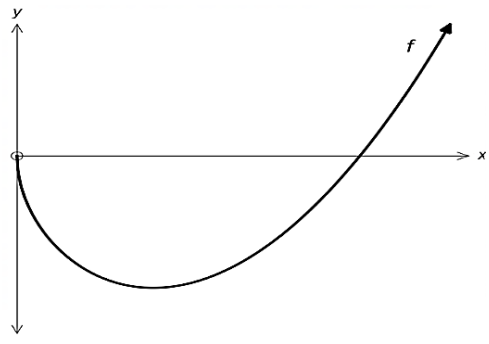
3.29.4 Write down the equation of h if h is a reflection of g about the y-axis. (1) **L1**

****IEB MAY 2026****

3.30

Given: $f(x)=\frac{3x^2}{4}-192\sqrt{x}$

Determine the coordinates of the point on the graph where the tangent is parallel to the x-axis.

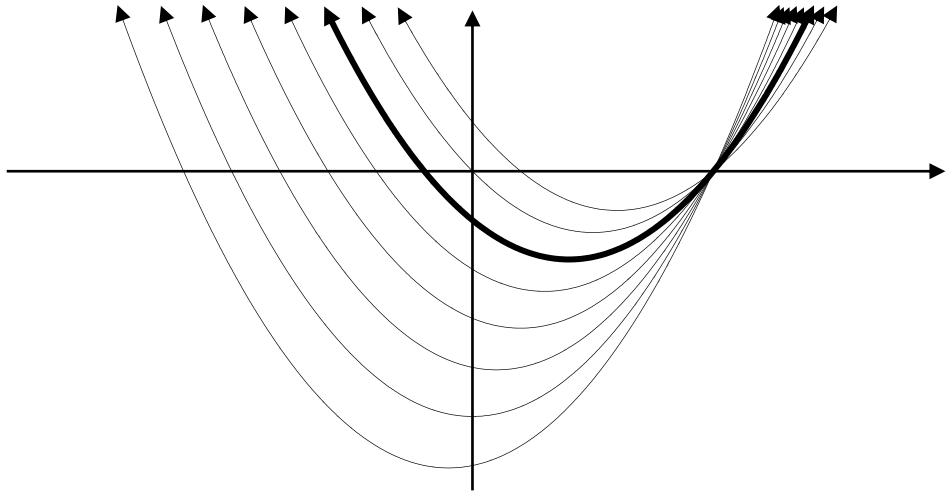


(6) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

****IEB MAY 2026****

3.31 In the diagram, a sequence of functions are generated from the function $r(x) = (x + 1)(x - 5)$, which is given in bold.
Each graph is generated by moving the left x – intercept of r, while the right x – intercept remains fixed.



3.31.1 Write the equation of the function when the left x – intercept of $r(x)$ is moved 2 units to the right. (1) **L2**

3.31.2 If the left x – intercept of $r(x)$ is moved n units to the left, show that the coordinates of the turning point is $\left(\frac{4-n}{2}; \frac{(n+6)^2}{-4}\right)$. (5) **L4**

3.31.3 Show that the function that goes through all the turning points of the graph created by this sequence is quadratic and determine the function's equation. (9) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

TOPIC: 4 FINANCIAL MATHEMATICS [15 ± 3 MARKS]

GUIDELINES, SUMMARY NOTES, & STRATEGIES (REFER TO THE 2025 LAST PUSH DOCUMENT)

EC SEP 2025

4.1 Brian deposits R700 at the end of each month into the savings account for 15 years. Exactly 5 years after the first payment, R5 200 was paid in the account as an additional payment. The interest rate on the savings account is 12% per annum, compounded monthly.
Determine the accumulated amount at the end of the investment period. (5) **L3**

4.2 Mr Faku was granted a loan of R900 000, over a period of 20 years. The loan is repaid in equal monthly instalments at the end of each month. The interest rate is 11,5% per annum compounded monthly. The first instalment is paid 4 months after the loan was granted.

4.2.1 Determine Mr Faku's monthly payments. (4) **L3**

4.2.2 The balance immediately after 16 years is R379 811,29. Determine how much interest he paid till the end of 16 years. (3) **L3**

DBE MAY/JUN 2025

4.3 John invests an amount of money in an account paying interest at a rate of 15% p.a., compounded monthly. Calculate the annual effective interest rate of this investment. (2) **L2**

4.4 Tino invests R500 000 in an account earning interest at a rate of 6% p.a., compounded quarterly. Tino decides to withdraw R11 250 at the end of every 3 months. Tino will continue making these regular withdrawals until there is no money in the account. How many withdrawals of R11 250 will Tino be able to make? (5) **L3**

4.5 On 1 March 2021, Abby made a once-off deposit of R12 000 into an account earning interest at a rate of 9,5% p.a., compounded monthly. She deposited R500 into the same account on 1 April 2023 and continues to make these monthly deposits of R500 on the first day of each month thereafter.
Calculate how much money was in the account immediately after the deposit of R500 is made on 1 March 2025, exactly four full years after her initial deposit. (6) **L3**

DBE MAY/JUN 2026

4.6 On 1 January 2025, the balance in Ben's savings account was R75 000. This savings account earns interest at $r\%$ p.a., compounded half-yearly. Calculate the value of r if Ben will have R85 000 in the account on 1 January 2027. (3) **L2**

4.7 William wants to buy a house. The bank requires William to pay a deposit of 10% of the purchase price of the house. The bank will grant William a loan to cover the outstanding amount, subject to the following conditions:

- The loan is to be repaid over 20 years.
- Interest on the loan is fixed at 11,5% p.a., compounded monthly.

An instalment of R10 665 is payable at the end of each month after the loan has been granted, starting on month after the loan has been granted.

4.7.1 Calculate the deposit required on the purchase price of the house. (5) **L3**

4.7.2 After negotiating with the seller, it was agreed that the purchase price of the house will be R1 100 000. William decides to pay a deposit of R120 000 and a monthly instalment of R11 000 at the end of each month, starting one month after the loan has been granted.

Study copy distributed by EduResource ZA | eduresourcza.com

(a) How long (in years) will it take for him to pay off the loan? (5) **L2**

(b) How much will William have paid in total towards the purchase of the house at the end of 16 years after the loan was granted? (3) **L3**

KZN SEP 2025

4.8 Johnson deposited an amount of R10 000 into a savings account that pays interest at 7,15% p.a., compounded quarterly.

4.8.1 Calculate the effective interest rate. (2) **L1**

4.8.2 How long will it take for this investment to grow to R17 628,78? (4) **L2**

4.9 Mr Simbine took out a home loan of R1 000 000 at an interest rate of 11,25% p.a., compounded monthly over 20 years. The loan is repaid via monthly instalments of R10 492,56.

4.9.1 Determine the outstanding balance after the 125th payment. (4) **L2**

4.9.2 Mr Simbine had financial difficulties and was unable to make the 126th, 127th, 128th, 129th and 130th payment. The bank agreed to restructure the loan so that it is paid off in the same amount of time. Determine the new monthly instalments. (5) **L3**

NW SEP 2018

4.10 William receives an amount of R2 000 000 upon his retirement. He invests this amount immediately at an interest rate of 9,3% p.a., compounded monthly.
He needs an amount of R35 000 per month to maintain his current lifestyle. He plans to withdraw the first amount by the end of the first month.
For how many months will he be able to live from his investment? (5) **L3**

4.11 John has just bought a new truck for R1 200 000. He has decided to replace the truck in 6 years' time, when the trade-in value of the truck would be R400 000. The replacement cost of the truck is expected to increase by 7,5% per annum.

4.11.1 Consider the trade-in value of his old truck and calculate how much cash John will need in 6 years' time to buy a new truck. (3) **L1**

4.11.2 John starts a sinking fund to provide for a new truck. Determine the quarterly deposit if his first deposit is made 6 months after he bought his truck and his last deposit 6 months before the end of the 6 years. The interest rate for the money in the sinking fund is 11% per annum, compounded quarterly. (5) **L3**

4.11.3 Suppose that 2 years after the purchase of his truck and every year thereafter for the next 3 years, John wants to withdraw R8000 from his account, to pay for services for his truck. If he makes four such withdrawals, what will the new quarterly deposit be during this period? (4) **L3**

DBE MAR 2017

4.12 On the 2nd day of January 2015, a company bought a new printer for R150 000.

- The value of the printer decreases by 20% annually on the reducing balance method.
- When the book value of the printer is R49 152, the company will replace the printer.

4.12.1 Calculate the book value of the printer on the 2nd day of January 2017. (3) **L1**

4.12.2 At the beginning of which year will the company have to replace the printer? Show ALL calculations. (4) **L2**

4.12.3 The cost of a similar printer will be R280 000 at the beginning of 2020. The company will use the R49 152 that it will receive from the sale of the old printer to cover some of the costs of replacing the printer. The company set up a sinking fund to cover the balance. The fund pays interest at 8,5% per annum, compounded quarterly. The first deposit was made on 2 April 2015 and every three months thereafter until 2 January **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

2020. Calculate the amount that should be deposited every three months to have enough money to replace the printer on 2 January 2020. (4)

DBE NOV 2017

4.13 Mbali invested R10 000 for 3 years at an interest rate of $r\%$ p.a., compounded monthly. At the end of this period, she received R12 146,72. Calculate r, correct to ONE decimal place. (5) **L2**

4.14 Piet takes out a loan from a bank to buy a car for R235 000. He agrees to repay the loan over a period of 54 months. The first instalment will be paid one month after the loan is granted. The bank charges interest at 11% p.a., compounded monthly.

4.14.1 Calculate Piet's monthly instalment. (4) **L2**

4.14.2 Calculate the total amount of interest that Piet will pay during the first year of the repayment of the loan. (6) **L3**

MP SEP 2025

4.15 A loan of R800 000 is taken out at an interest rate of 13% p.a., compounded bi-annually. It is repaid by equal bi-annual payments of R57 000 and a final payment less than R57 000.

4.15.1 How many payments are required to settle the loan? (5) **L3**

4.15.2 What is the outstanding balance on the loan after the final payment of R57 000? (2) **L3**

4.15.3 What is the value of the final payment? (1) **L1**

4.16 A company sold shares at a fixed price of R765,50 per share over 4 years. A minimum guaranteed interest rate earned over the 4-year period is 7,4% per annum compounded quarterly.
Charlene purchased 10 shares every quarter over the 4-year period. Her first purchase was at the beginning of the first quarter of the 4-year period, and then at the end of each quarter.

4.16.1 Calculate the minimum value of her investment at the end of the 4-year period. (4) **L3**

4.16.2 Calculate the percentage profit she made at the end of the 4-year period. (3) **L3**

GP SEP 2025

4.17 A business buys a machine that costs R120 000. The value of the machine depreciates at 9% per annum according to the diminishing-balance method.

4.17.1 Determine the scrap value of the machine at the end of 5 years. (2) **L1**

4.17.2 After 5 years the machine needs to be replaced. During this time, inflation remained constant at 7% per annum.
Determine the cost of the new machine at the end of 5 years. (2) **L1**

4.17.3 The business estimates that it will need approximately R90 000 by the end of 5 years. A sinking fund for approximately R90 000, into which equal monthly instalments must be paid, is set up. Interest on this fund is 8,5% per annum, compounded monthly. The first payment will be made immediately, and the last payment will be made at the end of the 5-year period.

Calculate the value of the monthly payments into the sinking fund. (3) **L2**

4.18 Kasala receives an amount of R900 000 upon her retirement. She invests this amount immediately at an interest rate of 10,5% per annum, compounded monthly. She needs an amount of R18 000 per month to maintain her current lifestyle and plans to withdraw the first amount at the end of the month.

For how many months will she be able to live off her investment? (5) **L3**

DBE MAY/JUN 2016

4.19 How long would the price of an asset take to reduce by a third of its original value if it depreciates on a reducing balance at a rate of 4,7% p.a.? (4) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

4.20 Lebogo bought a tractor for Rx on 1 April 2016.

- She will trade in this tractor when she replaces it with a similar one in 5 years' time on 1 April 2021.
- The tractor depreciates by 20% p.a. according to the reducing-balance method.
- The price of a similar tractor increases by 18% annually.
- Lebogo calculated that if she deposited R8 000 per month into a sinking fund, which paid interest at 10% p.a., compounded monthly, she would have enough money to cover the replacement cost of the tractor. She made the first deposit in this fund on

30 April 2016 and will continue to do so at the end of every month until 31 March 2021.

4.20.1 Determine, in terms of x, what the book value of the current tractor will be on 1 April 2021 (that is, 5 years after it was bought). Give your answer correct to FIVE decimal places. (2) **L1**

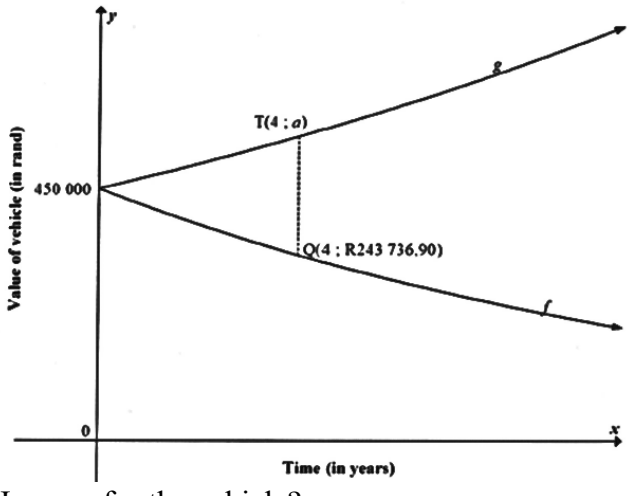
4.20.2 Determine, in terms of x, what the price of a similar new tractor will be on 1 April 2021 (that is, 5 years after it was bought). Give your answer correct to FIVE decimal places. (2) **L1**

4.20.3 Calculate the amount accumulated in the sinking fund on 1 April 2021. (4) **L2**

4.20.4 Calculate the value of x, the price of the current tractor. Round off your answer to the nearest thousand. (4) **L3**

DBE NOV 2015

4.21 The graph of f shows the book value of a vehicle x years after the time Joe bought it. The graph of g shows the cost price of a similar new vehicle x years later.



4.21.1 How much did Joe pay for the vehicle? (1) **L1**

4.21.2 Use the reducing-balance method to calculate the percentage annual rate of depreciation of the vehicle that Joe bought. (4) **L2**

4.21.3 If the average rate of the price increase of the vehicle is 8,1% p.a., calculate the value of a. (3) **L2**

4.21.4 A vehicle that costs R450 000 now, is to be replaced at the end of 4 years. The old vehicle will be used as a trade-in. A sinking fund is created to cover the replacement cost of this vehicle. Payments will be made at the end of each month. The first payment will be made at the end of the 13th month, and the last payment will be made at the end of the 48th month. The sinking fund earns interest at the rate of 6,2% p.a., compounded monthly.

Calculate the monthly payment to the fund. (5) **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

IEB NOV 2019

4.22 Busi opens a new credit card account that charges compound interest at 12,3% p.a. compounded weekly.

[<https://www.postoffice.co.uk/credit-card/platinum>]

Note: For the calculations in this question, assume that the relevant years have 52 weeks each.

4.22.1 Show that the balance owing on the credit card one week after the purchase will be R12 378,21 (to the nearest cent) (3) **L1**

4.22.2 Determine how long it will take Busi to pay off the money that she owes on her credit card if she repays R94,75 per week and she does not make any other purchases using this card. (4) **L2**

4.22.3 If the depreciation rate of her computer is 20% per annum on a straight-line basis, determine what its value will be after two years. (3) **L2**

4.22.4 She wants to sell the computer after two years. Will the depreciated value of the computer be sufficient to pay off the outstanding balance immediately after the 104th payment? Show all working. (6) **L3**

IEB MAR 2019

4.23 Greta purchased a car for R325 000, 9 years ago. The car depreciates on a reducing balance method at a rate of 7% per annum over the 9 years.

Determine what percentage this depreciation value would be of the car's original value. (3) **L2**

4.24 Greta was granted a loan of R1 825 000 to purchase a house which was repayable over 25 years. The bank offered her an interest rate of 9,5% per annum compounded monthly.
After 9 years of repaying her loan, she decided to purchase a second car.
Instead of applying for vehicle finance she decided to access money from her home loan.
Option 1:
She could access 75% of each of her monthly repayments over the 9 years without interest.
Option 2:
She could access five (5) times the monetary **difference** between the original loan and the balance on the loan after 9 years, immediately after the 108th payment.
Which option would yield the highest amount available to purchase a new car? **Show all working.** (10) **L4**

MIND ACTION SERIES 2nd EDITION

4.25 Robert invests money for 8 years. During the first 3 years, he deposits R5 000 at the end of each quarter. Thereafter, he invests R8 000 per quarter. For the first 2 years, interest is calculated at 12% p.a., compounded quarterly. The interest rate for the remainder of the term is 10,5% p.a., compounded quarterly. How much money will Robert have at the end of the 8-year period? (5) **L3**

4.26 Johann invests R10 000 per quarter. The first deposit is made 3 months from now and the last deposit at the end of 7 year. The interest rate is 12,5% p.a., compounded monthly. How much money will Johann have saved at the end of the 7 years? (4) **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

4.27 Yesheni deposits R1 000 per month, at the end of each month, into a bank account paying 5,4% p.a., compounded monthly over a period of 5 years. She makes the first deposit at the end of January of the first year. At the end of each year (31 December), an amount of R5 000 is paid to her father, from her account, as a new year's gift. How much money will Yesheni have in this account after ten years. (Take into account that she still makes the final R1 000 payment on the last day of the year and also that the R5 000 is paid to her father on this day.
Calculate the amount in the account after both these transactions have been completed. (5) **L4**

GP SEP 2023

4.28 Shimmy invests R4 000 000 into an account earning interest of 6% per annum, compounded monthly. She withdraws R30 000 per month. Her first withdrawal is exactly one month after she deposited the R4 000 000.

4.28.1 How many withdrawals of R30 000 will Shimmy be able to make? (5) **L2**

4.28.2 How many withdrawals will Shimmy be able to make if she changes the amount withdrawn per month to R20 000?
Substantiate your answer. (3) **L3**

ORIGINAL

4.29 Wandile bought a house costing R1 500 000 and used his savings as the deposit. He obtained a home loan for the balance of the purchase price at an interest of 12% p.a., compounded monthly over 25 years. He made his first monthly instalment of R11 058,85 towards the loan on 31 August 2017.

4.29.1 Calculate the percentage deposit paid by Wandile. (5) **L4**

4.29.2 What will the balance outstanding on the loan be on 31 July 2038? (3) **L3**

4.29.3 Calculate the interest Wandile will have paid over the last 4 years. (3) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

TOPIC : 4

DIFFERENTIAL CALCULUS [35 $\pm$ 3 MARKS]

GUIDELINES, SUMMARY NOTES, & STRATEGIES
TEACHING APPROACHES (CALCULUS)

1. **FIRST PRINCIPLES:**

The learners:

- ✓ Need to understand what is meant by determining the gradient from first principles and know the first principles formula.
- ✓ Be able to simplify the first principles expression (It seems as if learners handled this question better when they determine $f(x+h)$ separately and then bring it back to the formula).
- ✓ Need to be mindful of the notation and apply it correctly when they simplify the first principle expression.

2. **RULES FOR DIFFERENTIATION**

- ✓ Learners:
  - i. Need to revise how to simplify surds, rational, irrational exponents.
  - ii. Must know how to simplify expressions before differentiation.
  - iii. Must know how to tell which variable they are required to differentiating with respect to.
- ✓ Must always use of correct notation.

3. **CUBIC FUNCTIONS** $f(x) = ax^3 + bx^2 + cx + d$

The learners need to know and follow these steps when sketching a cubic function:

- ✓ Before learners can sketch a cubic function, they at least need to know the shape of their graph as guided by value of a where a could be $a > 0$ and $a < 0$.
- ✓ The learners must be able to factories a third-degree polynomial using any other method to determine the ***x*-intercepts** (the *x*-intercepts are known as the: zero, roots, $f(x) = 0$.) It would be an advantage if they can be able factories using a calculator.
- ✓ They must also be able to find the ***y*- intercept**, which is when $x = 0$, or given by the value of *d*.
- ✓ Learners must be able to use the first derivative, to find the coordinates of the turning points, which are also known as the Stationery points or local minima and local maxima. In simple terms, this is finding $f'(x) = 0$, solve for *x*, and then find the corresponding *y*-values to give the coordinate of the turning point.

4. **INTERPRETATION OF A CUBIC FUNCTION:**

The learners must be able to:

- ✓ Tell what the domain is, that $x \in R$
- ✓ Understand that the point of inflection is determined by equating the second derivative to zero and solving for *x*. An alternative method is to add up the *x*-coordinates of the turning points and divide by 2 (i.e. determining the midpoint of the two turning points).
- ✓ Tell for which values of *x* will $f(x)$ be concave up: $f''(x) > 0$ & Concave down: $f''(x) < 0$
- ✓ Tell where *f* is increasing or decreasing: increasing ($f'(x) > 0$, decrease $f'(x) < 0$).
- ✓ Determine the values of *x*, for which: $x.f(x) > 0$, $f'(x) > 0$, $f'(x).f(x) < 0$
- ✓ Tell when will f have three real roots, two real roots or one real root?

5. **OPTIMIZATION**

The learners need to develop the conceptual understanding on Optimization

- **Calculus of motion**
  - ✓ In this regard, the equation will be given.
  - ✓ The learners need to know that, Velocity is the derivative of displacement, and
  - ✓ Acceleration (2nd derivative) is the derivative of velocity

- **Rates of change**
  - ✓ Knowledge of formulae for the surface area and volume of right prisms is required from learners.

Study copy distributed by EduResource ZA | eduresourcza.com

- ✓ A list of relevant formulae will only be provided for the surface area and volume of cones, spheres and pyramids. Learners must select the correct one to use.

REVISION QUESTIONS

5.1 **FIRST PRINCIPLES**

Ultradeep.co.za SEP 2025

5.1.1 Determine $f^{/}(x)$ from first principles it is given that $f(x)=2x^2-1$. (5) **L2**

NW JUN 2026

5.1.2 Determine $f^{/}(x)$ from first principles if it is given that $f(x)=3+2x^2$. (5) **L2**

LP SEP 2025

5.1.3 If $f(x)=x^2-3x$,determine $f'(x)$ from first principle. (5) **L2**

KZN SEP 2025

5.1.4 Given: $f(x)=-5x^2$. Determine $f'(x)$ from first principles. (5) **L2**

FS SEP 2025

5.1.5 Given: $f(x)=-\frac{1}{x}$, determine $f^{/}(x)$ from first principles. (5) **L2**

NW SEP 2025

5.1.6 Given: $f(x)=x^2-x+3$, determine $f^{/}(x)$ from first principles. (5) **L2**

MIND ACTION GR12

5.1.7 Determine $f^{/}(x)$ from first principles if $f(x)=4$ (3) **L2**

MP SEP 2025

5.1.8 Determine $f^{/}(x)$ from first principles if $f(x)=-\frac{4}{x}$ (5) **L2**

FS SEP 2016

5.1.9 Determine $f^{/}(x)$ from first principles if $f(x)=x^3$ and hence find $f^{/}(-2)$ (5) **L2**

KZN JUN 2020

5.1.10 Determine $f^{/}(x)$ from first principles given $f(x)=x^2-bx$ (5) **L2**

5.2 **Rules for Differentiation:**

ULTRADEEP.CO.ZA SEP 2025

5.2.1 Determine $\frac{dy}{dx}$ if $y=(2x+1)(3x-3)$ (4) **L2**

FS SEP 23

5.2.2 Determine $\frac{dy}{dx}$ if $y=3x^4-\frac{7}{x}+2.\sqrt[3]{x^2}$ (4) **L3**

UTRADEEP.CO.ZA SEP 2025

5.2.3 Determine the derivative of $D_x\left[2x^5+\frac{4}{x^3}\right]$ (4) **L2**

MP SEP 2025

5.2.4 Determine $\frac{dy}{dx}$ if $y=(x^2-x-6)(x-3)^{-1}$ (2) **L2**

NW JUN 2026

5.2.5 Determine $\frac{dy}{dx}$ given that $yx=7x^3-5x^{\frac{2}{5}}$ (3) **L2**

KZN MAR 2025

5.2.6 Determine $\frac{dy}{dx}$ if $y=\frac{x^3-27}{x-3}$ (3) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

FS SEP 2025

5.2.7 $\frac{dy}{dx}$ if $y=\frac{\sqrt[4]{x^{-5}}+2x^{-3}-x}{x^{-3}}$ (6) **L2**

KZN SEPT 23

5.2.9 Determine $\frac{dy}{dx}$ if $\sqrt{y+x}=x+3$ (3) **L2**

NW SEP 2025

5.2.11 Differentiate with respect to x,
$f(x)=5x^2-7x+2$ (2) **L2**

NW JUN 2026

5.2.13 Determine
$\frac{dy}{dx}$ given that $yx=7x^3-5x^{\frac{2}{5}}$ (3) **L2**

FS SEP 2025

5.2.8 Determine $D_x\left[(4x+3)(2x-5)\right]$ (3) **L2**

ULTRADEEP.CO.ZA SEP 2025

5.2.10 Determine $f^{/}(x)$ if $f(x)=\sqrt{x}-3x$ (3) **L2**

NW SEP 2025

5.2.12 Determine $f^{/}(x)$ if $f(x)=\frac{2x+1}{\sqrt[3]{x}}$ (4) **L2**

KZN MAR 2025

5.2.14 Determine $\frac{dy}{dx}$ if $y=\frac{x^3-27}{x-3}$ (3) **L2**

5.3 **TANGENTS RELATED QUESTIONS**

NW JUN 2026

5.3.1 The tangent to the curve of $g(x)=x^3+px^2+qx-7$ at $x=1$ has the equation $y=5x-8$.

(a) Show that $(1;-3)$ is the point of contact of the tangent to the graph of g. (2) **L2**

(b) Hence, calculate the values of p and q (6) **L3**

Ultradeep.co.za SEP 2025

5.3.2 Given $f(x)=ax^3+bx^2+cx-5$ the gradient at any point on $f(x)$ is given as $6x^2-24$, determine the values of a,b and c. (4) **L3**

EC JUN 2026

5.3.3 Given that $h(x)=3x^4+4x$, determine the equation of the tangent to h at point $T(-1\,;\,y)$ (5) **L3**

5.3.4 Given: $h(x)=-x^3+bx^2+cx$, with $P\left(-1;-\frac{7}{2}\right)$ a turning point of h. the graph of h passes through the origin. Determine the values of b and c. (Show ALL your workings) (6) **L3**

PLATINUM MATHS GR12

5.3.5 Consider the graph $f(x)=-x^3-3x^2+4$ and $g(x)=\frac{23}{9}x^2-\frac{19}{3}x$

(a) State the point where the graphs share a common tangent. (5) **L4**

(b) Determine the equation of the common tangent at this point. (4) **L3**

NSC NOV 2010

5.3.6 A function $g(x)=ax^2+\frac{b}{x}$ has a minimum value at $x=4$. The function value at $x=4$ is 96. Calculate the value of a and b. (6) **L4**

5.3.7 The tangent to the curve of $g(x)=2x^3+px^2+qx-7$ at $x=1$ has the equation $y=5x-8$.

(a) Show that (2 ; 2) is the point of contact of the tangent to the graph. (1) **L2**

(b) Hence or otherwise, calculate the values of p and q. (6) **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

5.4 **CUBIC FUNCTIONS**

EC JUN 2026

5.4.1 Given: $f(x) = x^3 - 3x^2 - 9x - 5$

(a) Show that $(x+1)$ is a factor of $f(x)$ (1) **L2**

(b) Calculate the x and y-intercepts of f (3) **L2**

(c) Calculate the coordinates of the turning points of the graph of f (3) **L2**

(d) Draw a sketch graph of f. Clearly label all the intercepts with the axes and any turning points. (4) **L2**

(e) For which value of x is:

(i) f increasing? (2) **L2**

(ii) f concave up? (2) **L2**

(iii) $f(x).f^{/}(x) > 0$? (2) **L3**

EC JUN 2026

5.4.2 Given: $f(x) = 3x^3 - 3x^2 + 6x - 2$

For which values of x will f be concave down. (4) **L3**

FS SEP 2025

5.4.3 Given: $f(x) = (x-1)(x^2 - 4x + 4)$

$= x^3 - 5x^2 + 8x - 4$

(a) Determine the coordinates of the turning points of f. (5) **L1**

(b) Draw a sketch graph of f. Clearly label all the intercepts with the axes and any turning points. (4) **L2**

(c) For which value(s) of x is:

(i) $f^{/}(x) > 0$? (2) **L3**

(ii) $x.f^{/}(x) < 0$? (2) **L3**

NC SEP 2025

5.4.4 Given: $f(x) = x^3 - 2x^2 + x$

(a) Determine the value(s) of x for which $f(x) = 0$ (3) **L1**

(b) Calculate the coordinates of the turning points of f. (5) **L2**

(c) Sketch the graph of f. Clearly show all turning points of f. (3) **L2**

(d) Determine for which values of x:

(i) $f^{/}(x) < 0$ (2) **L2**

(ii) The graph of f concave down (4) **L2**

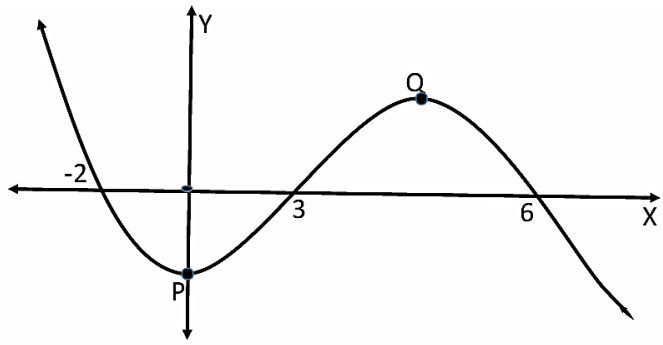
(e) A tangent is drawn to the graph of g. where $g(x) = f(x-2)$. Calculate the gradient of the tangent at $x = 0$. (3) **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

NW JUN 2026

5.4.6 Sketched alongside is the graph of $f(x) = -x^3 + ax^2 + bx + c$

- P and Q are the turning points of f
- $x = -2; x = 3$ and $x = 6$ and the x intercepts



(a) Show that $a = 7$, b = 0 and c = – 36 (2) **L3**

(b) Calculate the coordinates of the turning points P and Q (5) **L2**

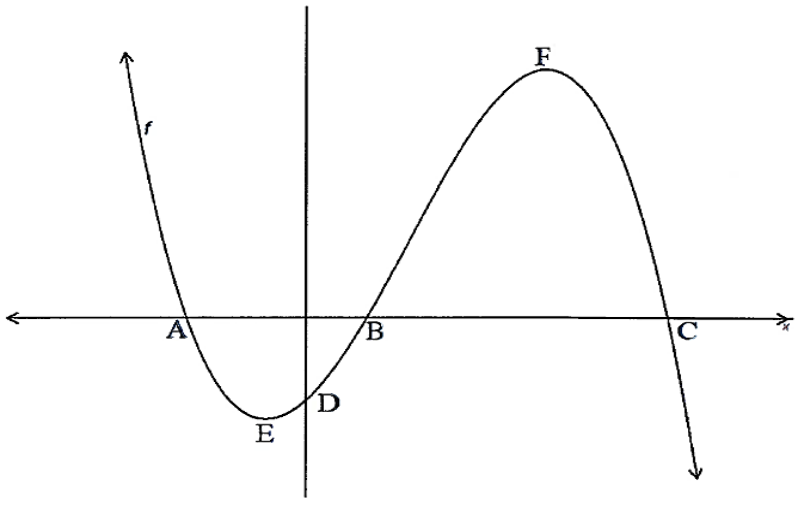
(c) Determine the x-coordinates of the point of inflection. (3) **L1**

(d) For which values of x will $x.f^{/}(x) > 0$? (2) **L3**

(e) For which values of k will $f(x) - 4 = k$ have one root? (2) **L3**

5.5 **FS JUN 2026**

The graph of cubic function with the equation $f(x) = -x^3 + 5x^2 + 8x - 12$ is drawn alongside. A, B, C and D are intercepts of f with axis. E and F are the turning points of f



5.5.1 Write down the coordinates of D (1) **L1**

5.5.2 Determine the coordinates of A, B and C. (5) **L2**

5.5.3 Calculate the coordinate of E, turning points of f (4) **L2**

5.5.4 If $g(x) = f^{/}(x)$ sketch the graph of g clearly showing the intercepts with axis. (3) **L2**

For which values of x will:

5.5.5 The graph of f be concave up? (2) **L2**

5.5.6 $f^{/}(x) > 0$ (2) **L3**

5.5.7 $f(x).f^{/}(x) \geq 0$ (4) **L3**

5.5.8 Determine the values of k for which $f(x) + k = 0$ has 3 unequal roots (3) **L4**

5.5.9 Determine the equation of the tangent to f at $x = 5$ (4) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

FS JUN 2026

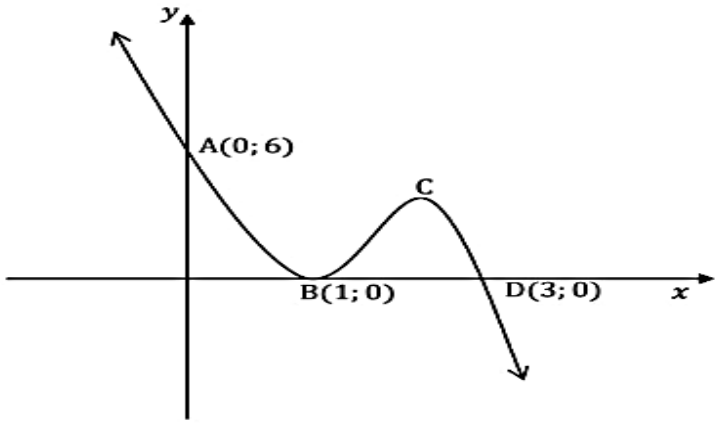
5.6 The graph of $h(x) = ax^3 + bx^2 + cx + d$ has two turning points.
The following information about *h* is also given:

- $h(2) = 0$
- $h^{/}(1) = 0$
- $h^{//}\left(\frac{1}{2}\right) > 0$
- The *x*-axis is a tangent to the graph of *h* at $x = -1$

Without calculating the equation of *h*, use the given information to draw a sketch of *h*, only indicating the *x*-coordinates of the *x*-intercepts and turning points. (4) **L4**

5.7 **Ultradeep.co.za Sep 2025**

The graph of $g(x) = -2x^3 + 10x^2 - 14x + 6$ is drawn below. The graph cuts the *y*-axis at A(0 ; 6) and the *x*-axis at B(1 ; 0) and D(3 ; 0). C is turning point of the graph.



5.7.1 Determine the co-ordinates of C (4) **L2**

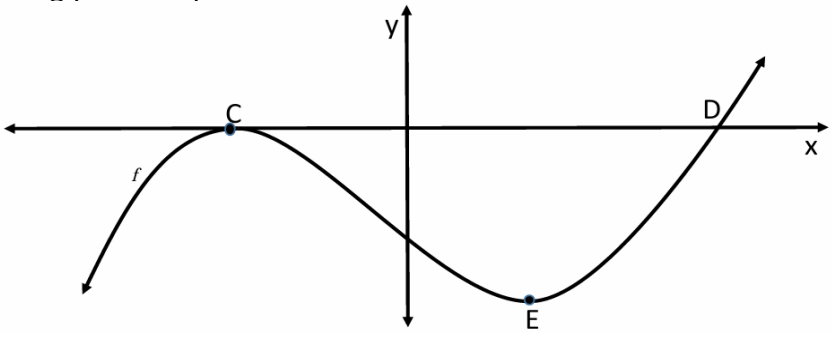
5.7.2 For which values of *x* is $g^{/}(x) > 0$? (1) **L3**

5.7.3 For which values of *x* is *g* concave up? (3) **L2**

5.7.4 Determine the values of *k* for which $g(x) = k$ will have two roots. (1) **L3**

KZN JUN 2026

5.8 The graph of $f(x) = x^3 - x^2 - 5x - 3$ is sketched below. The *x*-interception of *f* are C and D. C and E are the turning points of *f*.



5.8.1 Calculate $f(-1)$ (1) **L1**

5.8.2 Determine the *x*-coordinate of D. (3) **L2**

5.8.3 Determine the *x*-coordinate of E. (4) **L2**

For which values of *x* will:

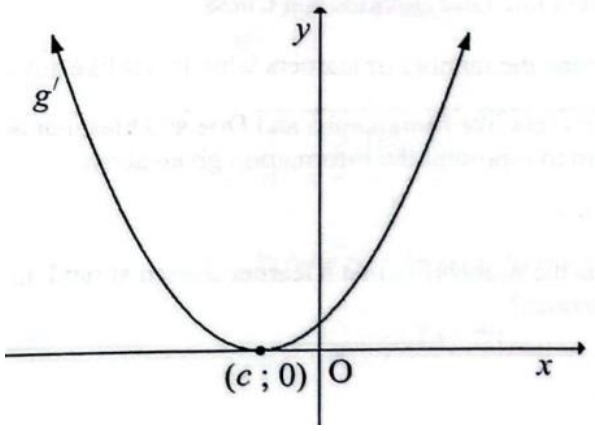
5.8.4 *f* be concave up? (3) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

5.8.5 $f'(x).f(x) < 0$? (3) **L3**

KZN JUN 2026

5.9 The graph of g' is sketched below. g' is the parabola that touches the x-axis at (c; 0)

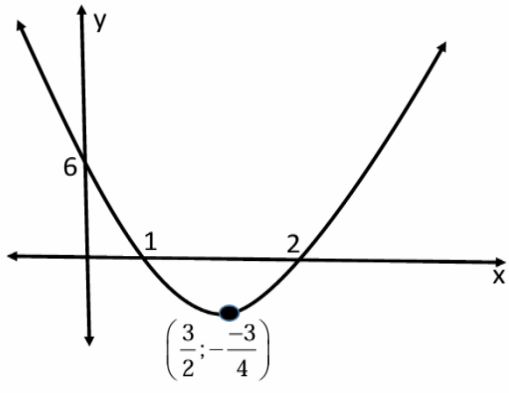


Draw a sketch graph of g. No coordinates of points on the graphs or intercepts with axis need to be shown. (2) **L2**

NW JUN 2026

5.10 The graph of $f'(x)$ is drawn below

- 6 is the y-intercept of f'.
- 1 and 2 are the x-intercepts and $\left(\frac{3}{2}; -\frac{3}{4}\right)$ is the turning point of f' respectively.



5.10.1 Write down the x-coordinates of the minimum turning point of $f(x)$. (1) **L2**

5.10.2 For which values of x will be decreasing? (2) **L2**

5.10.3 Write down the gradient of the tangent to f if $x = 0$. (1) **L2**

5.10.4 At which other x-value will a tangent to f be parallel to the answer in QUESTION 5.10.3 above (1) **L2**

5.10.5 Is $f(x)$ concave up or concave down for $x > \frac{3}{2}$? (1) **L1**

NW JUN 2026

5.11 A body moves according to the formulae $s(t) = 100t^2 - 200t + 300$, where S is the distance in meters, from a fixed point P after t seconds

5.11.1 Determine the distance from P after 10 seconds. (2) **L1**

5.11.2 Determine after how many seconds the body will reach a speed of 600m/s. (2) **L2**

5.11.3 After how many seconds will the body reach its minimum distance from P? (2) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

5.12 A ball is thrown into the air and its height, above the ground after t seconds is given by $h(t) = -5t^2 + 22t + 4$ meters.

5.12.1 Write down the initial height of the ball. (1) **L1**

5.12.2 Determine the maximum height of the ball above the ground. (4) **L2**

5.12.3 Determine the rate of change in velocity of the ball at any given time. (2) **L2**

IEB SUPPLIMENTARY 2014

5.13 The radioactive decay of a substance is given by the formula:

$m(t) = 500(0,92)^t$

where $m(t)$ is the mass in grams of the radioactive substance and *t* is its age in years.

5.13.1 Write down the initial mass of the radioactive material (1) **L1**

5.13.2 Write down the percentage by which the radioactive decreases each year (1) **L2**

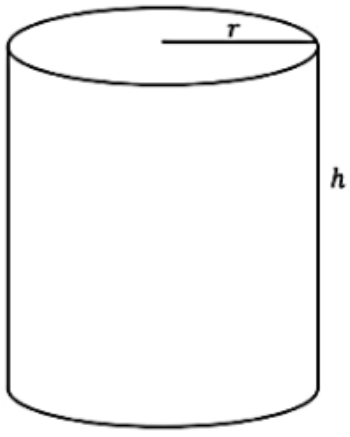
5.13.3 Determine the mass of the substance after 50 years. (2) **L2**

5.13.4 Determine the least number of years it takes for the substance's mass to be less than one gram. Give your answer to the nearest year. (3) **L3**

ULTRADEEP.CO.ZA SEP 2025

5.14 A **340ml** can with height *h* cm and radius *r* cm is shown below.

$\mathbf{1ml = 1cm^3}$



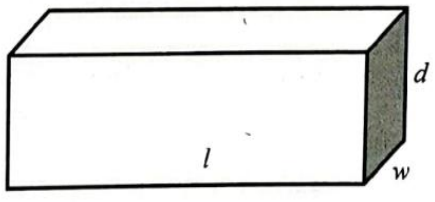
5.14.1 Determine the height of the can in terms of the radius r. (2) **L2**

5.14.2 Show that the surface area of the cylinder in terms of r can be expressed as $SA = 2\pi r^{20} + \frac{680}{r}$ (3) **L2**

5.14.3 Calculate the length of the radius of the can, in cm, if the surface area is to be minimum (3) **L2**

5.15 The measurements of a rectangular dam are as follows:

Length $(l) = 60 - 2x$, width $(w) = 2x$ and depth $(d) = \frac{x}{2}$



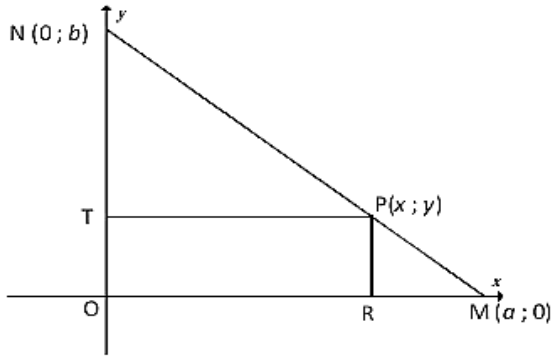
5.15.1 Determine the volume of the dam in terms of *x*. (2) **L2**

5.15.2 Determine the value of *x* for which the volume of the dam is a maximum. (3) **L2**

5.15.3 Calculate the maximum volume of the dam. (2) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

5.16 A farmer has a piece of land in the shape of a right-angled triangle OMN, as shown in the figure below. He allocates a rectangular piece of land PTOR to his daughter, giving her the freedom to choose P anywhere along the boundary MN. Let OM = a, ON = b and P(x ; y) be any point on MN.



5.16.1 Prove that the daughter's land will have a maximum area if she chooses P at the midpoint of MN. (7) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

TOPIC: 6 PROBABILITY [15 ± 3 MARKS]

GUIDELINES, SUMMARY NOTES, & STRATEGIES

The probability scale: $0 \le P \le 1$. If P (an event) = 0, the event is impossible; If P (an event) = 1, the event is certain to happen.

The **definition of probability**: $P(E) = \dfrac{n(E)}{n(S)}$

Addition Rule for any 2 events A and B: **P (A or B) = P(A) + P(B)−P (A and B)**
Mutually exclusive events A and B: **P (A or B) = P(A) + P(B)**
NOTE: Since P (A and B) = 0
Independent events A and B: **P (A and B) = P(A) × P(B)**
The complementary rule: **P (not A) = 1−P(A)**
Venn-Diagram, Tree diagram and Contingency Table

The fundamental counting principle: If one operation can be done in *m* ways and a second operation can be done in *n* ways then the total possible number of different ways in which both operations can be done is $m \times n$.

- *Pin codes and Passwords*
- *Arrangements [(a) Different/Selection (b) Identical)]*
- *Re-arrangements*

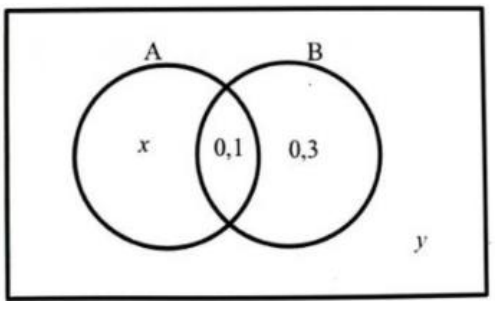
REVISION QUESTIONS
DBE NOV 2011

6.1 6.1.1 Let A and B be two events in a sample space.
Suppose that P(A) = 0,4 ; P(A or B) = 0,7 and P(B) = k

(a) For what value of k are A and B mutually exclusive? (2) **L2**

(b) For what value of k are A and B independent? (4) **L2**

6.1.2 If A and B are independent events, calculate the values of x and y in the diagram below. Show all calculations.



(5) **L3**

6.1.3. M and K are two events.
If P(K) = 0,5 ; P(K and M) = 0,2 and P(not K and not M) = 0,3

(a) Draw a Venn diagram to represent the above events (4) **L3**

(b) Are the events M and K independent or not? Explain (3) **L2**

6.2 6.2.1 A survey among 80 women regarding their reading preferences indicated the following:

- 44 read Home
- 33 read Glamour
- 39 read Women's Health
- 23 read Home and Women's Health
- 19 read Glamour and Women's Health
- 9 read all three magazines
- 69 read at least one of these magazines

Study copy distributed by EduResource ZA | eduresourcza.com

(a) How many women do not read any of these magazines? (1) **L1**

(b) Represent the women's reading preferences in a Venn diagram. Let x be the number of women who read Home and Glamour, but not Women's Health, but not Women's Health. (5) **L2**

(c) Hence, show that $x = 5$. (2) **L1**

(d) What is the probability that a woman who is randomly selected:

(i) only reads Home. (2) **L1**

(ii) reads at least two of these magazines? (2) **L2**

MATHS 4 AFRICA

6.3 You have seven snacks: one for each day of the week. There are two KitKats, three packets of wine gums and two chocolates Log

6.3.1 How many options are there to choose from on the first day (1) **L1**

6.3.2 If you randomly select one snack on each day of the week, in how many ways can these snacks be selected (3) **L2**

EC SEP 2022

6.4 A bag contains only two colours of tennis balls, red and green, in the ratio 1:3. Two balls are picked at random, one after the other, without replacement. The probability of picking first red and second green ball is $\frac{1}{5}$. Calculate the number of balls in the bag. (5) **L4**

MATHS 4 AFRICA

6.5 When creating an online profile with Titus Holdings, a unique login code is generated. The code is to be alphanumeric (include digits and letters) and to be 5 characters long. The first two characters are letters and the remaining characters are digits.

6.5.1 If letters may be repeated, but the digits may not, determine how many unique login codes can be created. (2) **L2**

6.5.2 A security alert is that codes are not safe enough and implements new restrictions on the codes. If Titus Holdings project they will have 1600 people create profiles, determine if they will have enough unique codes considering they need to meet the updated restriction below:

- No repetition allowed (letters and digits)
- All codes to begin with the letter "T"
- All digits to be $\in \{0; 2; 3; 5; 6; 8; 9\}$

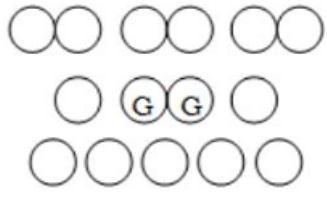
The 3-digit numeric portion to be divisible by 5. (3) **L3**

UNKNOWN SOURCE

6.6 The probability that Johan's favourite hiking spot is fully booked for the weekend is 30%. When his favourite hiking spot is fully booked the probability for him hiking is 20%. If his favourite hiking spot is not fully booked the probability that Johan will hike during the weekend becomes 60%. Calculate the probability of Johan hiking during the weekend. (4) **L3**

UNKNOWN SOURCE

6.7 The Lourens family wants to take family pictures. The photographer decides that the following set up will work the best for the three married couples, their 7 children and the one set of grandparents:



Study copy distributed by EduResource ZA | eduresourcza.com

Couples need to stand husband and wife together at the back, the grandparents in the middle and the children in the other positions

6.7.1 How many photos are possible? (5) **L3**

6.7.2 What is the probability of the mean all appearing on the same side their wives at the same time? (4) **L4**

DBE NOV 2023

6.8 6.8.1 A and B are independent events. $P(A) = \frac{1}{3}$ and $P(B) = \frac{3}{4}$.

Determine:

(a) P(A and B) (2) **L1**

(b) P(at least ONE event occurs) (2) **L2**

6.8.2 The probability that it will snow on the Drakensberg Mountains in June is 5 %.

- When is snows on the mountain, the probability that the minimum temperature in Central South Africa will drop below $0^{\circ}C$ is 72%.
- If it does not snow on the mountain, the diagram that the minimum temperature in Central South Africa will drop below $0^{\circ}C$ is 35%

(a) Represent the given information on a tree diagram. Clearly indicate the probabilities associated with EACH branch. (3) **L3**

(b) Calculate the probability that the temperature in Central South Africa will NOT drop below $0^{\circ}C$ in June 2024. (3) **L2**

6.8.3 Ten learners stand randomly in a line, one behind the other.

(a) In how many different ways can the ten learners stand in a line? (1) **L1**

(b) Calculate the probability that there will be 5 learners between the 2 youngest learners in the line (4) **L4**

ORIGINAL

6.9 The success rate of the SPAIN soccer team in the 2026 FIFA world cup depends on a number of factors. The fitness of the player is one of the factors that influence the outcome of the match.

- The probability that all the players are fit for the next match is 75%
- If all the players are fit to play the next match, the probability of winning the match is 88%

If there are players that are not fit to play the next match, the probability of winning the match is 51%. Based on fitness alone, calculate the probability that the SPAIN soccer team will win the next match. (5) **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

NW SEP 2025

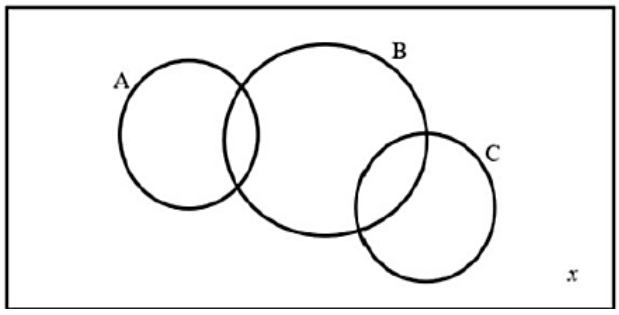
6.10 6.10.1 Three events A, B and C are represented on the Venn diagram below.

A and B are independent event

A and C are mutually exclusive events.

P(A or B) = 0,55

x lies outside the event A, B or C but inside the sample space.



(a) Show that P(B) = 0,4 (3) **L2**

(b) Calculate P(A and B) (2) **L2**

(c) If P(only C) = 0,25.
Calculate the value of x (2) **L3**

(d) If it is further given that P[(NOT A) and (NOT C)] = 0,38.
Calculate P(only B) (2) **L3**

6.10.2 Examination numbers for the grade 12 final examination are determined in the following way.

- The first digit is the province number. Northwest is number 9
- The 2nd and 3rd digits are the year. For this year it will be 25.
- The 4th digit is district number.
- Digits 5 to 8 are the centre or school number.
- The last 4 digits are the candidate's number in his / her school

The examination number may contain digits 0 to 9 and the digits may repeat.

Consider only examination numbers for the year 2025 of Northwest district number 4

An example of an examination number

9	25	4	4085	0001
Province	Year	year	Centre	Candidates number

(a) Determine the number of the different examination numbers (for the year 2025 of Northwest and district 4) that can be formed using this system (2) **L2**

(b) An examination number (For the year 2025 of Northwest and district 4) is randomly selected.
Calculate the probability that this examination number will have a certain number:

- That is an even number between 1000 and 5000
- Where the digits may not repeat (5) **L3**

ORIGINAL

6.11 The events T and R satisfy the following:

$P(T) = \frac{2}{5}$; P(T or R) = 2P(T and R) and P(T and R) = $\frac{3}{5}$P(R)

6.11.1 Determine the P(not R) (3) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

6.11.2 Represent the given information in a Venn Diagram (4) **L3**

6.11.3 Determine the P(not T or not R) (2) **L2**

6.11.4 Determine the P(not T and not R) (2) **L2**

ORIGINAL

6.12 Consider the letters of the word PROPORTIONALITY.

6.1.1 Calculate the total number of different arrangements of all the letters in the word. (3) **L2**

6.12.2 If all the vowels must be together, in how many ways can the letters be arranged? (4) **L3**

6.12.3 In how many ways can the letters be arranged if the 3 O's must NOT be together (4) **L3**

6.12.4 How many arrangements begin and end with a consonant? (3) **L3**

6.12.5 How many arrangements have the 2 P's together? (3) **L2**

6.12.6 If 4 letters are chosen from the word and arranged, how many different 4- letter words can be formed? Note: Remember repeats. (3) **L2**

UNKNOWN SOURCE

6.13 Sharon and Denice are retired teachers. Some days they forget to drink their medication. The probability that neither Sharon nor Denice will forget to take their medication is 0,6.

6.13.1 Calculate the probability that at least one of them will forget to take their medication. (1) **L1**

6.13.2 The probability that Sharon forgets to take medication at any random selected day is 0,24. The probability that both of them will forget to take their medication is 0,14. Calculate the probability that Denice will forget to take medication on that day. (3) **L2**

6.13.3 Determine whether the event of Sharon forgetting to take medication and the event of Denice forgetting to take medication are independent. (3) **L2**

UNKNOWN SOURCE

6.14 For a sample space S and events A and B, it is given that:

P(A) = 0,63

P[not (A and B)] – P(B) = 0,48

P(A or B) = 0,95

P(A or B) = P(A) + P(B) – P(A and B)

Determine P(A and B) (4) **L3**

IEB NOV 2022

6.15 You are given the following numbers 1, 3, 6, 8 and 9 to use in creating five- digit numbers.

6.15.1 How many unique five- numbers can be created if repetition is not allowed (1) **L1**

6.15.2 What is the probability that a unique five-digit number starts with an 8 and ends with a 3, if repetition is allowed? (2) **L2**

6.15.3 How many unique even numbers greater than 6500 can be created, if repetition is not allowed? (4) **L3**

DBE NOV 2021

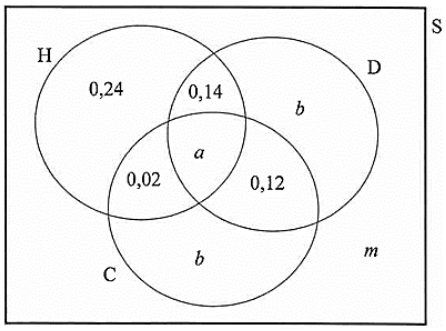
6.16 A survey was conducted among 150 learners at a school.
The following observations were made:

- The probability that a learner selected at random, will take part in:
  - Only hockey (H) is 0,24
  - Hockey and debating (D), but not chess (C) is 0,14
  - Debating and chess, but not hockey is 0,12
  - Hockey and chess, but not debating is 0,02
- The probability that a learner, selected at random, participates in at least one activity is 0,7

Study copy distributed by EduResource ZA | eduresourcza.com

- 15 learners participated in all three activities.
- The number of learners that participated only in debating is the same as the number of learners who participated only in chess.

The Venn diagram below shows some of the above information.



(a) Determine a, the probability that a learner, selected at random, participates in all three activities. (1) **L1**

(b) Determine m, the probability that a learner, selected at random, does NOT participate in any of the three activities. (1) **L1**

(c) How many learners play only chess? (4) **L2**

6.16.2 A three-digit number is made up by using three randomly selected digits from 0 to 9. NO digit may be repeated.

(a) Determine the total number of possible three-digit numbers, greater than 100, that can be formed. (2) **L2**

(b) Determine the total number of possible three-digit numbers, both even and greater than 600, that can be formed. (4) **L3**

UNKNOWN SOURCE

6.17 Each of the 200 employees of a company wrote a competency test. The results are indicated in the table below.

	Pass	Fail	Total
Males	46	32	78
Females	72	50	122
Total	118	82	200

6.17.1 Are the events Pass and Fail mutually exclusive? Explain your answer. (2) **L2**

6.17.2 Is passing the competency test independent of gender? Substantiate your answer with the necessary calculations. (4) **L3**

KZN SEP 2025

6.18 Tracy has 10 sweets in a bag. Some are green and some are red. She picks a sweet from the bag, takes note of the colour, and then puts it back into the bag. She does this four times. How many green sweets are there in the bag, if the probability that she picks at least one green sweet is 97,44%?

(4) **L4**

DBE MAY/JUN 2023

6.19 6.19.1 A group of people participated in a trial to test a new headache pill.
50% of the participants received the headache pill

- 50% of the participant received the headache pill
- 50% of the participant received a sugar pill
- $\frac{2}{5}$ of the participant receiving the headache pill were not cured
- $\frac{3}{10}$ of the group receiving the sugar pill were cured.

(a) Represent the given information on a tree diagram. Indicate on your diagram the probability associated with each branch as well as the outcomes. (3) **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

(b) Determine the probability that a person chosen at random from the group will NOT be cured. (2) **L2**

6.19.2 Three events A, B and C, are considered:

$P(A) = \frac{2}{5}$, $P(B) = \frac{1}{4}$ and $P(A \text{ or } B) = \frac{13}{20}$.

(a) Are the events A and B mutually exclusive? Support your answer with the necessary calculations. (2) **L2**

(b) Determine P(only C), if its further given that

$P(A \text{ or } C) = \frac{7}{10}$, $P(A \text{ and } C) = \frac{2}{5}$ and 2P(B and C). (2) **L2**

(c) Determine the probability that event A, B or C do NOT take place. (2) **L2**

6.19.3 Seven friends (4 boys and 3 girls) want to stand in a straight line next to each other to take a photo.

(a) In how many ways can the 3 girls stand next to each other in the photo? (2) **L2**

(b) In the next photo, determine the probability that Selwyn (a boy) and Lindiwe (a girl) will not stand next to each other in the photo. (3) **L3**

ORIGINAL

6.20 From his hotel room on the 11th floor overlooking the beach, Mr Sos observed nine different luxury cars parked in a straight line in the parking lot. Six of the cars are grey and three are white, as shown in the diagram below.

6.20.3 In how many different ways can nine cars be parked in a parking lot? (1) **L1**

6.20.2 In how many different ways can all cars of the same colour be parked next to each other? (3) **L2**

6.20.3 Calculate the probability that the first car parked at either end of the parking lot is red. (3) **L3**

6.20.4 Calculate the probability that white cars are NOT parked next to each other? (3) **L3**

6.20.5 Calculate the probability that a red car is parked in the middle position of the parking lot. (3) **L3**

6.20.6 Consider the letters of the word LUXURY. In how many ways can all the letters be arranged. (2) **L2**

ORIGINAL

6.21 A survey on the preference of maths topics was conducted among 140 learners of Hluzy High School. The information is summarized below.

Gender	Algebra	Trigonometry	Euclidean Geo	Total
Boys	a	28	c	80
Girls	b	21	d	60
Total	21	49	70	140

6.21.1 Calculate the probability that a girl prefers a trigonometry. (2) **L1**

6.21.2 The event of preferring Algebra and gender are independent of each other, calculate the value of a. (4) **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

HIGH FLYERS DOCUMENT 2021

6.22 Events A, B and C occur as follows where A and B are independent events:

- P(A) = 0,38
- P(B) = 0,42
- P(A and B) = 0,1596
- P(C) = 0,28

There are 456 people in event A.

6.22.1 Are A and B mutually exclusive? Motivate your answer. (2) **L1**

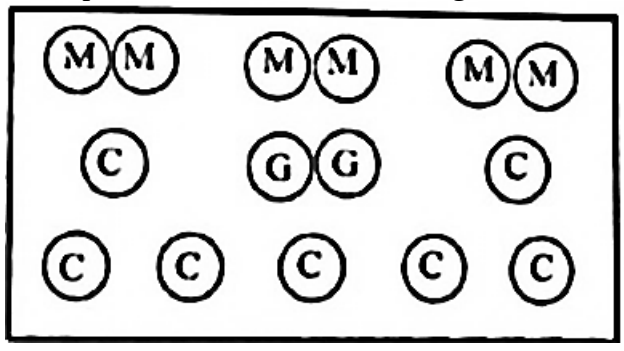
6.22.2 By using an appropriate formula, show that the value of P(A or B) = 0,64 (2) **L2**

6.22.3 Calculate the number of people in the sample space. (2) **L2**

6.22.4 Determine n(C). (2) **L2**

GPSEP 2018

6.23 The Ngcobo family takes family photos. The photographer arranges three married couples, seven children and two grandparents as follows:
The couples stand as husband and wife together at the back, the grandparents in the middle and the children in the other positions as shown in the diagram below.



M	Married Couples
G	Grandparents
C	Children

In how many different ways can the Ngcobo family be arranged for the photo? (5) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

TOPIC: 7 **STATISTICS [20 ± 3 MARKS]**

FOR TIPS AND NOTES REFER TO 2025 LAST PUSH DOCUMENT

WC SEP 2025

7.1 On JUN 25, the points of the top 10 Rugby 7s teams on the rankings were as follows:

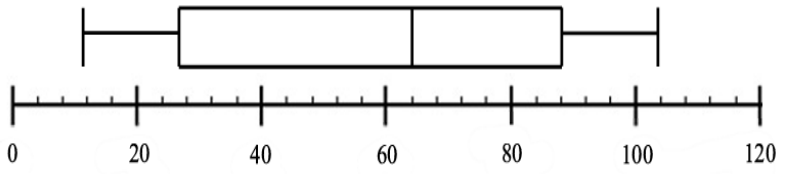
27	36	50	56	57	65	70	88	96	104

7.1.1 Calculate the Average(mean) of the 10 teams' points. (2) **L1**

7.1.2 Determine the standard deviation of the points. (2) **L2**

7.1.3 Determine the number of teams whose points lie within one standard deviation of the mean. (3) **L2**

7.2 If all 12 teams' points totals on the date are taken into account, the average is 59 points per team. The data is represented in the box -and-whisker diagram below:



Comment on the distribution of the data. Provide a reason for your answer. (2) **L2**

DBE NOV 2025

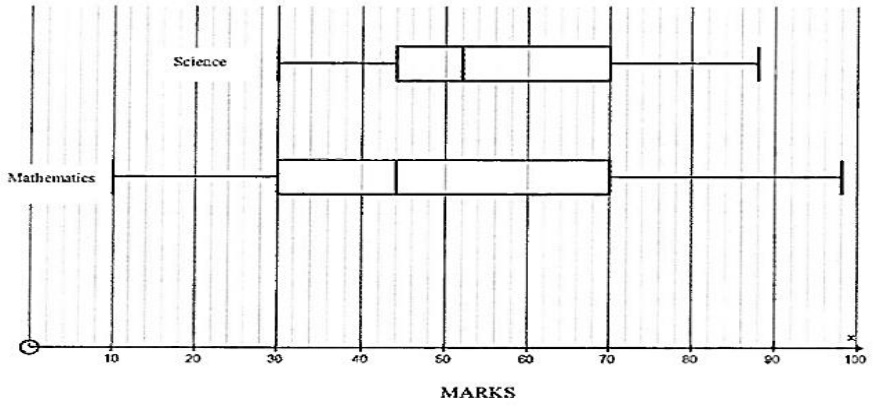
7.3 There are 9 players in a basketball team. The couch calculated that on average each player scored 12 points during a game. The points scored by 8 of the 9 players from the team is given below:

11	14	19	20	8	10	2	14

How many players' points score was outside ONE standard deviation of the mean points scores? (5) **L3**

FS SEP 2025

7.4 The box and whisker diagrams below show the distribution of test scores obtained by a sample of students in Mathematics and Science



Use the diagram to determine the following:

7.4.1 The range for Mathematics. (2) **L1**

7.4.2 The median for science. (1) **L1**

7.4.3 The interquartile range for science. (2) **L1**

7.4.4 Which examination was easier for the students? Motivate your answer. (2) **L2**

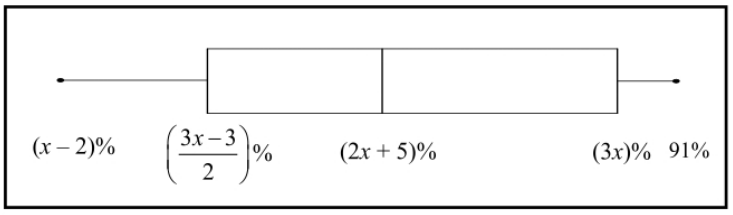
7.4.5 Which examination has a weaker spread of scores? Justify by giving TWO reasons. (2) **L3**

7.4.6 If a learner from the Science sample is selected randomly, find the probability that the learner achieved a mark greater than 70%. (1) **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

DBE MAY/JUN 2026

7.5 The marks as percentage, that grade12 learners at another school obtained in the same examination, are represented as in the box and whisker diagram below.



If it is given that the interquartile range (IQR) of the data is 45%, calculate the minimum percentage of learners who passed in this centre. (5) **L3**

GP SEP 2025

7.6 Research was conducted to measure the testosterone levels in runners at the Olympic Games. The table below shows the results of five runners who were tested for testosterone.

Runners	**1**	**2**	**3**	**4**	**5**
Level of testosterone	6,7	$2k+2,4$	p	3,8	4,3

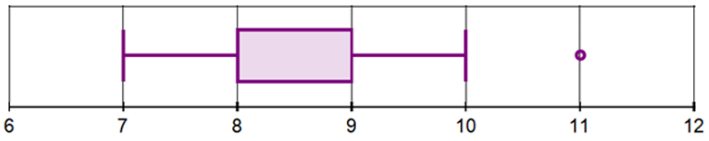
The mean of the five runners is 7.7 and the values of the last three runners form an arithmetic sequence. Determine the value of k and p. (5) **L3**

IEB MAY 2026

7.7 The shoe sizes of 11 players in a hockey team are given in ascending order as:

a; a; 8; 8; 9; 9; 9; 9; 9; b; c

The box and whisker plot below has been drawn using the shoes sizes.



7.7.1 Two true statements have been made about the data. Use calculations to explain how these statements are true.

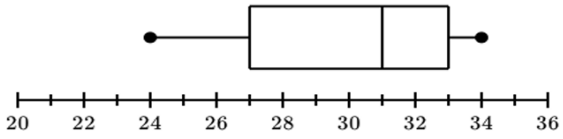
(a) The data is skewed to the left (negatively skewed). (5) **L4**

(b) Seven players shoe sizes fall within 1 standard deviation of the mean. (2) **L2**

7.7.2 Two more players are added to the team so that now there are no outliers. Both players have a shoe size of 10. Draw a new box and whisker plot for the team of 13 players. (3) **L2**

IEB/MAY 2024

7.8 The box and whisker plot below represents the times, in seconds, taken by a group of 60 students to run 200 metres.



7.8.1 How many students ran between 27 seconds and 31 seconds? (1) **L3**

7.8.2 Using $Q_1 - 1,5 \times \text{IQR}$, determine whether 24 seconds would be an outlier or not. (2) **L2**

7.8.3 The time for the fastest student was recorded incorrectly. It was 25 seconds. How would this affect the median? (1) **L2**

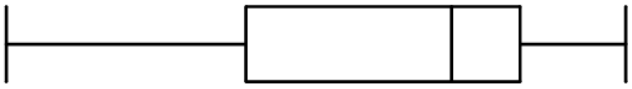
Study copy distributed by EduResource ZA | eduresourcza.com

MIND ACTION GRD 11

7.9 a, b, c and d are four numbers such that $a = b$ and $b < c < d$. The mode is 3, the range is 12 and the mean is 8. Determine the values of a, b, c and d (4) **L3**

IEB/NOV 2024

7.10 Study the box-and-whisker plot.

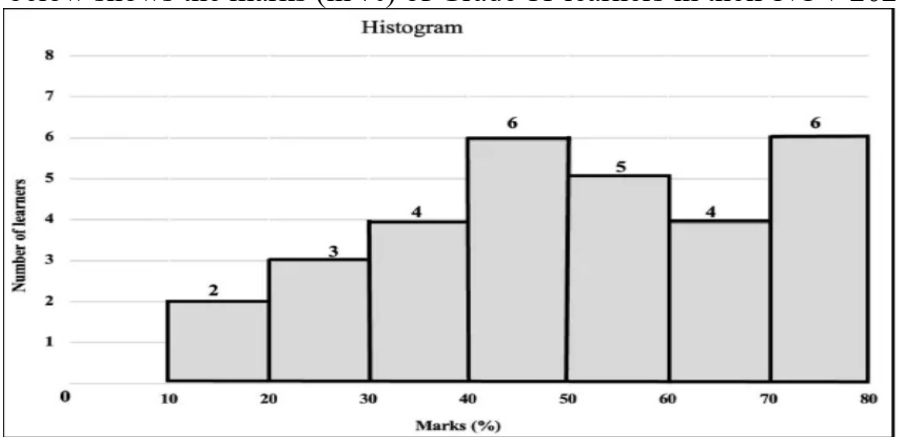


Will the mean be bigger or smaller than the median? Explain. (2) **L1**

7.11 **Refer to 2025 Last push learner revision document grade 12, page 49, question 5**

GP SEP 2025

7.12 The histogram below shows the marks (in %) of Grade 11 learners in their NOV 2024 examination.



7.12.1 Complete the table below:

Class interval (Marks in %)	Frequency	Cumulative Frequency
$10 < x \leq 20$		
$20 < x \leq 30$		
$30 < x \leq 40$		
$40 < x \leq 50$		
$50 < x \leq 60$		
$60 < x \leq 70$		
$70 < x \leq 80$		
TOTAL		

(2) **L1**

7.12.2 Draw an ogive (Cumulative frequency graph) to represent the data. (3) **L2**

7.12.3 Use the ogive to estimate how many learners obtained at least 54% in their examination. (2) **L1**

7.12.4 It is further given that the minimum mark is 12% and the range of the data is 66%, draw a box and whisker diagram. (4) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

DBE MAY/JUN 2026

7.13 The marks as a percentage, that Grade12 learners at a certain school obtained in the mathematics examination in JUN 2025was summarised in the frequency table below.

CLASS INTERVALS (AS %)	FREQUENCY
$20 < x \leq 30$	8
$30 < x \leq 40$	14
$40 < x \leq 50$	16
$50 < x \leq 60$	21
$60 < x \leq 70$	16
$70 < x \leq 80$	9
$80 < x \leq 90$	6
$90 < x \leq 100$	2

7.13.1 Write down the modal class interval. (1) **L1**

7.13.2 Complete the cumulative frequency table.

CLASS INTERVALS (AS %)	FREQUENCY	CUMULATIVE FREQUENCY
$20 < x \leq 30$	8	
$30 < x \leq 40$	14	
$40 < x \leq 50$	16	
$50 < x \leq 60$	21	
$60 < x \leq 70$	16	
$70 < x \leq 80$	9	
$80 < x \leq 90$	6	
$90 < x \leq 100$	2	

(2) **L1**

7.13.3 Draw a cumulative frequency curve(ogive) to represent the data above. (3) **L2**

7.13.4 Use the cumulative frequency curve(ogive) to estimate the median of the data. (3) **L2**

IEB/MAY 2024

7.14 A golfer recorded the distance (d) in metres of her 400 shots. The following is a summary of the data:

- $60 \leq d \leq 180$
- Lower quartile $(Q_1) = 96$
- Median $= 135$
- The Interquartile Range (IQR) $= 48$
- The 95th percentile $= 174$

7.14.1 Sketch the cumulative frequency curve (Ogive) to represent the given data. Show all key points on your curve clearly. (5) **L4**

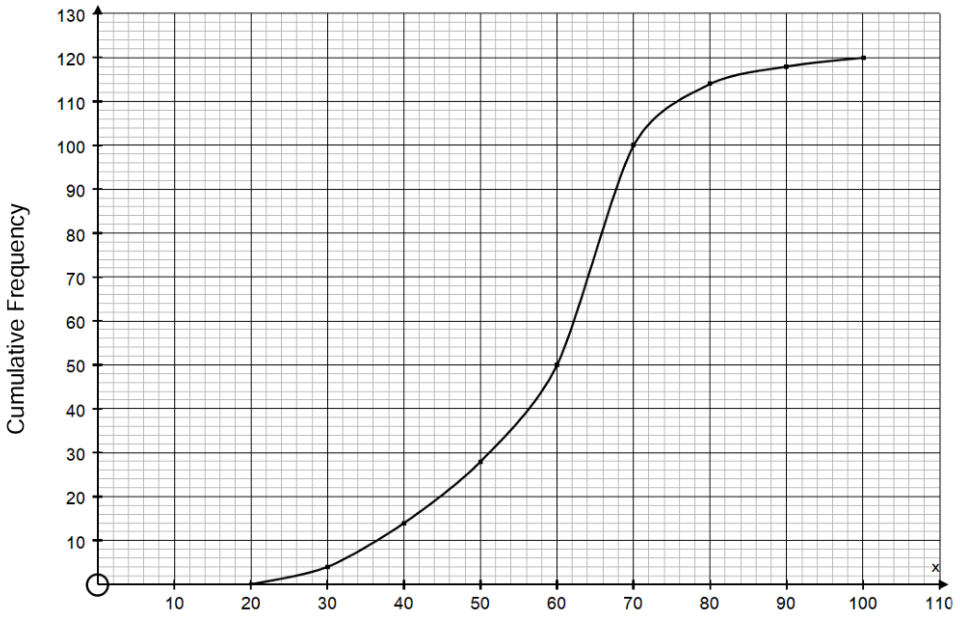
7.14.2 A friend of the golfer claims that the 40th percentile is 140. Is the friend correct? Explain. (1) **L2**

7.14.3 If the mean distance of the shots is 122 m, describe the skewness of the data giving a reason. (2) **L1**

Study copy distributed by EduResource ZA | eduresourcza.com

IEB/NOV 2024

7.15 Study the cumulative frequency curve below.



If the data for the class intervals above 70 are removed from the curve, determine the difference between the original median and the new median result. (3) **L2**

DBE NOV 2025

7.16 The cumulative frequency table below shows the amount of time that people spent on a particular website on a certain day.

TIME, t (in minutes)	CUMULATIVE FREQUENCY
$0 < t \leq 20$	16
$0 < t \leq 40$	40
$0 < t \leq 60$	59
$0 < t \leq 80$	67
$0 < t \leq 100$	70

7.16.1 How many people visited this website on that day? (1) **L1**

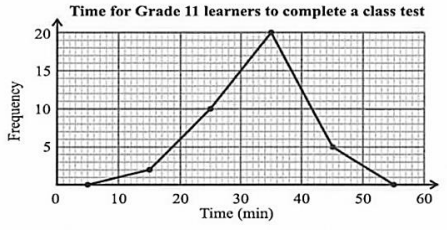
7.16.2 How many people spent more than 40 and up to 80 minutes on the website? (2) **L2**

7.16.3 Draw a histogram to represent the information provided in the cumulative frequency table. (3) **L2**

7.16.4 Comment on the skewness of data. (1) **L1**

MATHEMATICS HANDBOOK AND STUDY GUIDE GRD 11 KEVIN SMITH

7.17 The frequency polygon below shows the time that it took the learners in a grade 11 class to complete a test.



7.17.1 How many learners wrote the test? (2) **L1**

7.17.2 Determine the modal class interval. (1) **L1**

7.17.3 Comment on the distribution of data using the frequency polygon. (1) **L1**

Study copy distributed by EduResource ZA | eduresourcza.com

7.17.4 Complete the cumulative frequency table

TIME, t (in minutes)	FREQUENCY	CUMULATIVE FREQUENCY
$10 < x \leq 20$		
$20 < x \leq 30$		
$30 < x \leq 40$		
$40 < x \leq 50$		

(4) **L2**

7.17.5 Draw the cumulative frequency curve and show where the median, lower quartile and upper quartile lie. (6) **L2**

7.17.6 Use your ogive to estimate the numbers of learners who took at most 25minutes to complete the test. (2) **L1**

IEB/MAY 2025

7.18 Learners were surveyed to determine the distance between their homes and school. The shortest distance was recorded as 1,39 km and the furthest distance was 2,21 km. 80 responses are recorded in the table below**.**

Distance from home	Number of learners
$1,3 \leq x < 1,5$	12
$1,5 \leq x < 1,7$	36
$1,7 \leq x < 1,9$	16
$1,9 \leq x < 2,1$	11
$2,1 \leq x < 2,3$	5

7.18.1 Draw an ogive for the data. (4) **L2**

7.18.2 Using your ogive to estimate the quartiles, draw a box-and-whisker diagram for the data. (5) **L2**

7.18.3 Describe the skewness of the data. (1) **L1**

7.18.4 After careful analysis of the table above, it was noticed that 5 responses were never recorded. The missing values all lay in the interval $1,1 \leq x \leq 1,3$. The corrected table with 85 responses is given below.

Distance from home	Number of learners
$1,1 \leq x < 1,3$	5
$1,3 \leq x < 1,5$	12
$1,5 \leq x < 1,7$	36
$1,7 \leq x < 1,9$	16
$1,9 \leq x < 2,1$	11
$2,1 \leq x < 2,3$	5

What effect would the additional data have on the estimated mean and the median interval of the data? (2) **L2**

IEB/NOV 2025

7.19 A survey of 140 people was done to see how many WhatsApp messages they sent in a day. Complete the table of results and plot the ogive.

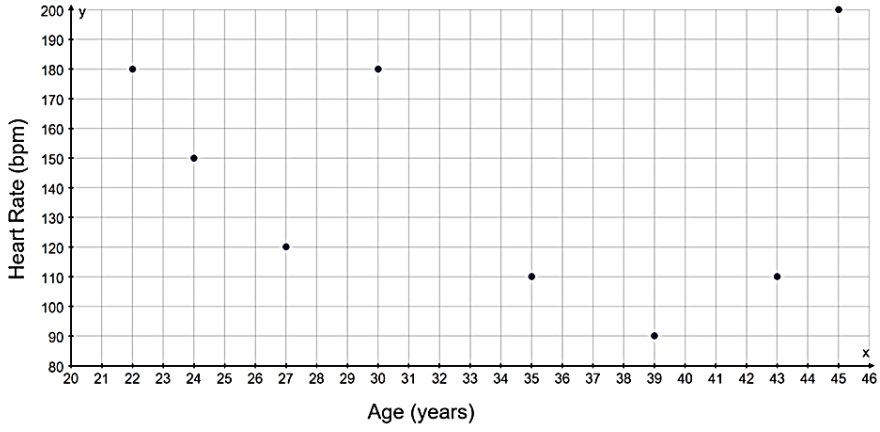
Number of WhatsApp messages sent	Frequency	Cumulative frequency
$20 < x \leq 40$		14
$40 < x \leq 60$	32	
$60 < x \leq 80$	54	
$80 < x \leq 100$	27	
$100 < x \leq 120$		140

(5) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

IEB/MAY 2024

7.20 The scatter graph shows the age of eight individuals and their maximum heart rate in beats per minute (bpm) after a particular exercise programme.



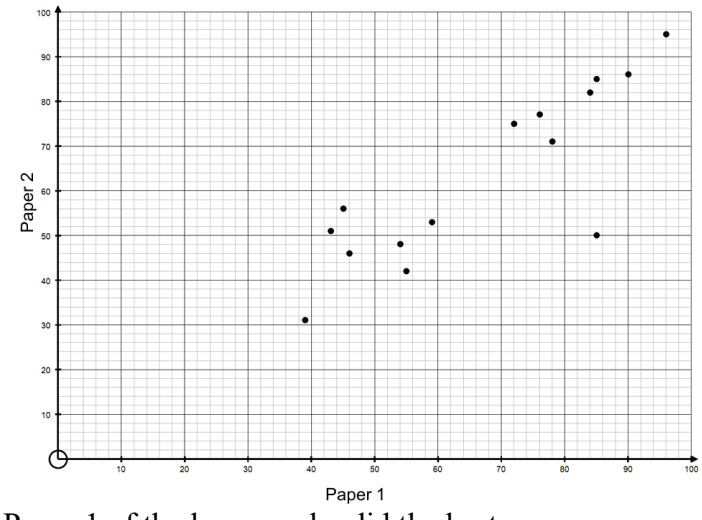
7.20.1 Determine the equation for the line of best fit in the form $y = a + bx$. (3) **L1**

7.20.2 Use your equation to predict the maximum heart rate of a typical 50-year-old individual. (2) **L1**

7.20.3 Give two reasons why the prediction in (7.21.2) may not be reliable. (2) **L2**

IEB/NOV 2025

7.21 The marks, as whole percentages, of 15 learners for their Grade 11 NOV Mathematics examinations are represented in the scatter plot below.



7.21.1 State the mark for Paper 1 of the learner who did the best. (1) **L1**

7.21.2 State the lowest mark for Paper 2. (1) **L1**

7.21.3 The line of best fit has the equation $y = 5,862 + 0,854x$.

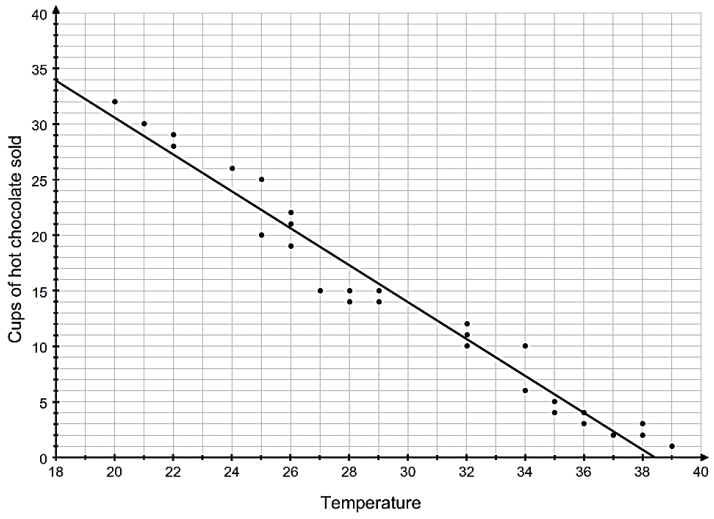
(a) If a learner got 67% for Paper 1, what could that learner expect to get for Paper 2? Round your answer to the nearest whole. (1) **L1**

(b) If it is further given that the mean for Paper 1 ($\bar{x}$) was 67%. **On the scatter plot above**, sketch the line of best fit, clearly indicating the y-intercept and the point $(\bar{x}\ ;\ \bar{y})$. (3) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

IEB/MAY 2026

7.22 A manager of a coffee shop does an analysis of sales over a 28-day period in January, of cups of hot chocolate per day and the maximum temperature (in degrees Celsius) that day.



7.22.1 Use the graph to determine the number of cups of hot chocolate the shop can expect to sell, if the temperature is $30°C$. Show on your graph how you found your answer. (2) **L1**

7.22.2 Choose the corelation coefficient that best describes the data represented in the diagram above:

A. $r = -0,9821$ B. $r = 0,9821$ (1) **L1**

7.22.3 The line of best fit drawn has an equation of $\hat{y} = 63,7883 - 1.6603x$.

(a) Use **the equation** to determine the temperature if 25 cups of hot chocolate were sold. (2) **L1**

(b) Use the equation to determine $\hat{y}$ if the temperature is 45°C. (2) **L1**

(c) Why is the answer in (b) not necessarily a reliable situation? (1) **L2**

7.22.4 The manager ran a special over three days on hot chocolate and left these sales off the data used to plot the graph and the line of best fit.

Temperature	33	35	35
Amount sold	32	30	28

(a) Plot these three days on **the graph**. (1) **L1**

(b) What will the effect of this extra data have on the value of a in the line of best fit's equation, $\hat{y} = 63,7883 - 1.6603x$? (1) **L2**

(c) How will the addition of these sales effect the new correlation co-efficient? (1) **L2**

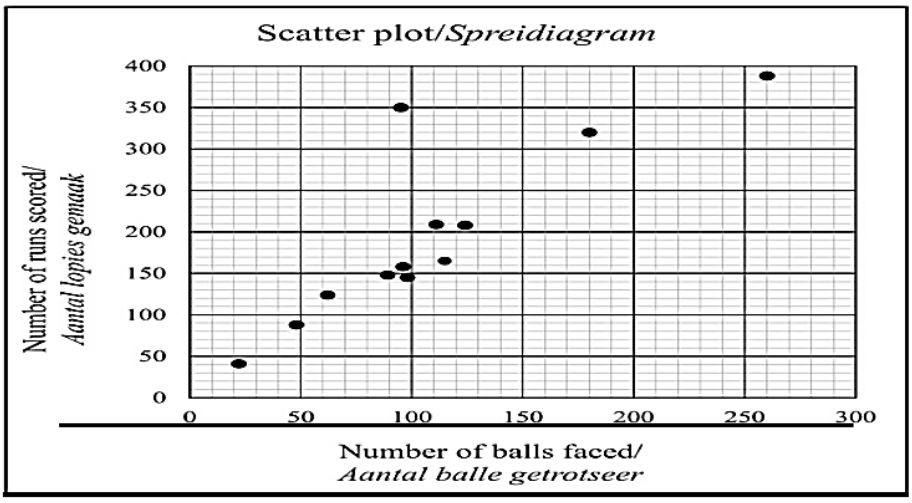
MAY/JUN 2026

7.23 There are 12 players in a cricket team. In a tournament, the number of balls each player faced and the corresponding number of runs each player scored are given in a table and a scatter plot below

NUMBER OF BALLS FACED (x)	62	111	22	48	124	180	89	96	95	260	98	115
NUMBER OF RUNS SCORED (y)	124	209	41	88	208	320	148	158	350	388	145	165

Study copy distributed by EduResource ZA | eduresourcza.com

7.23



7.23.1 Determine the equation of the least squares regression line. (3) **L1**

7.23.2 Estimate the number of runs a cricket player, who faced a total of 150 balls, made. (2) **L1**

7.23.3 Calculate how many players scored runs that were more than one standard deviation above the mean. (4) **L2**

7.23.4 Explain why (95;350), the outlier does not fit the trend of this set of data. (1) **L2**

FS SEP 2025

7.24 Refer to the statements below and answer the questions that follow:

- A least squares regression line (a line of best fit) has an equation of $y = 49 - 3x$
- There is one outlier that has been identified as $(9;4)$
- The correlation coefficient is very strong

7.24.1 If the outlier was removed, would the correlation coefficient be stronger? (Explain your answer). (1) **L2**

7.24.2 Is the correlation coefficient closer to one or negative one (explain your answer) (2) **L2**

7.24.3 If a line of best fit is used, would it be perfectly accurate? (Explain your answer) (2) **L2**

7.24.4 An individual predicts that if x has a value of 30, y will have a value of -41 . However, his friend explains that he is extrapolating and that his result is inaccurate. Explain what his friend is referring to. (2) **L3**

DBE NOV 2025

7.25 There were 11 cars of the same model for sale at a car dealership. The age (in years) of each car and its corresponding selling price (in rands) is provided in the table below.

AGE OF CAR (IN YEARS)	SELLING PRICE OF A CAR (IN RAND)
2	293 000
3	265 000
3	256 000
4	219 000
4	241 000
4	246 000
6	226 000
6	176 000
7	154 000
7	180 000
8	148 000

7.25.1 Determine the equation of the lest squares regression line. (3) **L1**

7.25.2 Predict the selling price of a similar car at this car dealership that is five years old (2) **L1**

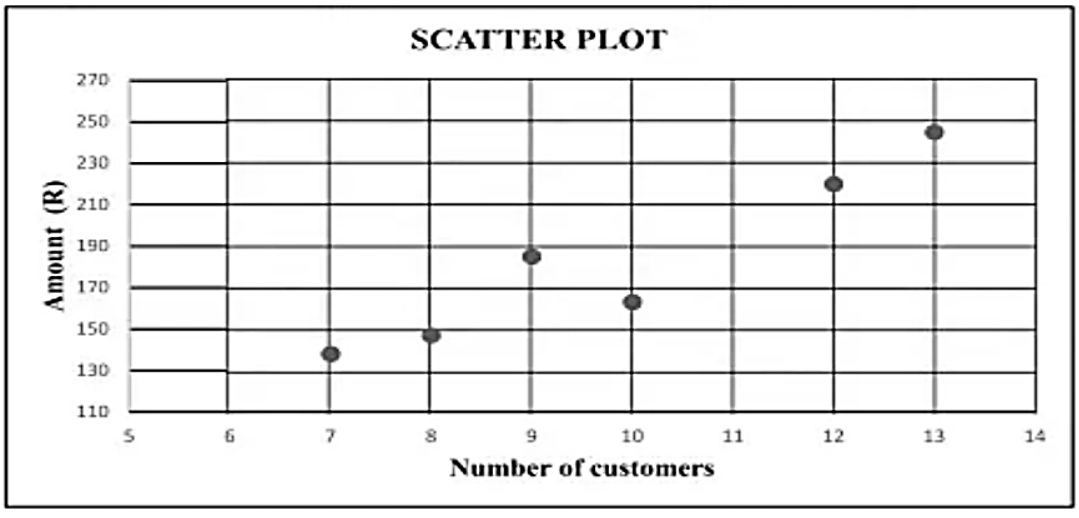
Study copy distributed by EduResource ZA | eduresourcza.com

7.25.3 Use the correlation coefficient to show whether the prediction made in QUESTION 7.16.2 is valid or not. (2) **L2**

7.25.4 Use the answer to QUESTION 7.14.1 to write down the estimated average yearly decrease in the selling price of these cars. (1) **L2**

7.26 During a netball tournament, there are several stalls selling food and refreshments. In a period of 10 minutes at 6 stalls, the following was observed:

Stall	**Number of customers**	**Amount of money spent (R)**
1	10	163
2	7	138
3	9	185
4	12	220
5	8	147
6	13	245



7.26.1 Determine the equation of the least squares regression line of the data. (3) **L2**

7.26.2 Predict the number of customers who spent R203,22 (2) **L2**

7.26.3 Determine the correlation coefficient of the data. (1) **L1**

7.26.4 The organiser of the event thinks that there is a very week positive correlation between the number of customers and the amount of money received at stalls. Motivate whether you agree or not. (1) **L2**

7.26.5 Comment on the trend of the data (2) **L1**

7.26.6 At another stall, 6 customers spent a total amount of R195. If this point is included in the data, will the gradient of the least squares regression line increase or decrease? Motivate your answer without any further calculations. (2) **L3**

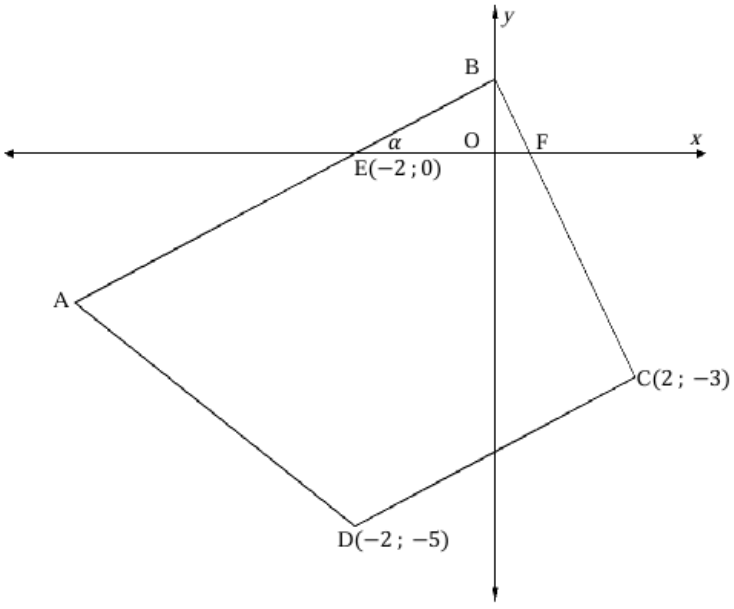
Study copy distributed by EduResource ZA | eduresourcza.com

TOPIC: 8 ANALYTICAL GEOMETRY [40 ± 3 MARKS]

GUIDELINES, SUMMARY NOTES & STRATEGIES REFER 2025 LAST PUSH DOCUMENT

NC JUN 2026

8.1 The diagram below shows a trapezium with vertices A, B, $C(2;\ -3)$ and $D(-2;-5)$. AB and CD are parallel. $E(-2\ ;\ 0)$ is the *x*- intercept of AB and α is angle of inclination of AB. The gradient of $BC = -2$.



8.1.1 Determine the gradient of DC. (2) **L1**

8.1.2 Determine the equation of AB in the form $y = mx + c$. (3) **L2**

8.1.3 Determine the size of α, the angle of inclination of AB. (2) **L2**

8.1.4 Calculate the length of BE. (2) **L2**

8.1.5 Show that $ABC = 90°$ (2) **L2**

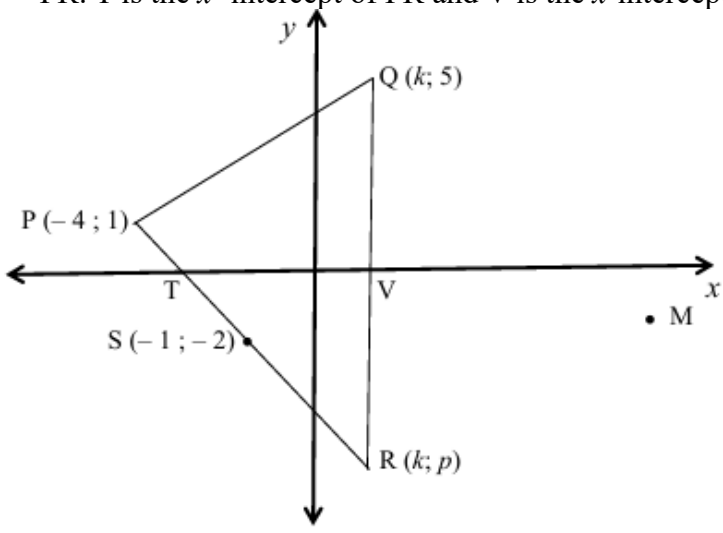
8.1.6 Describe the transformation that must be applied from D to D' so that $ABCD'$ will be a parallelogram. The coordinate of A is given as $(-6;-2)$. (4) **L4**

8.1.7 Hence, or otherwise, determine the point of intersection of the diagonals of the newly formed parallelogram. (3) **L3**

8.1.8 Determine the area of ΔBEF. (3) **L2**

NW JUN 2026

8.2 In the diagram below, $P(-4;1)$, $Q(k;5)$, and $R(k;p)$ are vertices of ΔPQR. $S(-1;-2)$ is a midpoint of PR. T is the *x*- intercept of PR and V is the *x*-intercept of QR. QR is perpendicular to the *x*- axis.



8.2.1 Show that $k = 2$ and $p = -5$. (2) **L2**

8.2.2 Calculate the length of PR. (2) **L1**

8.2.3 Determine the gradient of PR. (2) **L1**

8.2.4 Determine the equation of PR, in the form $y = mx + c$. (2) **L1**

8.2.5 Determine whether or not, the point $(-11;14)$ lies on the line determine in Question 8.2.4. (3) **L2**

8.2.6 Determine the equation of the line that passes through point R and is perpendicular to line PR. (3) **L3**

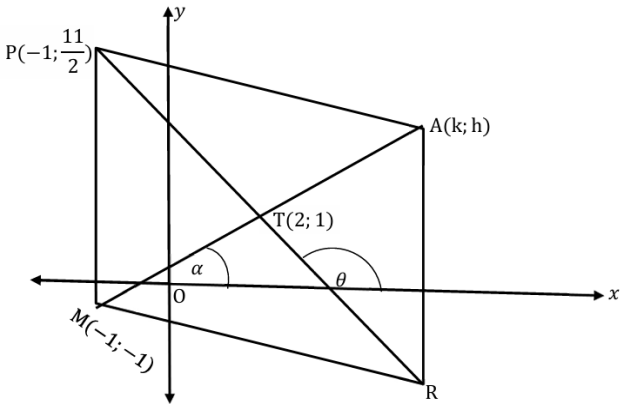
8.2.7 Determine the coordinates of M if PQMR is a parallelogram. (2) **L2**

8.2.8 Calculate the size of PRQ. (4) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

GP JUN 2026

8.3 In the diagram below, ARMP is a quadrilateral with vertices $P\left(-1\,;\ \frac{11}{2}\right)$, $A(k;h)$, R and $M\ (-1\,;\ -1)$. The diagonals of ARMP intersect at $T(2\,;\,1)$. α and θ are angles of inclination of AM and PR, respectively.



8.3.1 Calculate the gradient of MT. (2) **L1**

8.3.2 Determine the equation of the line that is parallel to AM and passes through P, in the form $y = mx + c$. (3) **L2**

8.3.3 If T is the midpoint of AM, calculate the values of k and h. (3) **L1**

8.3.4 Calculate the length of AM, leaving your answer **in surd form.** (2) **L2**

8.3.5 If it is given that $AP \parallel RM$ and $AR \parallel PM$, determine the coordinates of R. (3) **L2**

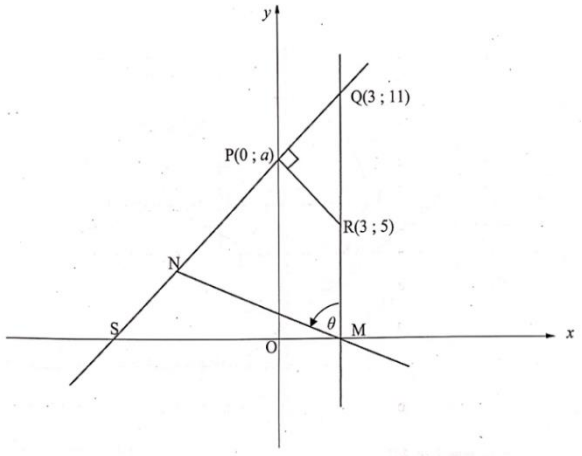
8.3.6 Calculate the size of PTA. (5) **L2**

8.3.7 Show that $PAM \neq 90°$. (3) **L3**

8.3.8 Calculate the area of ΔMPT. (4) **L2**

EC JUN 2026

8.4 In the diagram below, MQ is parallel to the y-axis. PR is perpendicular to QS. N lies on QS. MS lies on the x-axis. Area of triangle MQN is 45,38 square units.



8.4.1 Write down the coordinates of M. (1) **L1**

8.4.2 Write down the length of QM. (1) **L1**

8.4.3 Determine the gradient of PQ in terms of a. (2) **L2**

8.4.4 Show that the value of $a = 8$. (5) **L2**

8.4.5 Determine the equation of SQ in the form $y = mx + c$. (3) **L2**

8.4.6 Show that ΔSMQ is isosceles. (3) **L2**

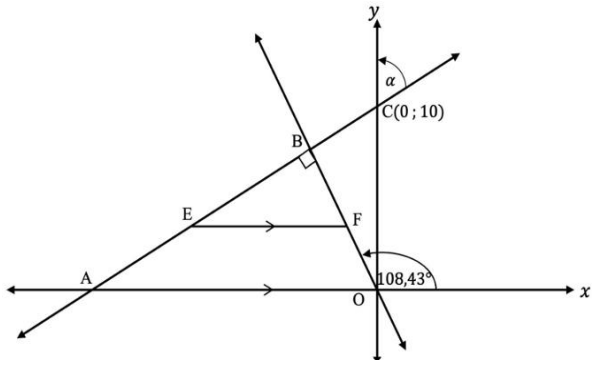
8.4.7 If it is further given that, $y = mx + 1$ is the equation of MN, determine the size of θ. (4) **L3**

8.4.8 Determine the length of MN. (3) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

GP SEP 2025

8.5 In the diagram, ΔABO is drawn. E is the midpoint of AB which is produced to C. A is the x-intercept and C$(0;10)$ is the y-intercept of AC. Line EF is drawn parallel to the x-axis. AB⊥OB and the angle of inclination of OB is $108,43°$. The angle formed by AC and the y-axis is α.



8.5.1 Determine the gradient of line OB. (2) **L2**

8.5.2 Determine the equation of line AB in the form $y = mx + c$ (2) **L2**

8.5.3 Calculate the coordinates of B. (3) **L2**

8.5.4 Calculate the length of BF. (4) **L3**

8.5.5 Calculate the area of ΔCBO. (4) **L2**

8.5.6 Determine the size of α. (3) **L3**

LP SEP 2025

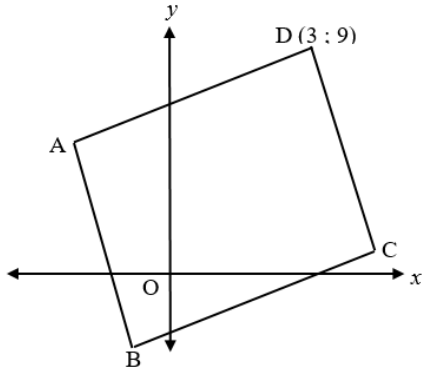
8.6 P$(-3;2)$ and Q$(5;8)$ are two points in a cartesian plane.

8.6.1 Calculate the length of PQ. (2) **L1**

8.6.2 Calculate the angle that PQ forms with the x-axis, rounded off to the nearest degree. (3) **L2**

8.6.3 Determine the equation of the perpendicular bisector of PQ in the form $y = ...$. (5) **L2**

8.7 In the diagram A, B, C and D (3 ; 9) are vertices of a rhombus. The equation of AC is $x + 3y = 10$.



8.7.1 Determine the equation of BD. (3) **L2**

8.7.2 Calculate the coordinates of K if the diagonals of the rhombus intersect at K. (4) **L2**

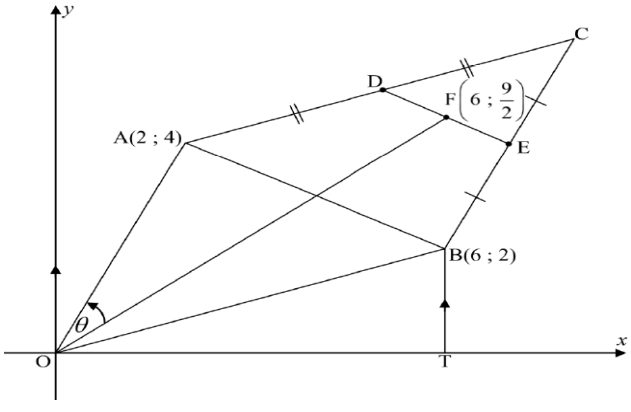
8.7.3 Determine the coordinates of B. (2) **L2**

8.7.4 Calculate the coordinates of A and C if $AD = 5\sqrt{2}$ units. (7) **L3**

8.7.5 If the rhombus is reflected about the line $x = 1$, find the coordinates of the image of A. (2) **L4**

FS SEP 2023

8.8 In the diagram below, A(2 ; 4), O, B(6 ; 2) and C are vertices of a quadrilateral. D and E are the midpoints of AC and BC, respectively. F$\left(6;\frac{9}{2}\right)$ is a point on DE. From B, the straight line drawn parallel to the y-axis cuts the x-axis in T. Angle AOF = θ.



8.8.1 Calculate:

(a) The length of AB. Leave your answer in surd form. (2) **L1**

(b) The gradient of AB. (2) **L1**

(c) Prove that OA⊥AB. (2) **L2**

(d) Determine the equation of DE. (4) **L2**

(e) Determine the coordinates of C such that AOBC, in that order, is a parallelogram. (3) **L2**

8.8.2 Calculate the:

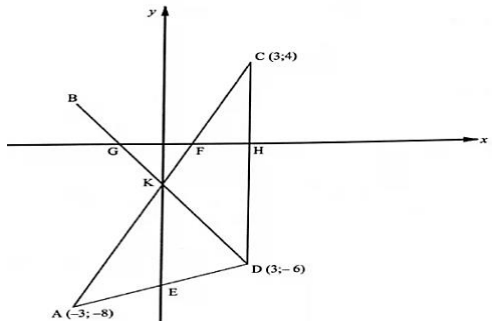
(a) Size of θ. (5) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

(b) Area of ΔABT, if A and T are joined to form ΔABT. (4) **L2**

EC SEP 2025

8.9 A(−3 ; −8), B, C(3 ; 4) and D(3 ; −6) are vertices of a quadrilateral. Diagonals BD and AC intersect at K. E and K are *y*-intercepts of AD and BD respectively. G, F and H are *x*-intercepts of BD, CA and CD respectively. Equation of line BD is given by $y = -\frac{4}{3}x - 2$



8.9.1 Calculate the length of AC. Leave your answer in simplest surd form. (2) **L1**

8.9.2 Calculate the gradient of AC. (2) **L1**

8.9.3 Determine the equation of line AD in the form $y = ...$ (3) **L2**

8.9.4 Determine the size of $C\hat{K}D$. (5) **L3**

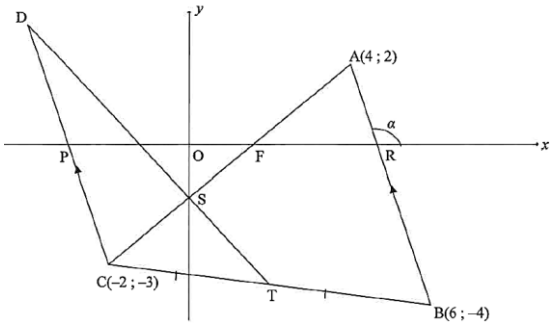
8.9.5 If it is further given that AC = 2KC, determine the area of ΔCKD. (5) **L4**

8.9.6 Write down the equation of CD. (1) **L1**

8.9.7 Determine the numerical value of $\frac{\text{Area of } \Delta CKD}{\text{Area of } \Delta EKCD}$. (5) **L4**

DBE NOV 2022

8.10 In the diagram, A(4 ; 2), B(6 ; −4) and C(−2 ; −3) are vertices of ΔABC. T is the midpoint of CB. The equation of line AC is $5x - 6y = 8$. The angle of inclination of AB is α. ΔDCT is drawn such that CD || BA. The lines AC and DT intersect at S, the *y*-intercept of AC. P, F and R are *x*-intercepts of DC, AC and AB respectively.



8.10.1 Calculate the:

a) Gradient of AB (2) **L1**

b) Size of α. (2) **L1**

c) Coordinates of T. (2) **L1**

d) Coordinates of S. (2) **L2**

8.10.2 Determine the equation of CD in the form $y = mx + c$. (3) **L2**

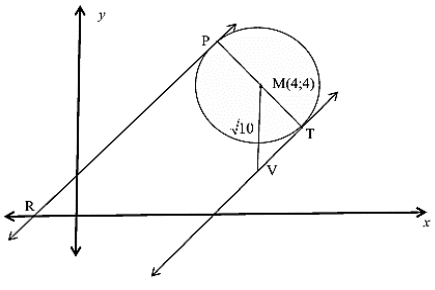
8.10.3 Calculate the:

a) Size of $D\hat{C}A$. (4) **L2**

b) Area of POSC. (5) **L3**

NW JUN 2026

8.11 In the diagram below, line PR with equation $y - x - 2 = 0$ is a tangent to the circle with centre M at P. The coordinates of M are $(4;4)$. PT is the diameter of the circle. V is a point the tangent at T such that $MV = \sqrt{10}$ units .



8.11.1 Calculate the gradient of the diameter. (2) **L1**

8.11.2 Determine the equation of the diameter in the form $y = mx + c$. (2) **L2**

8.11.3 Determine the coordinates of P. (3) **L2**

8.11.4 Determine the equation of the circle in the form $(x-a)^2 + (x-b)^2 = r^2$. (3) **L2**

8.11.5 Determine the coordinate of T. (2) **L2**

8.11.6 Calculate the length of VT. (2) **L2**

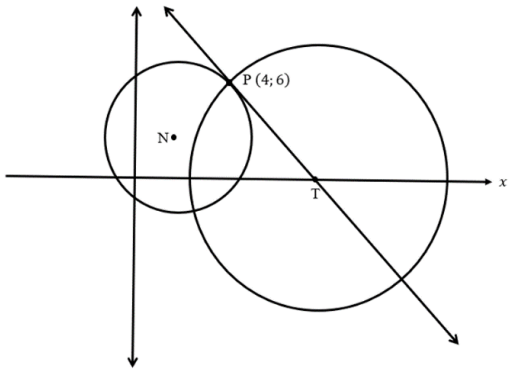
Study copy distributed by EduResource ZA | eduresourcza.com

8.11.7 If the equation of VT is $y = x - 2$, for which values of k will $y = x + k$ NOT be a secant to circle P? (2) **L3**

8.11.8 A second circle with centre $S(1;3)$ and radius q touches the circle with centre M. Determine the value(s) of q. (3) **L4**

WC JUN 2026

8.12 In the diagram, N is the centre of the smaller circle with the equation $(x-1)^2 + (x-2)^2 = 25$. Point T lies on the x-axis and is the centre of a second larger circle. The two circles intersect at $P(4;6)$. Line PT is a tangent to the smaller circle at P.



8.12.1 Write down the coordinates of N, the centre of the smaller circle. (2) **L1**

8.12.2 Write down the radius of the smaller circle. (1) **L1**

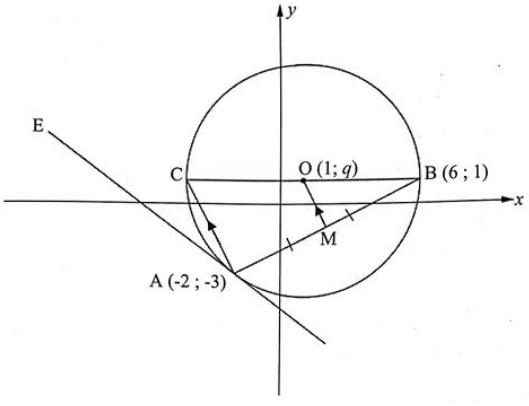
8.12.3 Determine the y- intercepts of the smaller circle N. (4) **L2**

8.12.4 Determine the equation of the circle with centre T in the form $(x-a)^2 + (x-b)^2 = r^2$. (7) **L3**

8.12.5 The radius of the smaller circle is reduced so that the circles only touch once. Determine the radius of the new smaller circle. (4) **L4**

EC JUN 2026

8.13 In the diagram below, $O(1;q)$ is the centre of the circle. M is the midpoint of chord AB. EA is the tangent to the circle at A.



8.13.1 Calculate the length of AB. (2) **L1**

8.13.2 If it is given that $AC = 2\sqrt{5}$, write down the length of OM. (1) **L1**

8.13.3 Write down the size of OMB. (1) **L1**

8.13.4 Determine the value of q. (3) **L2**

8.13.5 Calculate gradient of AO if $q = 1$. (2) **L2**

8.13.6 Determine the equation of the circle with centre O in the form $(x-a)^2 + (x-b)^2 = r^2$. (2) **L2**

8.13.7 Determine the equation of AE. (3) **L2**

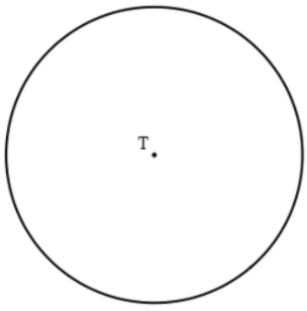
8.13.8 For which value(s) of t will $y = -\frac{3}{4}x + t$ be a secant to the circle? (2) **L3**

8.13.9 Does $\left(-\frac{22}{3};1\right)$ lie inside or outside the circle? Justify your answer. (3) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

NC JUN 2026 ADAPTED

8.14 In the diagram, the equation $x^2 + y^2 + 4x - 12y + 4 = 0$ describes the circle with the centre T and radius r.



8.14.1 Write the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$ to show that the coordinates of the centre T is (–2 ; 6) and the length of the radius *r* is equal to 6. (3) **L2**

8.14.2 Determine the coordinates of the *x*-intercepts of the circle. (3) **L2**

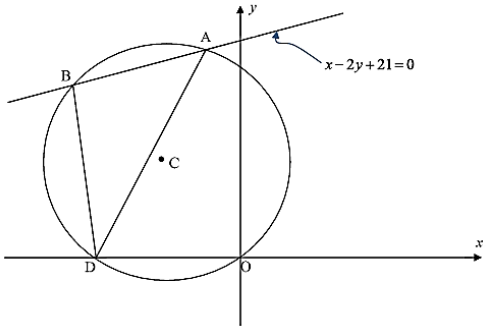
8.14.3 Calculate the coordinates of the *y*-intercepts . Leave your answer in simplified surd form. (4) **L2**

8.14.4 Determine the equation of the tangent to the circle at $\left(0; 6 + 4\sqrt{2}\right)$. (4) **L3**

8.14.5 For which values of k , $k \in R$, will the circle with centre $(-2; 6 + k)$ and radius 6, not touch or intersect the *x*-axis? (4) **L4**

MP JUN 2026

8.15 In the diagram below, A, B, D and the origin O lie on the circle with centre C. The equation of the circle is $x^2 + y^2 + 6x - 8y = 0$. The equation of AB is $x - 2y + 21 = 0$.



8.15.1 Determine the coordinates of the centre C, and the radius of the circle. (4) **L1**

8.15.2 Determine the coordinates of A and B. (6) **L3**

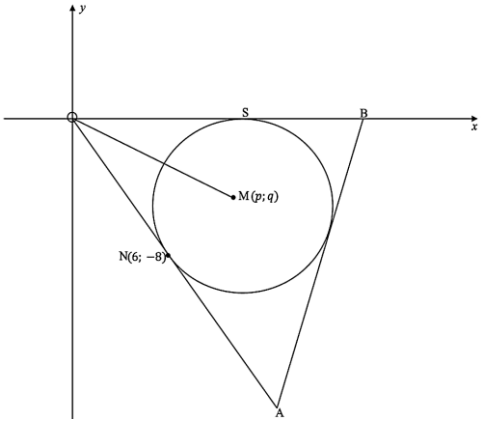
8.15.3 Determine the midpoint AD. (5) **L2**

8.15.4 Determine whether CB passes through the midpoint of line AD. (4) **L2**

8.15.5 A second circle with equation $(x-3)^2 + (x+4)^2 = 25$ is drawn on the same set of axes. A student claims that the circles touch each other in one point. Is the student correct? Show all calculations. (4) **L4**

GP JUN 2026

8.16 In the diagram below, a circle, centred a $M(p; q)$, touches the *x*-axis at S and line OA is a tangent to the circle at $N(6; -8)$.



8.16.1 Calculate the length of ON. (2) **L2**

8.16.2 Calculate the value of *p*. (2) **L2**

8.16.3 Calculate the gradient of MN. (2) **L2**

8.16.4 Calculate the value of *q*. (2) **L2**

8.16.5 Determine the equation of the circle in the form of $(x-a)^2 + (x-b)^2 = r^2$. (3) **L2**

8.16.6 $x = k$ is a tangent to the circle. Write down the value(s) of *k*. (2) **L2**

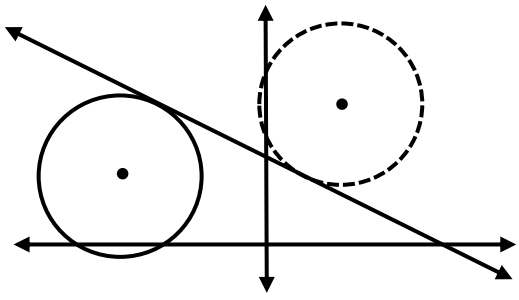
Study copy distributed by EduResource ZA | eduresourcza.com

8.16.7 The line $y=-\frac{4}{3}x+t$ is perpendicular to OA, passes through M and cuts the circle at two different points. Determine the values of *t*. (5) **L4**

8.16.8 Another circle with equation $(x-10)^2+(y-6)^2=25$ is given. Will the two circles touch, intersect at two distinct points or have no points of contact. Give a reason for the answer. (2) **L3**

LP SEP 2025

8.17 The diagram shows a circle with centre $P(-4;2)$ and a tangent at $A(a;b)$ with equation $y=-\frac{1}{2}x+\frac{5}{2}$.



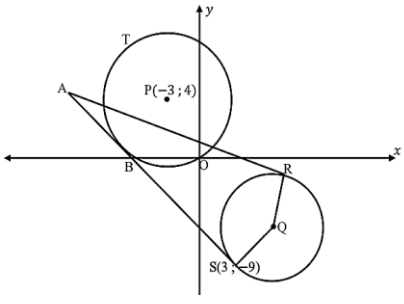
8.17.1 Determine the coordinates of A. (5) **L3**

8.17.2 Determine the equation of the circle P in the form $(x-a)^2+(y-b)^2=r^2$. (3) **L2**

8.17.3 It is further given that the circle P is reflected about the line AB and then translated along the line AB to centre R. Circle R has y-intercepts at 3 and 5. Find the coordinates of R. (4) **L4**

GP SEP 2025

8.18 In the diagram, $P(-3;4)$ is the centre of the circle passing through points B, T and the origin. A second circle is drawn, with centre Q and equation $x^2-12x+y^2+12y+54=0$. Points R and $S(3;-9)$ lie on the circle. AS and AR are both tangents to the ciecle with centre Q. B is the x-intercept of AS.



8.18.1 Determine the equation of the circle with centre P. (3) **L2**

8.18.2 OT is the diameter of the circle with centre P. Determine the coordinates of T. (3) **L2**

8.18.3 Calculate the coordinates of Q. (3) **L1**

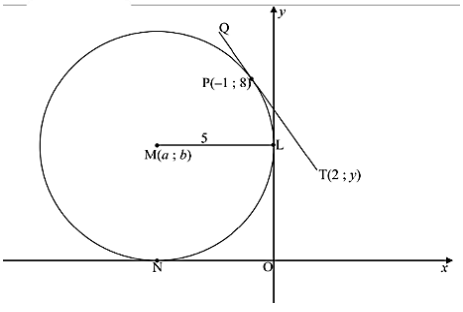
8.18.4 Determine the equation of AS in the form $y=mx+c$. (5) **L2**

8.18.5 Identify the type of quadrilateral that ASQR is and substantiate your answer. (2) **L3**

8.18.6 Determine the scale factor k by which the radius of the circle with centre Q should be increased such that the two circles touch once. (4) **L4**

FS SEP 2023

8.19 In the diagram, a circle centred at $M(a;b)$ with a radius of 5 units touches the x-axis and the y-axis at points N and L respectively. QPT is a tangent to this circle at $P(-1;8)$. The coordinates of T are $(2;y)$.



8.19.1 Give a reason why ML$\perp y$-axis. (1) **L1**

8.19.2 Determine the:

(a) Coordinates of M. (2) **L2**

(b) Equation of the circle having centre M. (2) **L2**

(c) Equation of the tangent QPT in the form $y=mx+c$. (5) **L2**

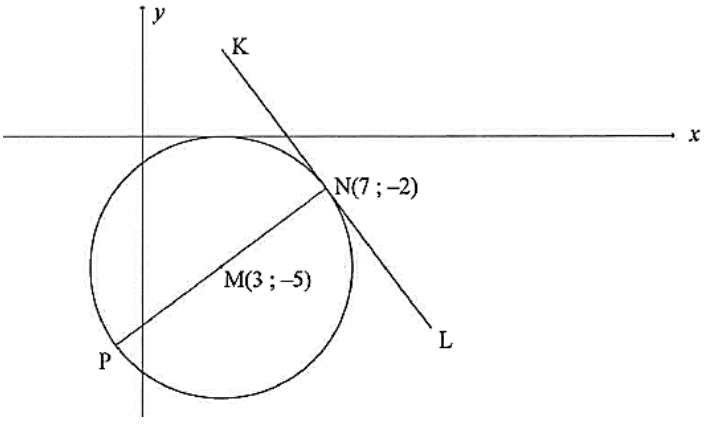
Study copy distributed by EduResource ZA | eduresourcza.com

8.19.3 Another circle, having point T as the centre, touches the circle, having M as the centre externally. Determine the equation of the circle centred at T in the form $(x-h)^2+(y-k)^2=r^2$. (6) **L4**

8.19.4 The circle with centre M is translated across the Cartesian plane in such a way that both horizontal and vertical axes remain tangents to the circle simultaneously. Write down all the possible coordinates of the centres of the newly translated circles, given that $\sqrt{xy}$ must be real for ALL values of x and y. (4) **L4**

DBE NOV 2022 ADAPTED

8.20 In the diagram, M(3 ; –5) is the centre of the circle having PN as its diameter. KL is a tangent to the circle at N(7 ; –2).



8.20.1 Determine the equation of KL in the form $y=mx+c$ (5) **L2**

8.20.2 For which values of k will $y=-\frac{4}{3}x+k$ be a secant to the circle? (4) **L3**

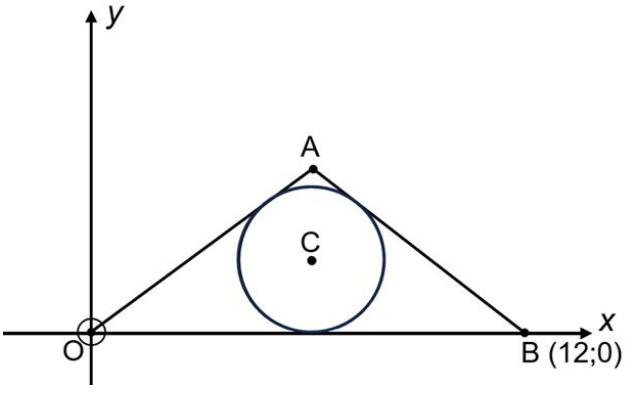
8.20.3 Point A $(t;t)$ and B are not shown on the diagram. From point A, another tangent is drawn to touch the circle with centre M at B.

a) Show that the length of tangent AB is given by $\sqrt{2t^2+4t+9}$. (2) **L4**

b) Determine the minimum length of AB. (4) **L4**

IEB NOV 2023

8.21 In the diagram below, AO, OB and AB are tangents to the circle with equation $x^2+y^2-12x-4y+p=0$. B is the point (12 ; 0).



8.21.1 Determine the value of p. (5) **L3**

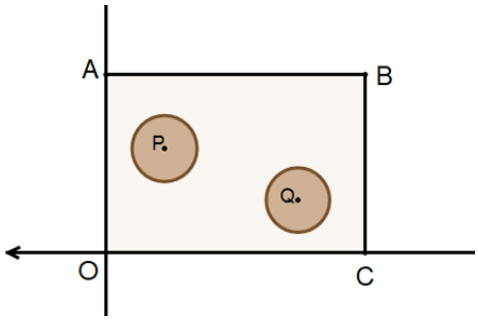
8.21.2 Determine the equation of AB. Show all your working. (7) **L4**

8.21.3 A second circle with centre (0 ; 9) and with point (2 ; 3) that lies on the circumference is drawn on the same set of axes. Do the two circles intersect? Show all working. (5) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

IEB NOV 2021

8.22 In the diagram below, P$(x;y)$ and Q(9 ; 3) are centres of two holes drilled into a rectangular piece of wood with base ABCO. The raddi of the holes are equal. The equation of the circle with centre P is $x^2+y^2-6x-12y+41=0$. The equations of AB and BC are $y=10$ and $x=14$ respectively.



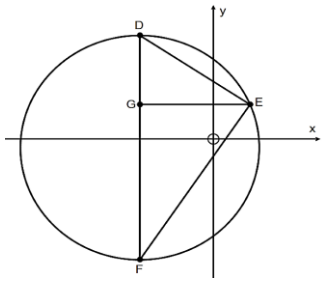
8.22.1 Determine the centre and radius of the hole with centre P. (4) **L2**

8.22.2 Determine the shortest distance between the circles. Leave your answer in surd form. (3) **L4**

8.22.3 Calculate the volume of the block with base ABCO and height 20 units. (4) **L4**

IEB MAY 2026

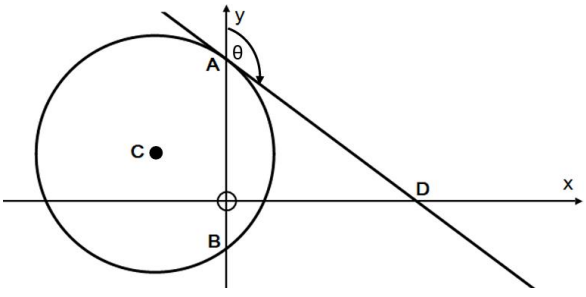
8.23 A circle with equation $x^2+16x+y^2+2y=104$ passes through D, E and F as shown below. DF is the diameter of the circle. DF || y – axis and GE || x – axis. DG : GF = 4 : 9



8.23.1 Determine the coordinates of E. **State all the relevant reasons with your statements** (10) **L4**

IEB MAY 2025

8.24 The diagram below shows circle centre C with equation $x^2+y^2+6x-4y=12$. A and B are the y – intercepts of the circle. AD is a tangent to the circle at A, and its x – intercept is D.



8.24.1 Determine the size of θ, the angle the tangent makes with the y – axis. (8) **L4**

ANSWER SERIES TEXTBOOK

8.25 Determine the maximum radius of the circle with the equation $x^2+y^2+6x\cos\theta-4y\sin\theta+3=0$ (5) **L3**

KZN SEP 2025

8.26 Prove that the radius of the circle having equation $x^2+y^2+4x\cos\theta+8y\sin\theta+3=0$ can never exceed $\sqrt{13}$ for any value of θ. (5) **L4**

ORIGINAL

8.27 Consider the two circles below:

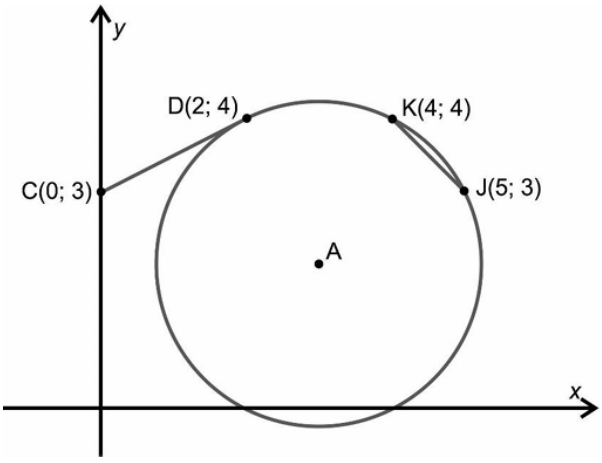
Study copy distributed by EduResource ZA | eduresourcza.com

$x^2 + y^2 - 4x + 6y - 12 = 0$ and $(x-5)^2 + (y-1)^2 = k$

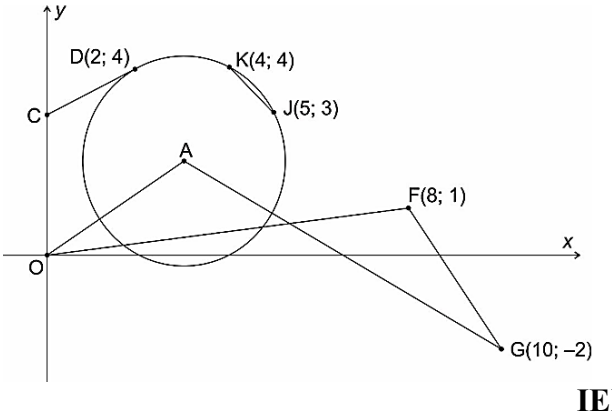
Determine the value(s) of k for which the two circles will intersect at exactly two distinct points. (6) **L4**

IEB MAY MAY 2021

8.28 In the diagram, the line from K(4 ; 4) to J(5 ; 3) is a chord of the circle with centre A. CD is a tangent to the circle at D(2 ; 4). C(0 ; 3) lies on the y – axis.



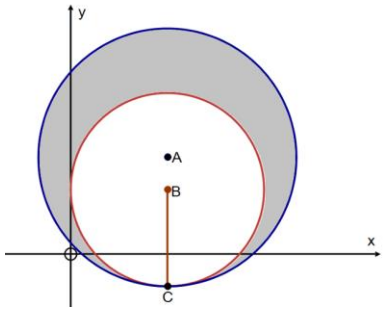
8.27.1 Determine the coordinates of A (show all of your workings) (8) **L4**



8.27.2 Two additional points F(8 ; 1) and G(10 ; –2) are added to the diagram with $A\hat{O}F = 26,6°$ and A(3 ; 2) as shown on the diagram alongside.
Prove that AFGO is a cyclic quadrilateral. (8) **L4**

IEB NOV 2025

8.29 Given circles with centre A and B. The two circles are tangent at point C. Circle with centre B is tangent to the y-axis. BC || y-axis. A, B and C are co-linear. The equation of the circle with centre B is $x^2 + y^2 - 4(3x + 2y) = -16$.



8.29.1 Determine the equation of the tangent to the circle at C. (6) **L4**

8.29.2 Determine the equation of circle centered A. If the shaded region has an area of 28π units^2. (6) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

TOPIC: 9 TRIGONOMETRY [± 50 MARKS]

GUIDELINES, SUMMARY NOTES, & STRATEGIES REFER 2025 LAST PUSH DOCUMENT

General Trigonometry

STUDY & MASTER (2013) Adapted

9.1 If $\tan A = 0,5$ and $\cos A < 0$, determine without using a calculator and with the aid of a sketch,

The value of $2\sin\frac{A}{2}\cos\frac{A}{2}$ (3) **L3**

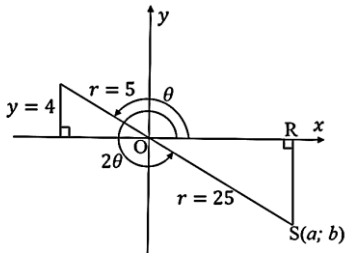
NW JUN 2026

9.2 Given $\cos 22,5° = \frac{a}{c}$ and $a^2 + b^2 = c^2$, with the aid of a diagram, or otherwise, show that

$\frac{2ab}{c^2} = \frac{\sqrt{2}}{2}$ (5) **L3**

EC JUN 2026

9.3 In the diagram below, point $\sin\theta = \frac{4}{5}$ where $90° < \theta < 180°$ and point $S(a;b)$ is in the 4th quadrant with reflex where $180° < 2\theta < 360°$. Determine **without using a calculator,** the value of:



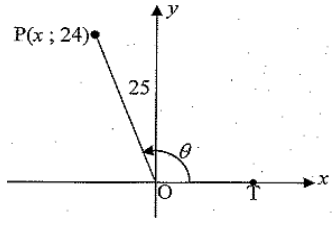
9.3.1 $\cos\theta$ (2) **L1**

9.3.2 $\cos(90° - \theta)$ (2) **L1**

9.3.3 Hence or otherwise determine the coordinates of S (5) **L3**

NW SEP 2024

9.4 In the diagram below, point P(x ; 24) is 25 units from the origin O. T is a point on the x-axis such that . $T\hat{O}P = \theta$.



9.4.1 Calculate the value of x. (2) **L1**

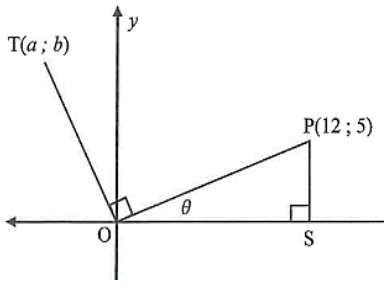
9.4.2 **Without using the calculator**, determine the value of $\tan(360° - \theta)$ (2) **L2**

9.4.3 Calculate the size of $P\hat{O}T$. (2) **L2**

GP SEP 2023

9.5 In the diagram below, P is a point $(12\ ;\ 5)$ and $T(a\ ;\ b)$. $OT \perp OP$; $PS \perp$ x-axis and $P\hat{O}S = \theta$.

Without using the calculator, determine the value of:



9.5.1 $\tan\theta$ (2) **L1**

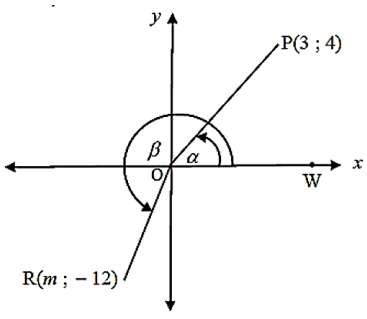
9.5.2 $\sin\theta$ (2) **L2**

9.5.3 a, if $TO = 19,5$ units. (3) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

LP SEP 2023

9.6 In the diagram below $P(3\,;4)$ and $R(m\,;\,-12)$ are two points as indicated. $P\hat{O}W=\alpha$ and $R\hat{O}W=\beta$. Answer the following questions without using a calculator.



9.6.1 Write down the value of $\tan\alpha$ (1) **L1**

9.6.2 Determine the value of $\sin(90°+\alpha)$ (3) **L2**

9.6.3 Determine the value of *m* if it is given that $12+13\sin\beta=0$. (4) **L2**

9.6.4 Determine the value of $\cos(\alpha+\beta)$. (3) **L3**

9.7 If $\sqrt{5}\sin\theta+2=0$ and $\theta\in[90°\,;270°]$, determine **without the use of a calculator** the value of the following:

9.7.1 $\cos 2\theta$ (3) **L2** 9.7.2 $\cos(\theta-135°)$ (4) **L2**

NSC MAY-JUN 2026

9.8 9.8.1 Show, without using a calculator, that $\sin 260° = -\sqrt{\dfrac{\cos 20°+1}{2}}$ (4) **L2**

9.8.2 Given: cos 330°.cos 365°+ sin 175°.sin 210°
Without using a calculator, simplify the expression above to a single trigonometric ratio (5) **L2**

9.9 If $\sin 2\theta = \dfrac{-4\sqrt{2}}{9}$ and $2\theta\in[90°\,;270°]$, determine the value of the following **without the use of a calculator.**

9.9.1 $\cos 2\theta$ (3) **L1** 9.9.2 $\sin\theta$ (4) **L2** 9.9.3 $\tan\theta$ (4) **L2**

MP SEP 2023

9.10 If θ is a reflex angle, and $\tan\theta=-\dfrac{3}{4}$, determine without the use of a calculator and with the aid of a sketch, the value of:

9.10.1 $\cos 2\theta$ (3) **L2** 9.10.2 $\cos(\theta+30°)$ (3) **L2**

NC SEP 2020

9.11 If $13\cos\theta=5$, and $\theta\in[180°\,;360°]$, **without the use of a calculator**, determine the value of the following:

9.11.1 $\sin\theta$ (2) **L1** 9.11.2 $\sin(\theta+45°)$ (3) **L2**

STUDY AND MASTER (2013) Adapted

9.12 9.12 (a) If $\sin 33° = \sqrt{k}$ determine the value of $\sin 51°\cos 15° + \cos 51°\sin 15°$ in terms of k (3) **L2**

9.12 (b) If $\cos x = 2\sin y\cos y$ and $0°\le y\le 90°$, show that: $x+2y=90°$ (4) **L3**

9.12 (c) Determine the integer values of *a*, *b* and *c* if $3\cos^2\theta+\sin^2\theta-6\sin\theta\cos\theta$ is written in the form $a\sin 2\theta+b\cos 2\theta+c$ (4) **L2**

KZN SEP 2024

9.13 If $\tan 58° = n$, determine the following in terms of *n* without using a calculator.

9.13.1 $\sin 58°$ (3) **L2** 9.13.2 $\sin 296°$ (4) **L2** 9.13.3 $\cos 2°\cos 2°$ (3) **L3**

9.14 If $\cos 21° = p$, determine the following in terms of *p*.

9.14.1 $\tan 201°$ (3) **L2** 9.14.2 $\sin 42°$ (3) **L2** 9.14.3 $\cos 51°$ (3) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

NC SEP 2024

9.15 Given that $\sin 10° = \sqrt{k}$, write each of the following in terms of *k* **without using a calculator**.

9.15.1 $\sin 190°$ (2) **L2** 9.15.2 $\cos 20°$ (3) **L2** 9.15.3 $\cos 50°$ (4) **L2**

FS SEP 2024

9.16 If $\sin\frac{x}{2} = p$, express $\sin x - 1$ in terms of *p*. (4) **L3**

9.17 Given that $\cos 42° = \sqrt{k}$, determine the value of $\sin^2 69°$ in terms of *k*. (3) **L3**

9.18 If $\sin 38° = p$, determine the value of the following in terms of *p* **without the use of a calculator.**

9.18.1 $\cos 218°$ (3) **L2** 9.18.2 $\cos 14°$ (3) **L3** 9.18.3 $\sin 26° \cos 26°$ (2) **L3**

WC SEP 2023

9.19 Given $\sin 64° = p$. **Without using the calculator** determine the value of the following in terms of *p*.

9.19.1 $\tan 64°$ (2) **L1** 9.19.2 $\cos 334°.\sin 244°$ (3) **L2** 9.19.3 $8\sin 16° \cos 16°.\cos 32°$ (3) **L2**

DBE NOV 2024

9.20 If $\cos\alpha = p$ express the following expression in terms of *p*.

$$\frac{\cos\left(\frac{\alpha}{2} - 45°\right).\sin\left(\frac{\alpha}{2} - 45°\right)}{2}$$

(4) **L3**

IEB NOV 2023

9.21 Given that $\cos\theta = \frac{a^2 - b^2}{a^2 + b^2}$ where $0 < b < a$ and $\sin\theta < 0$.

Determine the value of $\tan\theta$ in terms of a and b. (4) **L4**

2021 HIGHFLYER DOCUMENT

9.22 Given $\cos A.\sin A = k$, and A is acute

9.22.1 Determine the value of $\tan A + \frac{1}{\tan A}$ in terms of *k*. (4) **L4**

9.22.2 Prove that: $\sin A + \cos A = \sqrt{1 + 2k}$ (3) **L4**

STUDY & MASTER (2013) Adapted

9.23 Without the use of a calculator, determine the value of:

9.23.1 $\frac{\cos 300° \sin 140°}{\sin 45° \sin 290° \sin 160°}$ (6) **L2** 9.23.2 $\frac{\tan(-60°)\cos(-156°)\cos 294°}{\sin 492°}$ (6) **L2**

9.23.3 $\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x}$ (5) **L2**

ORTCD MAR 2026

9.24

9.24 (a) If $\cos(A + B) = m$ and $\cos(A - B) = n$, determine the value of $\cos A \cos B$ in terms of *m* and *n*. (3) **L2**

9.24 (b) If $\cos 25° = \sqrt{1 - k^2}$, determine the value of $\cos 70°$ in terms of k: (5) **L3**

UNDERSTANDING PURE MATHEMATICS

9.24 (c) Given $x - 2\sin\theta$ and $3y = \cos\theta$: Show that $x^2 + 36y^2 = 4$ (3) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

MP JUN 2026

9.25 Given that $\sin\theta = \frac{1}{3}$, calculate the numerical value of $\sin 3\theta$, WITHOUT using a calculator. (5) **L3**

KZN JUN 2026

9.26 The expression $\sqrt{3}\cos\beta + \sin\beta$ can be rewritten in the form $k\sin(m° + \beta)$. Determine the values of *k* and *m*. Show all your working details. (4) **L3**

WC JUN 2026

9.27 If $\sin\theta = 2p$ and $\cos\theta = p$, determine the numerical value(s) of p. (4) **L3**

WC SEP 2025

9.28 Prove that $\sqrt{-\cos^2(90° - M) - \cos M.\cos(-M)}$ is non-real for any real value of M. (4) **L2**

HIGH-FLYERS DOCUMENT

9.29 9.29.1 Given that *P* and *Q* are both acute, solve for P and Q if: $\sin P\sin Q - \cos P\cos Q = \frac{1}{2}$

and $\sin(P - Q) = \frac{1}{2}$ (5) **L3**

9.29.2 Simplify: $\frac{\sin 6A}{\sin 2A} - \frac{\cos 6A}{\cos 2A}$ (4) **L3**

9.29.3 If $\cos\beta + \sin\beta = T$, express $\frac{\cos 2\beta}{\sin(\beta - 45°)}$ in terms of T (4) **L3**

9.30 Simplify the following **without the use of a calculator.**

9.30.1 $\frac{\sin^2 35° - \cos^2 35°}{4\sin 10°\cos 10°}$ (4) **L3**

9.30.2 $\frac{[2\cos^2(180° + x) - 1].\cos 67°}{(6\sin^2 x - 3).\tan 23°}$ (6) **L3**

9.30.3 $\cos 330°.\cos 365° + \sin 175.\sin 210°$ (5) **L2**

9.30.4 $\frac{\sin 210°.\cos 510°}{\cos 315°.\sin(-135°)}$ (7) **L2**

9.30.5 $\frac{\cos 105°.\cos 15°}{\cos(45° - x).\cos x - \sin(45° - x).\sin x}$ (5) **L3**

9.30.6 $\frac{\cos 330°.\sin 140°}{\sin(-160°).\tan 405°.\sin 290°}$ (6) **L2**

9.30.7 $\frac{\cos^2 208^{O}}{\tan 118^{O}.\sin 124^{O}}$ (6) **L3**

9.30.8 $\frac{\left(\sqrt{2}\cos 15° + 1\right)\left(\sqrt{2}\cos 15° - 1\right)\sin(-2x)}{4\sin x\cos x}$ (4) **L3**

9.30.9 $\frac{\cos 66^{O}}{\cos 22^{O}} - \frac{\sin 66^{O}}{\sin 22^{O}}$ (6) **L3**

9.31 Simplify the following expressions.

9.31.1 $\frac{\cos(90° + x).\sin(x - 180°) - \cos^2(180° - x)}{\cos(-2x)}$ (6) **L2**

9.31.2 $\frac{\sin(360° + \theta).\sin(90° + \theta).\tan(-\theta)}{(\cos 90° - 2\theta).\tan(180° + \theta)}$ (6) **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

9.31.3 $\tan(-x).\sin(90°+x)+\dfrac{\sin 2x}{2\cos(360°+x)}$ (6) **L2**

9.31.4 $\dfrac{1-\sin(-A).\cos(90°+A)}{\cos(A-360°)}$ (5) **L2**

9.31.5 $\dfrac{\sin 10°}{\cos 440°}+\tan(360°-\beta).\sin 2\beta$ (6) **L2**

9.31.6 $\dfrac{\sin 140°.\sin(360°-\theta)}{\cos 50°.\tan(-\theta)}$ (6) **L2**

9.31.7 $-\cos(-\beta).\cos^3(90°+\theta)-\tan(180°-\beta).\cos(\beta).\cos^3(-\beta)$ (5) **L2**

9.31.8 $\dfrac{\sin x}{\cos x.\tan x}+\sin(180^{\circ}+x).\cos(90^{\circ}-x)$ (6) **L2**

9.31.9 $\dfrac{\sin 2x}{2\cos^2 x}+\dfrac{\sin x}{\tan x[1+\cos(90^{\circ}-x)]}$ (5) **L3**

9.31.10 $\dfrac{\sin x}{\sin 2x}+\dfrac{\sin(180^{\circ}-x)}{\cos x}-\dfrac{\sin^2(x-180^{\circ})+\cos^2(x+360^{\circ})}{2\sin(90^{\circ}+x)}$ (4) **L3**

DBE MAY/JUN 2024

9.32 Given the expression: $\dfrac{\sin 150°+\cos^2 x-1}{2}$

9.32.1 **Without using a calculator,** simplify the expression to a single trigonometric term in terms of $\cos 2x$ (6) **L3**

9.32.2 Hence, determine the general solution for $\dfrac{\sin 150°+\cos^2 x-1}{2}=\dfrac{1}{25}$ (5) **L3**

Trig Identities and General Solution
STUDY & MASTER (2013) Adapted

9.33 9.33.1 Prove that $\sin(45°+\theta)\sin(45°-\theta)=\dfrac{1}{2}\cos 2\theta$ (5) **L2**

9.33.2 Hence, determine the value of $\sin 75°\sin 15°$ (3) **L2**

9.34 Prove that $2\cos^2(x-45°)-2x=1$ and hence deduce, without using a calculator, that

$\cos^2(-15°)=\dfrac{1}{2}\left(1+\dfrac{\sqrt{3}}{2}\right)$ (5) **L3**

9.35 Show that $\sin^2 20°+\sin^2 40°+\sin^2 80°=\dfrac{3}{2}$ (6) **L4**

LP MAR 2026

9.36 Prove that $2\cos^2(45°+x)=1-\sin 2x$ (4) **L2**

9.37 If it is given that: $\sin(A-B)-\sin(A+B)=-2\cos A\sin B$

9.37.1 Simplify the expression $\sin 4x-\sin 10x$ to a single term (2) **L2**

9.37.2 Hence, determine the solution for $\sin 4x-\sin 10x=\sin 3x$, $x\in[0°;30°]$ (5) **L2**

9.38 Prove the following identities:

9.38.1 $\dfrac{\sin 5x-\sin 3x}{\cos 3x+\cos 5x}=\tan x$ (6) **L3**

9.38.2 $\dfrac{3\sin x+2\sin 2x}{2+3\cos x+2\cos 2x}=\tan x$ (4) **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

9.38.3 $\tan^2 x\left(\frac{1}{\tan^2 x}-1\right)=\frac{\cos 2x}{\cos^2 x}$ (3) **L3**

9.38.4 $\frac{1-\cos 2x}{\sin 2x}=\tan x$ (3) **L2**

9.38.5 $\frac{\sin 5x-\sin 3x}{\cos 3x+\cos 5x}=\tan x$ (6) **L3**

9.38.6 $\frac{\cos\theta-\cos 2\theta+2}{3\sin\theta-\sin 2\theta}=\frac{1+\cos\theta}{\sin\theta}$ (5) **L2**

9.38.7 $\frac{\cos 2x+\cos^2 x+3\sin^2 x}{2-2\sin^2 x}=\frac{1}{\cos^2 x}$ (4) **L3**

9.38.8 $\frac{\sin\theta.\tan\theta}{\tan 2\theta.\left(1-\tan^2\theta\right)}=\frac{\sin\theta}{2}$ (5) **L3**

9.38.9 $\tan x\left(1-\cos^2 x\right)+\cos^2 x=\frac{\left(\sin x+\cos x\right)\left(1-\sin x.\cos x\right)}{\cos x}$ (5) **L3**

9.38.10 $\frac{\cos^4 x+\sin^2 x.\cos^2 x}{1+\sin x}=1-\sin x$ (4) **L3**

9.38.11 $\frac{\tan x+1}{\sin x\tan x+\cos x}=\sin x+\cos x.$ (5) **L2**

9.38.12 $\frac{\cos 2x+\sin 2x-\cos^2 x}{-3\sin^2 x+6\sin x\cos x}=-\sin x$ (3) **L3**

9.38.13 $\frac{\sin\ 2x}{\cos x\ \left(1-\cos 2x\right)\left(1+\frac{1}{\tan^2 x}\right)}=\sin x$ (5) **L3**

9.38.14 $\frac{2\cos 2\theta.\cos\theta}{\cos^2\theta-\sin^2\theta}+2\tan\theta.\sin\theta=\frac{2}{\cos\theta}$ (3) **L3**

9.38.15 $\frac{\sin\left(90°+\theta\right).\sin^2\left(\theta-180°\right)-\cos^3\left(180°+\theta\right)}{\cos\theta}-2\sin^2\theta=\cos 2\theta$ (5) **L2**

9.38.16 $\frac{\sin 2\theta+\cos 2\theta+1}{\cos 2\theta}=\frac{2\cos\theta}{\cos\theta-\sin\theta}$ (4) **L3**

9.38.17 $\frac{\cos\left(360°-2x\right)-\cos\left(180°+x\right)}{\sin 2x-\cos\left(90°-x\right)}=\frac{\cos x+1}{\sin x}$ (7) **L2**

9.38.18 $\frac{\sin 5x.\cos 3x-\cos 5x.\sin 3x}{\tan 2x}-1=-2\sin^2 x$ (4) **L2**

9.38.19 $\frac{1-\cos 2x-\sin x}{\sin 2x-\cos x}=\tan x$ (4) **L3**

9.38.20 $\sin\theta=\frac{2\tan\left(\frac{\theta}{2}\right)}{1+\tan^2\left(\frac{\theta}{2}\right)}$ (3) **L3**

DBE NOV 2024

9.39 Given the identity: $\cos\left(x-y\right)=\cos x.\cos y+\sin x.\sin y$

9.39.1 Use the compound angle identity to derive a formula for $\cos\left(x+y\right)$. (2) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

9.39.2 Hence, or otherwise, show that :

$$\frac{\cos(90°-x).\cos y+\sin(-y).\cos(180°+x)}{\cos x.\cos(360°+y)+\sin(360°-x).\sin y}=\tan(x+y)$$ (6) **L3**

2021 HIGHFLYER DOCUMENT

9.40 Given the following identity: $\frac{\cos x-\sin x.\sin 2x}{\cos 2x}=\cos x$

9.40.1 Prove the identity (2) **L2**

9.40.2 For which values of x is the identity undefined? (3) **L2**

STUDY AND MASTER (2013) Adapted

9.41 Calculate the value(s) of x if

9.41.1 $\tan 2x+2\sin x=0$ for $x\in[0°;360°]$ (5) **L2**

9.41.2 $\tan 2x+2\sin x$ is undefined for $x\in[0°;360°]$ (2) **L1**

9.42 Determine the general solution of

9.42.1 $\cos 2x=1-3\cos x$ (5) **L3**

9.42.2 $\sin x=\cos 2x-1$ (5) **L3**

9.43 Solve for x:

9.43.1 $\tan x=\sin 2x$ for $x\in[-90°;180°]$ (5) **L3**

9.43.2 $6\sin^2 x+2\sin 2x=1$, for $x\in[-180°;200°]$ (6) **L2**

9.43.3 $\cos 3x\cos x+\sin 3x\sin x=-\frac{1}{\sqrt{2}}$, $x\in[-90°;180°]$ (5) **L3**

9.43.4 $\sin^2 x+\sin 2x-\sin x-2\cos x=0$, $x\in[-270°;90°]$ (6) **L3**

2021 HIGHFLYER DOCUMENT

9.44 Given the following identity: $\frac{\sin 7x+\sin x}{2\cos 3x}=\sin 4x$

9.44.1 Prove the identity (4) **L3**

9.44.2 For which values of x is the identity above undefined? (3) **L2**

9.45 Determine the general solution for the following equations.

9.45.1 $\cos 5x=\cos 3x$ (5) **L2**

9.45.2 $\sin 5x=\cos 3x$ (5) **L2**

9.45.3 $\cos 2x-\frac{1}{3}=\frac{1}{3}\sin x$ (6) **L4**

9.45.4 $6\sin\frac{3\theta}{2}+\cos\frac{3\theta}{2}.\cos\theta=-\frac{\sqrt{3}}{2}$ (4) **L3**

9.45.5 $\cos(x-60°)=-\frac{\sqrt{3}}{2}\sin x$ (5) **L3**

9.45.6 $\frac{\cos 2x+1}{\sin 2x}=1$ (7) **L3**

9.45.7 $2\sin\alpha\cos\alpha=2\cos^2\alpha-2\sin^2\alpha$ (5) **L3**

9.45.8 $\cos 2x-5x-2=0$ (6) **L2**

9.45.9 $\sqrt{6\sin^2 x-11\cos(90°+x)+7}=2$; where $x\in(0°\ ;\ 360°)$. (6) **L2**

9.45.10 $\cos 54°\cos x+\sin 54°\sin x=\sin 2x$ (5) **L3**

9.45.11 $6\cos 2x-\cos x+6=0$ (5) **L3**

9.45.12 $1-\sin x.\cos x-3\cos^2 x=0$ (5) **L3**

9.45.14 $4\sin x+6\sin x\cos x-3\cos x=2$ (5) **L3**

DBE MAY/JUN 2023

9.46 Given: $3\tan 4x=-2\cos 4x$

9.46.1 **Without using a calculator**, show that $\sin 4x=-0,5$ is the only solution to the above equation. (4) **L3**

9.46.2 Hence, determine the general solution of x in the equation $3\tan 4x=-2\cos 4x$ (3) **L2**

9.47 Given that $\cos x=2m$ and $\cos 2x=7m$, determine the value(s) of m. (5) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

Trigonometric graphs

STUDY AND MASTER (2000) Adapted

9.48 Consider the functions $f(x) = \tan x - 1$ and $g(x) = \cos 2x$.

9.48.1 On the same set of axes, draw sketch graphs of f and g for $x \in [-180°;180°]$.
Clearly show all intercepts with the axes, and the co-ordinates of the turning points.
Show the asymptotes of $f(x) = \tan x - 1$ (5) **L2**

9.48.2 Use the graph to determine the value of x if
$\cos 2x + 1 \leq \tan x$; in the interval $[-180°;0°]$ (3) **L2**

9.48.3 Determine the equation of h if $h(x)$ is the graph formed when $f(x) = \tan x - 1$ is moved upwards by 3 units and reflected about the x-axis. (2) **L2**

9.48.4 Determine the period of $h(x)$ (1) **L1**

9.48.5 Use the graphs in 9.48.1 to solve the equation:

$$\cos B + 1 = \frac{\sin\left(\frac{B}{2}\right)}{\cos\left(\frac{B}{2}\right)} \text{ if B } \in [-360°;180°]$$ (4) **L4**

ORIGINAL

9.49 A trigonometric function $f(x)$ has the following properties:

- maximum value is 5
- minimum value is –1
- period is $180°$
- passes through the point $(15°;2)$
- is increasing at $x = 15°$

9.49.1 Determine the equation of the function. (4) **L4**

9.49.2 Draw a sketch graph of the function for $0° \leq x \leq 270°$ (3) **L2**

9.50 The graph of a trigonometric function f has the following properties:

- Domain: $0° \leq x \leq 360°$
- Range: $-1 \leq y \leq 5$
- Maximum turning point: $(60°;5)$
- Minimum turning point: $(240°;-1)$
- Passes through the point $(0°;2)$
- Has no x-intercepts

9.50.1 Use the given information to sketch the graph of the function. (3) **L4**

9.50.2 State the amplitude. (1) **L1**

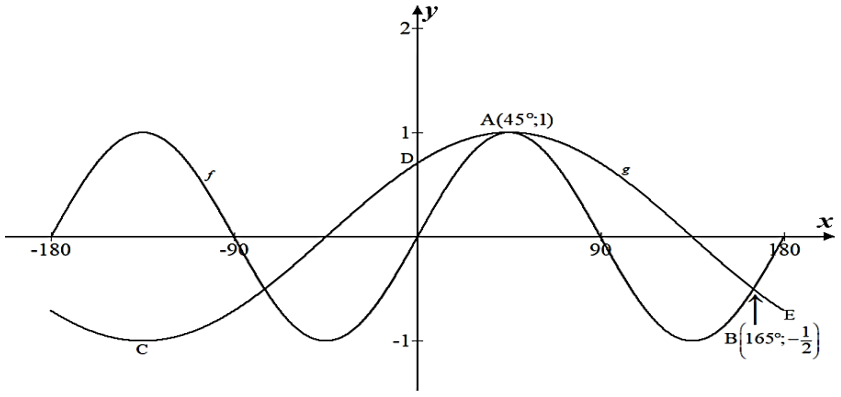
9.50.3 State the period. (1) **L1**

9.50.4 Determine the value(s) of x for which $f(x) = 4$ (2) **L2**

9.50.5 Hence determine the range of $g(x) = 2f(x) - 3$ (1) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

9.51 In the diagram below, the graphs of $f(x)=\sin ax$ and $g(x)=\cos(x+b)$ where $-180° \le x \le 180°$ are drawn.



9.51.1 Determine the values of a and b (2) **L2**

9.51.2 Write down the period of g. (1) **L1**

9.51.3 Determine the coordinates of C, the turning point of g. (2) **L2**

9.51.4 Determine the coordinates of D and E. (2) **L2**

9.51.5 Determine the equation of f if the y-axis is moved $30°$ to the left. (2) **L2**

9.51.6 Use the graph to solve the following equation: $\sqrt{2}\sin 2x = \cos x + \sin x$ (3) **L4**

9.52 Given: $f(x)=\cos(x+45°)$ and $g(x)=-2\sin x$ where $-180° \le x \le 180°$

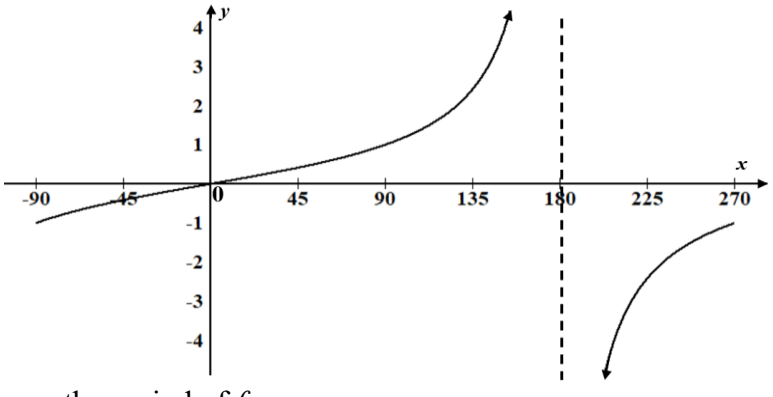
9.52.1 Draw the graph of f and g on the same set of axes, showing all the intercepts with the axes, turning points and endpoints. (2) **L1**

9.52.2 What is the period of g ? (3) **L2**

9.52.3 Determine by means of calculation, the values of x if $f(x)=g(x)$ in the interval above. (5) **L3**

NC SEP 2024

9.53 The graph of $f(x)=\tan\frac{1}{2}x$ for the interval $x \in [-90°;270°]$, is drawn below.



9.53.1 Write down the period of f. (1) **L1**

9.53.2 Write down the equation of the asymptote of f in the interval $x \in [-90°;270°]$ (2) **L2**

9.53.3 On the grid provided in the ANSWER BOOK, draw the graph of $g(x)=2\cos x$ for the interval $x \in [-90°;270°]$. Clearly show all intercepts with the axes, turning points and end points. (3) **L2**

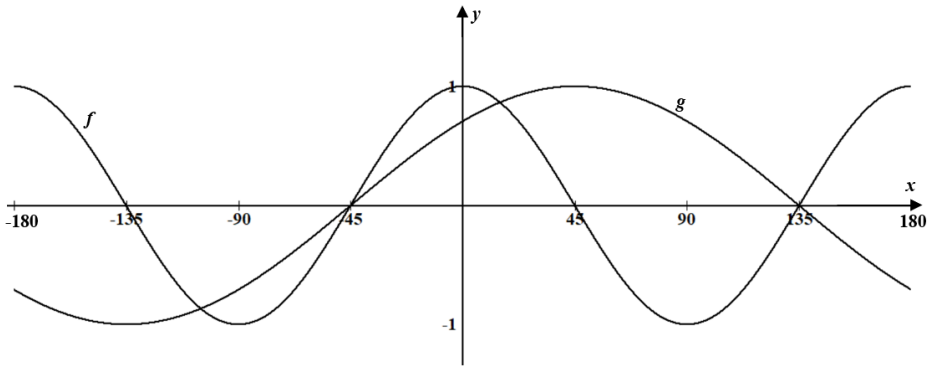
9.53.4 Give 2 values of x, in the interval $x \in [-90°;270°]$, for which $g(x)-f(x)=1$ (2) **L2**

9.53.5 Write down the range of g(x) – 3 in the interval $x \in [-90°;270°]$. (2) **L2**

9.53.6 Determine the maximum value of, $[5-2\sin(90°-x)]^2$ for $x \in \mathbb{R}$. (3) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

9.54 In the diagram below the graphs of $f(x) = a\cos bx$ and $g(x) = \sin(x+p)$ are drawn for $x \in [-180° ; 180°]$.



9.54.1 Write down the values of *a*, *b* and *p*. (3) **L2**

9.54.2 For which values of x in the given interval does the graph of f increase as the graph of g increases? (2) **L1**

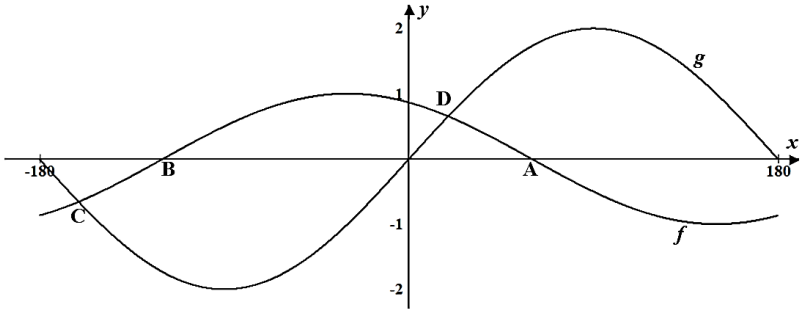
9.54.3 Write down the period of $f(3x)$. (2) **L1**

9.54.4 Determine the minimum value of h if $h(x) = 2f(x) - 5$. (2) **L2**

9.54.5 Describe how the graph g must be transformed to form the graph h, where $h(x) = -\cos x$ (2) **L3**

9.54.6 Determine the value(s) of x for which $f'(x).g(x) < 0$ (6) **L3**

9.55 In the diagram, the graphs of $f(x) = \cos(x - 30°)$ and $g(x) = 2\sin x$ are drawn for the interval $x \in [-180° ; 180°]$, A and B are the x-intercepts of f. The two graphs intersect at C and D, the minimum and maximum turning points respectively of f.



9.55.1 Write down the coordinates of

a) A and B (2) **L1**

b) C and D (2) **L1**

9.55.2 Determine the values of x in the interval $x \in [-180° ; 180°]$, for which:

a) $g(x) > f(x)$ (2) **L2**

b) $f(x - 20°) > g(x - 20°)$ (2) **L2**

c) $x.f(x) < 0$ (3) **L2**

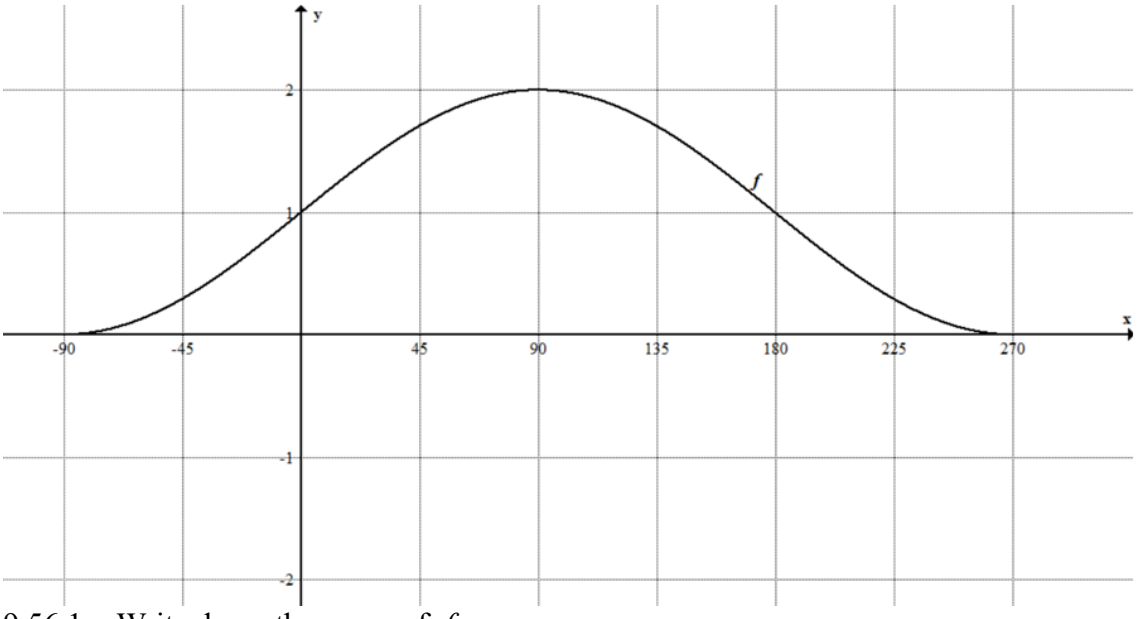
d) $x.\sqrt{f(x) - g(x)} > 0$ (2) **L3**

9.55.3 Determine the range of $y = 2^{\cos(x-30°)+1}$ (4) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

THE ANSWER SERIES 2-in-1

9.56 In the diagram, the graph of $f(x)=\sin x+1$ is drawn for $-90° \leq x \leq 270°$.



9.56.1 Write down the range of *f*. (1) **L1**

9.56.2 Show that $\sin x+1=\cos 2x$ can be written as $(2\sin x+1)\sin x=0$ (2) **L2**

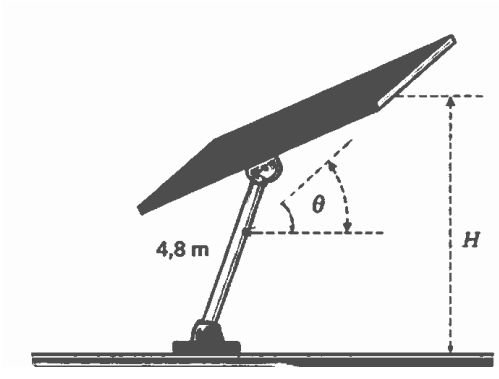
9.56.3 Hence or otherwise, determine the general solution of $\sin x+1=\cos 2x$ (4) **L2**

9.56.4 Redraw the graph above or use the grid above to draw on the same system of axes, the graph of $g(x)=\cos 2x$ for $-90° \leq x \leq 270°$. (3) **L2**

9.56.5 Determine the value(s) of x for which $f(x+30°)=g(x+30°)$. in the interval $-90° \leq x \leq 270°$ (3) **L3**

9.56.6 Consider the geometric series: $1+2\cos 2x+4\cos^2 2x+...$
Refer to the graph of g to determine the value(s) of x in the interval $0° \leq x \leq 90°$ for which this series will converge. (5) **L4**

9.57



A solar panel rotates during the day to follow the sun. The support arm of the panel has a length of 4,8 m as shown in the diagram. The angle of rotation at any time *t* hours after sunrise satisfies the equation:

$\theta=30°+15°\sin\left(\frac{180°t}{6}\right)$, where θ is the angle of rotation.

Determine:

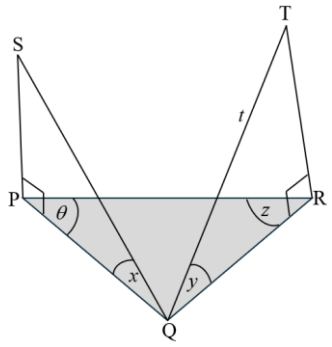
9.57.1 the value of *t* when the panel reaches its maximum height . (3) **L4**

9.57.2 Determine the maximum height reached by the solar panel. (3) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

2D and 3D Trigonometry
GUIDELINES, SUMMARY NOTES, & STRATEGIES: Refer 2025 Last Push Document
STUDY AND MASTER (2000)

9.58

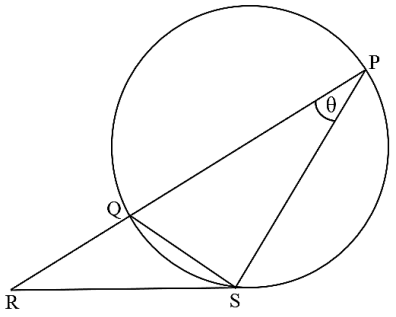


In the diagram P,Q, and R, are three points in the same horizontal plane. PS and RT are the two vertical poles. Wires are strung from Q to the top of the poles. The wire from Q to S forms an angle of $x°$ with the ground. The other wire forms an angle of $y°$ with the horizontal plane and is t metres long. $Q\hat{P}R = \theta$ and $P\hat{R}Q = z$.

Prove that: $PS = \dfrac{t \sin z \tan x \cos y}{\sin \theta}$

(4) **L3**

9.59

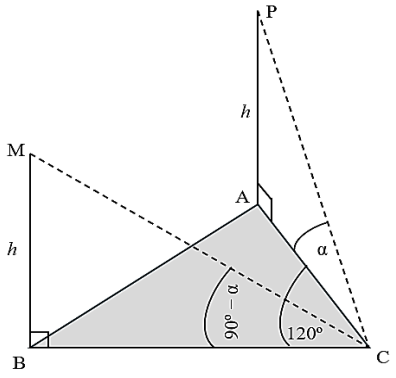


In the diagram PQ is the diameter of the circle. PQR is a straight line and $QR = QS = x,\ Q\hat{P}S = \theta$.

Prove that $RS^2 = 2x^2(1 + \sin\theta)$

(4) **L3**

9.60

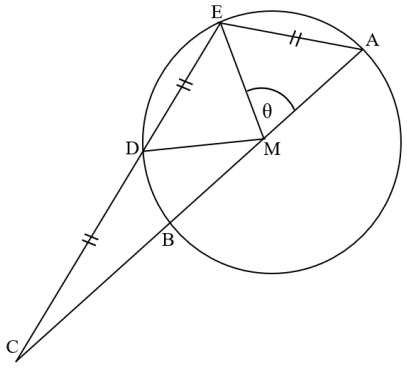


A, B and C are the three points in the same horizontal plane. $A\hat{C}D = 120°$. MB and PA are two vertical towers with equal heights, h metres. From point C the angle of elevation to the tops M and P of the towers are $90° - \alpha$ and α respectively: Prove that

$$AB = h\sqrt{\frac{1}{\cos^2\alpha} + \frac{1}{\tan^2\alpha}}$$

(5) **L3**

9.61



In the diagram AB is the diameter of the circle with centre M and radius $= r$. If $BC = r$, $CD = DE = AE$ and $A\hat{M}E = \theta$:

Prove that $\cos\theta = \dfrac{1}{4}$

(6) **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

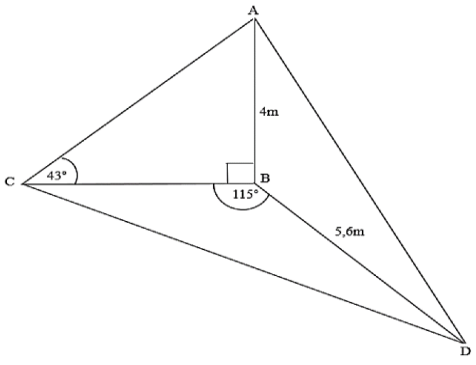
UNDERSTANDING PURE MATHEMATICS

9.62 In $\Delta DEF, E\hat{D}F = \theta$ and $D\hat{E}F = 4\theta$. If EF = x, show that $DF = 4x\cos\theta\cos 2\theta$ (4) **L3**

9.63 The three points A, B and C all lie on horizontal ground and form a triangle right angled at C with $B\hat{A}C = x$. A point V, lies vertically above B and the angle of elevation of V from A is x. If the length of AV is $2x$, show that: $BC = x\sin 2x$ (4) **L3**

FS SEP 2024

9.64



B, C, and D, are points in the same horizontal plane. AB is a vertical pole of length 4 m,
BD = 5.6 m, $C\hat{B}D = 115°$ and the angle of elevation of A from C is $43°$.

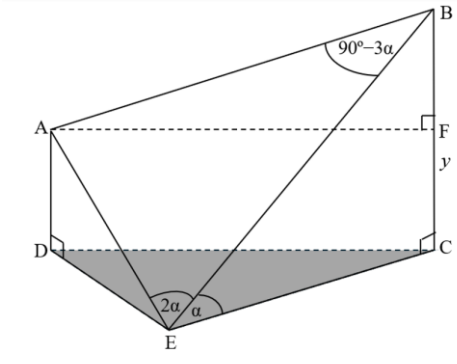
9.64.1 Calculate the length of CB (3) **L2**

9.64.2 Calculate the length of CD (3) **L2**

9.64.3 Determine the area of ΔBCD (3) **L2**

GP SEP 2024

9.65 The diagram alongside shows two vertical poles, AD and BC. Point E lies on the same horizontal plane as bases D and C of poles AD and BC.
$A\hat{E}B = 2\alpha;\ B\hat{E}C = \alpha;\ A\hat{B}E = 90° - 3\alpha$ and $BC = y$ metres



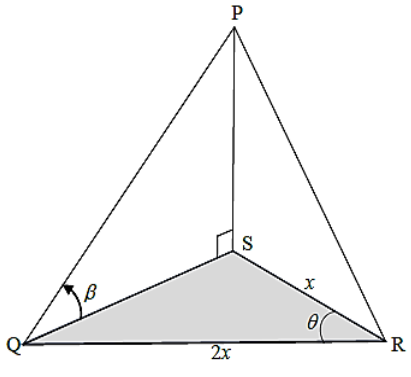
9.65.1 Determine BE in terms of α and y (2) **L1**

9.65.2 Show that $AB = 2y$ (5) **L2**

9.65.3 It is further given that $AF = \frac{7}{4}BC$.
Determine $B\hat{A}F$, the angle of elevation of B from A. Give your answer to the nearest degree. (3) **L2**

NC SEP 2024

9.66 In the diagram, PS is a vertical flagpole. Q, R and S lie in the same horizontal plane. PQ and PR are two cables, anchored at Q and R. $P\hat{Q}S = \beta$ and $Q\hat{R}S = \theta$. SR = x and QR = $2x$.



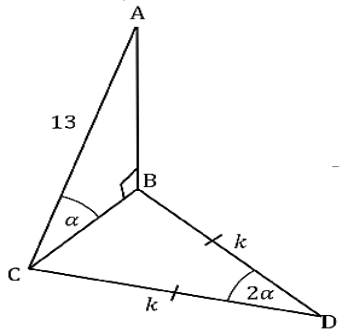
9.66.1 Show that $PQ = \dfrac{x\sqrt{5 - 4\cos\theta}}{\cos\beta}$ (4) **L2**

9.66.2 The area of ΔQRS is $57,36\ m^2$ and $\theta = 35°$. Calculate the value of x. (2) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

WC SEP 2024

9.67 In the diagram below, ABC is a vertical triangular wall on the floor CBD.
CA = 13 meters, CD = BD = k meters, $A\hat{C}B = \alpha$ and $B\hat{D}C = 2\alpha$



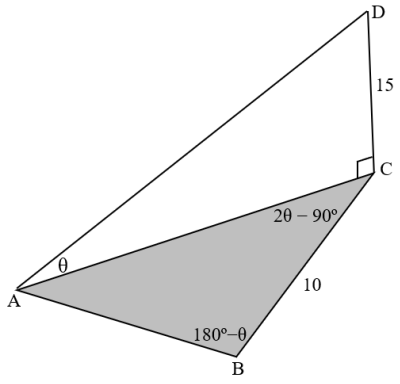
9.67.1 Show that $CB = 13\cos\alpha$ (1) **L1**

9.67.2 Show that $k = \dfrac{13}{2\tan\alpha}$ (4) **L3**

9.67.3 Calculate the area of the floor ΔBCD if $\alpha = 35°$ (2) **L2**

IEB NOV 2024

9.68 In the diagram below: ΔABC is in the horizontal plane, with $BC = 10$ units. $A\hat{C}B = 2\theta - 90°$ and $A\hat{B}C = 180° - \theta$. The angle of inclination from A to D is θ. ΔACD is in the vertical plane, with $CD = 15$ units

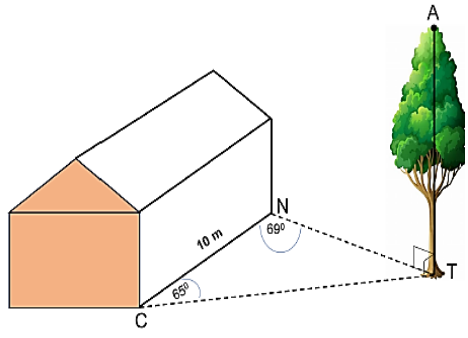


9.68.1 Show that $AC = 10\tan\theta$ (4) **L3**

9.68.2 Hence, determine the size of θ if $\theta \in [0°; 90°]$ (4) **L3**

IEB NOV 2023

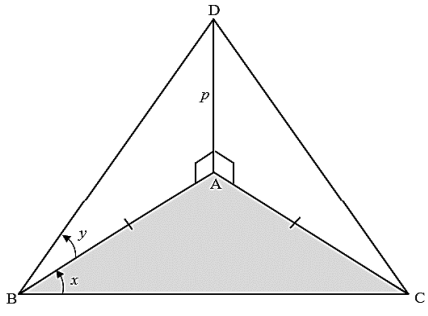
9.69



In the diagram below, a house is in the shape of a prism with a triangular shaped roof with one side CN = 10m. A vertical tree is growing at a point T in the garden. $N\hat{C}T = 65°$ and $C\hat{N}T = 69°$.
The angle of elevation of the top of the tree from C is $43{,}5°$. Is it possible for the tree to hit the house if it falls? Show all working. (8) **L4**

NSC MAY/JUN 2025

9.70



In the diagram, A, B and C lie in the same horizontal plane with AB = AC. D is directly above A such that 2AD = BC. Also, $AD = p$, $A\hat{B}C = x$ and $D\hat{B}A = y$

9.70.1 Show that $\cos x = \tan y$ (4) **L3**

9.70.2 If $x = 60°$, calculate the size of y. (2) **L2**

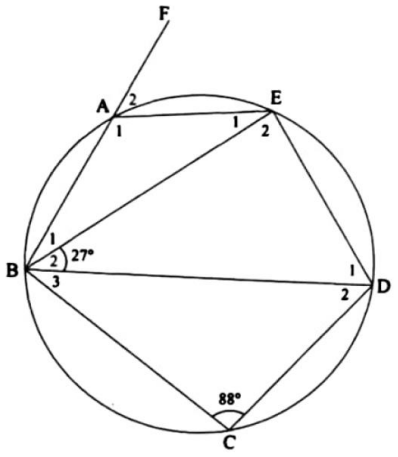
Study copy distributed by EduResource ZA | eduresourcza.com

TOPIC: 10 EUCLIDEAN GEOMETRY [40 ± 3 MARKS]

GUIDELINES, SUMMARY NOTES & STRATEGIES REFER SLAST PUSH 2025 DOCUMENT

NSC JUN 2026

10.1 In the diagram, A, B, C, D and E lie on the circle. BA is produced to F. $\hat{B}_2 = 27°$ and $\hat{C} = 88°$. AE, ED, DC, BC, BE and BD are drawn.



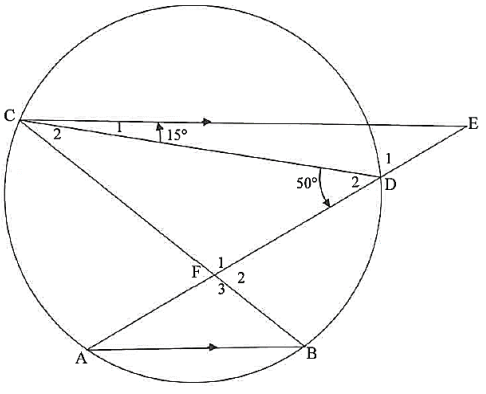
10.1.1 Calculate, with a reason, the size of $\hat{E}_2$ (2) **L1**

10.1.2 If it is given that $ED = DC$, write down, with reasons, the size of $\hat{B}_3$. (2) **L1**

10.1.3 Calculate, with a reason, the size of $\hat{E}_1$. (3) **L2**

ADAPTED NSC NOV 2017

10.2 In the diagram below, points A, B, D and C lie on a circle with E on AD produced. Chords CB and AD intersect at F. $\hat{D}_2 = 50°$ and $\hat{C}_1 = 15°$.



10.2.1 Calculate, with reasons, the size of:

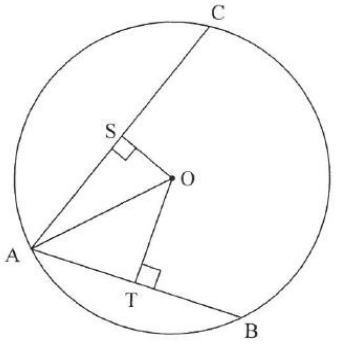
(a) $\hat{A}$ (3) **L2**

(b) $\hat{C}_2$ (2) **L1**

10.2.2 Prove, with reasons, that CF is a tangent to the circle passing through points C, D and E. (2) **L3**

NSC NOV 2016

10.3



A, B and C are points on the circle with centre O. S and T are points on AC and AB respectively such that $OS \perp AC$ and $OT \perp AB$. $AB = 40$ cm and $AC = 48$ cm.

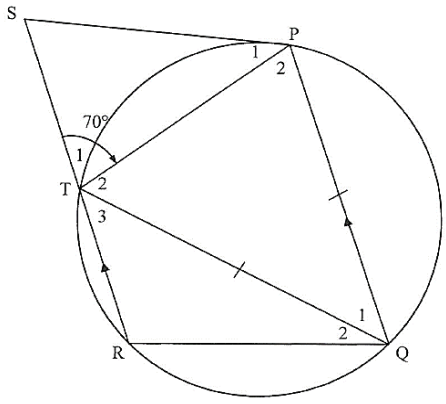
10.3.1 Calculate AT. (1) **L1**

10.3.2 If $OS = \frac{7}{15}OT$, calculate the radius OA of the circle (5) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

ADAPTED NSC NOV 2016

10.4 In the diagram below, PQRT is a cyclic quadrilateral with RT || QP. The tangent at P meets RT produced at S. QP = QT and $P\hat{T}S = 70°$.



10.4.1 Give a reason why $\hat{P}_2 = 70°$ (1) **L1**

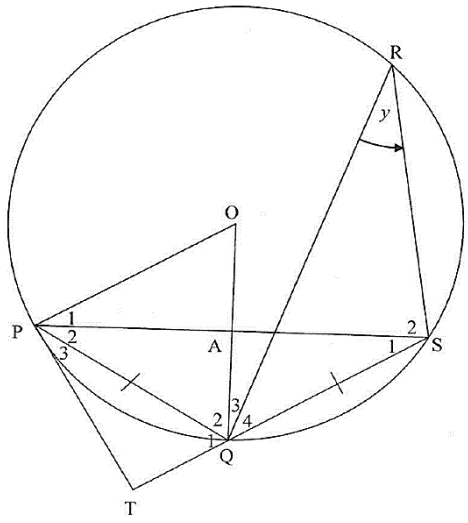
10.4.2 Calculates, with reasons, the size of:

(a) $\hat{Q}_1$ (3) **L1**

(b) $\hat{R}$ (2) **L1**

NSC FEB/MAR 2017

10.5 In the diagram, O is the centre of the circle and P, Q, S and R are points on the circle. PS = QS and $Q\hat{R}S = y$. The tangent at P meets SQ produced at T. OQ intersects PS at A



10.5.1 Give a reason why $\hat{P}_2 = y$. (1) **L1**

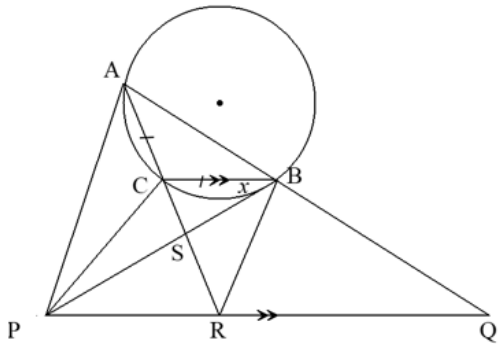
10.5.2 Prove that PQ bisects $T\hat{P}S$. (4) **L2**

10.5.3 Determine $P\hat{O}Q$ in terms of y. (2) **L2**

10.5.4 Prove that PT is a tangent to the circle that passes through points P, O and A. (2) **L3**

10.5.5 Prove that $O\hat{A}P = 90°$ (5) **L2**

10.6



PA and PB are tangents to the circle ABC. AC = CB. CB || PR. ACSR and BSP are straight lines.

If $P\hat{B}C = x$, prove that:

10.6.1 BC bisects $A\hat{B}S$. (5) **L2**

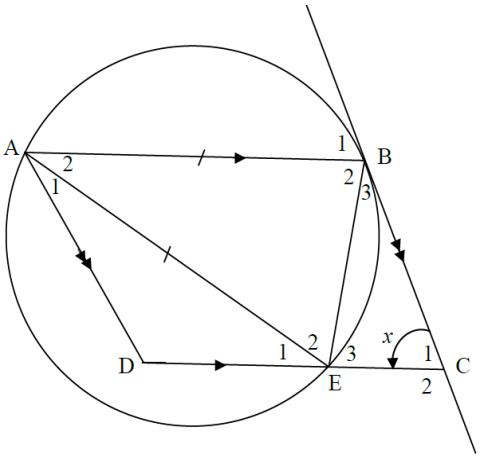
10.6.2 ABRP is a cyclic quadrilateral. (4) **L3**

10.6.3 AP = BQ. (5) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

NSC MAY 2016

10.7 In the diagram below, the circle passes through A, B and E. ABCD is a parallelogram. BC is a tangent to the circle at B. AE = AB. Let $\hat{C}_1 = x$.



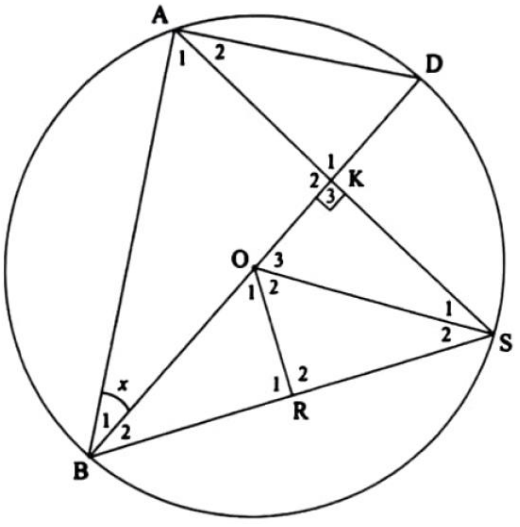
10.7.1 Give a reason why $\hat{B}_1 = x$ (1) **L1**

10.7.2 Name, with reasons, THREE other angles equal in size to x. (6) **L2**

10.7.3 Prove that ABED is a cyclic quadrilateral (3) **L3**

ADAPTED NSC JUN 2026

10.8 In the diagram below, O is the centre of the circle. Chord AS is perpendicular to diameter BOD at K. OR is drawn with R on BS, AB, AD and OS are drawn. $\hat{B}_1 = x$.



10.8.1 Determine, with reasons, the size of the following angles in terms of x:

(a) $\hat{D}$ (3) **L2**

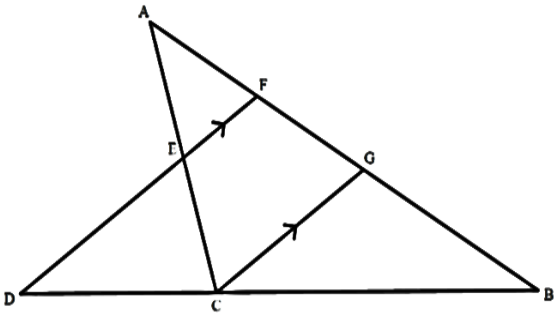
(b) $\hat{B}_2$ (3) **L2**

(c) $\hat{S}$ (2) **L1**

10.8.2 It is further given that BR = BS. Prove that OKSR is a cyclic quadrilateral. (3) **L2**

KZN PRE-SEP 24

10.9



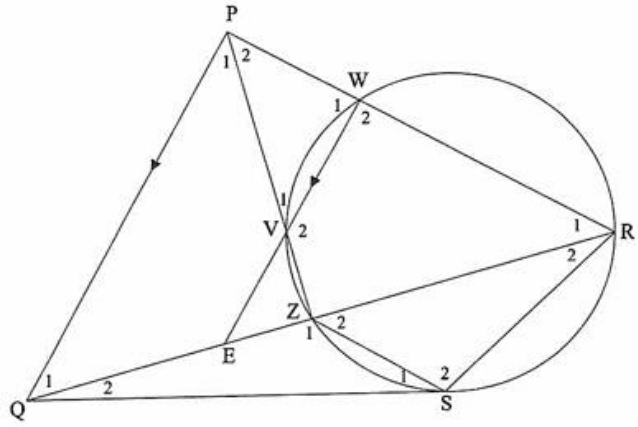
In the diagram alongside, ΔABC is drawn. E and F are points on AC and AB respectively such that $\frac{AE}{EC} = \frac{3}{2}$ and $\frac{AF}{FB} = \frac{2}{5}$. BC produced meet EF produced in D. G is a point on FB such that FD || GC .

Determine with reasons, the ratio $\frac{BC}{CD}$ **(5)** **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

DBE NOV 25

10.10 In the diagram below, WVZR is a cyclic quadrilateral. RZ is produced to Q. A tangent is drawn from Q to touch the circle at S. WV is produced to E, a point on ZQ. RW produced meets ZV produced at P. PQ || WE. RS and ZS are drawn.

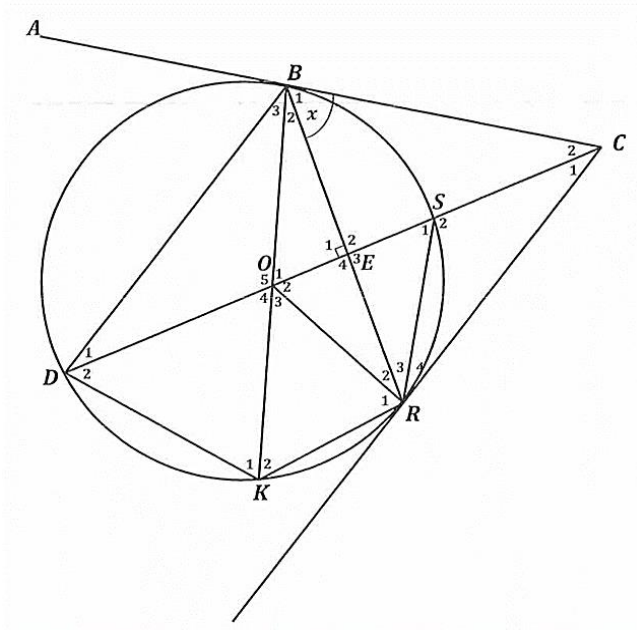


Prove, giving reasons, that:

10.10.1	$PR = \dfrac{PW.QR}{QE}$	(3)	**L2**
10.10.2	$PQ^2 = RQ.QZ$ if $\Delta PQZ \,|||\, \Delta RQP$	(1)	**L1**
10.10.3	$\Delta QSZ \,|||\, \Delta QRS$	(3)	**L2**
10.10.4	$PQ = QS$	(3)	**L3**
10.10.5	$PW = \dfrac{QE.PZ}{\sqrt{QR.QZ}}$	(4)	**L4**

SACS 2018

10.10

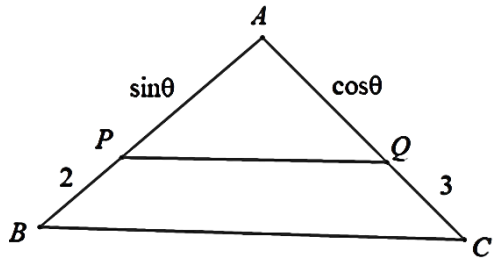


In the diagram alongside AC is a tangent to the circle at B. O is the centre of the circle. D, K, R and S are points on the circumference of the circle. BE ⊥ CD and $\hat{B}_1 = x$.

10.10.1	Prove that DC || KR	(4)	**L2**
10.10.2	Prove that $\hat{B}_3 = \frac{1}{2}x$.	(4)	**L3**
10.10.3	Prove BCRO is a cyclic quadrilateral	(4)	**L3**

ST STITHIANS SEP 21

10.11



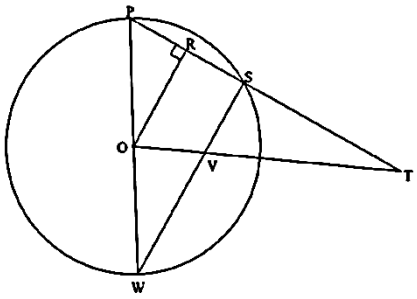
In the triangle alongside, PQ || BC, $AP = \sin\theta$, PB = 2 units, $AQ = \cos\theta$, and QC = 3 units. Determine the values of θ for which the diagram is valid, where $x \in [0° ; 360°]$.

(6) **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

NSC NOV 24

10.12 In the diagram, O is the centre of the circle. is drawn with P, W and S on the circle. PRS is produced to T. SW and OT intersect at V.

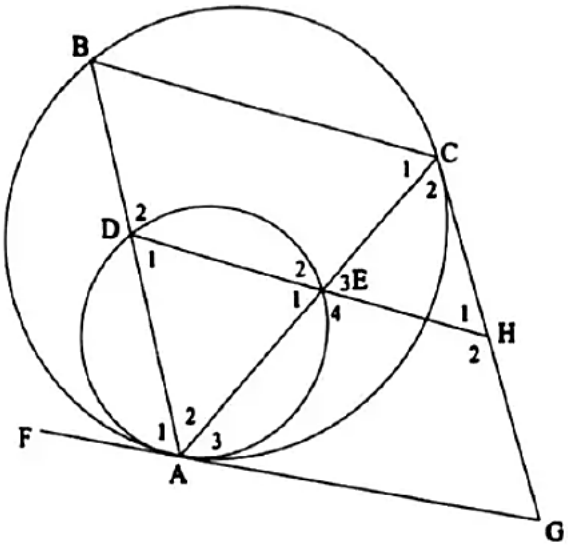


10.12.1 Prove, with reasons, that $OR:WS = 1:2$ (5) **L3**

10.12.2 Calculate the length of PT if $ST = 15$ units. (4) **L2**

DBE MAY/JUN 26

10.13 In the diagram below FA is a common tangent to circle ABC and circle ADE. Chords AB and AC of the larger circle cut the smaller circle at D and E respectively. Another tangent is drawn from C to intersect FA produced at G. DE produced intersects tangent GC at H.



10.13.1 Prove that $\Delta ABC \,|||\, \Delta ADE$ (4) **L1**

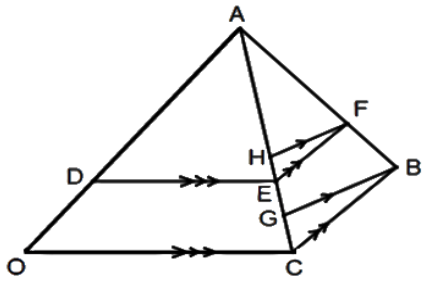
10.13.2 It is further given $AD:AB = 1:3$ and $AE = 4$ cm.
Calculate the length of EC. (4) **L2**

10.13.3 Prove that $\frac{EH}{EC} = \frac{AC}{CB}$ (3) **L2**

10.13.4 Prove that $EC = \sqrt{\frac{EH.CB.DB}{3AD}}$ (5) **L4**

IEB NOV 21

10.14 In the diagram, ΔAOC and ΔABC are given. D lies on OA and E on AC with $DE \parallel OC$. F lies on AB with $EF \parallel CB$. H and G lie on AC with $HF \parallel GB$. AO = 7 units and AD = 4 units. Determine the numerical value of the following:



10.14.1 AF:FB (3) **L2**

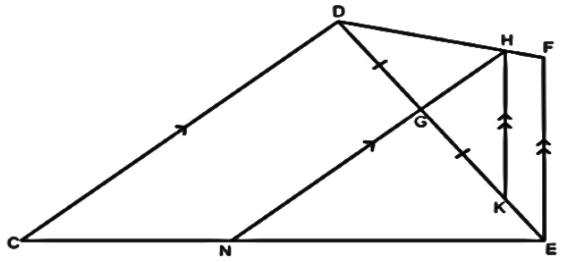
10.14.2 GB:HF (3) **L3**

10.14.3 If it is given that G is the midpoint of EC, determine the numerical value of $AE:EG$. (2) **L2**

Study copy distributed by EduResource ZA | eduresourcza.com

IEB NOV 19

10.15 In the diagram below, CD || NH, HK || FE, DG = GK and CN : CE = 2 : 5. Determine:



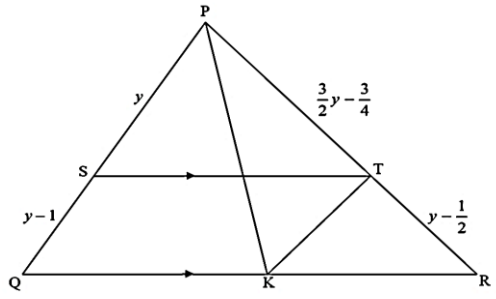
10.15.1 $\dfrac{EK}{KG}$ (3) **L2**

10.15.2 $\dfrac{\text{Area of } \Delta DGH}{\text{Area of } \Delta DEF}$ (4) **L4**

NW SEP 24

10.16 In the diagram below ΔPQR with ST || QR is drawn. K is a point on QR joined to T and P.

PS = y, SQ = $y - 1$, TR = $y - \frac{1}{2}$ and PT = $\frac{3}{2}y - \frac{3}{4}$ units.

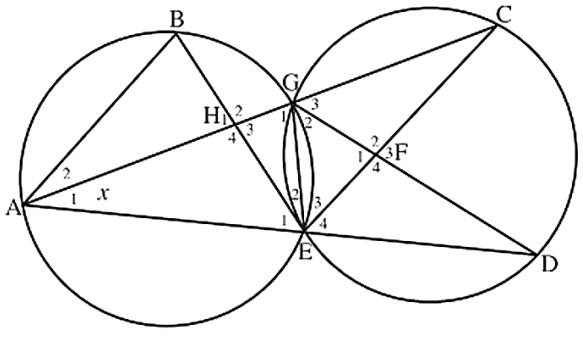


10.16.1 Calculate, with reasons, the value of y. (5) **L2**

10.16.2 If $P\hat{K}T = \hat{Q}$, prove that PSKT is a cyclic quadrilateral. (2) **L2**

MIND ACTION 2022

10.17 In the sketch below, BE is a tangent to the circle CEG at E. AE = EC. $\hat{A}_1 = x$.



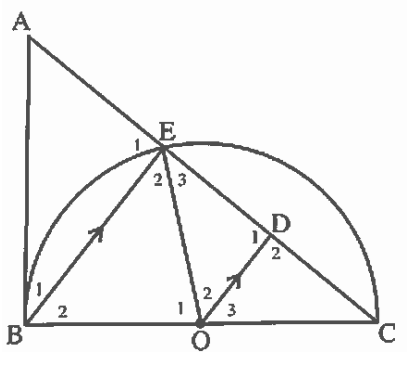
10.17.1 Name, with reasons, four other angles equal to x. (4) **L2**

10.17.2 Prove that AG = GD. (3) **L3**

10.17.3 Prove that $EG^2 = GD.GH$ (4) **L4**

MIND ACTION 2022

10.18



In the sketch below, O is the centre of the semi-circle. EB || BO. AB is a tangent to the circle at B. Prove that:

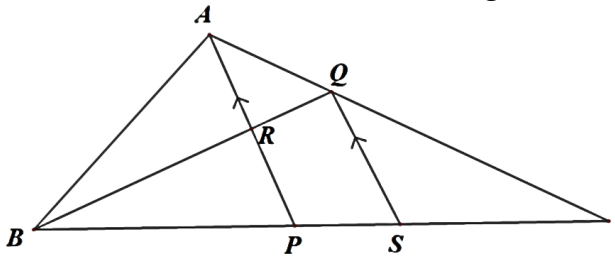
10.18.1 $BC^2 = CE.CA$ (3) **L2**

10.18.2 $CA.DE = 2OE^2$ (4) **L4**

Study copy distributed by EduResource ZA | eduresourcza.com

KZN JUN 22

10.19 In ΔABC below $AQ : QC = 1:3$. $AP \parallel QS$ with P and S on BC and Q on AC. BQ intersects AP in R. Determine, with reasons, the following:



10.19.1 $\dfrac{BP}{PS}$ (2) **L2**

10.19.2 $\dfrac{BR}{QR}$ (2) **L2**

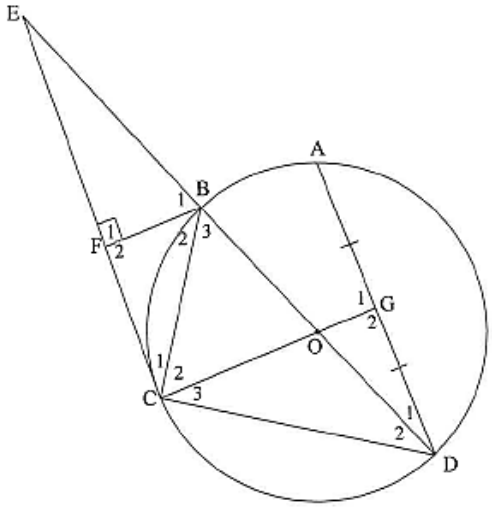
10.19.3 $\dfrac{\text{Area } \Delta APC}{\text{Area } \Delta BQC}$ (3) **L3**

10.19.4 $\dfrac{\text{Area } \Delta ABC}{\text{Area } \Delta APC}$ (3) **L3**

10.19.5 $\dfrac{\text{Area } \Delta QSC}{\text{Area } \Delta ABP}$ (5) **L4**

DBE MAY 2022

10.20



In the diagram below O is the centre of the circle passing A, B, C and D. EC is a tangent to the circle at C. Diameter DB produced meets the tangent EC at E. F is a point on EC such that $BF \perp EC$. Radius CO produced bisects AD at G. BC and CD are drawn. With reasons, prove that:

10.20.1 $FB \parallel CG$ (3) **L2**

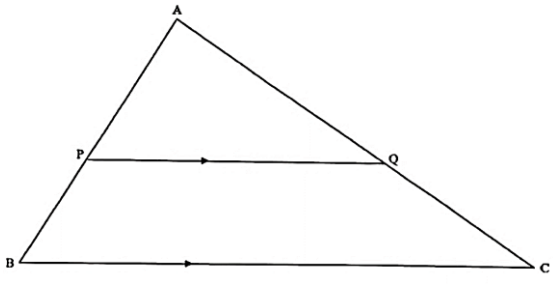
10.20.2 $\Delta FCB \,|||\, \Delta CDB$ (5) **L2**

10.20.3 $CD^2 = CG.DB$ (5) **L3**

10.20.4 $DB = CG + FB$ (5) **L4**

FS SEPT 24

10.21 In the diagram below ΔABC is drawn with $PQ \parallel BC$ and $AP:AB = 4:7$.



Determine $\dfrac{\text{area of } \Delta APQ}{\text{area of } \Delta ABC}$ **(5)** **L3**

Study copy distributed by EduResource ZA | eduresourcza.com

ANSWERS
TOPIC: ALGEBRA & EQUATIONS

1.1 **Solve for x**

1.1.1 $x=0$ and $x=-\frac{1}{2}$

1.1.2 $x=4$ and $x=-3$

1.1.3 $x=\frac{1}{2}$ and $x=-5$

1.1.4 $x=6$ and $x=-1$

1.1.5 $x=0$ and $x=-7$

1.1.6 $x=0$ and $x=6$

1.1.7 $x=6,\ x=-2$ and $x=2$

1.1.8 $x=3$ and $x=-3$

1.1.9 $x=0$ and $x=\frac{1}{2}$

1.1.10 $x=\frac{5}{2},\ x=\sqrt{2}$ and $x=-\sqrt{2}$

1.1.11 $x=0$ and $x=\frac{4}{3}$

1.1.12 $x=-2$ and $x=\frac{7}{3}$

1.1.13 $x=0$ and $x=\pm 2$

1.1.14 $x=\frac{5}{2}$ and $x=-1$

1.2 Solve for x (Correct to TWO decimal places)

1.2.1 $x=0.91$ and $x=-1.31$

1.2.2 $x=3.44$ and $x=-0.44$

1.2.3 $x=1.71$ and $x=0.29$

1.2.4 $x=0.44$ and $x=-3.78$

1.2.5 $x=1.27$ and $x=-2.77$

1.2.6 $x=1.54$ and $x=-0.87$

1.2.7 $x=2.37$ and $x=-0.17$

1.2.8 $x=0.88$ and $x=-0.68$

1.3 **INEQUALITIES**

1.3.1 $x\le -\frac{1}{2}$ or $x\ge 2$

1.3.2 $-1<x<4$

1.3.3 $x<-7$ or $x>5$

1.3.4 $x\le \frac{4}{5}$ or $x\ge 1$

1.3.5 $3^x>0$ for all x values, and $1\le x\le 2$

1.3.6 $x\le -1.5$ or $x\ge 4$

1.3.7 $3<x<7$

1.3.8 $x<-1.67$ or $x>4$

1.3.9 $-\frac{1}{2}<x<4$

1.3.10 $x<3$, and $\left(\frac{1}{2}\right)^x>0$, for x values

1.4 **Simplification of EXPONENTIAL and SURD expressions without using a calculator:**

1.4.1 10

1.4.2 8

1.4.3 $-21\sqrt{2}$

1.4.4 2.7^{1009}

1.4.5 $\frac{21}{5}$

1.4.6 $\frac{2}{3}$

1.4.7 27

1.4.8 3

1.4.9 9

1.4.10 $-1+2\sqrt{3}$

1.5 **Solve the following EXPONENTIAL equations:**

1.5.1 $x=\frac{1}{2}$

1.5.2 $x=3$ and $x=-1$

1.5.3 $x=1$

1.5.4 $x=2$ and $3^x\ne 0$

1.5.5 $x=2$

1.5.6 $x=3$

1.5.7 $x=-2$

1.5.8 $x=3$ and $2^x=-2$ no real solution

1.5.9 $x=2$

1.5.10 $x=1$ and $x=1.46$

1.6 **Solve the following SURD equations:**

1.6.1 $x=1$ and $x\ne -4$

1.6.2 $x=84$

1.6.7 $m=-1$ and $x\ne 4$

1.6.8 $x=3.10$ and $x\ne 5$

Study copy distributed by EduResource ZA | eduresourcza.com

1.6.3 $x = 9$ and $x \neq 2$

1.6.4 $x = 2$ and $x \neq 7$

1.6.5 $x = 4.48$

1.6.6 $x = 6 + 2\sqrt{7}$ and $x \neq 6 - 2\sqrt{6}$

1.6.9 $x = 3$ and $x = 7$

1.6.10 $x = 2$ and $x \neq -2$

1.6.11 $x = 9$ and $x = 16$

1.6.12 $x = 10$ and $x \neq -2$

1.7 **Solve the following equations simultaneously:**

1.7.1 $x = -1$ and $x = 4$
$y = 3$ and $y = -2$

1.7.2 $x = 2$ and $x = -1$
$y = 2$ and $x = 0.5$

1.7.3 $x = 1$ and $x = \frac{2}{3}$
$y = 1$ and $y = \frac{4}{3}$

1.7.4 $x = -9$ and $x = 7$
$x = -6$ and $x = 2$

1.7.5 $x = 10$ and $x = -10$
$y = 7$ and $xy = -3$

1.7.6 $x = 2$ and $x = -1$
$y = 0.5$ and $y = -1$

1.7.7 $x = 4$ and $x = 1$
$y = 1$ and $y = -5$

1.7.8 $x = 1$ and $x = \frac{11}{7}$
$y = 0$ and $y = \frac{16}{7}$

1.7.9 $x = -\frac{1}{6}$ and $y = -\frac{1}{2}$

1.7.10 $x = 1$ and $x = 0.2$
$y = 1$ and $y = -1.4$

1.7.11 $x = 5$ and $y = 15$

1.7.12 $x = 6$ and $x = -1$
$y = 3$ and $y = -0.5$

1.8 **Mathematical modelling**

1.8.1 8

1.8.2 -17

1.8.3 $y \leq -2$ or $y \geq 2$

1.8.4 $k = 14$

1.8.5 11

1.8.6 12

1.8.7 12

1.8.8 *proof*

1.8.9 $ED = 6cm$

1.8.10 $x = 6.63cm$

1.9 **Nature of roots**

1.9.1 $Non-real$

1.9.2 $p = \pm 3$

1.9.3 $1 < m < 4$

1.9.4 *proof*

1.9.5 *proof*

1.9.6 (a) $x = 3$
(b) $\frac{x+2}{3}$

1.9.7 $r > 9$

1.9.8 *non*-real

1.9.9 (a) $x = -4$ and $x = 0$
(b) $x < -4$

1.9.10 (a) *proof*
(b) $\frac{3}{2}$

TOPIC: SEQUENCES AND SERIES

2.1

2.1.1 $T_n = n^2 + 4n$

2.1.2 725

2.1.3 $n = 32$

2.1.4 $T_{72} - T_{71} = 147$

2.2

Study copy distributed by EduResource ZA | eduresourcza.com

2.2.1 $x=-2$
2.2.2 proof
2.2.3 $n=24$
2.3 2.3.1 6
2.3.2 $T_n=3n^2-2n+5$
2.4
2.4.1 $T_{150}=602$
2.4.2 proof
2.4.3 25602
2.5 2.5.1 $T_n=185-4n$
2.5.2 $n=46$
2.5.3 4171
2.5.4 $\sum_{n=1}^{43}(185-4n)$
2.6
2.6.1 30 terms
2.6.2 $n=54$
2.7
2.7.1 $T_n=x\left(\frac{x}{2}\right)^{n-1}$
2.7.2 $-2<x<2$
2.7.3 $\frac{2}{3}$
2.8
2.8.1 12, 16, 20
2.8.2 $T_n=4n-8$
2.8.3 $n=16$
2.8.4 $T_{12}=40$
2.8.5 $T_n=2(2n+4)$, when you multiply any number by 2 the answer will always be even.
2.9
2.9.1 The series will converge
2.9.2 $S_7=190,5$
2.9.3 $S_\infty=192$
2.9.4 1,5
2.10 3069
2.11
2.11.1 1620
2.11.2 77
2.12
2.12.1 $x=6$
2.12.2 $T_7=63$
2.12.3 T_{28} and T_{29}
2.13 $n=10$
2.14 $p=\frac{197}{926}$
2.15
2.15.1 $r=2$
2.15.2 No, r is not between -1 and 1
2.15.3 $n=10$
2.15.4 4603,5
2.16
2.16.1 $T_2=21$ and $T_3=24$
2.16.2 1638
2.16.3 840
2.17
2.17.1 Time on clock B will read 09:05
2.17.2 Clock B will never reach the time 09:06 because the sum to infinity is 5
2.18 $a=1\,;b=2$ and $c=-4$
2.19
2.19.1 $\frac{15}{16}$ / 0,9375
2.19.2 $\frac{23211}{4096}$ / 5,67
2.20
2.20.1 $T_n=3(2)^{n-1}$
2.20.2 $n=9$
2.21
2.21.1 proof
2.21.2 $T_{10}=13\,;T_{11}=30$
2.21.3 $T_{500}=8343$
2.22 $n=10$
2.23
2.23.1 -2
2.23.2 $19\,terms$
2.23.3 -555
2.23.4 0
2.24
2.24.1 $8x+4$
2.24.2 $T_n=(5x-2)+(n-1)(n+1)$
2.24.3 $x=\frac{62}{19}$
2.55 $a=2\,;c=6$
2.26
2.26.1 $T_9=\frac{1}{9}$
2.26.2 $m=8$
2.27 $2^{nd}=\frac{1}{3}\,;3^{rd}=\frac{1}{9}$
2.28 $T_n=n^2+3n+3$
2.29 $n=7$
2.30 $S_n=\frac{2^n-1}{2}$
2.31
2.31.1 proof
2.32.2 $r=\frac{1}{3}\,;a=\frac{3}{2}$
2.32
2.32.1 57
2.32.2 435
2.32.3 proof

Study copy distributed by EduResource ZA | eduresourcza.com

2.33

2.33.1 $6p$

2.33.2 $6\sqrt{3}p$

2.33.3 $48\sqrt{3}p^2$

TOPIC: FUNCTIONS & INVERSES

3.1.1 $x = 2$ $y = 2$

3.1.2 $x = 3$

3.1.3 Graph

3.1.4 $y = -x + 4$

3.1.5 $y \in R; y \neq -1$

3.1.6 $x \in (2;3]$ **OR** $2 < x \leq 3$

3.2.1 y – intercept of f: (0 ; –3) & g: (0 ; 3)

3.2.2 Graph

3.2.3 $-4 < k < -3$

3.2.4 $x \geq 1$

3.3.1 (a) $x = 5$

3.3.1 (b) $x = 3$ or $x = 7$

3.3.1 (c) Graph

3.3.1 (d) $y \in (-4;\infty]$ **OR** $y \geq -4; y \in \mathbb{R}$

3.3.1 (e) $y = -(x-5)^2 + 4$

3.3.2 $x = \pm 4$

3.4.1 Graph

3.4.2 Distance = $\sqrt{2q}$

3.5 Graph

3.6.1 (a) $f^{-1}(x) = -\log_3 x$ **OR** $f^{-1}(x) = \log_{\frac{1}{3}} x$

3.6.1 (b) Graph

3.6.2 $p^{-1}(x) = -\frac{1}{2}x + 8$

3.6.1 $y = -\log_5 x$ **OR** $y = \log_{\frac{1}{5}} x$

3.6.2 Graph

3.6.3 $x > 0; x \in \mathbb{R}$

3.6.4 $0 < x \leq 1; x \in \mathbb{R}$

3.6.5 $y < -3, y \in \mathbb{R}$

3.8.1 $y = \log_8 x$

3.8.2 Proof

3.8.3 Graph

3.9.1 (a) $x = -\log_a + k$

3.9.1 (b) Graph

3.9.2 (a) Yes, each input has exactly one output.

3.9.2 (b) {2; –1; 7; 1; 6}

3.9.2 (c) $p = 2$

3.10.1 $k = 3$

3.10.2 $x = 0$ $y = -1$

3.10.3 $g^{-1}(x) = \log_3 x$

3.10.4 $x = -2$

3.10.5 Graph

3.10.6 $x = \pm\sqrt{2}$

3.10.7 $-2 < x \leq 1$

3.11.1 M(3 ; 4)

3.11.2 $D\left(0;\frac{8}{3}\right)$

3.11.3 $t = 1$

3.11.4 $2 \leq x < 3$

3.11.5 A(5 ; 6)

3.11.6 $AA^{/} = 10$ units

3.12.1 Proof

3.12.2 $k = -2$

3.13.1 $B(-6;0)$ & $D(4;0)$

3.13.2 $c = 8$ & $m = -2$

3.13.3 $A^{/}(24;-8)$

3.13.4 $k = 36$

3.14.1 $A(-1;-8)$, $B(-3;0)$ & $C(1;0)$

3.14.2 Proof

3.14.3 $(2;-10)$

3.14.4 $k = -8$

3.14.5 $y \in (-\infty;8]$ **OR** $y \leq 8; y \in \mathbb{R}$

3.14.6 OT = 4,46 units

3.15.1 $E(-5;0)$ & $P(1;0)$

3.15.2 $m = 1$ & $c = 5$

3.15.3 NT = 6 units

3.15.4 $S(-4;5)$

3.16.1 $x_A = -4$ & $x_B = 3$

3.16.2 $y \leq \frac{49}{4}; y \in \mathbb{R}$

3.16.3 1,32

3.16.4 $x < -4$ or $0 < x < 3$

3.16.5 $12 < k < \frac{49}{4}$

3.16.6 $h(x) = -\left(x - \frac{5}{2}\right)^2 + \frac{49}{4}$

3.17.1 $K(3;2)$

3.17.2 FE = $\frac{29}{3}$

3.17.3 $a = -1$ & $b = 6$

3.17.4 $y \in \mathbb{R}; y \neq 5$

3.17.5 $-7 < p < 2$ **OR** $p \in (-7;2)$

3.17.6 $y = -x + 5$

3.17.7 Reflection about the x –axis then shifted 2units up **OR**
Shifted 2 units down then reflected about the x – axis.

3.17.8 $x = 2$ or $x = 4$

3.18.1 Proof

Study copy distributed by EduResource ZA | eduresourcza.com

3.18.2 $(0;4)$

3.18.3 $\left(-1;\frac{9}{2}\right)$

3.18.4 $h(x)=-x+2$

3.18.5 $g(x)=\frac{1}{x+1}+3$

3.18.6 $\left(-\frac{4}{3};0\right)$

3.18.7 $x\in\left[-\frac{4}{3};-1\right)\cup\left(-1;\infty\right)$ **OR**

$x\in\left[-\frac{4}{3};\infty\right);x\neq-1$

3.18.8 $k(x)=\frac{-1}{x-9}+3$

3.18.9 $-\frac{9}{2}\leq p<-4$

3.19.1 C $(0;3)$

3.19.2 B $(2;1)$

3.19.3 $a=-4,\ r=-2\ \&\ t=1$

3.19.4 $x=-1$ and $y=0$

3.19.5 Proof

3.19.6 $-7<k<-5$

3.19.7 $-8<d<2$

3.20.1 A $(1;0)$

3.20.2 Proof

3.20.3 $g(x)=\frac{9}{x+2}-3$

3.20.4 Proof

3.20.5 $x=4,46$ or $x=-2,46$

3.20.6 $x<-2,46$ or $1<x<4,46$

3.20.7 Max. value = 1

3.20.8 $4,32<m<12,25$

3.21.1 $p=2\quad q=-1$

3.21.2 $g(x)=-\frac{4}{x+2}-1$

3.21.3 $y=-x-3$

3.21.4 $y=-4$

3.21.5 $f(x)=3^x-4$

3.21.6 $h(x)=3^x$

3.21.7 $y=h^{-1}(x)=\log_3 x$

3.21.8 $x>-2$

3.22.1 Proof

3.22.2 $k<-4$

3.22.3 Graph

3.23.1 $a=-\frac{2}{9}$

3.23.2 $y=-\sqrt{-\frac{9}{2}x}$

3.23.3 $y\leq 0$

3.23.4 Graph

3.23.5 $y=\sqrt{-\frac{9}{2}x}$

3.24.1 $x=\frac{3}{2}$

3.24.2 $\frac{3}{2}<x\leq 3$

3.24.3 $y=-\log_2 x$ **OR** $y=\log_{\frac{1}{2}} x$

3.24.4 $-4\leq y\leq 3$

3.24.5 $(1\ ;\ 1)$

3.25.1 $a=\frac{1}{4}$

3.25.2 $y=\sqrt{4x}$

3.25.3 $x=6$

3.26.1 $q=2$

3.26.2 $y=\log_2 x$

3.26.3(a) $0<x\leq 1$

3.26.3(b) $x>6$

3.26.4 $h(x)=-2^{x+3}+2$

3.27.1 Proof

3.27.2 $f^{-1}(x)=\log_3 x$

3.27.3 Graph

3.27.4 21 522 960,44

3.28.1 D $(1;0)$

3.28.2 $b=\frac{1}{4}$

3.28.3 $m=3$

3.28.4 $x>0$ **OR** $x\in(0;\infty)$

3.28.5 $y=\left(\frac{1}{4}\right)^x$ **OR** $y=4^{-x}$

3.28.6 $-2\leq x\leq 1$

3.28.7 Graph

3.29.1 $y=0$

3.29.2 $p=-1$

3.29.3 $y=-x^2;\ x\leq 0$

3.29.4 $h(x)=-\sqrt{x}$

3.30 $(16;-576)$

3.31.1 $y=(x-1)(x-5)$

3.31.2* Proof

3.31.3* Use $n=1\ \ldots\ n=4$ to find a sequence of turning points.

$-16\ ;\ -9\ ;\ -4\ ;\ -1$

$y=-x^2+10x-25=-(x-5)^2$

TOPIC:FINANCIAL MATHEMATICS

Study copy distributed by EduResource ZA | eduresourcza.com

4.1 R366 868,15
4.2 4.2.1 R9908,90
4.2.2 R1352593,39
4.3 16,08%
4.4 73 withdrawals
4.5 R30 679,83
4.6 6,36%
4.7.1 R111 323,77
4.7.2 (a) 16,75 years
(b) R2 232 000
4.8 4.8.1 7,34% p.a.
4.8.2 8 years
4.9 4.9.1 R736 498,33
4.9.2 R11 273,50
4.10 75
4.11 4.11.1 R1 451 961,83
4.11.2 R49 261,76
4.11.3 R51 181,12
4.12 4.12.1 R96 000
4.12.2 n=5 (Beginning of 2020)
4.12.3 R9 383,26
4.13 6,5%
4.14.1 R 5 536,95
4.14.2 R23 739,60
4.15 4.15.1 39 payments
4.15.2 R34 865,59
4.15.3 R37 131,85
4.16 4.16.1 R151 297,64
4.16.2 16,26%
4.17 4.17.1 R74 883,86
4.17.2 R168 306,21
4.17.3 R1 184,68
4.18 66 or 67 months
4.19 8,42 years
4.20.4 x=R316 000
4.21 4.21.1 R450 000
4.21.2 14,21% p.a.
4.21.3 R614 490,66
4.21.4 R9397,11
4.22 4.22.1 Proof
4.22.2 3 years
4.22.3 R7 409,40
4.22.4 It would be sufficient
4.23 52%
4.24 Option 1
4.25 R324 680,86
4.26 R439 567,94
4.27 R53 387,19
4.28 4.28.1 220 withdrawals
4.28.2 Infinite number of withdrawals
4.29 4.29.1 30,48%
4.29.2 R419 870,37
4.29.3 R110 954,43

TOPIC: DIFFERENTIAL CALCULUS

5.1
5.1.1 $4x$
5.1.2 $4x$
5.1.3 $2x-3$
5.1.4 $-10x$
5.1.5 $\frac{1}{x^2}$
5.1.6 $2x-1$
5.1.7 0
5.1.8 $\frac{4}{x^2}$
5.1.9 $3x^2$ and 12
5.1.10 $2x-b$

5.2
5.1.1 $12x-3$
5.1.2 $\frac{dy}{dx}=12x^3+7x^{-2}+\frac{4}{3}x^{\frac{-1}{3}}$
5.1.3 $=10x^4+\frac{12}{x^4}$
5.1.4 $\frac{dy}{dx}=1$
5.1.5 $\frac{dy}{dx}=14x+3x^{\frac{-8}{5}}$
5.1.6 $\frac{dy}{dx}=2x+3;\quad x\neq 3$
5.1.7 $\frac{dy}{dx}=\frac{7}{4}x^{\frac{3}{4}}-4x^3$
5.1.8 $=16x-4$
5.1.9 $\frac{dy}{dx}=2x+5$
5.1.10 $f'(x)=\frac{1}{2}x^{\frac{-1}{2}}-3$
5.1.11 $f'(x)=10x-7$
5.1.12 $f'(x)=\frac{4}{3}x^{\frac{-1}{3}}-\frac{1}{3}x^{\frac{-4}{3}}$
5.1.13 $f'(x)=14x-3x^{\frac{-8}{5}}$
5.1.14 $\frac{dy}{dx}=2x+3$
5.1.15 $f'(x)=\frac{5}{2}x^{\frac{-3}{2}}-5x^4$
5.1.16 $f'(x)=10x^{-3}+4\pi$

5.3
5.3.1 a. substitute x and get y
b. $p=-3\quad q=5$
5.3.2 $a=2\quad b=0\quad c=-24$
5.3.3 $y=-8x-9$
5.3.4 $c=6\quad b=\frac{3}{2}$
5.3.5 a. $x=\frac{3}{2}$
b.
5.3.6 $a=2\qquad b=256$
5.3.7 a. substitute x and solve for y

Study copy distributed by EduResource ZA | eduresourcza.com

b. $p = 3$ $q = 5$

5.4

5.4.1 a. substitution of -1 and getting to 0
b. $x = -1$ or $x = -1$ or $x = 5$
c. $P(-1;0)$ Q(3 ; -32)
d. graph
e. $x < -1$ or $x > 3$
ii $x > 1$
iii $-1 < x < 3$

5.4.2 $x < \frac{1}{3}$

5.4.3 a. $\left(\frac{4}{3};\frac{4}{27}\right)$
b. graph
c. $x < \frac{4}{3}$ $x > 2$
ii) $x < 0$ $\frac{4}{3} < x < 2$

5.4.4 a. $x = 0$ or $x = 1$
b. $(1;0)$ $\left(\frac{1}{3};\frac{4}{27}\right)$
c. graph
d. i) $x \in \left(\frac{1}{3};\ 1\right)$
ii) $x \in \left(\infty;\ \frac{2}{3}\right)$
e 21

5.4.6 a. $a = 7;\ b = 0;\ c = -36$
b. $\left(\frac{14}{3};\ \frac{400}{27}\right)$
c. $x = 2.33$
d. $x < \frac{14}{3};\ x \neq 0$
e. $k > \frac{292}{27}$ or $k < -40$

5.5

5.5.1 $D(0\ ;\ -12)$
5.5.2 A(–2 ; 0) B(1 ; 0) C(6 ; 0)
5.5.3 $E\left(\frac{-2}{3};\frac{400}{27}\right)$
5.5.4 graph
5.5.5 $x < \frac{5}{3}$
5.5.6 $\frac{-2}{3} < x < 4$
5.5.7 $x \geq 6$
5.5.8 $-36 < k < \frac{400}{27}$
5.5.9 $y = -17x + 113$

5.6 Graph

5.7

5.7.1 $C\left(\frac{7}{3};\frac{64}{27}\right)$
5.7.2 $1 < x < \frac{7}{3}$
5.7.3 $x < \frac{5}{3}$
5.7.4 $0 < k < \frac{64}{27}$

5.8

5.8.1 0
5.8.2 $x = 3$
5.8.3 $x = \frac{5}{3}$
5.8.4 $x > \frac{1}{3}$
5.8.5 $x < -1$ or $\frac{5}{3} < x < 3$

5.9 graph

5.10

5.10.1 $x = 2$
5.10.2 $1 < x < 2$
5.10.3 $x = 0$ $\therefore m = 6$
5.10.4 $x = 3$
5.10.5 Concave up

5.11

5.11.1 $8300m$
5.11.2 $t = 4s$
5.11.3 $t = 1s$

5.12

5.12.1 4m
5.12.2 28.2m
5.12.3 $10m/s^2$

5.13

5.13.1 500g
5.13.2 8%
5.13.3 $\approx 7.7g$
5.13.4 75 years

5.14

5.14.1 $h = \frac{340}{\pi r^2}$
5.14.3 $r \approx 3.78cm$

5.15

5.15.1 $V(x) = 60x^2 - 2x$
5.15.2 $x = 20$
5.15.3 8000

5.16

5.16.1 $P\left(\frac{a}{2};\frac{b}{2}\right)$ which is the midpoint of MN

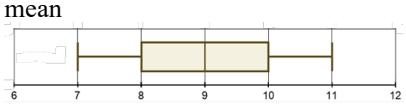
TOPIC: PROBABILITY

6.1 6.1.1
a) $k = 0{,}3$

Study copy distributed by EduResource ZA | eduresourcza.com

	b)	$k = 0,5$
	6.1.2	$x = 0,15$ and $y = 0,45$
	6.1.3	a) diagram
		b) independent
6.2		
	6.2.1	a) 11
		b) diagram
		c)proof
		d) i) 0,20
		ii) 0,475
6.3		
	6.3.1	7 options
	6.3.2	210
6.4		16 balls
6.5	6.5.1	486 720
	6.5.2	Not enough
6.6		0,48
6.7	6.7.1	483 840
	6.7.2	0,25
6.8	6.8.1	a) 0,25
		b) 5/6
	6.8.2	a) diagram
		b) 0,6315
	6.8.3	a) 3 628 800
		b) 4/45
6.9		0,7875
6.10	6.10.1	a) proof
		b) 0,10
		c) 0,20
		d) 0,18
	6.10.2	a) 100 000 000
		b) 0,1008
6.11	6.11.1	0,5
	6.11.2	Diagram
	6.11.3	0,7
	6.11.4	0,4
6.12	6.12.1	13 621 608 000
	6.12.2	27 216 000
	6.12.3	13 232 419 200
	6.12.4	359 251 200
	6.12.5	1 816 214 400
	6.12.6	4 796
6.13	6.13.1	0,4
	6.13.2	0,30
	6.13.3	No independent
6.14		0,1
6.15	6.15.1	120
	6.15.2	0,04
	6.15.3	48
6.16	6.16.1	a) $a = 0.10$
		b) $m = 0,30$
		c) 6
	6.16.2	a) 648
		b) 144
6.17	6.17.1	Yes
	6.17.2	Not independent
6.18		6
6.19	6.19.1	a) diagram
		b) 0,55
	6.19.2	a) there are mutually exclusive
		b) 1/10
		c) ¼
	6.19.3	a) 720
		b) 5/7
6.20	6.20.1	362 880
	6.20.2	8 640
	6.20.3	11/12
	6.20.4	5/12
	6.20.5	2/3
	6.20.6	720
6.21	6.21.1	0,35
	6.21.2	$a = 12$
6.22	6.22.1	No
	6.22.2	Proof
	6.22.3	1200
	6.22.4	336
6.23		483 840

TOPIC: STATISTICS

7.1.1	$\overline{X} = 64.9$
7.1.2	$\sigma_X = 23,86$
7.1.3	6 players
7.2	Skewed to the left because $\overline{x} < Q_2$
7.3	3 players
7.4.1	88
7.4.2	52
7.4.3	26
7.4.4	Science, the median and the minimum in Science is higher than Mathematics
7.4.5	Mathematics. Range is 88% and the IQR IS 40 compared to Science with a range of 58% and IQR of 26
7.4.6	25% or 0,25
7.5	27%
7.6	$p = 3,3$
	$k = 9$
7.7.1(a)	$\overline{x} < Q_2$
(b)	$\sigma = 1,14$
	$(7,59; 9,87)$
	7 players fall within one std of the mean
7.7.2	
7.8.1	15 students
7.8.2	$27 - 1,5 \times 6$
	$= 18$
	It is not an outlier since 24>18
7.8.3	No effect on the median

Study copy distributed by EduResource ZA | eduresourcza.com

7.9 $a = 3$

$b = 3$

$c = 11$

$d = 15$

7.10 Data is skewed left

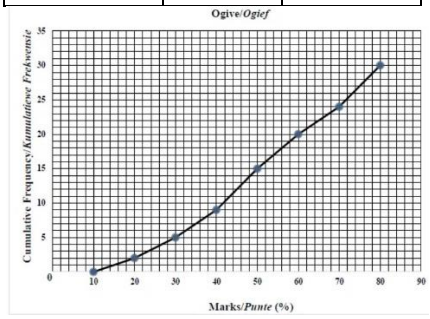
$\overline{x} < Q_2$

7.11 Refer to 2025 last push revision document

7.12.1

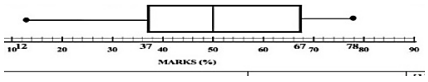
Class interval (Marks in %)	Frequency	Cumulative Frequency
$10 < x \le 20$	2	2
$20 < x \le 30$	3	5
$30 < x \le 40$	4	9
$40 < x \le 50$	6	15
$50 < x \le 60$	5	20
$60 < x \le 70$	4	24
$70 < x \le 80$	6	30
TOTAL	30	

7.12.2



7.12.3 12 -14

7.12.4

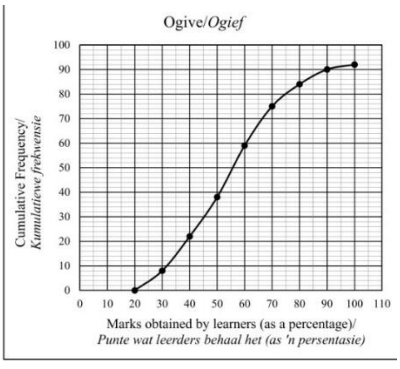


7.13.1 $50 < x \le 60$

7.13.2

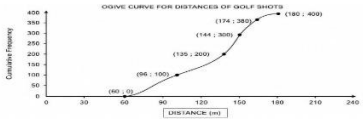
CLASS INTERVALS (AS %)	FREQUENCY	CUMULATIVE FREQUENCY
$20 < x \le 30$	8	8
$30 < x \le 40$	14	22
$40 < x \le 50$	16	38
$50 < x \le 60$	21	59
$60 < x \le 70$	16	75
$70 < x \le 80$	9	84
$80 < x \le 90$	6	90
$90 < x \le 100$	2	92

7.13.3



7.13.4 Median = 54

7.14.1

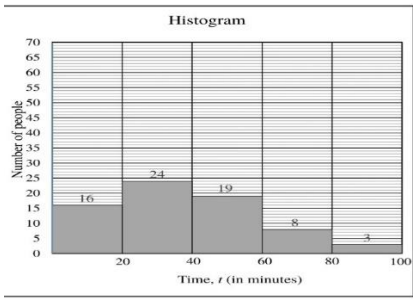


7.14.2 No, the distance is close to 120 not 140.

7.14.3 Since the mean < median, the data is negatively skewed

7.15 Original Q_2 – New $Q_2 = 62 - 60 = 2$

7.16.3



7.16.4 Skewed to the right or positively skewed

7.17.1 37

7.17.2 $30 < x \le 40$

7.17.3 Symmetrical

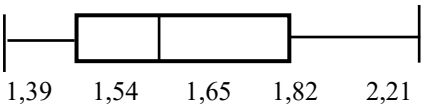
7.17.4 Table

7.17.5 **Graph**

7.17.6 6 learners

7.18.1 Graph

7.18.2



7.18.3 Skewed positively/to the right

7.19 Table & Graph

7.20.1 $y = 171,983 - 0,890x$

7.20.2 $y \approx 127,5$ bpm

7.20.3 Extrapolation – 50 is outside of the given range

There is a weak correlation of –0,2

7.21.1 96%

7.21.2 31%

7.21.3 63%

(a)

(b) Graph

7.22.1 14 cups

7.22.2 $r = -0,9821$

7.22.3

(a) $x = 23,36$

(b) $\hat{y} = -10,93$

(c) Extrapolation OR You can't sell a negative number of cups

7.22.4 Graph

(a)

(b) a will decrease

(c) r will increase OR r will become less negative OR r will not be as strong

Study copy distributed by EduResource ZA | eduresourcza.com

7.23.1 $y = 39,47 + 1,44x$

7.23.2 $y = 255,47$

7.23.3 3 players

7.23.4 This player scored many more runs than the other players who faced a similar number of balls

7.24.1 Yes it will strengthen the correlation coefficient

7.24.2 Closer to negative one as the gradient is negative or the relationship is indirect

7.24.3 No, the correlation is not equal to negative 1

7.24.4 He is saying that the value for x is outside of the lowest and highest values used to find the line of best fit, so he cannot be sure if it is accurate

7.25.1 $y = 333197,20 - 22988,32x$

7.25.2 $= 216455,61$

7.25.3 The strong correlation ($r = -0,95$) suggest that the data points lie close to the regression line. Therefore, the prediction will be valid

7.25.4 The average decrease per year is R22 988,32.

7.26.1 $y = 12,60 + 17,33x$

7.26.2 $x = 11$

7.26.3 $r = -0,95$

7.26.4 As the number of customers increases, the amount of money spent also increases

7.26.5 No, the correlation shows a positive strong correlation between the money spent and the number of customers.

7.26.6 The gradient will decrease as the line will become less steep. The point is far away from the line. Or the point is above the data.

TOPIC: ANALYTICAL GEOMETRY

8.1

8.1.1 $m_{CD} = \frac{1}{2}$

8.1.2 $y = \frac{1}{2}x + 1$

8.1.3 $\alpha = 26,57°$

8.1.4 $BE = \sqrt{5}$ units

8.1.6 2 units left & 1 unit down

8.1.7 $\left(-2; -\frac{5}{2}\right)$

8.1.8 1,25 units^2

8.2

8.2.2 $PR = 6\sqrt{2}$ units

8.2.3 $m_{PR} = -1$

8.2.4 $y = -x - 3$

8.2.5 It does not lie on the line

8.2.6 $y = x - 7$

8.2.7 $M(8;-1)$

8.2.8 $P\hat{R}Q = 45°$

8.3

8.3.1 $m_{MT} = \frac{2}{3}$

8.3.2 $y = \frac{2}{3}x + \frac{37}{6}$

8.3.3 $k = 5$ and $h = 3$

8.3.4 $AM = 2\sqrt{13}$ units

8.3.5 $R\left(5; -\frac{7}{2}\right)$

8.3.6 $P\hat{T}A = 90°$

8.3.8 9,75 units^2

8.4

8.4.1 $M(3;0)$

8.4.2 $QM = 11$ units

8.4.3 $m_{PQ} = \frac{11-a}{3}$

8.4.5 $y = x + 8$

8.4.7 $\theta = 71,57°$

8.4.8 $MN = 8,7$ units

8.5

8.5.1 $m_{OB} = -3$

8.5.2 $y = \frac{1}{3}x + 10$

8.5.3 $B(-3;9)$

8.5.4 $BF = \frac{3\sqrt{10}}{2}$

8.5.5 15 units^2

8.5.6 $\alpha = 71,57°$

8.6

8.6.1 $PQ = 10$

8.6.2 $\theta = 37°$

8.6.3 $y = -\frac{4}{3}x + \frac{19}{3}$

8.7

8.7.1 $y = 3x$

8.7.2 $K(1;3)$

8.7.3 $B(-1;-3)$

8.7.4 $A(-2;4)$ and $C(4;2)$

8.7.5 $A'(4;4)$

8.8

8.8.1

a) $AB = 2\sqrt{5}$

b) $m_{AB} = -\frac{1}{2}$

d) $y = -\frac{1}{2}x + \frac{15}{2}$

Study copy distributed by EduResource ZA | eduresourcza.com

e) $C(8;6)$

8.8.2

a) $\theta = 26,56°$

b) 4 units^2

8.9

8.9.1 $AC = 6\sqrt{5}$

8.9.2 $m_{AC} = 2$

8.9.3 $y = \frac{1}{3}x - 7$

8.9.4 $C\hat{K}D = 116,57°$

8.9.5 15 units^2

8.9.6 $x = 3$

8.9.7 $\frac{2}{3}$

8.10

18.10.1

a) $m_{AB} = -3$

b) $\alpha = 108,43°$

c) $T\left(2;-\frac{7}{2}\right)$

d) $S\left(0;-\frac{4}{3}\right)$

8.10.2 $y = -3x - 9$

8.10.3

a) $D\hat{C}A = 68,62°$

b) $5,83$ units^2

8.11

8.11.1 $m_{dia} = -1$

8.11.2 $y = -x + 8$

8.11.3 $P(3;5)$

8.11.4 $(x-4)^2 + (y-4)^2 = 2$

8.11.5 $T(5;3)$

8.11.6 $VT = 2\sqrt{2}$

8.11.7 $k \le -2$ or $k \ge 2$

8.11.8 $q = 1,75$

8.12

8.12.1 $N(1;2)$

8.12.2 $r = 5$

8.12.3 $y - 2,9$ or $y = 6,9$

8.12.4 $(x-12)^2 + y^2 = 100$

8.12.5 New radius = $1,18$

8.13

8.13.1 $AB = 4\sqrt{5}$

8.13.2 $OM = \sqrt{5}$

8.13.3 $O\hat{M}B = 90°$

8.13.4 $q = 1$

8.13.5 $m_{AO} = \frac{4}{3}$

8.13.6 $(x-1)^2 + (y-1)^2 = 25$

8.13.7 $y = -\frac{3}{4}x - \frac{9}{2}$

8.13.8 $-\frac{9}{2} < t < 8$

8.13.9 Point lies outside the circle

8.14

8.14.2 $(-2;0)$

8.14.3 $\left(0;6+4\sqrt{2}\right)$ or $\left(0;6-4\sqrt{2}\right)$

8.14.4 $y = -0,35x + 11,66$

8.14.5 $k < -12$ or $k > 0$

8.15

8.15.1 $C(-3;4)$ and radius = 5

8.15.2 $A(-3;9)$ and $B(-7;7)$

8.15.3 $\left(-\frac{9}{2};\frac{9}{2}\right)$

8.15.4 It does not.

8.15.5 The student is correct.

8.16

8.16.1 $ON = 10$

8.16.2 $p = 10$

8.16.3 $m_{MN} = \frac{3}{4}$

8.16.4 $q = -5$

8.16.5 $(x-10)^2 + (y+5)^2 = 25$

8.16.6 $k = 5$ or $k = 15$

8.16.7 $0 < t < \frac{50}{3}$

8.16.8 No point of contact,

8.17

8.17.1 $A(-3;4)$

8.17.2 $(x+4)^2 + (x-2)^2 = 5$

8.17.3 $R(2;4)$

8.18

8.18.1 $(x+3)^2 + (y-4)^2 = 25$

8.18.2 $T(-6;8)$

8.18.3 $Q(6;-6)$

8.18.4 $y = -x - 6$

8.18.5 ASQR is a kite.

8.18.6 $k = 1,99$ or $k = 4,35$

8.19

8.19.1 Tangent $\perp$ radius

8.19.2

a) $M(-5;5)$

b) $(x+5)^2 + (x-5)^2 = 25$

Study copy distributed by EduResource ZA | eduresourcza.com

c) $y = -\frac{4}{3}x + \frac{20}{3}$

8.19.3 $(x-2)^2 + (y-4)^2 = 4,29$

8.19.4 $M^{/}(5;5)$ and $M^{//}(-5;-5)$

8.20

8.20.1 $y = -\frac{4}{3}x + \frac{22}{3}$

8.20.2 $-\frac{28}{3} < k < \frac{22}{3}$

8.20.3 $AB = \sqrt{7}$

b)

8.21

8.21.1 $p = 36$

8.21.2 $y = -\frac{3}{4}x + 9$

8.21.3 Do not intersect.

8.22

8.22.1 $P(3;6)$ and radius = 2

8.22.2 $3\sqrt{5} - 4$

8.22.3 2297,3 units^3

8.23

8.23.1 $E(4;4)$

8.24

8.24.1 $\theta = 143,1°$

8.25 $\sqrt{6}$

8.27 $k > 16$

8.28

8.28.1 $A(3;2)$

8.29

8.29.1 $y = -2$

8.29.2 $(x-6)^2 + (y-6)^2 = 64$

TOPIC: TRIGONOMETRY

9.12 (c) $a = 1,\ b = 3,\ c = 0$

9.13.1 $\frac{n}{\sqrt{1+n^2}}$

9.13.2 $-\frac{2n}{1+n^2}$

9.13.3 $\frac{1-n}{\sqrt{2(1+n^2)}}$

9.14.1 $\frac{\sqrt{1-p^2}}{p}$

9.14.2 $2p\sqrt{1-p^2}$

9.14.3 $\sqrt{\frac{1+p}{2}}$

9.15.1 $-k$

9.15.2 $\sqrt{1-4k^2(1-k^2)}$

9.15.3 $\sqrt{\frac{1+\sqrt{1-4k^2}}{2}}$

9.16 $-2p\sqrt{1-p^2}$

9.1 $-\frac{4}{5}$

9.3.1 $-\frac{3}{5}$

9.3.2 $\frac{4}{5}$

9.3.3 $S(-15;-20)$

9.4.1 7

9.4.2 $\frac{7}{24}$

9.4.3 $73,74°$

9.5.1 $\frac{5}{12}$

9.5.2 $\frac{5}{13}$

9.5.3 -18

9.6.1 $\frac{4}{3}$

9.6.2 $-\frac{3}{5}$

9.6.3 -5

9.6.4 $-\frac{63}{65}$

9.7.1 $-\frac{7}{25}$

9.7.2 $-\frac{4\sqrt{2}+\sqrt{2}}{10}$

9.8.2 $-\cos 35°$

9.9.1 $\frac{1}{9}$

9.9.2 $-\frac{\sqrt{5}}{3}$

9.9.3 $-2\sqrt{5}$

9.10.1 $\frac{7}{25}$

9.10.2 $\frac{3\sqrt{3}+4}{10}$

9.11.1 $-\frac{12}{13}$

9.11.2 $\frac{5\sqrt{2}-12\sqrt{2}}{26} = -\frac{7\sqrt{2}}{26}$

9.12(a) $\sqrt{1-k^2}$

9.17 $\frac{1+k}{2}$

9.18.1 $-p$

9.18.2 $\sqrt{\frac{1+\sqrt{1-p^2}}{2}}$

9.18.3 $\frac{1}{2}p$

9.19.1 $\frac{p}{\sqrt{1-p^2}}$

9.19.2 $-p^2$

9.19.3 $2p(1-p^2)$

9.20 $\frac{1-p^2}{2}$

9.21 $\frac{2ab}{a^2-b^2}$

9.22.1 $\frac{1+2k}{2k}$

9.23.1 -1

9.23.2 $\sqrt{3}$

9.23.3 -1

Study copy distributed by EduResource ZA | eduresourcza.com

9.24(a)	$\frac{m+n}{2}$
9.24(b)	$\sqrt{2k-k^2}$
9.25	$\frac{23}{27}$
9.27	$p=0,\ \pm\frac{\sqrt{3}}{2}$
9.29.1	$P=45°,\ Q=15°$
9.29.2	$2\cos\ 4A$
9.29.3	$\frac{T-1}{\sqrt{2}}$
9.30.1	$-\frac{1}{2}$
9.30.2	$-\frac{1}{3}\cos 23^{\circ}$
9.30.3	$\cos 35^{\circ}$
9.30.4	$-\frac{\sqrt{3}}{2}$
9.30.5	$-\frac{\sqrt{4}}{2}$
9.30.6	$\sqrt{3}$
9.30.7	$-\frac{1}{2}$
9.30.8	$-\frac{\sqrt{3}}{4}$
9.30.9	-2
9.31.1	-1
9.31.2	$-\frac{1}{2}$
9.31.3	0
9.31.4	$\cos A$
9.31.5	$\cos 2\beta$
9.31.6	$\cos\theta$
9.31.7	$\sin\beta\cos\beta$
9.31.8	$\cos^2 x$
9.31.9	$\frac{1}{\cos x}$
9.31.10	$\tan x$
9.32.1	$\frac{1-\cos\ 2x}{4}$
9.32.2	$x=30°+180°k$ or $x=150°+180°k,\ k\in Z$
9.33.2	$\frac{1}{4}$
9.37.1	$2\cos\ 7x\sin\ 3x$
9.37.2	$x=0°,\ 15°,\ 30°$
9.40.2	$x=45°+180°k,\ k\in Z$
9.41.1	$0°,\ 180°$
9.41.2	$90°,\ 270°$
9.42.1	$x=2k(180°)\pm 60°,\ k\in Z$
9.42.2	$x=90°+360°k$ or $x=330°+360°k,\ k\in Z$
9.43.1	$135°$
9.43.2	$-210°,\ -195°$
9.43.3	$150°$
9.43.4	$-180°,\ 0°$
9.44.2	$x=30°+60°k,\ k\in Z$
9.45.1	$x=90°k$ or $x=45°+90°k,\ k\in Z$
9.45.2	$x=15°+180°k$ or $x=75°+180°k,\ k\in Z$
9.45.3	$x=30°+360°k$ or $150°+360°k,\ k\in Z$
9.45.4	$\theta=45°+180°k,\ k\in Z$
9.45.5	$x=0°+360°k$ or $120°+360°k,\ k\in Z$
9.45.6	$x=90°+360°k$ or $270°+360°k,\ k\in Z$
9.45.7	$\alpha=45°+180°k,\ k\in Z$
9.45.8	$x=0°+360°k,\ 180°+360°k,\ k\in Z$
9.45.9	$150°,\ 210°$
9.45.10	$x=18°+360°k,\ 126°+360°k,\ k\in Z$
9.45.11	$\theta=120°+360°k,\ 240°+360°k,\ k\in Z$
9.45.12	$x=60°+360°k,\ 300°+360°k,\ k\in Z$
9.45.13	$240°,\ 300°$
9.45.14	$x=60°+360°k,\ 180°+360°k,\ k\in Z$
9.46.2	$x=-7,5°+90°k,\ k\in Z$
9.47	$m=1$ or $m=-2$
9.48.2	$-45°$
9.48.3	$h(x)=-\tan\ x-2$
9.48.4	$180°$
9.48.5	$-270°,\ -90°,\ 90°,\ 270°$
9.49.1	$f(x)=3\sin(2x-30°)+2$
9.50.1	Graph of: $f(x)=3\sin(x+30°)+2$ **OR** $f(x)=3\cos(x-60°)+2$
9.50.2	3
9.50.3	$360°$
9.50.4	$x=130,53°$ or $x=349,47°$
9.50.5	$y\in[-5;7]$
9.51.1	$a=2;\ b=-45°$
9.51.2	$360°$
9.51.3	$C(-135°;-1)$
9.51.4	$D\left(0°;\frac{\sqrt{2}}{2}\right),\ E\left(165°;-\frac{1}{2}\right)$
9.51.5	$f(x)=\sin(2x+60°)$
9.51.6	$x=75°;45°,165°$
9.52.2	$360°$
9.52.3	$x=-28{,}68°,\ 151{,}32°$
9.53.1	$180°$
9.53.2	$x=90°$
9.53.4	$-60°,\ 180°$
9.53.5	$y\epsilon[-5;\ -1]$
9.53.6	25
9.54.1	$a=2,\ b=2,\ p=30°$
9.54.2	$0°<x<90°$
9.54.3	$180°$
9.54.4	-9
9.54.5	Reflect in the ***x*-axis**
9.54.6	$30°<x<150°$
9.55.1 (a)	$A=(-60°,0),\ B=(120°,0)$

Study copy distributed by EduResource ZA | eduresourcza.com

9.55.1 (b) $C = (-30°, -1)$, $D = (150°, 1)$

9.55.2 (a) $-30° < x < 150°$

9.55.2 (b) $-150° < x < 30°$

9.55.2 (c) $0° < x < 120°$

9.55.2 (d) $-30° < x < 0°$

9.55.3 $y \in \left[\frac{1}{3}; \infty\right)$

9.56.1 $y \in [0; 2]$

9.56.3 $x = 180°k$ or $x = 210° + 360°k$

9.56.5 $x = -60°; -30°; 150°; 180°$

9.56.6 $30° < x < 60°$

9.57.1 3 hours

9.57.2 3,39 metres

9.64.1 $CB \approx 4{,}29$ m

9.64.2 $CD \approx 8{,}69$ m

9.64.3 $11{,}08\ m^2$

9.65.1 $BE = \frac{y}{\sin(2\alpha)}$

9.65.3 $29°$

9.66.2 $x \approx 10$ m

9.67.3 $66{,}9\ m^2$

9.68.2 $56,3°$

9.69 **Yes**, the tree **will hit the house**.

9.70.2 $y \approx 35,3°$

TOPIC: EUCLIDEAN GEOMETRY

10.1

10.1.1 $88°$

10.1.2 $27°$

10.1.3 $65°$

10.2

10.2.1 $a)35°$

10.2.1 $b)35°$

10.2.1 $c)35°$

10.3

10.3.1 20 cm

10.3.2 25 cm

10.4

10.4.1 Alt. ∠s

10.4.2 $40°$

$a)110°$

10.5

10.5.1 ∠s on same segment

10.5.3 $2y$

10.7

10.7.1 corresp. ∠s

10.7.2 $\hat{E}_2$, $\hat{B}_2$, $\hat{E}_3$ & $D\hat{A}B$

10.8

10.8.1 $a)90° - x$

$b)\ x$

$c)90° - x$

10.10

10.11 $\theta = 33.69°$ or $213.69°$

10.12

10.12.2 $\frac{45}{2}$

10.13

10.13.2 8 cm

10.14

10.14.1 $\frac{4}{3}$

10.14.2 $\frac{7}{4}$

10.14.3 $\frac{8}{3}$

10.15

10.15.1 $\frac{1}{2}$

10.16

10.16.1 $y = 3$ or $y \neq \frac{1}{2}$

10.17

10.17.1 $\hat{C}$, $\hat{D}$, $\hat{E}_2$ & $\hat{A}_2$

10.19.5 $\frac{9}{8}$

10.21 $\frac{16}{49}$

10.19.5 $\frac{9}{8}$

10.19.1 2

10.19.2 2

10.19.3 $\frac{8}{9}$

10.19.4 $\frac{3}{2}$

Study copy distributed by EduResource ZA | eduresourcza.com

Study copy distributed by EduResource ZA | eduresourcza.com