

Grade 12

Mathematics



MSI

Maths and Science Infinity

CAPS Syllabus

Functions

074 772 9926

Thulani Mjikwa

Senior Facilitator & Strategic Interventionist

CONCEPT	HOW TO LEARN IT?	RELEVANT FORMULAE AND KEYWORDS
Straight line	Standard form: $y = mx + c$ Calculating the gradient Substitute the gradient and given point. $x = a$: vertical line $y = b$: horizontal line	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $y - y_1 = m(x - x_1)$ $\frac{x}{a} + \frac{y}{b} = 1$ a: x – intercept b: y – intercept
	x – intercept and y – intercept Plotting the graph Finding the equation	Intercept, coordinates, x – axis, y – axis Sketch, plot, graph paper, relationship Standard form, derive Linear function
	Plot co-ordinates from a table then move on to dual intercept or any other method.	For every function that you have drawn check if you can interpret it yourself. Ask yourself questions like: What is the gradient of this function? What are the intercepts of this function?
	This is asked more on analytical geometry and in calculus. Expect this topic integrated in any section.	If you can get the equation of a tangent from calculus then you have mastered this section.



Hyperbola	Standard form: $y = \frac{a}{x} \text{ (NB: } x \text{ is the denominator)}$ Axes of symmetry: $y = -x$ and $y = x$ If $a > 0$, plot the graph in the first and third quadrants. If $a < 0$, plot the graph in the second and fourth quadrants. $x = -p$: vertical asymptote $y = q$: horizontal asymptote If $p > 0$, the graph will shift p units to the left. If $p < 0$, the graph will shift p units to the right. If $q > 0$, the graph will shift q units upwards. If $q < 0$, the graph will shift p units downwards. There are two axes of symmetry. $y = \frac{a}{x+p} + q$ Domain: $x \in (-\infty ; \infty)$ where $x \neq -p$ Range: $y \in (-\infty ; \infty)$ where $y \neq q$	$y = \frac{a}{x+p} + q$ p: vertical asymptote q: horizontal asymptote $y = -(x+p) + q$: axis of symmetry with positive gradient. $y = x+p+q$: axis of symmetry with negative gradient.
	If the sketch is given:	Find the equation of asymptotes:



	Horizontal asymptotes: $y = q$ (dotted line) Vertical asymptote: $x = -p$ (dotted line) Identify any intercepts.	Find the equation of the $f(x)$. Find the equation of symmetry: They are specific in terms of the gradient if gradient is not mentioned then give both a negative gradient and the positive one.
	Plot coordinates from a table then move on to dual intercept or any other method.	For every function that you have drawn check if you can interpret it yourself. Ask yourself questions like: What are the asymptotes of this function? What are the intercepts of this function?
	Study different examples. Know how to find the equation of a hyperbola.	This function is on its own. But be careful because sometimes they can give you this function in one quadrant and hide the other part.
Parabola	Standard form: $y = ax^2 + bx + c$ Note: the exponent of x is always 2. If $a > 0$: the graph becomes concave up. If $a < 0$: the graph becomes concave down. Axis of symmetry: $x = -\frac{b}{2a}$ Turning point: $\left[-\frac{b}{2a} ; f\left(-\frac{b}{2a}\right)\right]$ or $(-p ; q)$	$y = a(x - x_1)(x - x_2)$ $y = a(x + p)^2 + q$ $y = ax^2$



	Completing the square.	
	Identify if what is given in the graph: usually one point and the turning point will be given: Or x and y intercepts will be given. Domain: $x \in (-\infty ; \infty)$ Range: If $a > 0$ then you have a minimum value at $q, y \geq q$ If $a < 0$ then you have a maximum value at $q, y \leq q$	Find the equation of parabola if passes through point $(a; b)$ and turning point $(p; q)$. The root of the graph is $(a; 0)$ and $(b; 0)$ $(0; y)$
	Plot co-ordinates from a table then move on to dual intercept or any other method.	For every function that you have drawn check if you can interpret it yourself. Ask yourself questions like: What is the axis of symmetry and the turning of this function? What are the intercepts of this function?
	Know how to find the equation of a parabola. You must be able to read off from the graph.	One needs to be very careful because they can give you a function which is restricted here. • And this can be asked in Physical Sciences. • Check where the function increases and decreases. • At the turning point the gradient is zero.
Exponential function	Standard form: $y = a.b^x$; $a > 1$ or $0 < a < 1$	$y = a.b^x + q$; $a > 1$ or $0 < a < 1$



	There is one horizontal asymptote: $y = q$ If $p > 0$, the graph will shift p units to the left. If $p < 0$, the graph will shift p units to the right. If $q > 0$, the graph will shift q units upwards. If $q < 0$, the graph will shift p units downwards. For any exponential function: $y = a \cdot b^{x+p} + q$; $b > 0$, $a > 0$ and $b \neq 1$ Domain: $x \in (-\infty; \infty)$ Range: $y \in (q; \infty)$	$y = a \cdot b^{x+p} + q$; $a > 1$ or $0 < a < 1$
	Mostly only horizontal asymptotes that is identified: $y = q$	Mostly in the exam they will give you a graphs / functions and you need to know how to interpret them.
	Plot co-ordinates from a table then move on to dual intercept or any other method.	For every function that you have drawn check if you can interpret it yourself. Ask yourself questions like: What are the asymptotes of this function? What are the intercepts of this function?
	Know how to find the equation of a parabola. You must be able to plot and interpret.	Check where the function increases and decreases.



Theory on functions	Definition of a function. One-to-one relation and many-to-one. Difference between graph and function. Vertical and horizontal line test. The f – Notation. Increasing and decreasing functions. Domain and the range. End-points of the function.	
	Identify shape of each graph and label them e.g. straight line, exponential, log, hyperbola and parabola. Identify all the points given. Know the asymptotes in case you are given an exponential or hyperbola graph usually are identified by dotted line. Know the axis of symmetry or the turning point if parabola is given.	Vertical line test Horizontal line test One-to-one function Many – to – one function Relation
	Plot co-ordinates from a table then move on to dual intercept or any other method.	For every function that you have drawn check if you can interpret it yourself. Ask yourself questions like: What is the domain and range of this function? Is this a one to one or many to one?



	Know the horizontal and vertical line test to test for a function. One – to – one function Many – to – one function	
Theory on inverses	Reflection about the line $y = x$. Write the equation in the form $y = ...$ Change exponential to logarithm form. Restrict the domain to make the inverse of parabola to be a function.	Exponential to logarithm: $y = a^x \rightarrow x = a^y \rightarrow y = \log_a x$ Inverse of a parabola: $y = x^2 \rightarrow x = y^2 \rightarrow y = \pm\sqrt{x}$
	It is the reflection about the line $y = x$.	Inverse f^{-1}
	Plot co-ordinates from a table then move on to dual intercept or any other method.	Try to restrict all the domain of the parabola that you have drawn in Grade 11 so that its inverses can be a function.
	The reflection about the line $y = x$, $(x; y) = (y; x)$	Interchange x and y values
Logarithmic function	Remember that this is an exponential inverse.	$y = \log_a x$
	Know that the inverse of log is exponents.	Exponential equation will be given and you will be asked to give the inverse equation in the form: $y = ...$
	Know how to change exponents to logs.	
	Plot co-ordinates from a table then move on to dual intercept or any other method.	For every function that you have drawn check if you can interpret it yourself. Ask yourself questions like: What are the asymptotes of this function? What are the intercepts of this function?
	Know that it is the inverse of an exponential function. Know your theory well and how to interchange between logs and exponents.	Check where the function increases and decreases.

Function

Definition:

A function is a relationship between x and y , where for every x – value there is only one y – value

Test for function:

One way to decide whether or not a graph represents a function is to use the vertical line (any line parallel to the y –axis or perpendicular to x –axis) test, such that this line cuts the graph at once (at only one point).

If this line cuts the graph more than once (at more than one point), then the graph is not a function.

Function notation:

$f(x)$ means the each y –value in the function f has an x –value.

Type of function:

One – To – One: Where an x –value is associated with only one y –value.

Vertical line test cuts the graph once (at only one point)

Horizontal line test cuts the graph once (at only one point)

Many – To – One: Where more than one x –value are associated with only one

y –value. For an example: a parabola is a many-to-one function.

Vertical line test cuts the graph once (at only one point).

Horizontal line test cuts the graph more than once (more than one point)

Not a function:

One – To – Many: Where an x –value is associated with more than one y –value.

Vertical line test cuts the graphs more than once (at more than one point).



Terminology:

Term	Description
Asymptote	
Axis of symmetry	
Domain	Is a set of all possible x – values for which the function or graph is defined.
Intercept	
Maximum	Is the maximum y – value in the range of a function or a graph, for which above it the function will not be defined.
Minimum	Is the minimum y – value in the range of a function or a graph, for which below it the function will not be defined.
Range	Is a set of all possible y – values for which the function or graph is defined.
Turning point	

✚ Key words:

- ✓ **Domain:** the set of possible x -values
- ✓ **Range:** the set of possible y -values
- ✓ **Axis of symmetry:** an imaginary line that divides a graph into two mirror images of each other.
- ✓ **Maximum:** the highest possible y -value of a function.
- ✓ **Minimum:** the lowest possible y -value of a function.
- ✓ **Asymptote:** an imaginary line that a graph approaches but never touches.
- ✓ **Turning point:** The point at which a graph reaches its maximum or minimum value and changes direction.
- ✓ **Asymptote** – a line which the function approaches but never touches



Key notes:

$m > 0$	[kwv right-hand rule : both x and y increases]
$m < 0$	[kwv left-hand rule : as x increases ,y decreases]
$m = 0$	[horizontal line]
$m = \text{undefined}$	[vertical line]
$m = \frac{y_2 - y_1}{x_2 - x_1}$	[average gradient]
$m_1 = m_2$	[parallel lines]
$m_1 \cdot m_2 = -1$	[perpendicular lines]
$[\leq / \geq]$	[value included]
$(< / >)$	[value excluded]
The point is given by:	[<i>smallest : largest</i>]

For which values of x will:

$f'(x) > 0$	Positive gradient	$f(x)$ is increasing
$f'(x) < 0$	Negative gradient	$f(x)$ is decreasing
$f(x) \cdot g(x) > 0$	Case 1: $f(x) > 0$ & $g(x) > 0$ Case 2: $f(x) < 0$ & $g(x) < 0$	Positive product
$f(x) \cdot g(x) < 0$	Case 1: $f(x) < 0$ & $g(x) > 0$ Case 2: $f(x) > 0$ & $g(x) < 0$	Negative product
$f(x) > 0$	y – values are greater than zero	y is above the x – axis
$f(x) < 0$	y – values are less than zero	y is below the x – axis



$f'(x) = 0$	Gradient is zero	At the turning point
$f'(x) > 0$ and $g'(x) > 0$	Both gradients are positive	Both graphs are increasing
$f'(x) < 0$ and $g'(x) < 0$	Both gradients are negative	Both graphs are decreasing
$f(x) = g(x)$	Both graph are equal	Both graphs have the same x – value and the y – value
$x.f'(x) > 0$	Case 1: $x \times m$ Case 2: $-x \times -m$	Positive product Value of x multiplied by the gradient
$x.f'(x) < 0$	Case 1: $x \times -m$ Case 2: $-x \times m$	Negative product Value of x multiplied by the gradient
$f(x).f'(x) > 0$	Case 1: $y \times m$ Case 2: $-y \times -m$	Positive product Value of y multiplied by the gradient
$f(x).f'(x) < 0$	Case 1: $-y \times m$ Case 2: $y \times -m$	Negative product Value of y multiplied by the gradient

HYPERBOLIC FUNCTIONS

TUTORIAL 1

Activity 1

Given: $f(x) = \frac{x+3}{x+1}$

1.1 Calculate the coordinates of the x -intercept and y -intercept of f . (4)

1.2 Show that $f(x) = \frac{2}{x+1} + 1$. (2)

1.3 Write down the equations of the vertical and horizontal asymptotes of f . (2)



1.4 Draw a sketch graph of $f(x)$ showing clearly the intercepts and asymptotes on the axes provided on DIAGRAM SHEET attached. (4)

1.5 Use your graph to solve: $\frac{2}{x+1} \geq 1$. (3)

[15]

Activity 2

Given: $f(x) = \frac{2}{x-3} + 1$

2.1 Write down the equation of the asymptotes of f . (2)

2.2 Calculate the coordinates of the x -intercept and y -intercept of f . (3)

2.3 Sketch f on the grid provided on the DIAGRAM SHEET. Show all intercepts with the axes and the asymptotes. (3)

[8]

Activity 3

3.1 The function defined as $y = \frac{a}{x+p} + q$ has the following properties:

- The domain is $x \in \mathcal{R}, x \neq -2$.
- $y = x + 6$ is an axis of symmetry.
- The function is increasing for all $x \in \mathcal{R}, x \neq -2$.

Draw a neat sketch of the graph of this function. Your sketch must include the if any.

[4]

Activity 4

Given the function: $f(x) = \frac{3}{x-2} - 3$.

4.1 Write down the equation of the asymptotes of f . (2)

4.2 Determine the coordinates intercepts of the graph of f . (3)

4.3 Make a neat sketch of f , using the DIAGRAM SHEET provided. Indicate all intercepts with the axes. (4)

4.4 Write down the equation of the axis of symmetry of f with a negative gradient. (2)

4.5 Write down the range of $f(x) + 2$. (1)



[12]

Activity 5

Consider the function: $f(x) = \frac{3}{x-1} - 2$

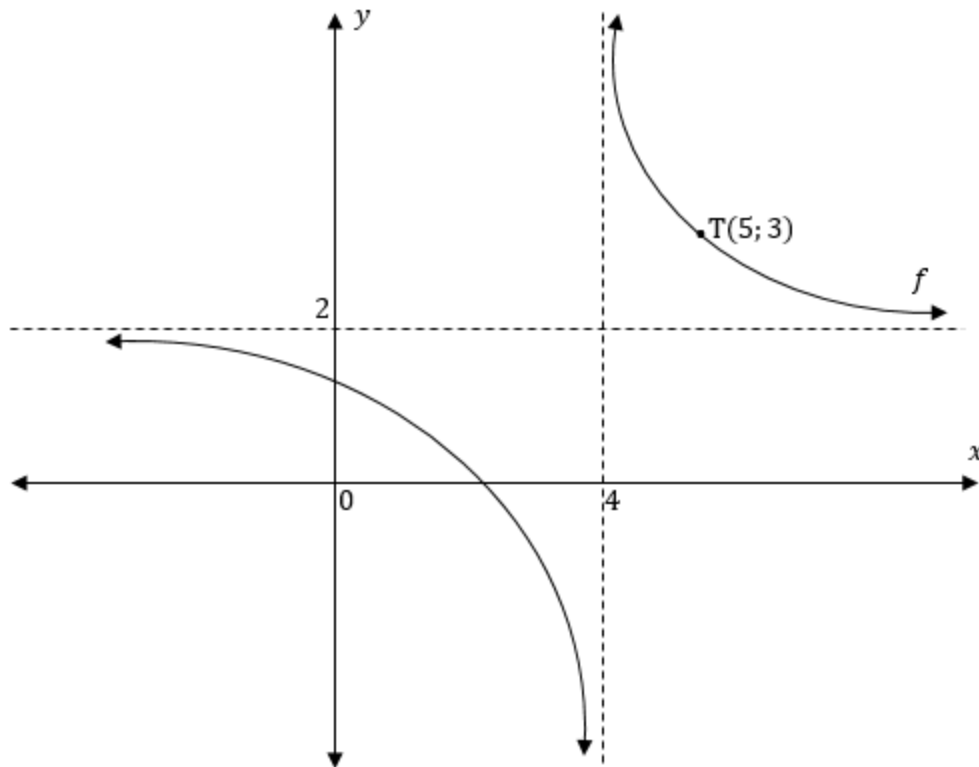
- 5.1 Write down the equation of the asymptotes of f . (2)
- 5.2 Determine the coordinates of the intercepts of the graph of f with the axes. (3)
- 5.3 Sketch the graph of f on DIAGRAM SHEET provided. (3)
- 5.4 Write down the range of $y = -f(x)$. (1)
- 5.5 Describe, in words, the transformation of f to g if $g(x) = \frac{-3}{x+1} - 2$. (2)

[11]

TUTORIAL 2

Activity 1

The diagram below represents the graph of $f(x) = \frac{a}{x-p} + q$. T(5; 3) is a point of f .

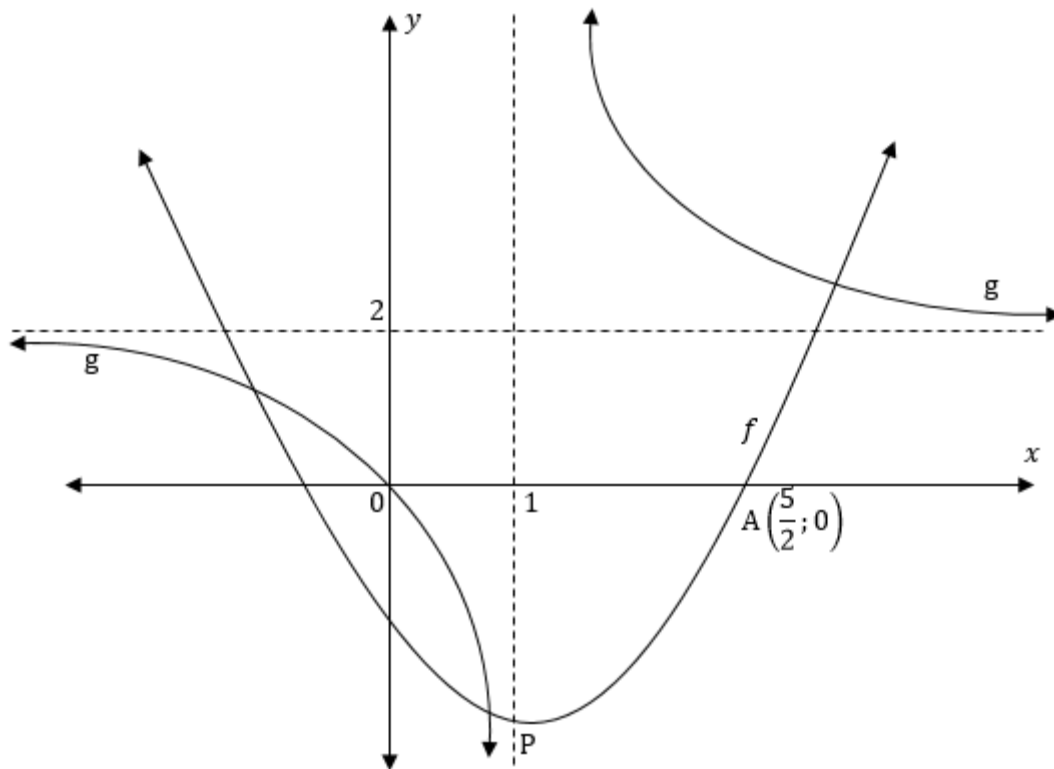


- 1.1 Determine the values of a , p and q . (4)
 - 1.2 If the graph of f is reflected across the line having equation $y = -x + c$, the new graph coincides with the graph of $y = f(x)$. Determine the value of c . (3)
- [7]

Activity 2

Sketched below are the graphs of $f(x) = (x - p)^2 + q$ and $g(x) = \frac{a}{x-b} + c$.

$A\left(\frac{5}{2}; 0\right)$ is a point of the graph of f . P is the turning point of f . The asymptotes of g are represented by the dotted lines. The graph of g passes through the origin.



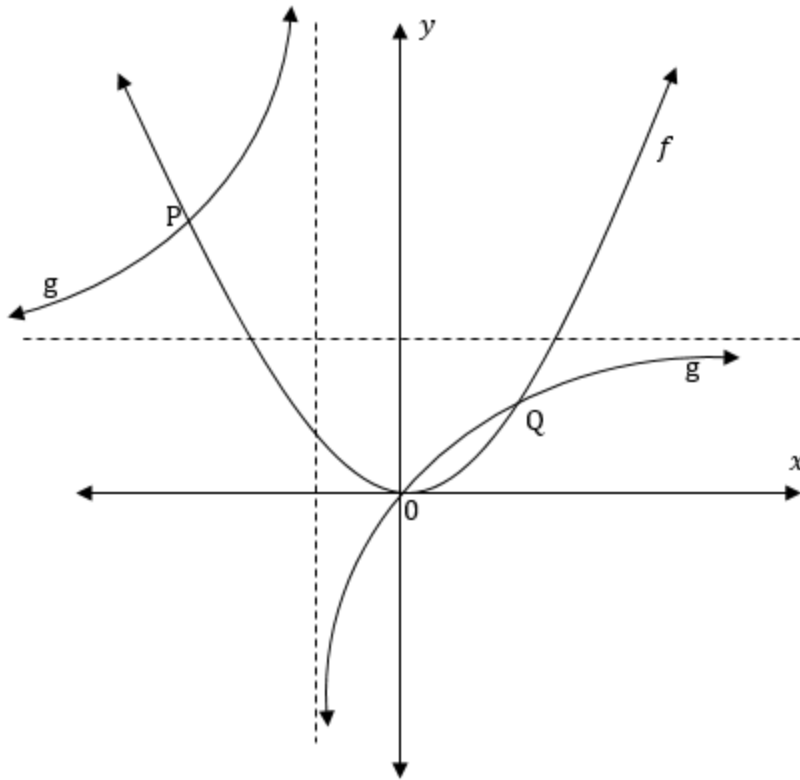
- 2.1 Determine the equation of g . (4)
- 2.2 Determine the coordinates of P , the turning point of f . (4)
- 2.3 Write down the equations of the asymptotes of $g(x - 1)$. (2)
- 2.4 Write down the equation of h , if h is the image of f reflected in the x -axis. (1)

[11]

Activity 3

Sketched below are the graphs of $f(x) = \frac{1}{2}x^2$ and $g(x) = -\frac{1}{x+1} + 1$.

P and Q are the points of intersection of f and g .



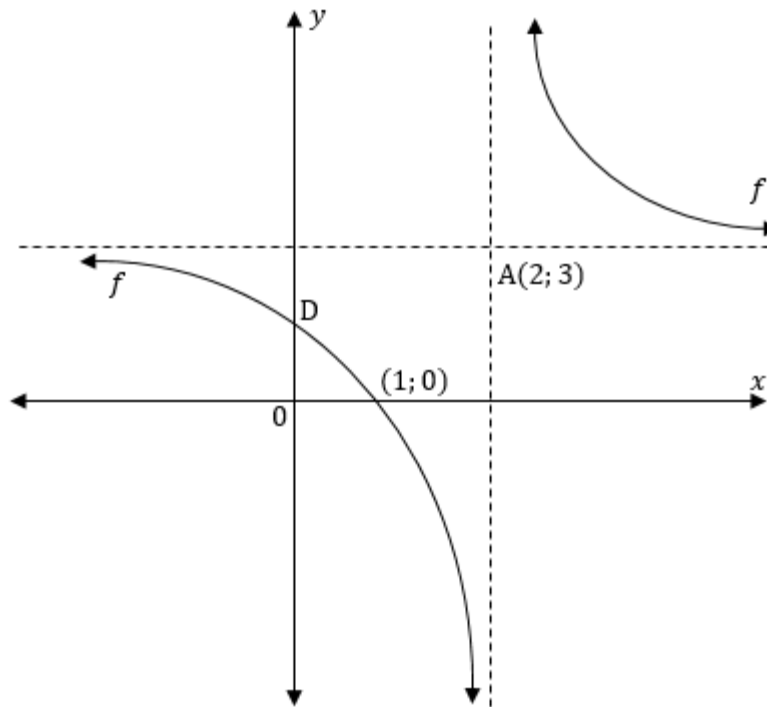
- 3.1 Show that the coordinates of P and Q are $P(-2; 2)$ and $Q(1; \frac{1}{2})$ respectively. (6)
- 3.2 An axis of symmetry of the graph of g is a straight line defined a $y = mx + c$, where $m > 0$. Write down the equation of this straight lie in the form $y = h(x) = \dots$ (2)
- 3.3 Determine the equation of h^{-1} in the form $y = \dots$ (2)
- 3.4 Show algebraically that $g(x) + g\left(\frac{1}{x}\right) = g(-x) \cdot g(x - 1)$. ($x \neq 0$ or $x \neq 1$). (3)

[13]

Activity 4

Given: $f(x) = \frac{a}{x-p} + q$. The point A(2; 3) is the point of intersection of the asymptotes of f .

The graph of f intersects the x -axis at (1; 0). D is the y -intercept of f .

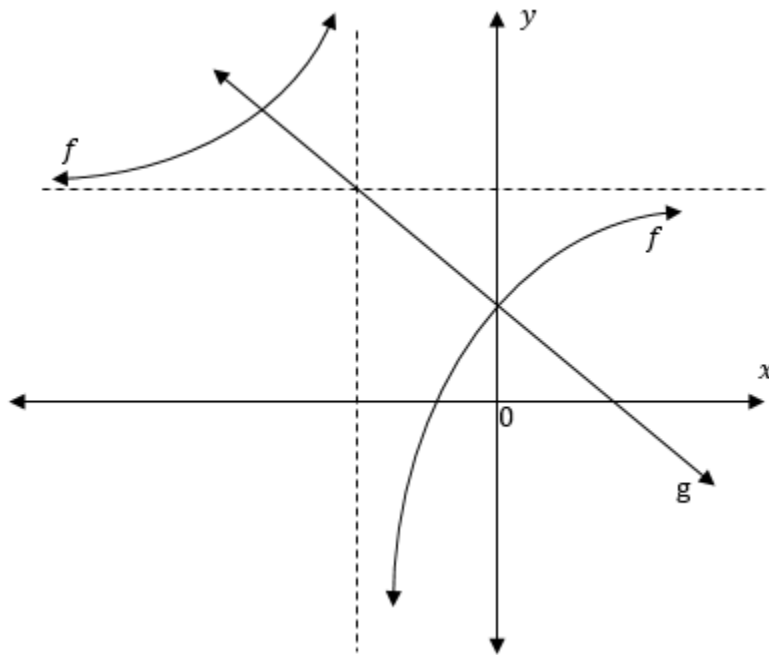


- 4.1 Write down the equations of the asymptotes of f . (2)
- 4.2 Determine the equation of f . (3)
- 4.3 Write down the coordinates of D. (2)
- 4.4 Write down the equation of g if g is the straight line joining A and D. (2)
- 4.5 Write down the coordinates of the other point of intersection of f and g . (4)

[14]

Activity 5

The graphs of $f(x) = \frac{-3}{x+1} + 5$ and $g(x) = -3x + 2$ are sketched below.



- 5.1 Write down the range of f . (1)
- 5.2 Determine the coordinates of the points of intersection of f and g . (7)
- 5.3 Describe the transformation of f to h if $h(x) = \frac{3}{x+1} + 5$. (1)

[9]

EXPONENTIAL FUNCTIONS & INVERSES

TUTORIAL 3



Activity 1

Consider the function: $f(x) = \left(\frac{1}{3}\right)^x$.

- 1.1 Is f an increasing or decreasing function? Give a reason for your answer. (2)
- 1.2 Determine $f^{-1}(x)$ in the form $y = \dots$ (2)
- 1.3 Write down the equation of the asymptote of $f(x) - 5$. (1)
- 1.4 Describe the transformation from f to g if $g(x) = \log_3 x$. (2)

[7]

Activity 2

Given: $f(x) = 3^x$

- 2.1 Determine an equation for f^{-1} in the form $f^{-1}(x) = \dots$ (1)
- 2.2 Sketch, in the DIAGRAM SHEET, the graphs of f and f^{-1} , showing clearly ALL intercepts with the axes. (4)
- 2.3 Write down the domain of f^{-1} . (2)
- 2.4 For which values of x will $f(x) \cdot f^{-1}(x) \leq 0$? (2)
- 2.5 Write down the range of $h(x) = 3^{-x} - 4$. (2)
- 2.6 Write down the equation of g if g is the image of the graph of f after f has been translated two units to the right and reflected about the x -axis. (2)

[13]

Activity 3

Given: $f(x) = 2^{-x}$

- 3.1 Write down the equation that defines f^{-1} in the form $y = \dots$ (2)
- 3.2 Draw the sketch graphs of f and f^{-1} on the same system of axes. Clearly label the graphs as well as all possible intercepts with the axes. (4)
- 3.3 By means of a dotted line, draw the axis of symmetry of g and f^{-1} . (1)
- 3.4 Write down the axis of symmetry. (1)
- 3.5 For which value(s) of x if $f(x) = f^{-1}(x)$? (1)

[9]

TUTORIAL 4

Activity 1

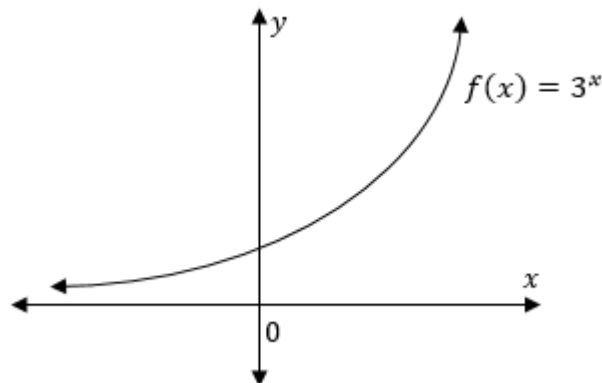


- 1.1 Given: $f(x) = \log_5 x$
Determine f^{-1} . (2)
- 1.2 Given: $h(x) = x^2$
- 1.2.1 Determine the inverse of h in the form $y = \dots$ (2)
- 1.2.2 Give a reason why the inverse of h is not a function. (2)
- 1.2.3 Write down TWO ways in which you can restrict the domain of h so that its inverse is a function. (2)
- 1.2.4 Hence, sketch the graph of the function h^{-1} . (4)
- 1.2.5 Determine the value(s) of x for which $h^{-1}(x) \leq 2$. (2)

[14]

Activity 2

The graph of $f(x) = 3^x$ is drawn below.

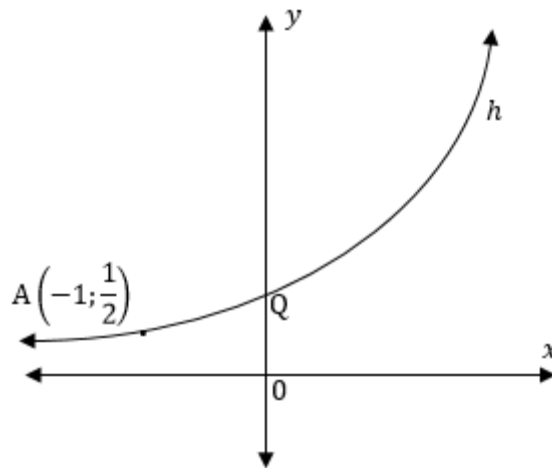


- 2.1 Write f^{-1} in the form $y = \dots$ (1)
- 2.2 Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x - 2)$ on the grid provided in the DIAGRAM SHEET. (4)
- 2.3 Use your graphs to solve for x if $\log_3(x - 2) < 1$. (2)

[7]

Activity 3

The graph of $h(x) = a^x$ is sketched below. A $\left(-1; \frac{1}{2}\right)$ is a point on the graph of h .

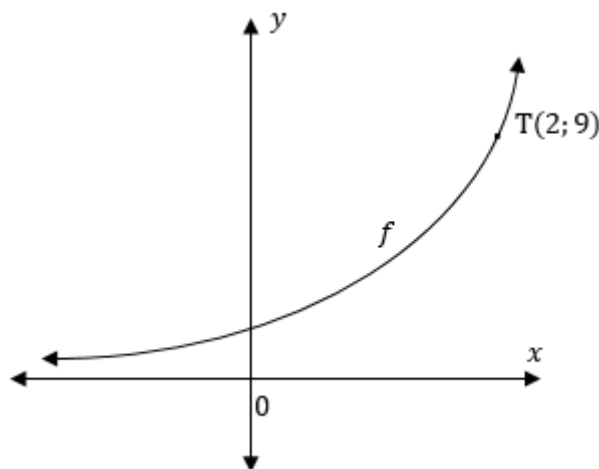


- 3.1 Explain why the coordinates of Q are (0; 1). (2)
- 3.2 Calculate the value of a . (2)
- 3.3 Write down the equation for the inverse function, h^{-1} , in the form $y = \dots$ (2)
- 3.4 Draw a sketch graph, on DIAGRAM SHEET, of h^{-1} . Indicate on this graph the coordinates of two points that lie on this graph. (3)
- 3.5 Read off from your graph the values of x for which $\log_2 x > -1$. (2)
- 3.6 If $g(x) = (100)3^x$, determine values of x for which $h(x) = g(x)$. (3)

[14]

Activity 4

The graph of $f(x) = a^x$, $a > 1$ is shown below. T(2; 9) lies on f .



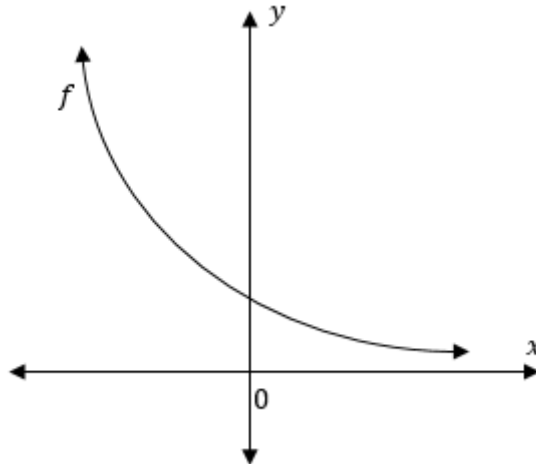
- 4.1 Calculate the value of a . (2)
- 4.2 Determine the equation of $g(x)$ if $g(x) = f(-x)$. (1)



- 4.3 Determine the value(s) of x for which $f^{-1}(x) \geq 2$. (2)
- 4.4 Is the inverse of f a function? Explain your answer. (2)
- [7]**

Activity 5

The graph of $f(x) = \left(\frac{1}{3}\right)^x$ is sketched below.



- 5.1 Write down the domain of f . (1)
- 5.2 Write down the equation of the asymptote of f . (1)
- 5.3 Write down the equation of f^{-1} in the form $y = \dots$ (2)
- 5.4 Sketch the graph of f^{-1} in the DIAGRAM SHEET. (3)
- Indicate the x -intercept and ONE other point.
- 5.5 Write down the equation of the asymptote of $f^{-1}(x + 2)$. (2)
- 5.6 Prove that: $[f(x)]^2 - [f(-x)]^2 = f(2x) - f(-2x)$ for all values of x . (3)

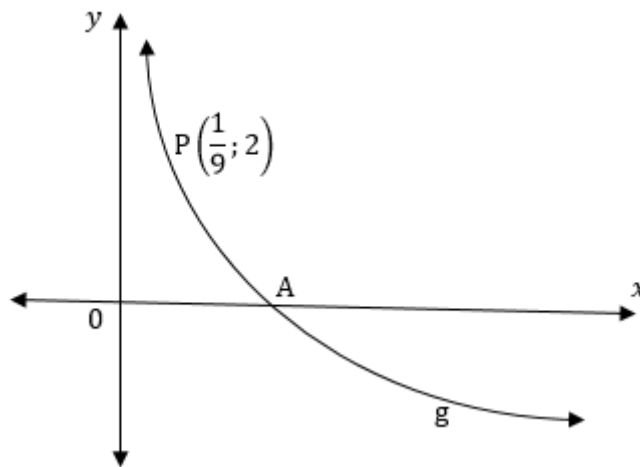
[12]

TUTORIAL 5

Activity 1



Given the graph of $g(x) = \log_{\frac{1}{3}} x$. A is the x -intercept of g . $P\left(\frac{1}{9}; 2\right)$ is a point on g .

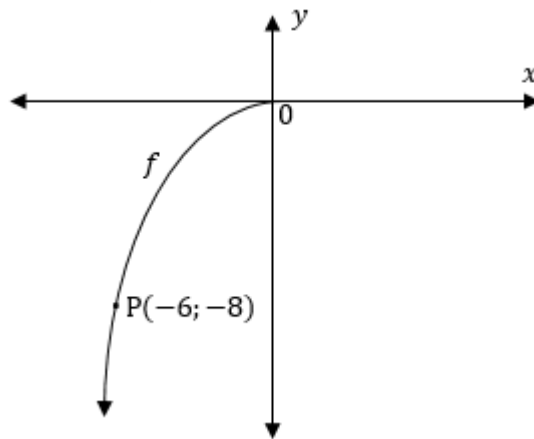


- 1.1 Write down the coordinates of A. (1)
- 1.2 Sketch the graph of g^{-1} indicating an intercept with the axes and ONE other point on the graph. (3)
- 1.3 Write down the domain of g^{-1} . (1)

[5]

Activity 2

The graph of $f(x) = ax^2$, $x \leq 0$ is sketched below. The point $P(-6; -8)$ lies on the graph of f .



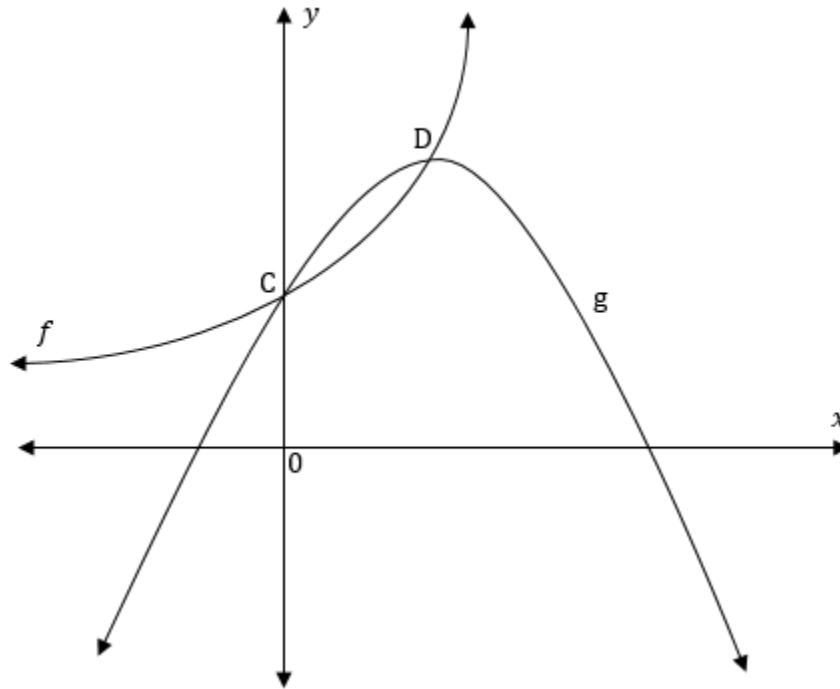
- 2.1 Calculate the value of a . (2)
- 2.2 Determine the equation of f^{-1} , in the form $y = \dots$ (3)
- 2.3 Write down the range of f^{-1} . (1)
- 2.4 Draw the graph of f^{-1} on the DIAGRAM SHEET. Indicate the coordinates of a point on the different from $(0; 0)$. (2)
- 2.5 The graph of f is reflected across the line $y = x$ and thereafter it is reflected across the x -axis. Determine the equation of the new function in the form $y = \dots$ (3)

[11]

Activity 3



Sketched below are the graphs of $f(x) = 2^x$ and $g(x) = -(x - 1)^2 + b$, where b is a constant. The graphs of f and g intersect the y -axis at C . D is the turning point of g .



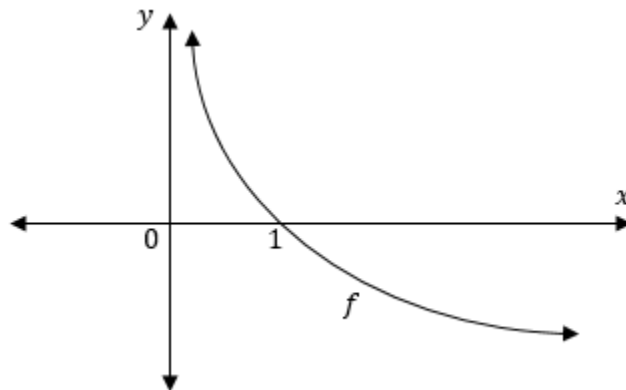
- 3.1 Show that $b = 2$. (2)
- 3.2 Write down the coordinates of the turning point of g . (2)
- 3.3 Write down the equation $f^{-1}(x)$ in the form $y = \dots$ (1)
- 3.4 Sketch the graph of f^{-1} on the system of axes on DIAGRAM SHEET.
Indicate the x -intercept and the coordinates of ONE other point on your graph. (3)
- 3.5 Write down the equation of h if $h(x) = g(x + 1) - 2$. (2)
- 3.6 How can the domain of h be restricted so that h^{-1} will be a function? (1)
- 3.7 Determine the maximum value of $2^{2-(x-1)^2}$. (2)

[13]

Activity 4



Sketched below is the graph of $f(x) = -\log_2 x$.



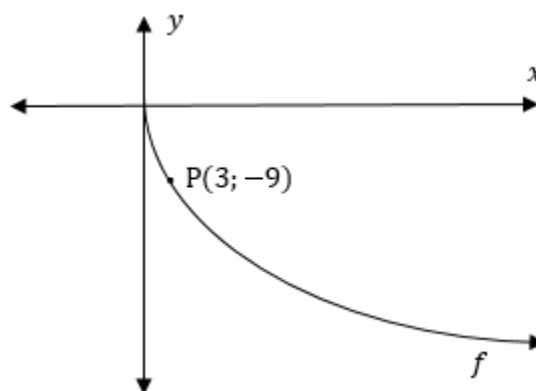
- 4.1 Write down the domain of f . (1)
- 4.2 Write down the equation of f^{-1} in the form $y = \dots$ (1)
- 4.3 Write down the equation of the asymptote of f^{-1} . (1)
- 4.4 Explain how, using the graph of f , you would sketch the graphs of:
 - 4.4.1 $g(x) = \log_2 x$ (1)
 - 4.4.2 $h(x) = 2^{-x} - 5$ (3)
- 4.5 Use the graph of f to solve for x if $\log_2 x < 3$. (3)

[10]

Activity 5

The graph of $f(x) = -\sqrt{27x}$ for $x \geq 0$ is sketched below.

The point $P(-3; 9)$ lies on the graph of f .



- 5.1 Use your graph to determine the values of x for which $f(x) \geq -9$. (2)
- 5.2 Write down the equation of f^{-1} in the form $y = \dots$ Include ALL restrictions. (3)
- 5.3 Sketch f^{-1} , the inverse of f , on the DIAGRAM SHEET. Indicate the

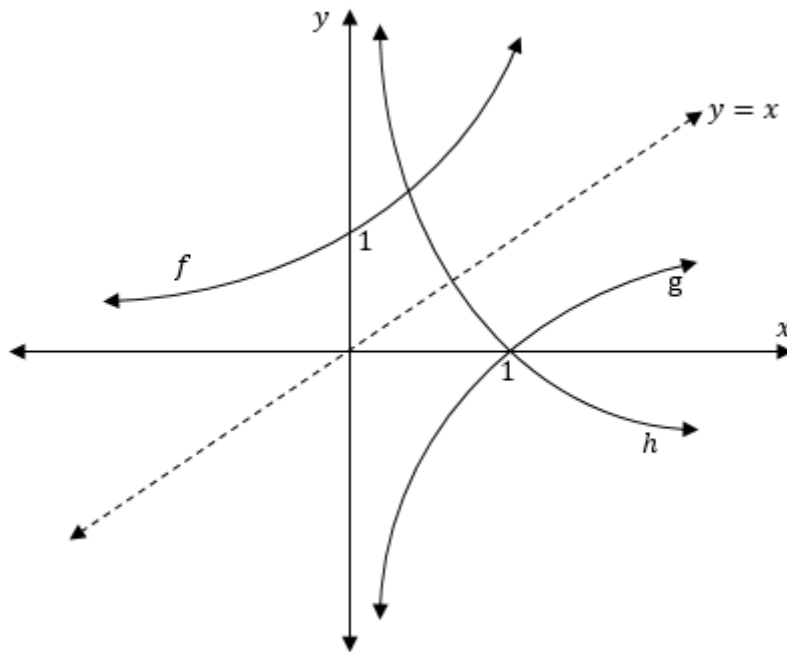


intercept(s) with the axes and the coordinates of ONE other point. (3)

[8]

Activity 6

In the sketch, the following functions are represented: $f(x) = 3^x$, $g(x)$, the reflection of f in the line $y = x$. $h(x)$, the reflection of g in the line x -axis.



6.1.1 Determine the defining equations of g and h in the form $y = \dots$ (5)

6.1.2 Determine with the aid of the above sketch the value(s) for which:

a) $f(x) > g(x)$ (1)

b) $\frac{g(x)}{h(x)} = 1$ (1)

6.2 The following function is given: $f(x) = 2x^2$

6.2.1 Determine $f^{-1}(x)$. (2)

6.2.2 Is f^{-1} a function? Give a reason for your answer. (2)

6.2.3 How can the domain be restricted so that f^{-1} is a function? (2)

[13]

QUADRATIC FUNCTIONS



TUTORIAL 6

Activity 1

Given: $h(x) = 4^x$ and $f(x) = 2(x - 1)^2 - 8$.

- 1.1 Sketch the graphs of h and f on the DIAGRAM SHEET. Indicate ALL intercepts with the axes and any turning points. (8)
- 1.2 Without any further calculations, sketch the graph of $y = \log_4 x = g(x)$ on the same system of axes. (2)
- 1.3 The graph of f is shifted 2 units to the LEFT. Write down the equation of the new graph. (2)
- 1.4 Show, algebraically, that $h\left(x + \frac{1}{2}\right) = 2h(x)$. (3)

[15]

Activity 2

Given: $f(x) = 2x^2 - 10x - 28$ and $g(x) = mx + k$.

- 2.1 Write down the y -intercept of f . (1)
- 2.2 Determine the x -intercepts of f . (3)
- 2.3 Determine the coordinates of the turning point of f . (2)
- 2.4 Sketch the graph of f . Clearly show the intercepts with both axes as well as the coordinates of the turning point. (2)
- 2.5 Determine the coordinates of point P , a point on f , where the gradient of the tangent of f at P is equal to 6. (4)
- 2.6 Determine the equation of g , the straight line passing through the points $(-2; 0)$ and $(4; -36)$. (3)
- 2.7 Write down the equation of h in the form $h(x) = a(x + p)^2 + q$ if $h(x) = f(x + 2) - 3$. (3)

[18]

Activity 3



Given: $f(x) = -(x + 2)^2 + 6$ and $g(x) = 2^{-x} + 1$.

3.1 Draw the graphs of f and g on the same set of axes. Clearly show the intercepts with both axes, as well as the asymptote(s) where applicable. (8)

3.2 Write down the value(s) of t if $f(x) = t$ has:

3.2.1 equal roots. (2)

3.2.2 one root equal to 0. (2)

3.3 Write down the equation of the asymptote of h if $h(x) = g(x) + 1$. (2)

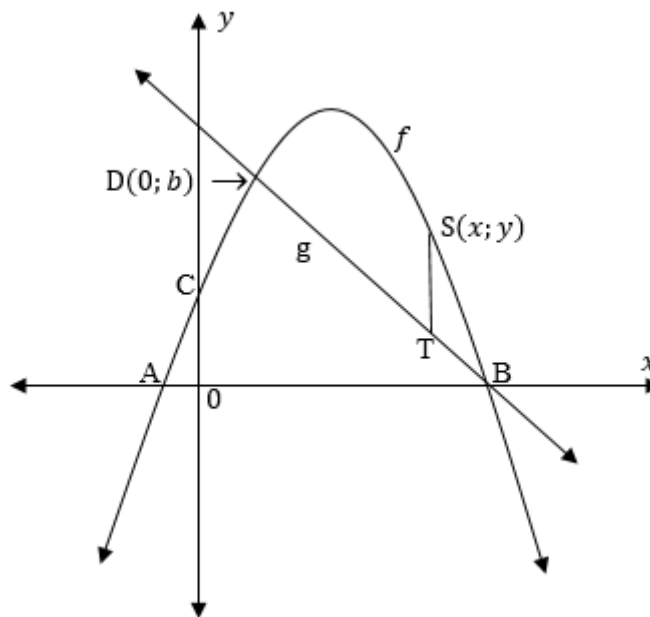
[14]

TUTORIAL 7

Activity 1

The graphs of $f(x) = -x^2 + 7x + 8$ and $g(x) = -3x + 24$ are sketched below.

f and g intersect in D and B. A and B are the x -intercepts of f .



1.1 Determine the coordinates of A and B. (4)

1.2 Calculate a , the x -intercept of D. (4)

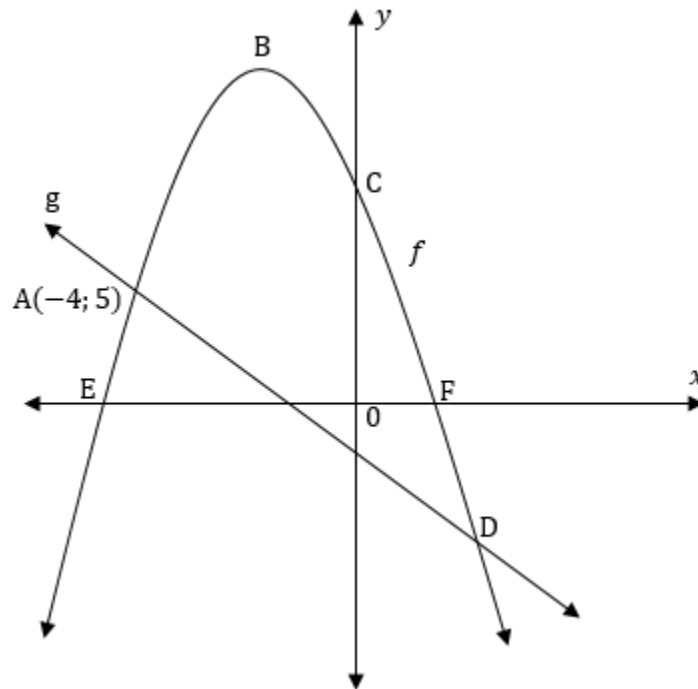
1.3 $S(x; y)$ is a point on the graph of f , where $a \leq x \leq 8$. ST is drawn parallel to the y -axis with t on the graph of g . Determine ST in terms of x . (2)

1.4 Calculate the maximum height of ST. (2)

Activity 2



The graph of $f(x) = a(x - p)^2 + q$ where a , p and q are constants, is given below. Points E, F(1; 0) and C are its intercepts with the axes. A(-4; 5) is the reflection of C across the axis of symmetry of f . D is the point on the graph such that the straight line through A and D has equation $g(x) = -2x - 3$.



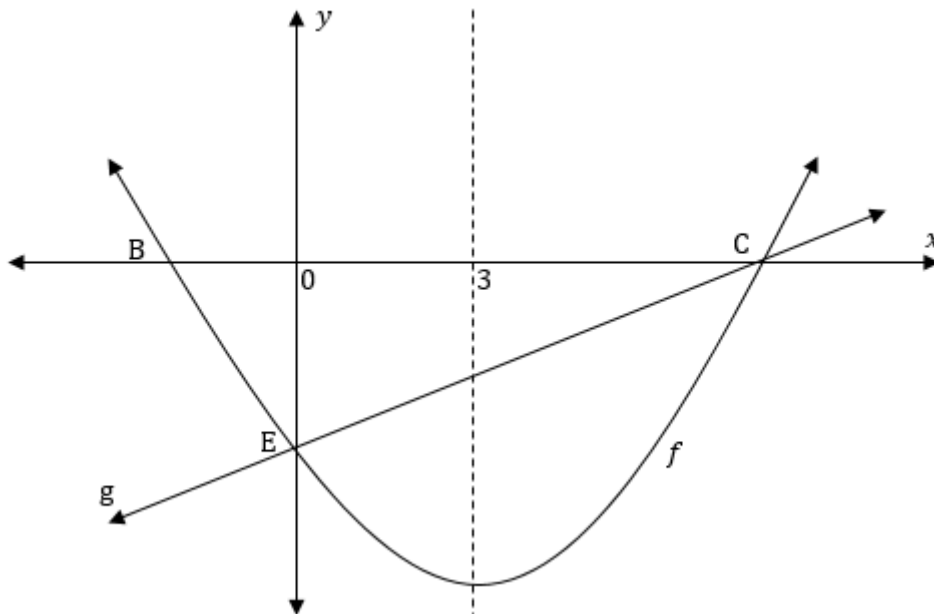
- 2.1 Write down the coordinate of C. (1)
- 2.2 Write down the equation of the axis of symmetry of f . (1)
- 2.3 Calculate the values of a , p and q . (6)
- 2.4 If $f(x) = -x^2 - 4x + 5$, calculate the x -coordinate of D. (4)
- 2.5 The graph of f is reflected about the x -axis. Write down the coordinates of the turning point of the new parabola. (2)

[14]

Activity 3



A parabola f intersects the x -axis at B and C and the y -axis at E. the axis of symmetry of the parabola has equation $x = 3$. The line through E and C has equation $g(x) = \frac{x}{2} - \frac{7}{2}$.



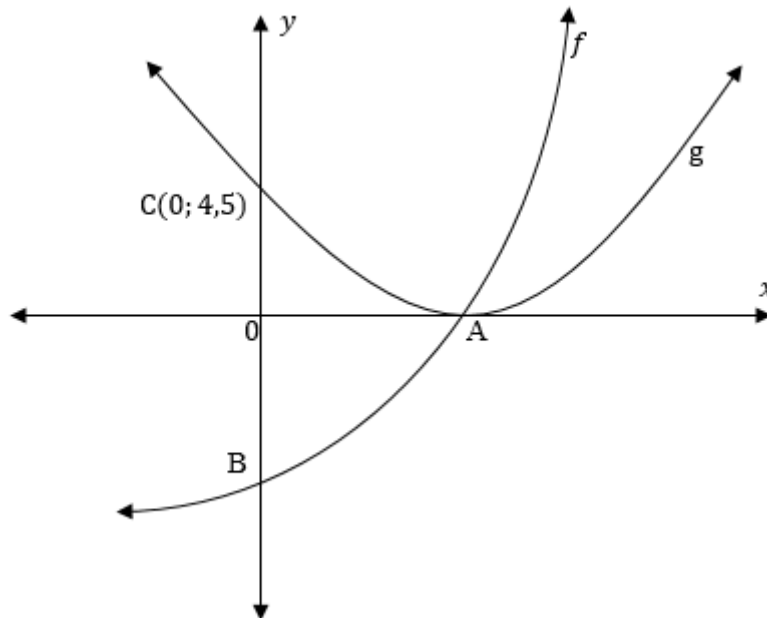
- 3.1 Show that the coordinates of C are (7; 0). (1)
- 3.2 Calculate the x -coordinate of B. (1)
- 3.3 Determine the equation of f in the form $y = a(x - p)^2 + q$. (6)
- 3.4 Write down the equation of the graph of h , the reflection of f in the x -axis. (1)
- 3.5 Write down the maximum value of $t(x)$ if $t(x) = 1 - f(x)$. (2)
- 3.6 Solve for x if $f(x^2 - 2) = 0$. (4)

[15]

Activity 4



The graph of $f(x) = 2^x - 8$ and $g(x) = ax^2 + bx + c$ are sketched below. B and C(0; 4,5) are the y -intercepts of the graphs of f and g respectively. The two graphs intersect at A, which is the turning point of the graph of g and the x -intercept of the graph of f and g .



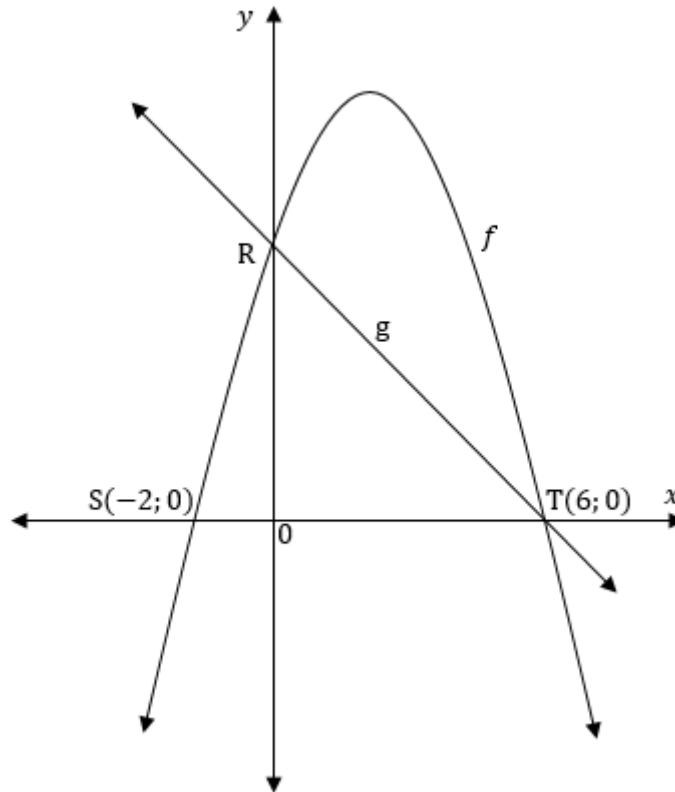
- 4.1 Determine the coordinates of a and B. (4)
- 4.2 Write down an equation of the asymptote of the graph of f . (1)
- 4.3 Determine an equation of h if $h(x) = f(2x) + 8$. (2)
- 4.4 Determine the equation of h^{-1} in the form $y = \dots$ (2)
- 4.5 Write down the equation of p , if p is the reflection of h^{-1} about the x -axis. (1)

[10]

TUTORIAL 8

Activity 1

$S(-2; 0)$ and $T(6; 0)$ are the x -intercepts of the graph of $f(x) = ax^2 + bx + c$ and R is the y -intercept. The straight line through R and T represents the graph of $g(x) = -2x + d$.



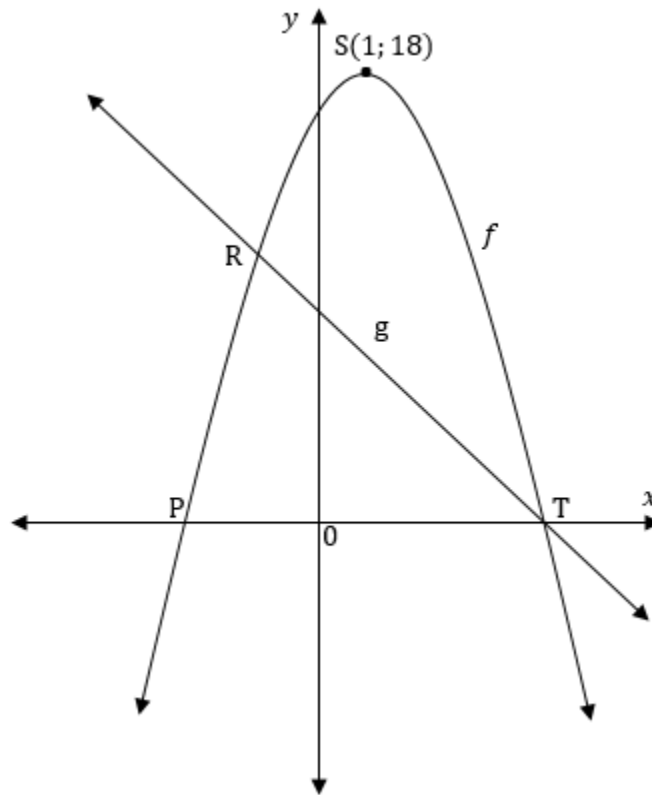
- 1.1 Determine value of d . (2)
- 1.2 Determine the equation of f in the form $f(x) = ax^2 + bx + c$. (4)
- 1.3 If $f(x) = -x^2 + 4x + 12$, calculate the coordinates of the turning point of f . (2)
- 1.4 Determine the maximum value of $h(x) = 3^{f(x)-12}$. (3)

[11]

Activity 2



$S(1; 18)$ is the turning point of the graph of $f(x) = ax^2 + bx + c$. P and T are x -intercepts of f . The graph of $g(x) = -2x + 8$ has an x -intercept at T. R is the point intersection of f and g .



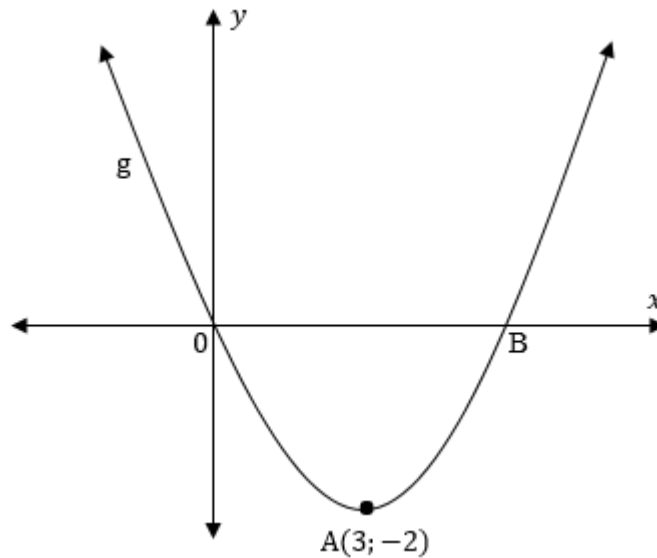
- 2.1 Calculate the coordinates of T. (2)
- 2.2 Determine the equation for f in the form $f(x) = ax^2 + bx + c$. Show ALL your working. (4)
- 2.3 If $f(x) = -x^2 + 4x + 16$, calculate the coordinates of R. (4)
- 2.4 Use your graph to solve for x where:
 - 2.4.1 $f(x) \geq g(x)$. (2)
 - 2.4.2 $-x^2 + 4x - 2 < 0$. (4)

[16]

Activity 3



The diagram below shows the curve of g which is a quadratic function with a turning point at $A(3; -2)$ and passes through the origin.



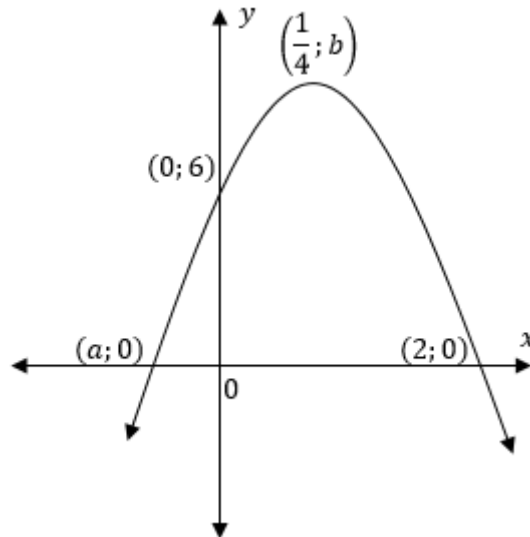
- 3.1 Show that the equation of $g(x) = \frac{2}{9}(x - 3)^2 - 2$. (3)
- 3.2 Write down the coordinates of B. (1)
- 3.3 Is g a one-to-one function or a one-to-many function? Substantiate your answer. (2)
- 3.4 Draw a sketch graph of g^{-1} . On this graph, indicate the coordinates of the turning point as well as ALL intercepts with the axes. (3)
- 3.5 Determine the range of g^{-1} so that g^{-1} will be a function. (1)
- 3.6 Determine the equation of the graph in the form $y = \dots$ if the graph of g is shifted horizontally to the left by 2 units. (2)

[12]

Activity 4



In the figure, the graph of the parabola f , intersecting the x -axis at $(a; 0)$ and $(2; 0)$ is given. Furthermore $(\frac{1}{4}; b)$ are coordinates of the turning point of the curve of f , while its point of intersection with the y -axis is $(0; 6)$. The point $k; 6$ lies on the curve of f .



Determine:

- 4.1 the value of k . (1)
- 4.2 the value of a . (1)
- 4.3 the value of b (show the necessary calculations) (6)
- 4.4 the equation of the tangent to the curve of f at the point $(0; 6)$. (3)

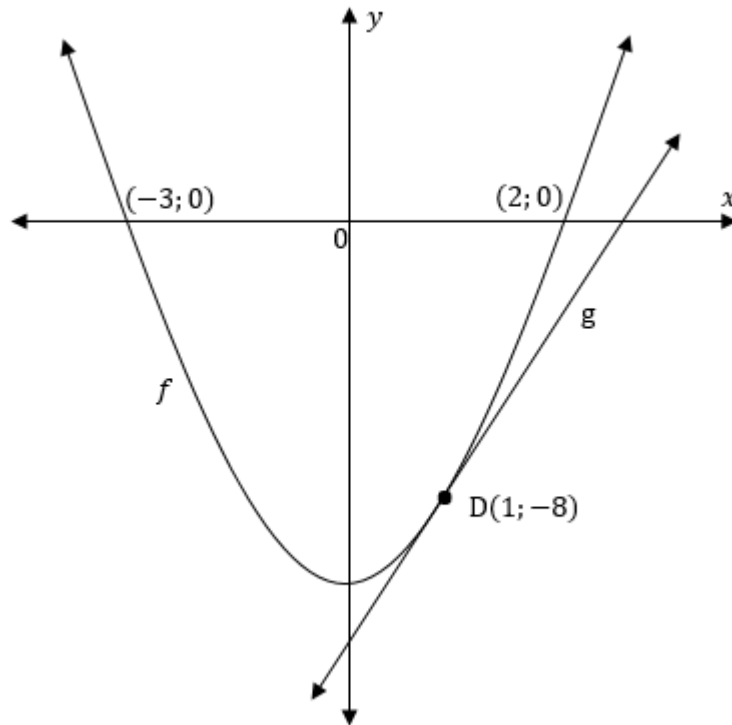
[11]

Activity 5



The graphs of $f(x) = ax^2 + bx + c$; $a \neq 0$ and $g(x) = mx + k$ are drawn below.

$D(1; -8)$ is a common point of f and g . f intersects the x -axis at $(-3; 0)$ and $(2; 0)$. g is the tangent of f at D .



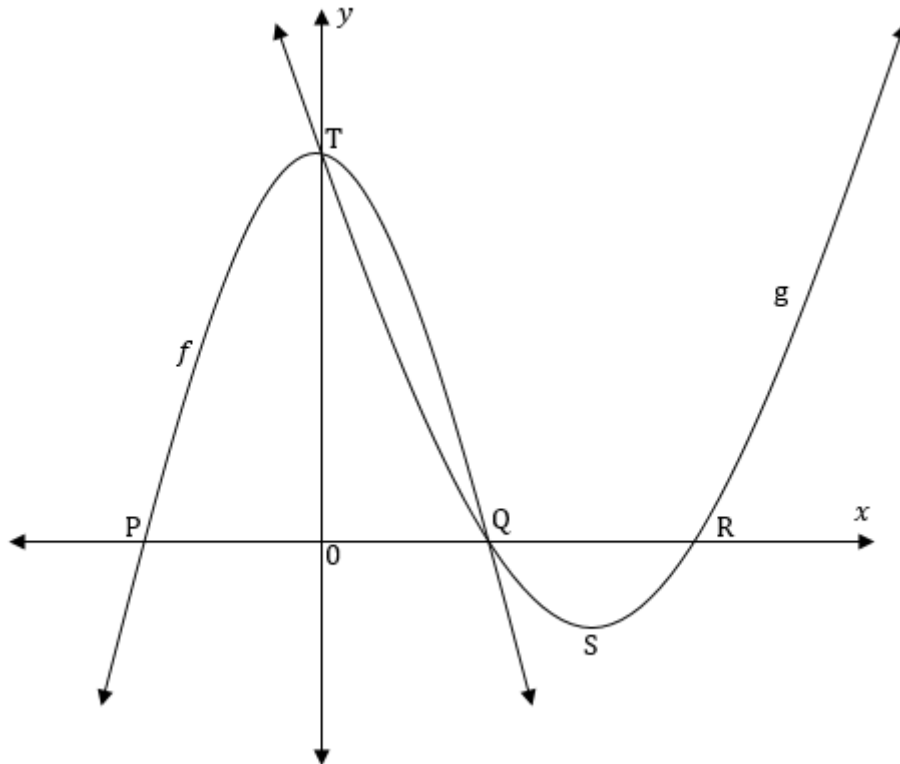
- 5.1 For which values of x is $f(x) \leq 0$? (2)
- 5.2 Determine the values of a , b and c . (5)
- 5.3 Determine the coordinates of the turning point of f . (3)
- 5.4 Write down the equation of the axis of symmetry of h if $h(x) = f(x - 7) + 2$. (2)
- 5.5 Calculate the gradient of g . (3)

[15]

Activity 6

The graphs of $f(x) = -2x^2 + 18$ and of $g(x) = ax^2 + bx + c$ are sketched below.

Points P and Q are the x -intercepts of f . Points Q and R are the x -intercepts of g . S is the turning point of g . T is the y -intercepts of both f and g .



- 6.1 Write down the coordinates of T. (1)
- 6.2 Determine the coordinates of Q. (3)
- 6.3 Given that $x = 4,5$ at S, determine the coordinates of R. (2)
- 6.4 Determine the value(s) of x for which $g''(x) > 0$. (2)

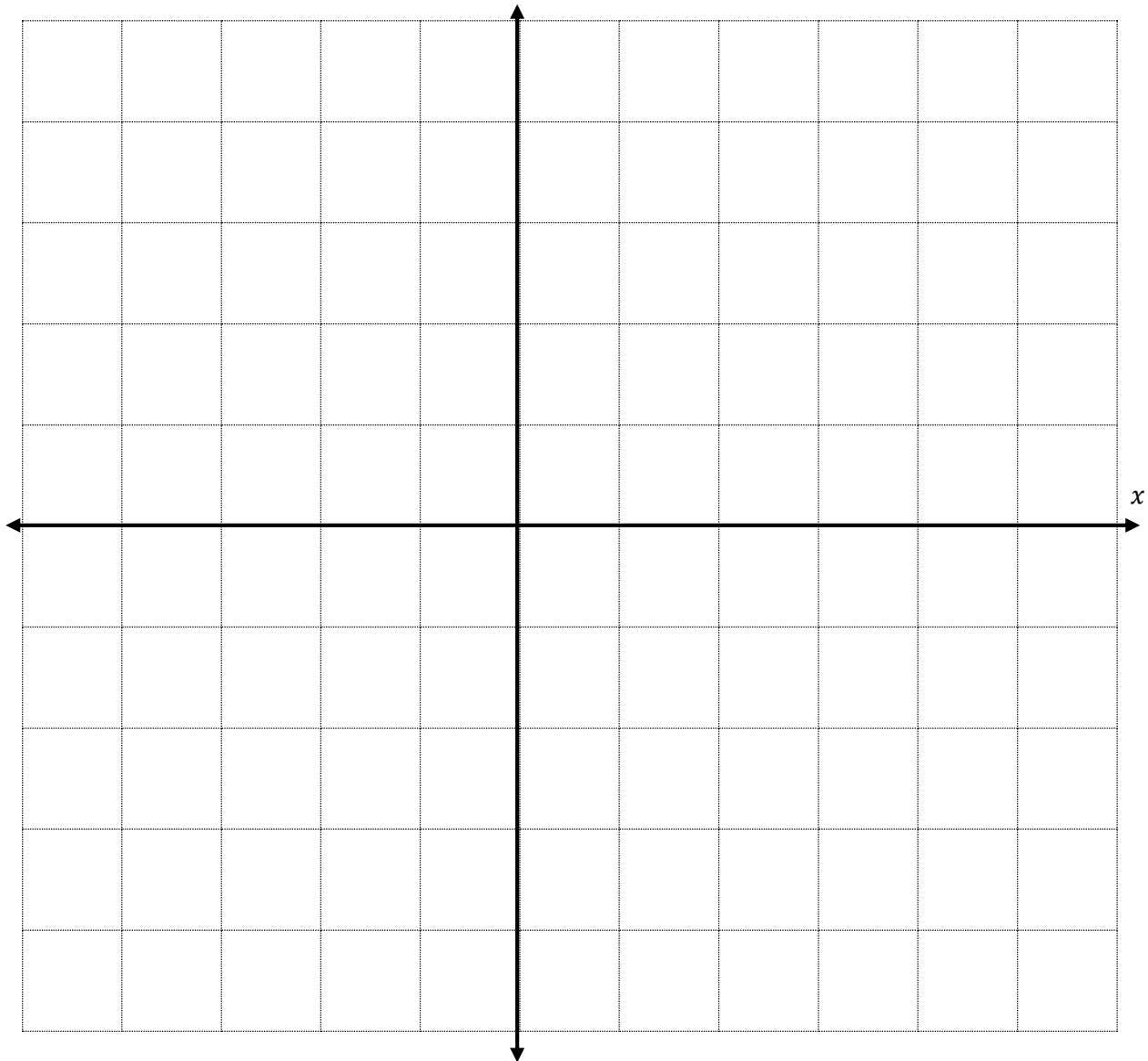
[8]



Hyperbola

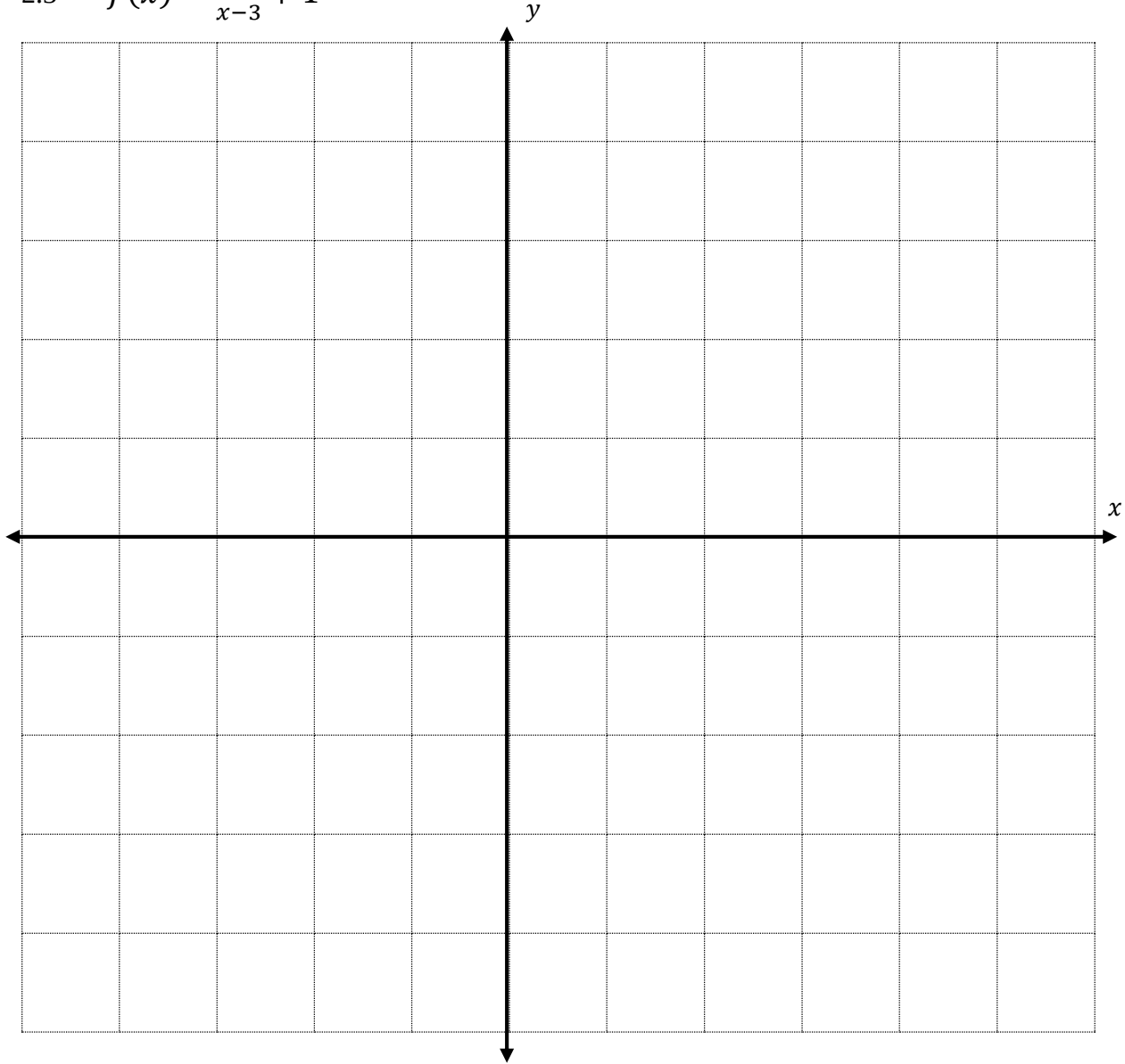
Tutorial 1

1.4 $f(x) = \frac{2}{x+1} + 1$



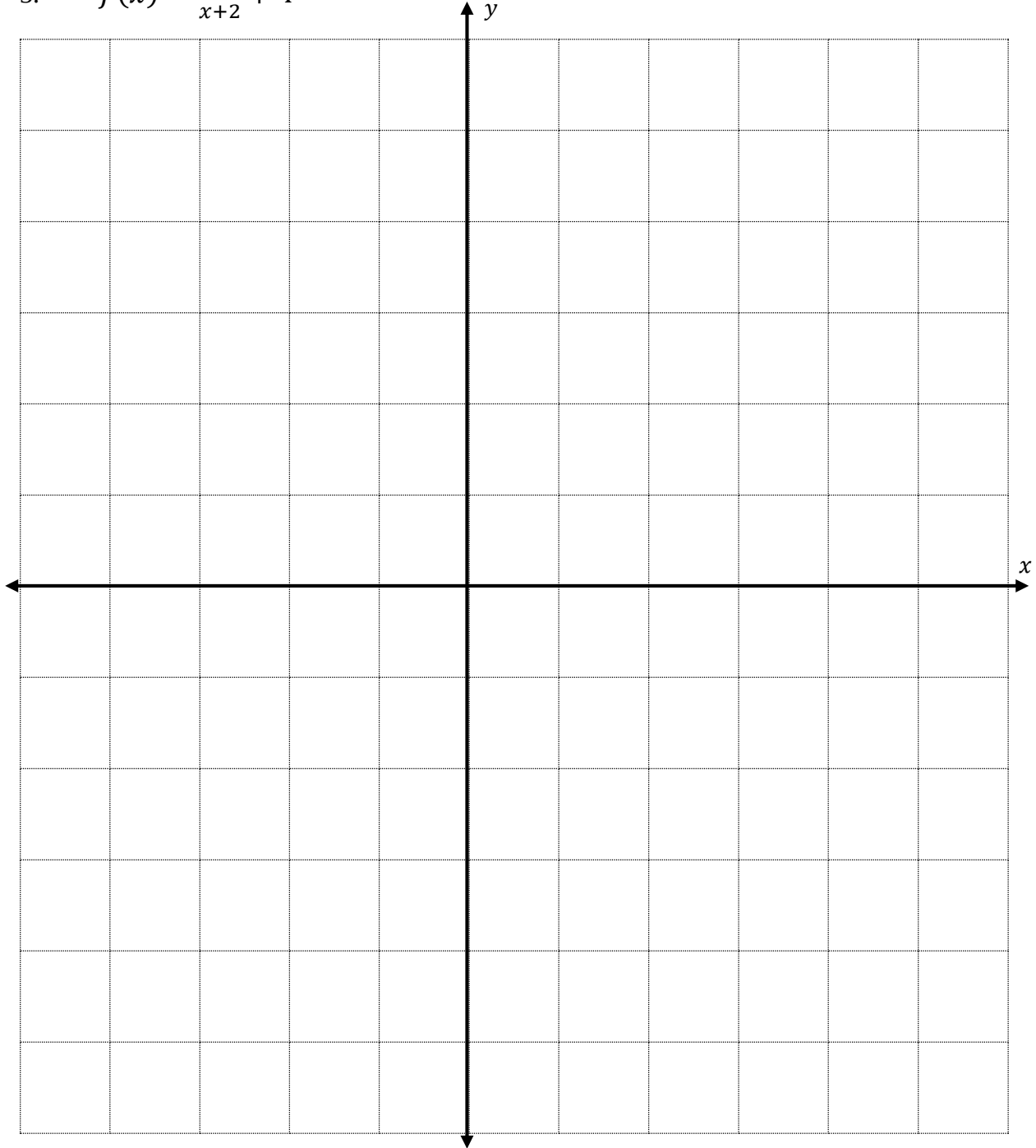


2.3 $f(x) = \frac{2}{x-3} + 1$



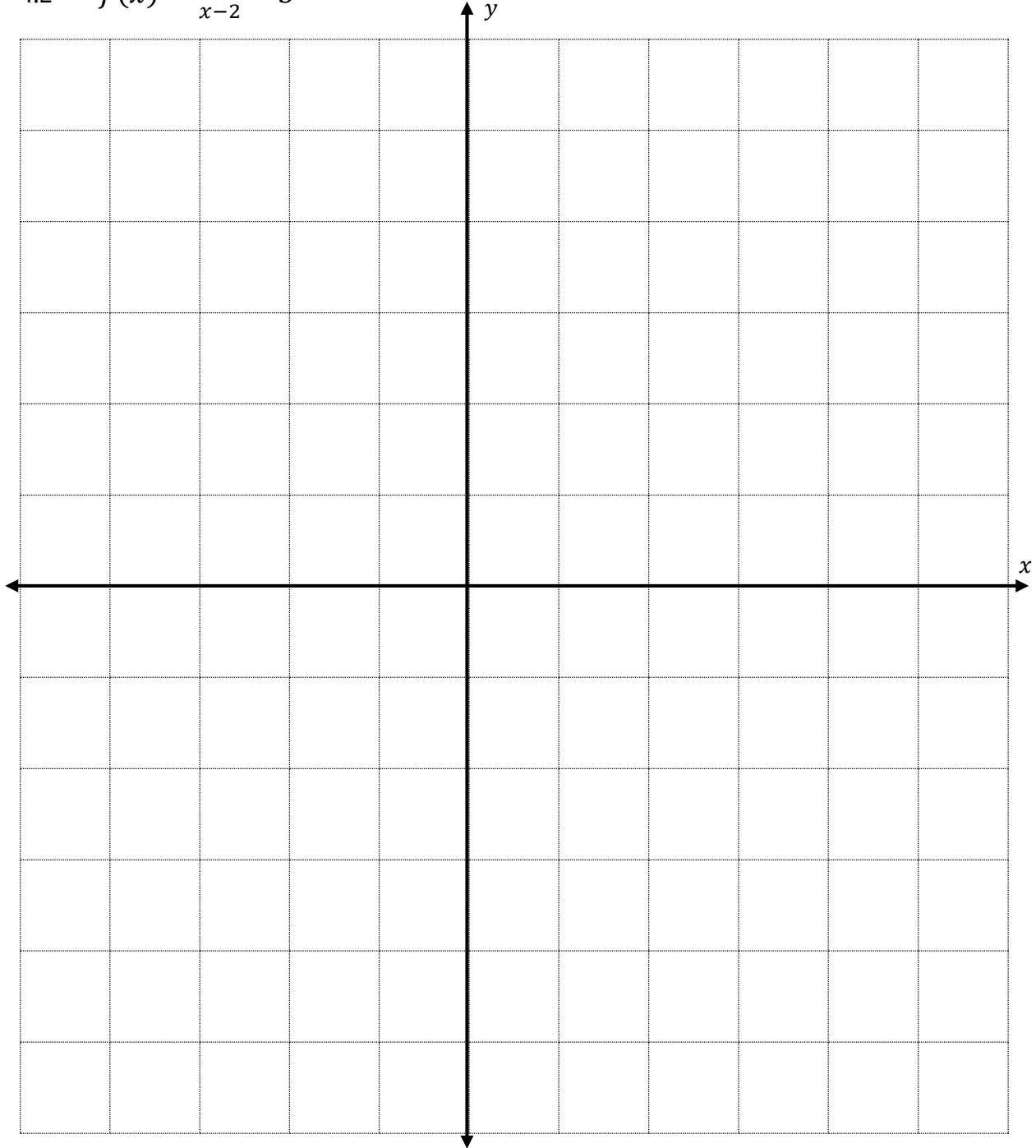


3. $f(x) = \frac{3}{x+2} + 4$



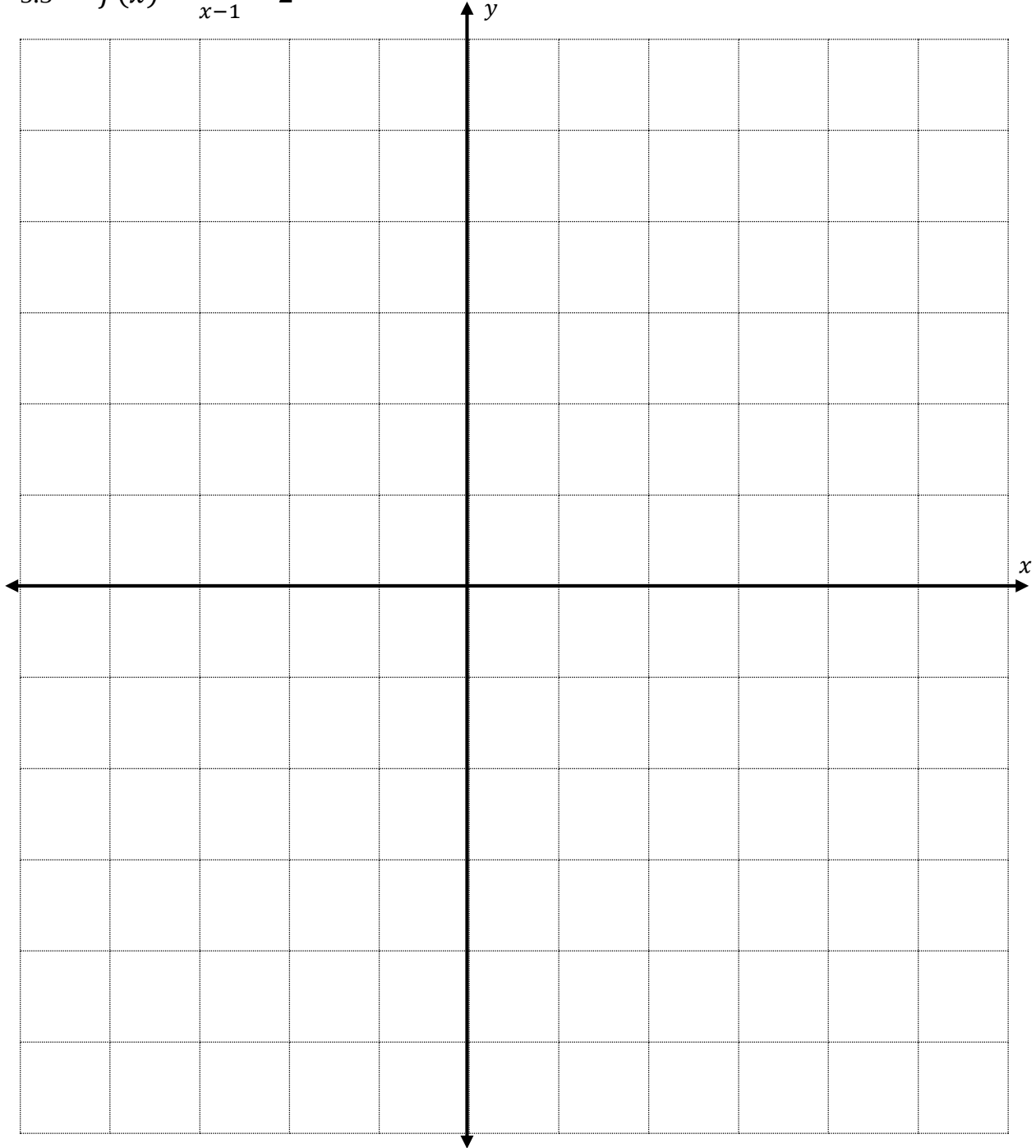


4.2 $f(x) = \frac{3}{x-2} - 3$





5.3 $f(x) = \frac{3}{x-1} - 2$

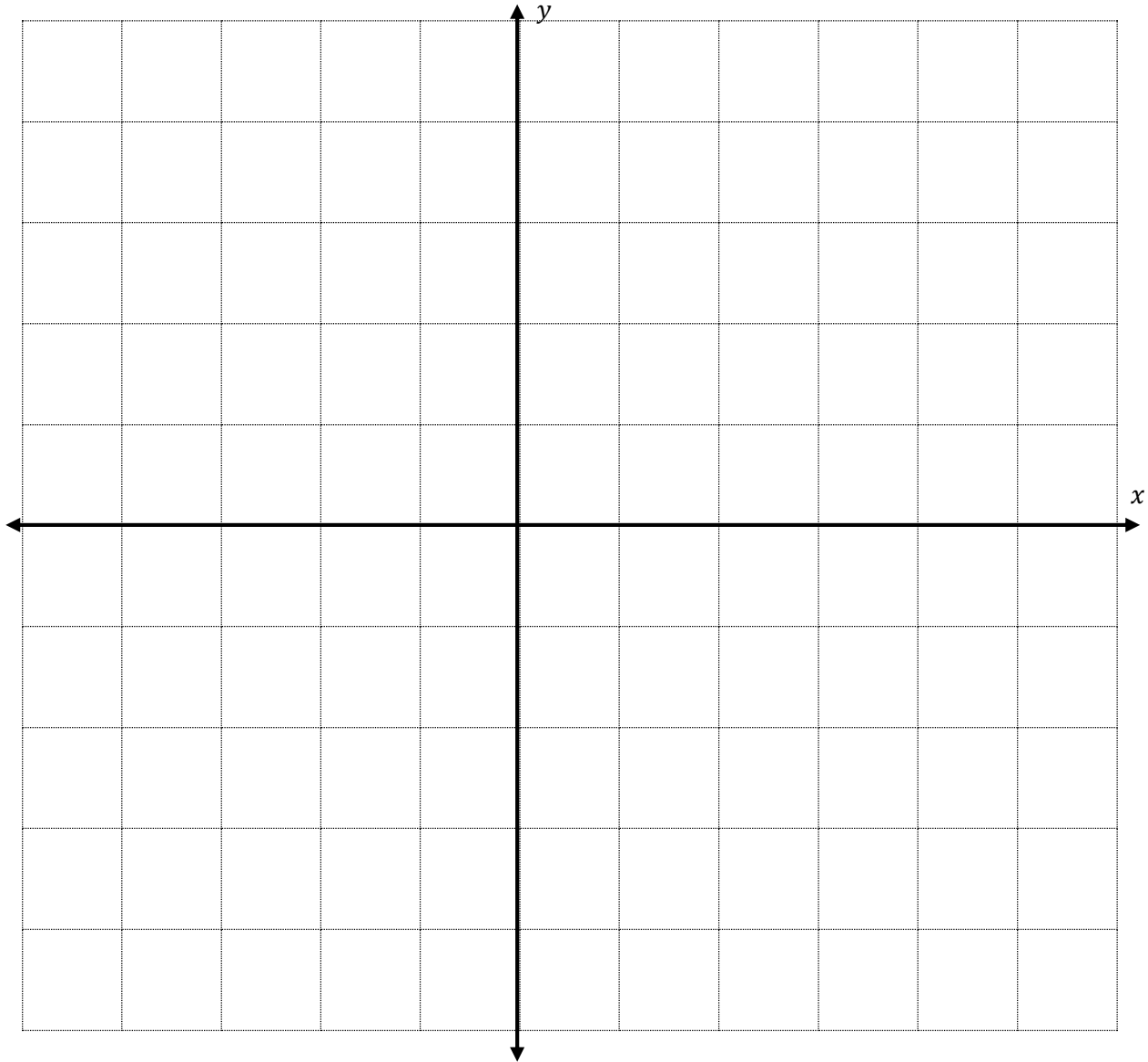




Exponential functions

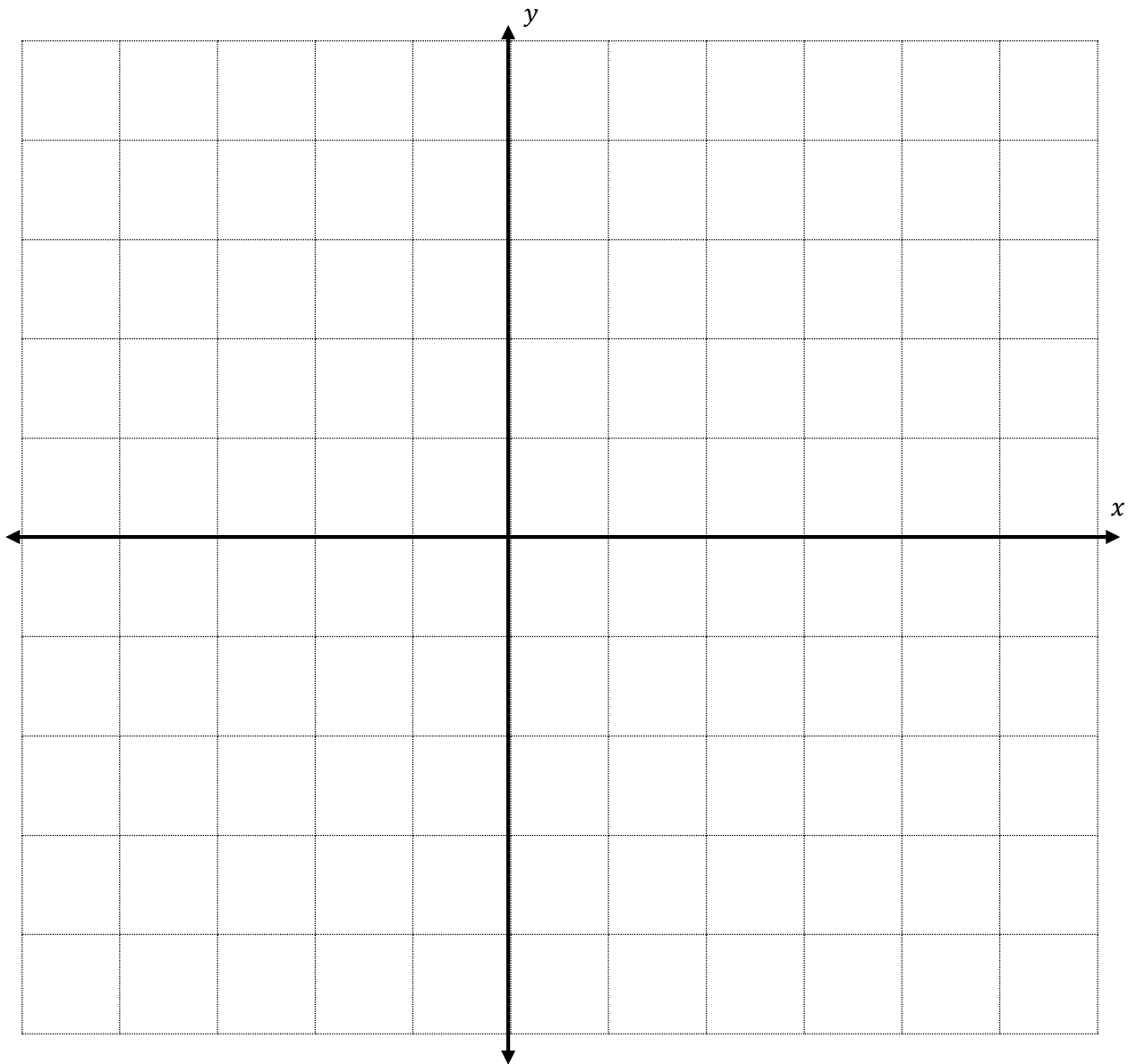
Tutorial 3

2.2 $f(x) = 3^{-x}$ and $f^{-1}(x) = \log_3 x$





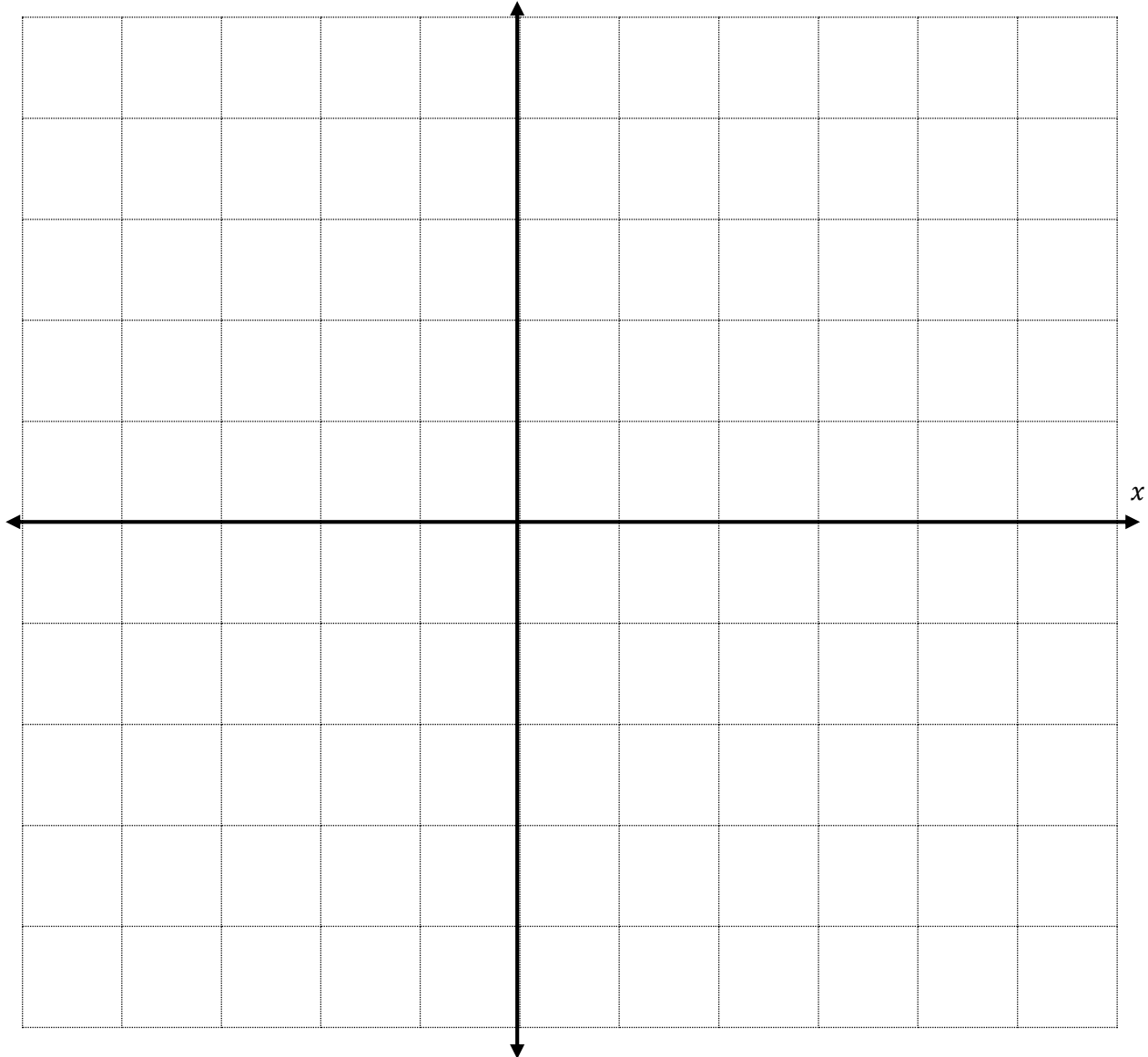
3.2 $f(x) = \left(\frac{1}{2}\right)^x$ and $f^{-1}(x) = \log_{\left(\frac{1}{2}\right)} x$





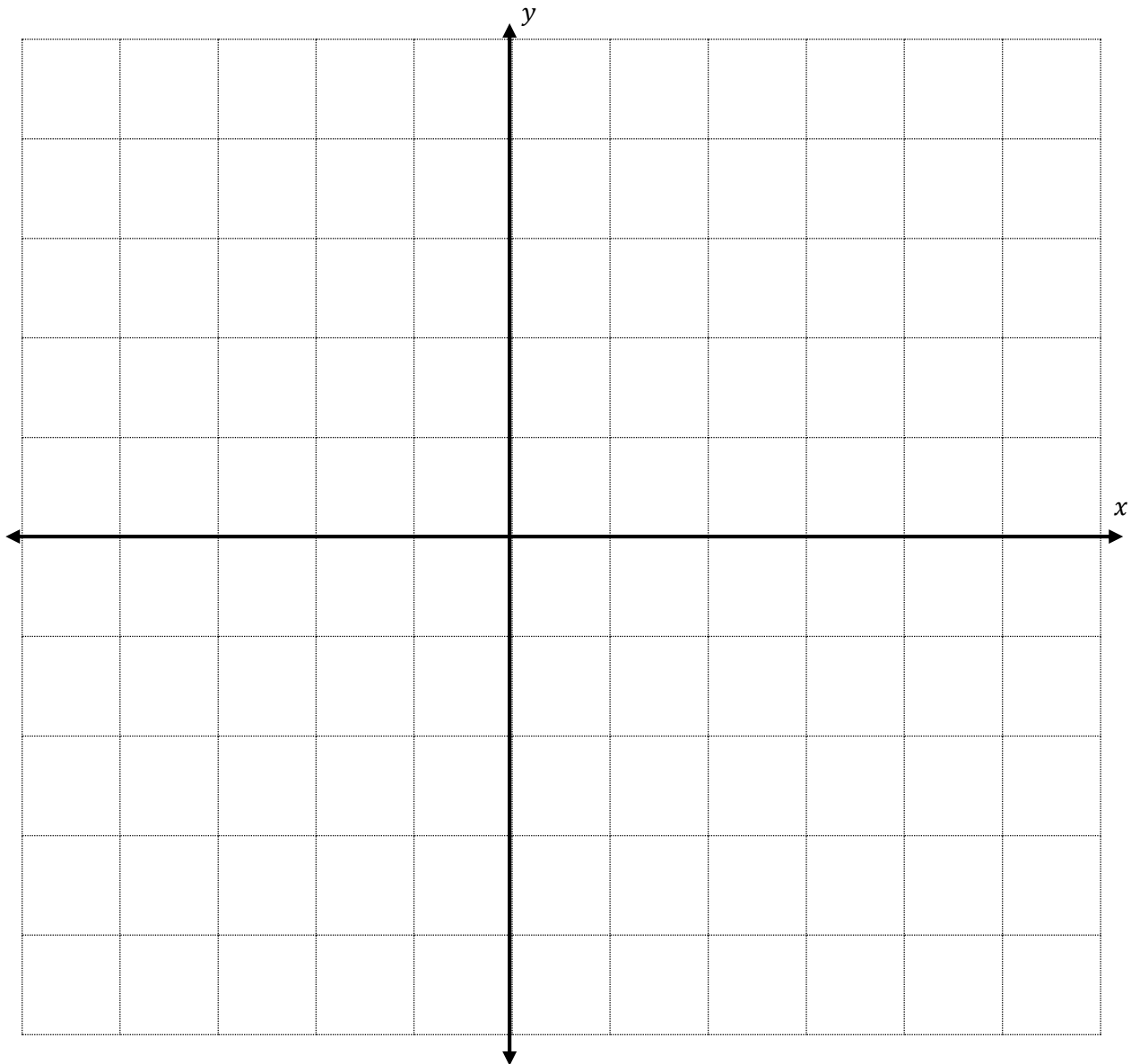
Tutorial 4

1.2.4 $h^{-1}(x) = +\sqrt{x}$ for $y \geq 0$



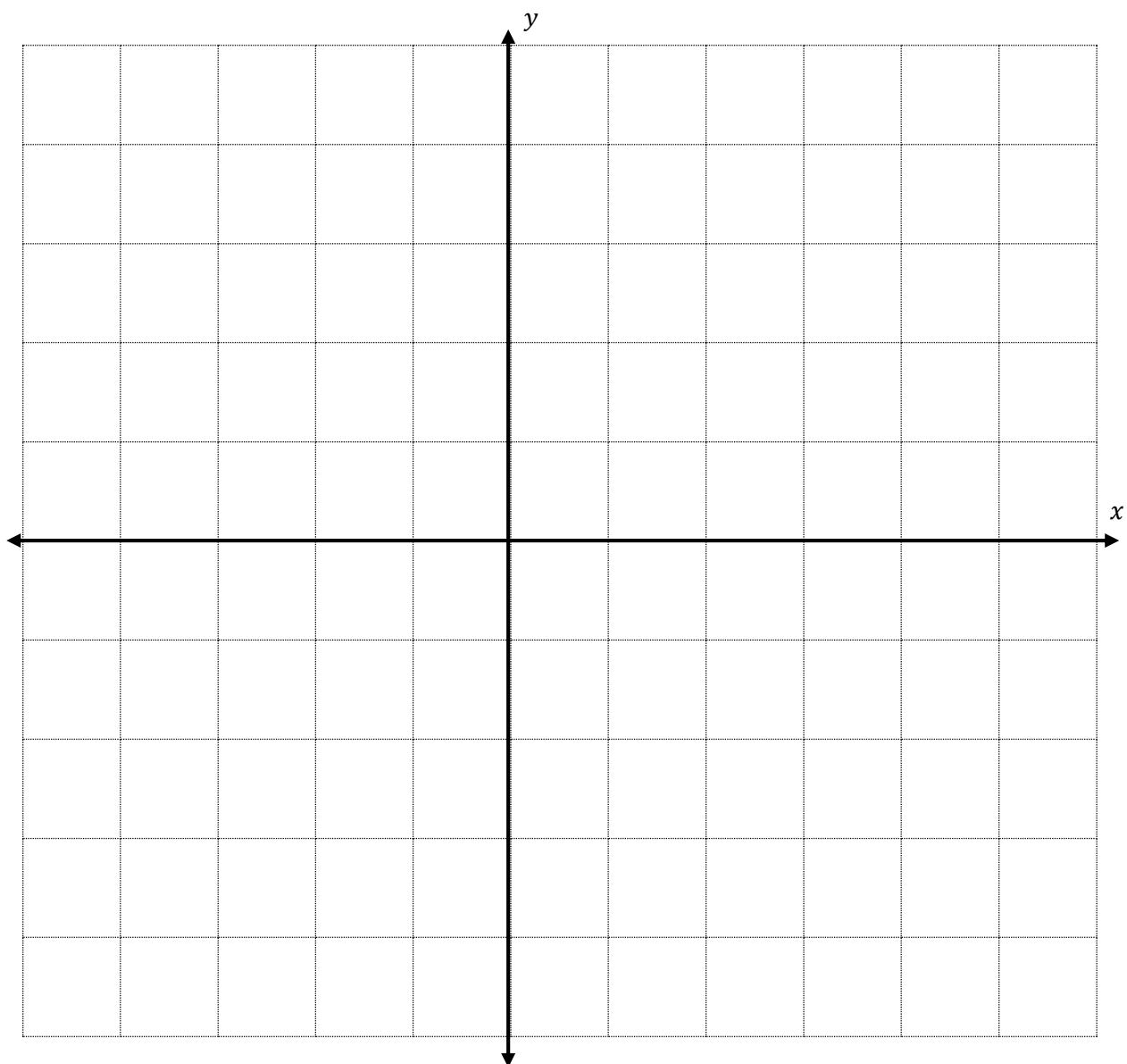


2.2 $f^{-1}(x) = \log_3(x - 2)$



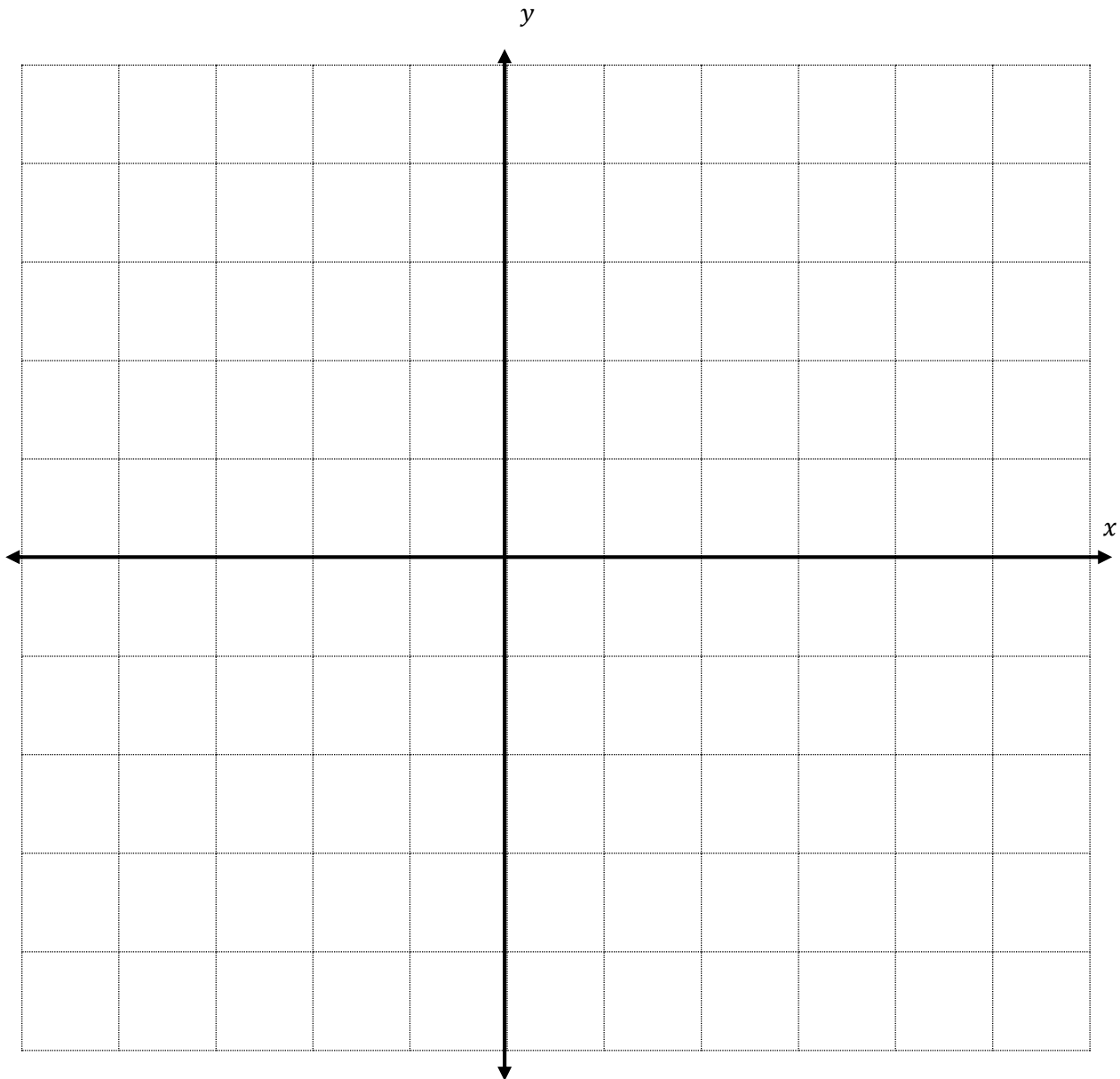


3.4 $f^{-1}(x) = \log_2 x$





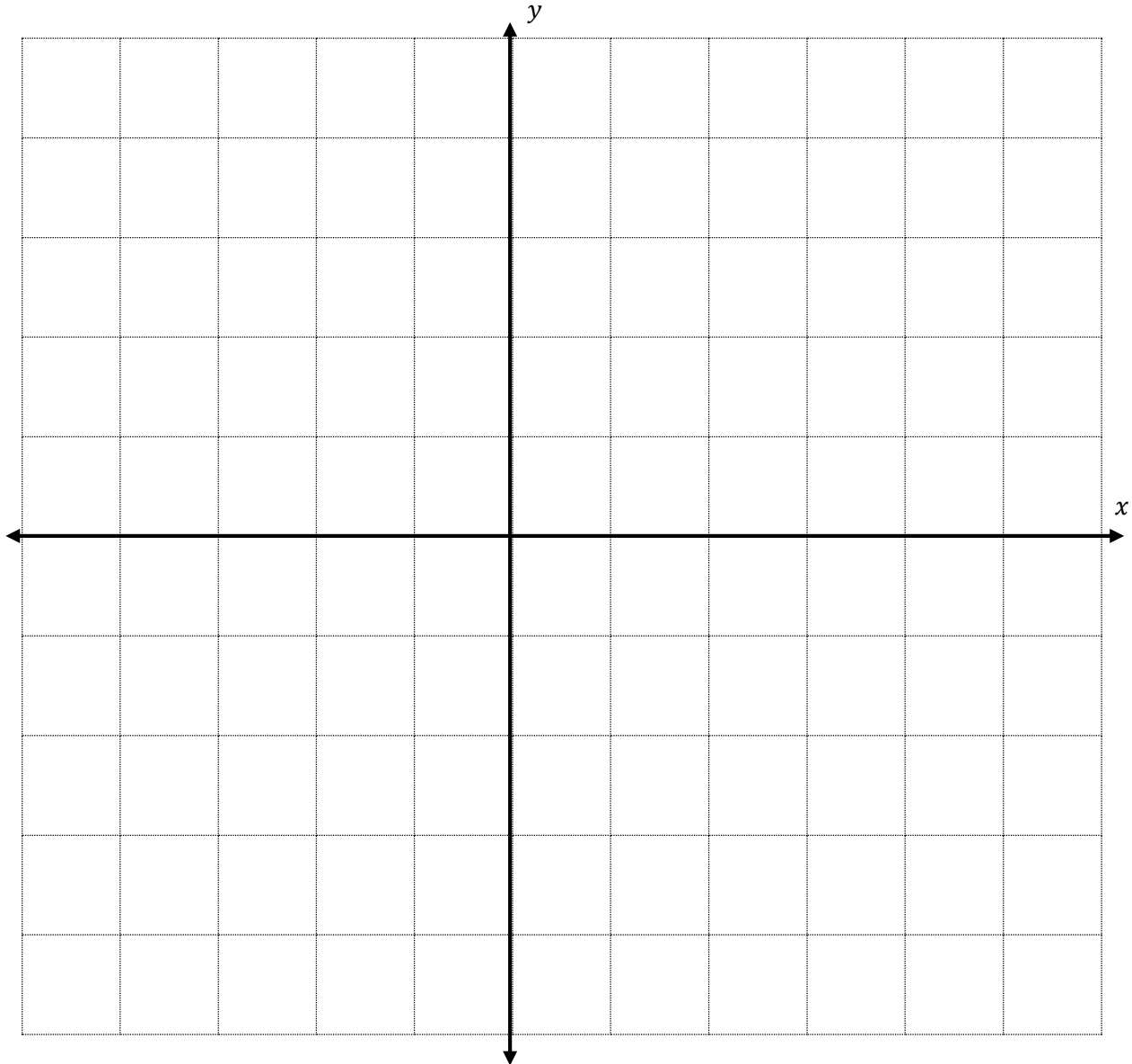
5.4 $f^{-1}(x) = \log_{\left(\frac{1}{3}\right)} x$





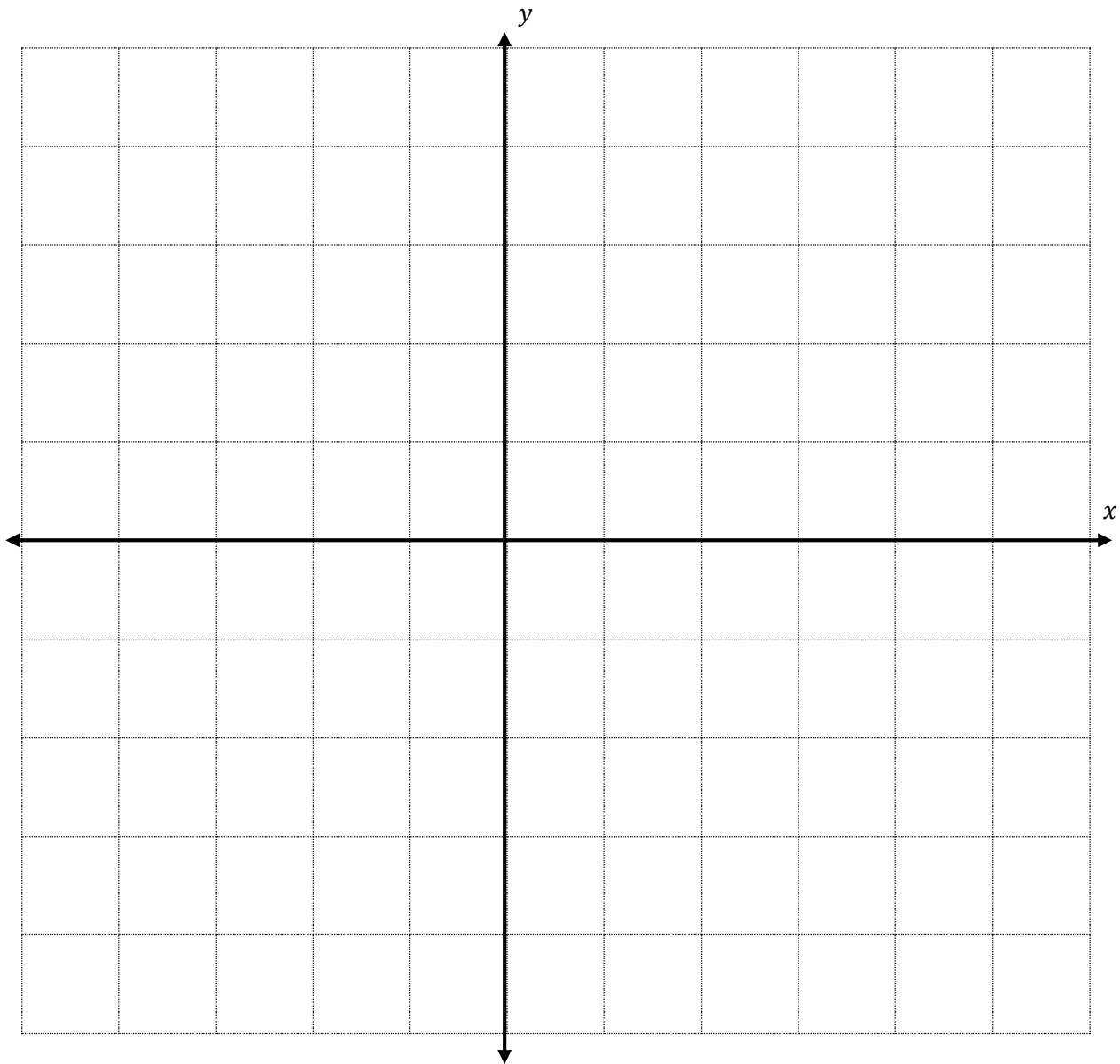
Tutorial 5

1.2 $g^{-1}(x) = \left(\frac{1}{2}\right)^x$



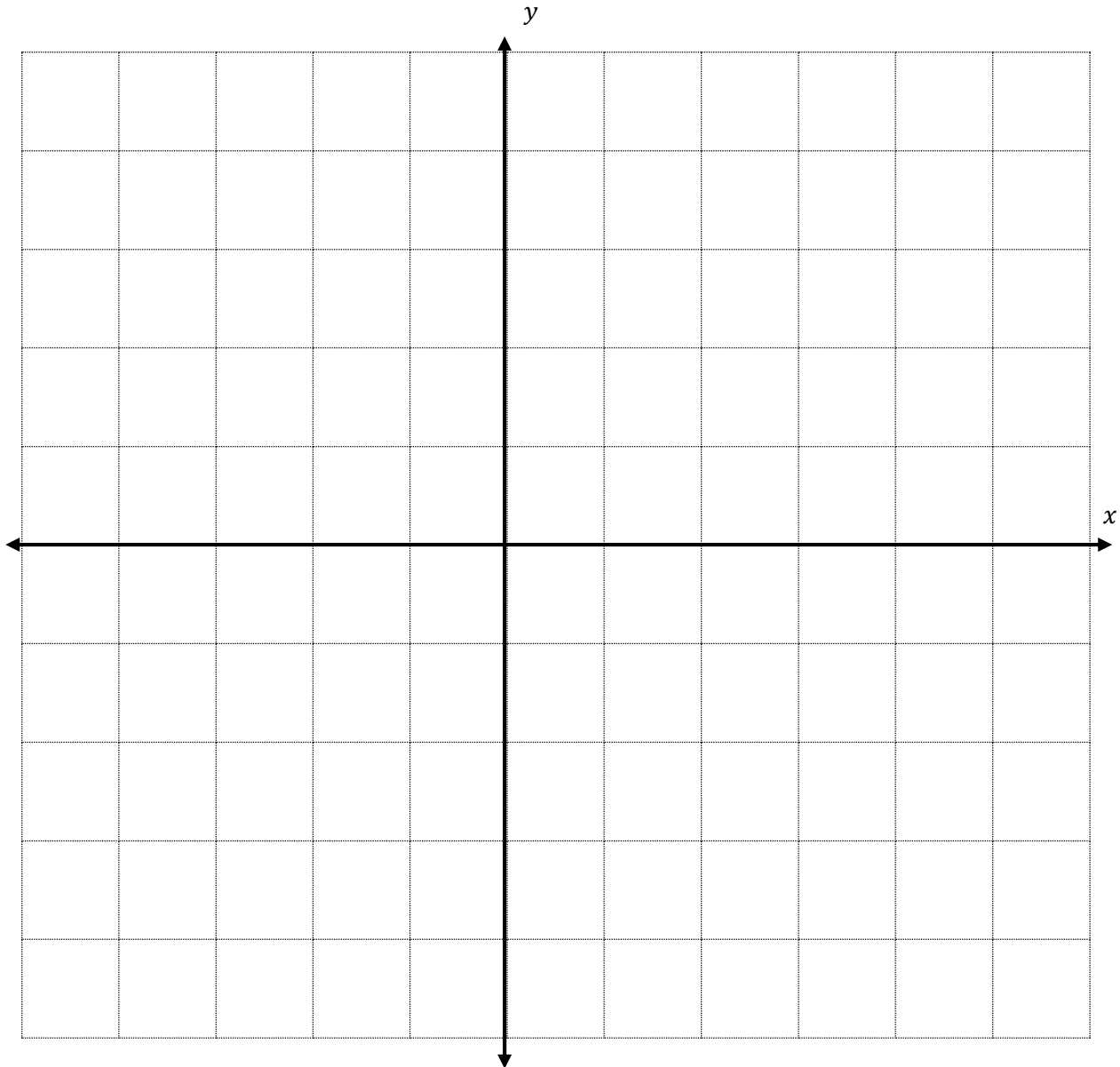


2.4 $f^{-1}(x) = -\sqrt{x}$ for $y \leq 0$



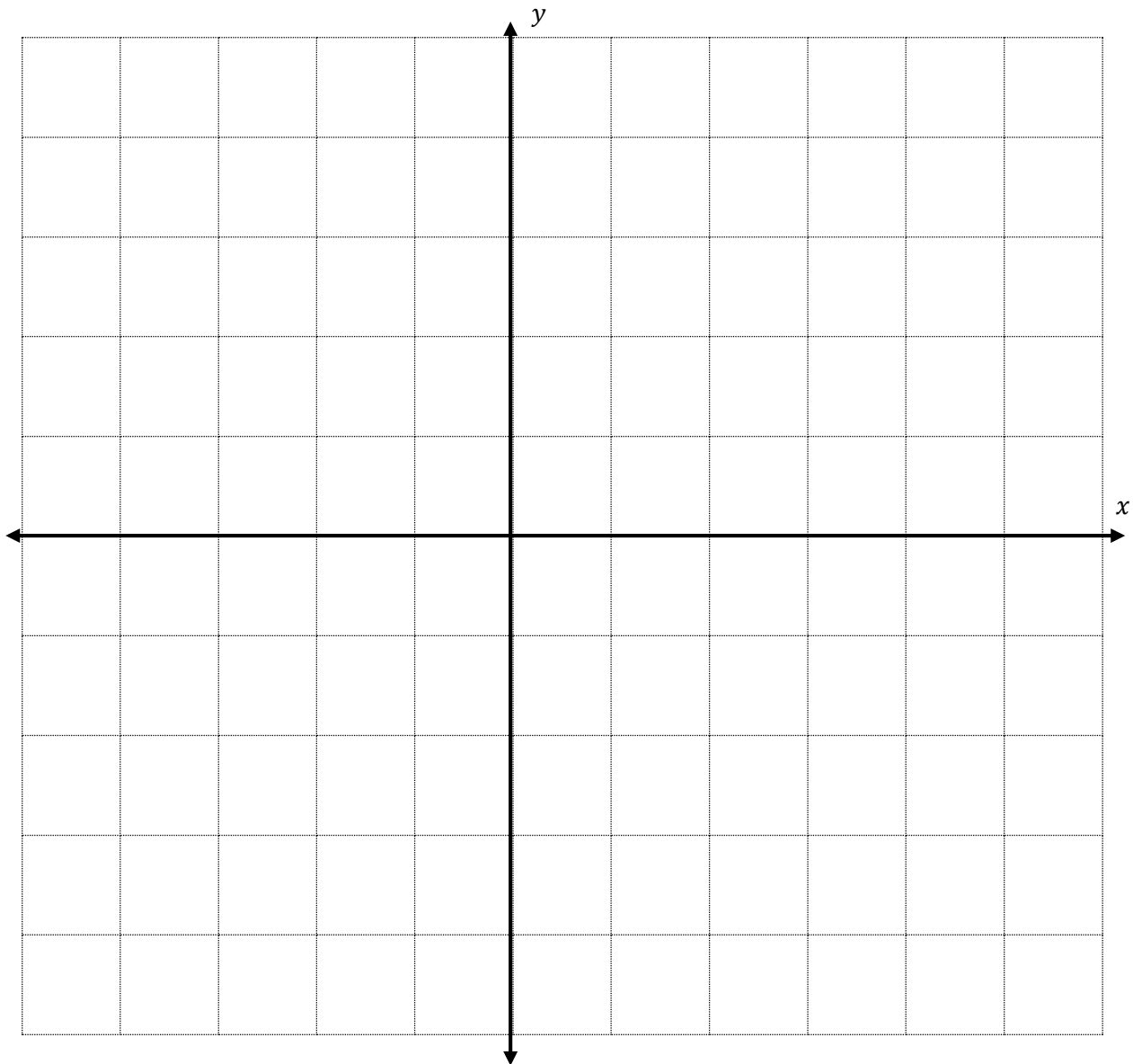


3.4 $f^{-1}(x) = \log_2 x$





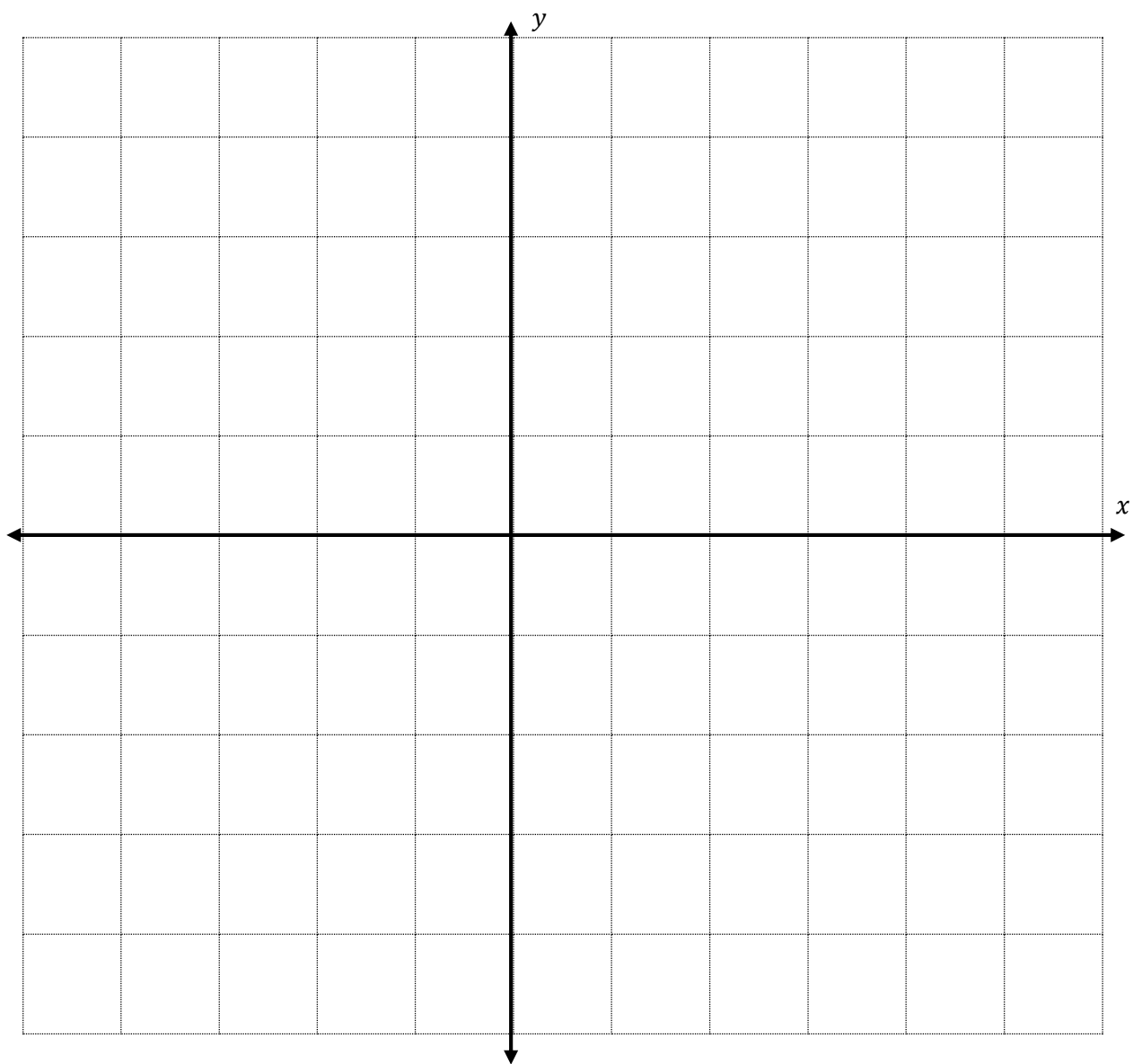
5.3 f^{-1} for $x \geq -2$





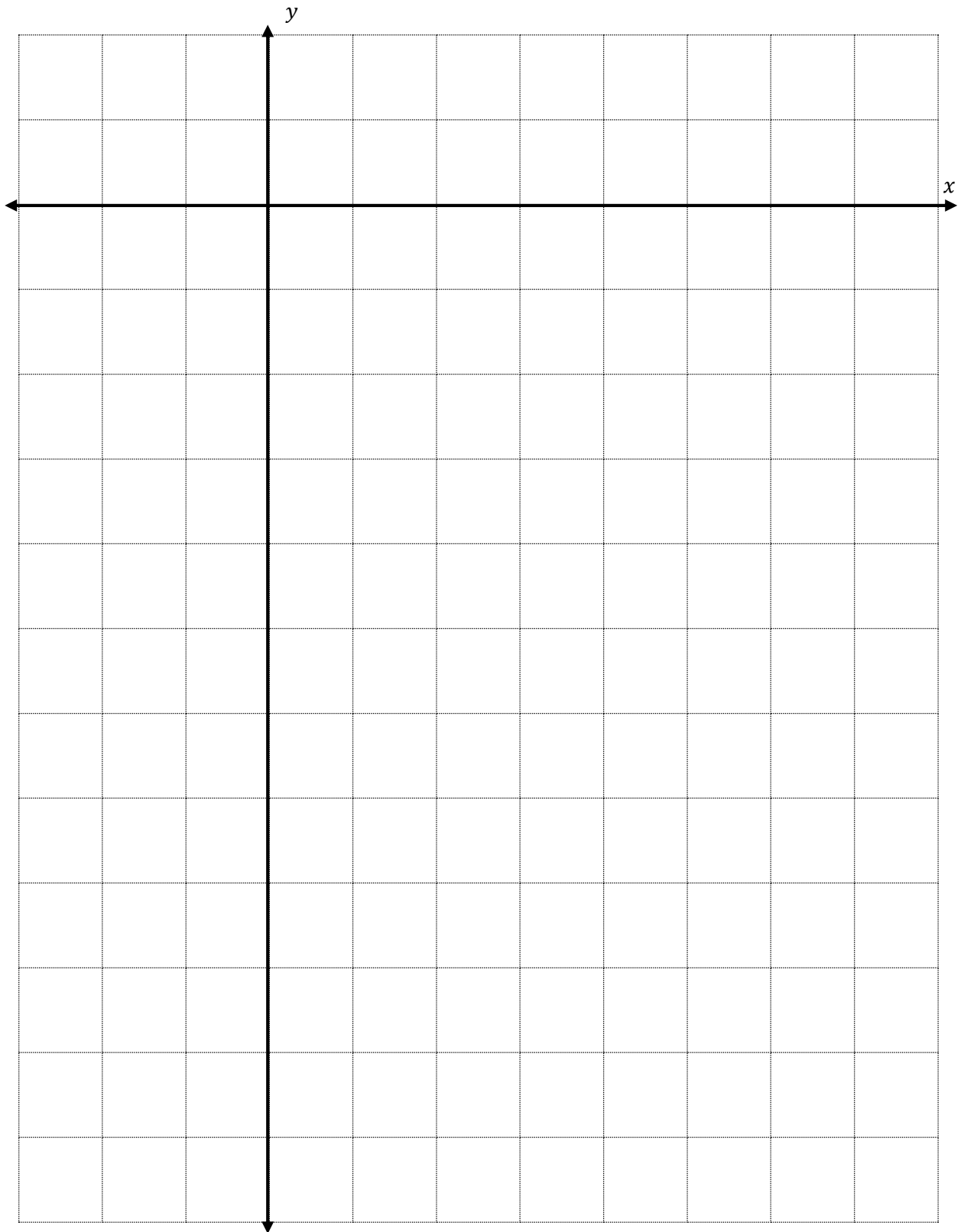
Tutorial 6

1.1



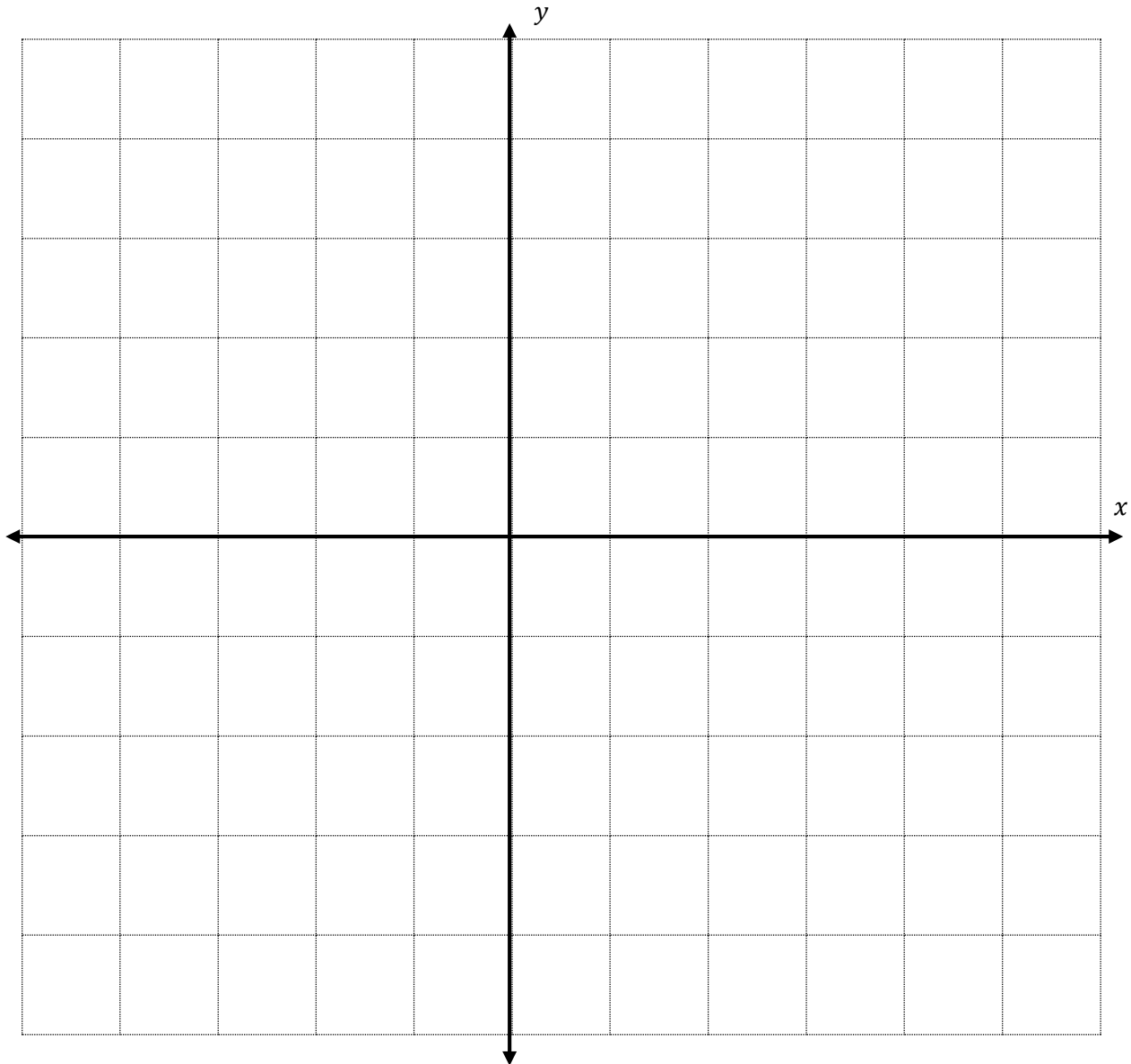


2.4 $f(x) = 2x^2 - 10x - 28$





3.1 $f(x) = -(x + 2)^2 + 6$ and $g(x) = 2^{-x} + 1$





Tutorial 8

3.4 g^{-1}

