

## Prepared by Lemniscates Academy ∞

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1 hour 15 minutes

You will need: List of formulae (MF19)

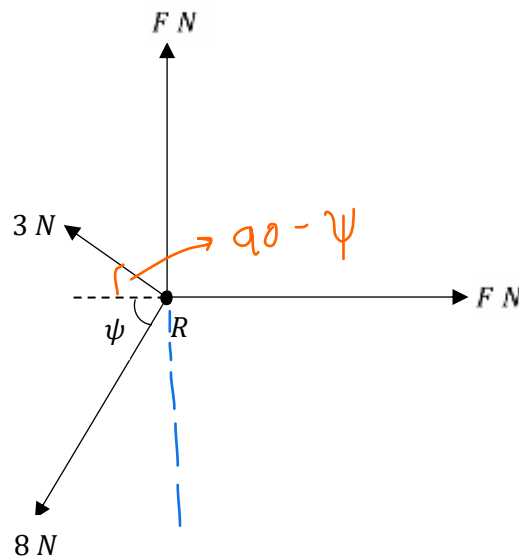
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity ( $g$ ) is needed, use  $10\text{ m s}^{-2}$ .

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

- AS & A Level Mathematics
- AS & A Level Physics
- AS & A Level Further Mathematics

1. [YouTube](#) lessons covering past papers, topic-wise practice, exam strategies, and more
2. Dedicated [WhatsApp doubt-solving groups](#) with support from both experts and peers
3. High-quality, reliable resources and guidance throughout your As and A Level preparations
4. Regular motivation and mindset support to help you stay focused and exam-ready

1.



A particle  $R$  of mass  $5 \text{ kg}$  is in equilibrium on a smooth horizontal table and four horizontal forces of magnitudes  $8 \text{ N}$ ,  $3 \text{ N}$ ,  $F \text{ N}$  and  $F \text{ N}$  acts on  $R$  in the directions shown. Given that  $3 \text{ N}$  force is perpendicular to  $8 \text{ N}$  force, find the values of  $\psi$  and  $F$ . [6]

→ Resolving Horizontally:  $3 \cos(90 - \psi) + 8 \cos(\psi) = F$   
 $\rightarrow 3 \sin(\psi) + 8 \cos(\psi) = F$  — (1)

→ Resolving Vertically:  $F + 3 \sin(90 - \psi) = 8 \sin(\psi)$   
 $\rightarrow F + 3 \cos(\psi) = 8 \sin(\psi)$  — (2)

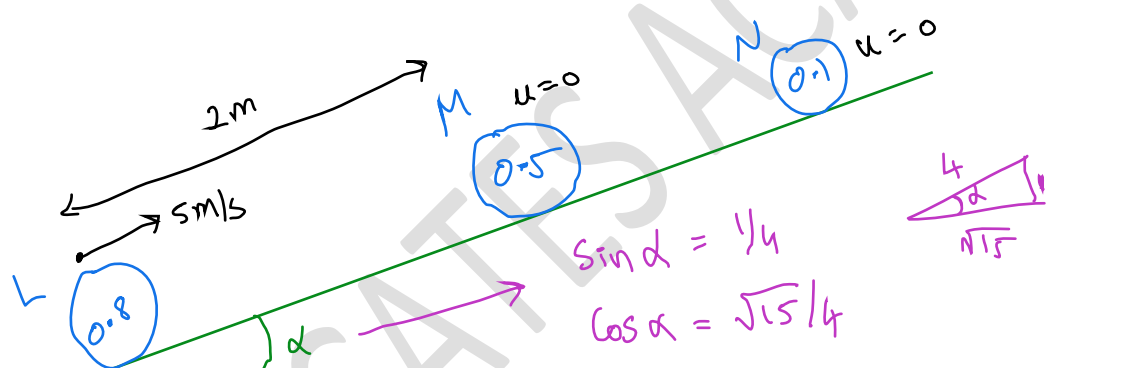
→ Substitute  $F$  from (1) in (2):  
 $\rightarrow 3 \sin(\psi) + 8 \cos(\psi) + 3 \cos(\psi) = 8 \sin(\psi)$   
 $\rightarrow 11 \cos(\psi) = 5 \sin(\psi)$   
 $\rightarrow \tan(\psi) = 11/5$   
 $\therefore \psi = \tan^{-1}(11/5)$

→ Substitute  $\psi$  in (1):  
 $\rightarrow F = 3 \sin(\tan^{-1}(11/5)) + 8 \cos(\tan^{-1}(11/5))$   
 $\therefore F = 6.0415 \dots \text{ N}$

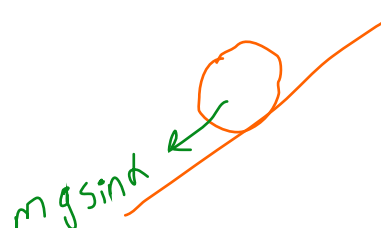
⇒  $\psi = 65.6^\circ (1 \text{ dp})$ ,  $F = 6.04 \text{ N} (3 \text{ sf})$

2. Three particles  $L$ ,  $M$  and  $N$ , have respective masses  $0.8 \text{ kg}$ ,  $0.5 \text{ kg}$ , and  $0.1 \text{ kg}$ . The three particles are placed on a smooth inclined plane such that  $\sin(\alpha) = \frac{1}{4}$  where  $\alpha$  is the angle of inclination of the plane to the horizontal. It is given that the particle  $L$  is at the foot of the plane, particle  $M$  is at a distance of  $2 \text{ m}$  up the plane and particle  $N$  is the furthest from  $L$  along the plane. Particle  $L$  is projected up the plane from the foot of the plane with speed of  $5 \text{ m s}^{-1}$  while particles  $M$  and  $N$  are released from rest at the same time as  $L$  was projected.

- (a) Find the acceleration down the plane on all the particles  $L$ ,  $M$  and  $N$ . [2]
- (b) Find the time taken for particles  $L$  and  $M$  to collide. [3]
- (c) Given that  $L$  and  $M$  coalesce into a single particle  $J$  after the collision. Determine the velocity of  $J$  just after the collision. [3]
- (d) Given that  $J$  and  $N$  collide and coalesce into a single particle  $K$ .  $K$  is formed  $0.6$  seconds after the formation of particle  $J$ . Determine the velocity of  $K$  just after the collision. [3]
- (e) Given that the entire inclined plane has a length of  $3 \text{ m}$ . Determine if the particle  $K$  comes to rest before it reaches the end of the inclined plane. [4]



(a) Only force acting on  $L$ ,  $M$  and  $N$  is the component of gravity down the plane.

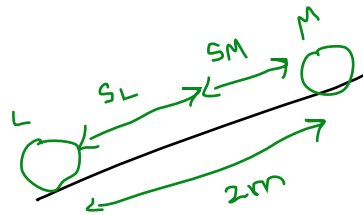


$$\begin{aligned} \rightarrow -mg \sin \alpha &= ma & [\text{Taking up the plane as positive}] \\ \rightarrow a &= -10(1/4) \\ \therefore a &= -2.5 \text{ m/s}^2 \end{aligned}$$

$\Rightarrow$  Acceleration down the plane for  $L$ ,  $M$  and  $N$  is  $2.5 \text{ m/s}^2$

$$(b) S_L = 5t - \frac{1}{2}(2.5)t^2$$

$$\rightarrow S_M = -\frac{1}{2}(2.5)t^2$$



L collides with M when  $S_L = 2 + S_M$

$$\therefore 5t - 1.25t^2 = 2 - 1.25t^2$$

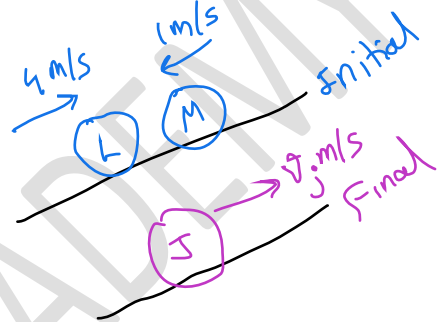
$$\Rightarrow t = 0.4 \text{ s}$$

$$(c) V_{Li} = 5 - 2.5(0.4) = 4 \text{ m/s}$$

$$\rightarrow V_{Mi} = -2.5(0.4) = -1 \text{ m/s}$$

$$\therefore (0.8)(4) - (0.5)(1) = (0.8+0.5)V_j$$

$$\Rightarrow V_j = \frac{27}{13} \text{ m/s}$$

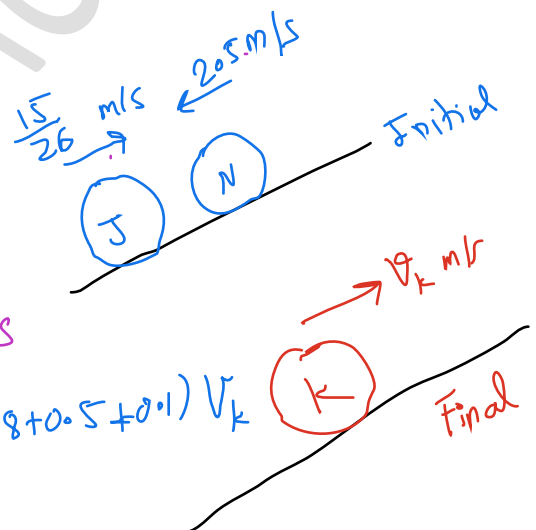


$$(d) V_j = \frac{27}{13} - (2.5)(0.6) = \frac{15}{26} \text{ m/s}$$

$$\rightarrow V_N = 0 - 2.5(0.4+0.6) = -2.5 \text{ m/s}$$

$$\therefore (0.8+0.5)\left(\frac{15}{26}\right) - (0.1)(2.5) = (0.8+0.5+0.1)V_k$$

$$\Rightarrow V_k = \frac{5}{14} \text{ m/s}$$



$$(e) \text{ Distance from foot to J formation: } S = 5(0.4) - 1.25(0.4)^2 = 1.8 \text{ m}$$

$$\rightarrow \text{Distance from J to K formation: } S = \left(\frac{27}{13}\right)(0.6) - 1.25(0.6)^2 = \frac{207}{260} \text{ m}$$

$$\rightarrow \text{Distance from K till } V_k = 0: 0^2 = \left(\frac{5}{14}\right)^2 - 2(2.5)S \Rightarrow S = \frac{5}{146} \text{ m}$$

$$\rightarrow \text{Total distance} = 1.8 + \frac{207}{260} + \frac{5}{146} = 2.6216... \text{ m}$$

$\therefore$  K comes to instantaneous rest before the inclined plane ends.

3. The velocity with which the particle  $M$  is moving in a straight line is given by

$$v = 3t^2 + at + b, \quad 0 \leq t \leq 5$$

- (a) It is given that at time  $t = 2$  s and  $t = 4$  s,  $M$  is instantaneously at rest.  
Find the time at which the velocity is a minimum and find this minimum value. [3]
- (b) Prove that the maximum deceleration experienced by  $M$  is when the speed of  $M$  is greatest.  
You must show the values for that deceleration, speed and the time when this happens. [4]
- (c) Given that  $M$  is at point  $A$  when  $t = 0$  s and at point  $B$  when  $t = 5$  s.  
Calculate the total distance that  $M$  has covered in the entire journey from  $A$  to  $B$ . [5]
- (d) Calculate how far is point  $B$  from point  $A$ . [2]

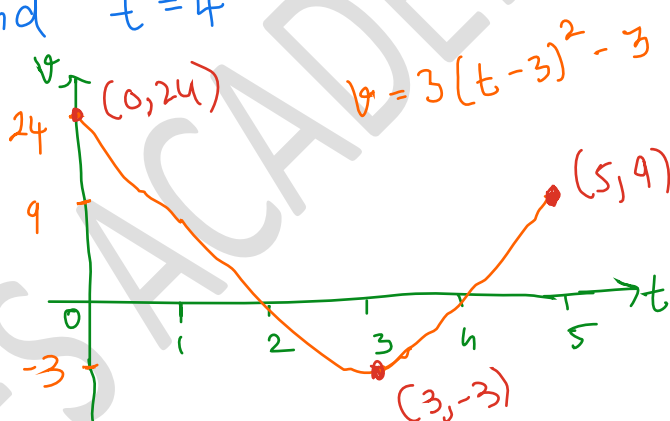
(a)  $v = 0$  at  $t = 2$  and  $t = 4$

$$\rightarrow 3t^2 + at + b = 0$$

$$\rightarrow 3(t-2)(t-4) = 0$$

$$\rightarrow 3t^2 - 18t + 24 = 0$$

$$\therefore v = 3t^2 - 18t + 24$$



From the graph:

$$\rightarrow v_{\min} = -3 \text{ m/s at } t = 3 \text{ s}$$

(b) At  $t = 0$ , the speed of  $M$  is maximum of  $24 \text{ m/s}$

$$\rightarrow a = \frac{dv}{dt} = 6t - 18$$

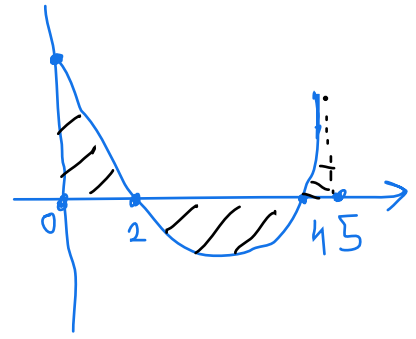
$$\rightarrow a = -18 \text{ m/s}^2 \text{ at } t = 0 \text{ s}$$

$\rightarrow$  The steepness of gradients at every point on the curve is smaller than at  $t = 0$ .

$$\therefore \text{At } t = 0 \text{ s, Max speed} = 24 \text{ m/s, Max deceleration} = 18 \text{ m/s}^2$$

$$(c) v = 3t^2 - 18t + 24$$

$$\rightarrow \text{Distance}_{AB} = \int_0^2 v \cdot dt + \left| \int_2^4 v \cdot dt \right| + \int_4^5 v \cdot dt$$



$$\rightarrow \int (3t^2 - 18t + 24) dt = t^3 - 9t^2 + 24t + C$$

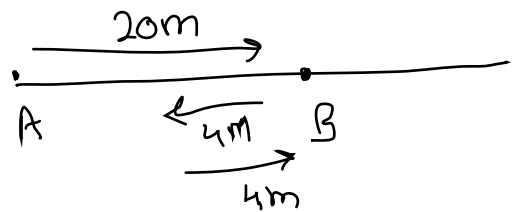
$$\rightarrow \left[ t^3 - 9t^2 + 24t \right]_0^2 = 20 - 0 = 20 \text{ m}$$

$$\rightarrow \left| \left[ t^3 - 9t^2 + 24t \right]_2^4 \right| = |16 - 20| = 4 \text{ m}$$

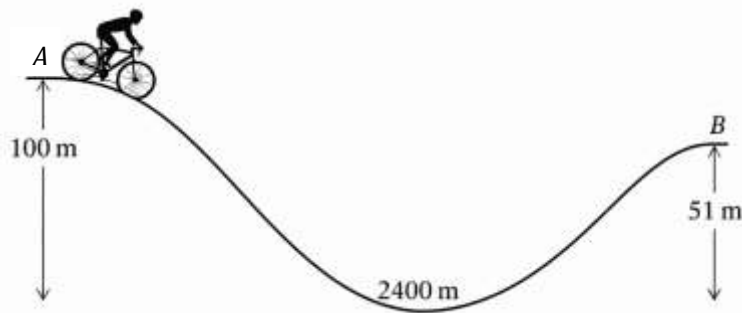
$$\rightarrow \left[ t^3 - 9t^2 + 24t \right]_4^5 = 20 - 16 = 4 \text{ m}$$

$\therefore$  Distance Covered by M from A to B = 28 m

(d) Point B is 20 m away from Point A.



4.



The diagram shows the path a cyclist takes from point A to point B which is a total of 2400 m stretch. The cyclist has a speed of  $k$  m/s as it leaves the point A and descends a vertical height of 100 m. He then climbs a vertical height of 51 m and reaches the point B as shown in the diagram.

Assume that the cyclist is working at a constant rate of 40 W throughout the motion from A to B and experience a constant resistance of 20 N. The time taken for the journey is 220 seconds.

Given that the combined mass of the cyclist and his cycle is 80 kg, find the speed of the cyclist in terms  $k$  as he reaches the point B.

[5]

$$\rightarrow \Delta W_D = \Delta KE + \Delta PE$$

$$\rightarrow W_{Df} - W_{Rf} = (KE_B - KE_A) + (PE_B - PE_A)$$

$$\rightarrow W_{Df} = P \times t = 40(220) = 8800 \text{ J}$$

$$\rightarrow W_{Rf} = F_r \times d = 20(2400) = 48000 \text{ J}$$

$$\rightarrow \Delta KE = \frac{1}{2} m \Delta v^2 = \frac{1}{2} (80) (v_B^2 - k^2) \text{ J}$$

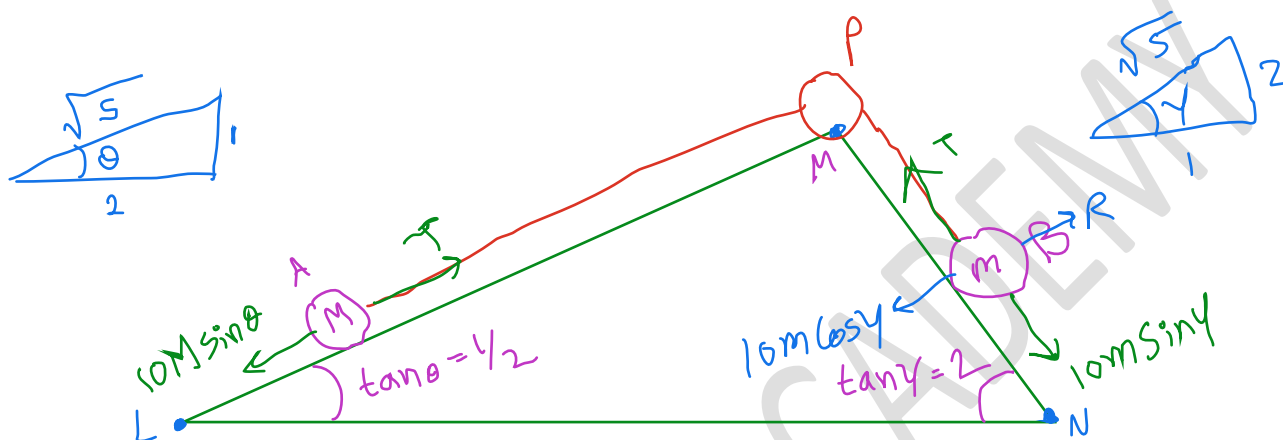
$$\rightarrow \Delta PE = mg \Delta h = (80)(10)(51 - 100) = -39200 \text{ J}$$

$$\therefore 8800 - 48000 = 40(v_B^2 - k^2) - 39200$$

$$\Rightarrow -39200 = 40(v_B^2 - k^2) - 39200$$

$$\Rightarrow v_B = k \text{ m/s}$$

5. Two particles A and B of masses  $M$  and  $m$  respectively, are attached to the ends of the light inextensible string. The particle A lies on a smooth inclined plane LM and the particle B lies on a rough inclined plane NM. The LM is inclined at  $\theta^\circ$  above the horizontal such that  $\tan\theta = \frac{1}{2}$  and NM is inclined at  $\gamma^\circ$  above the horizontal such that  $\tan\gamma = 2$ . The inclined planes are connected at point M where a smooth and small pulley P is fixed over which the string that connects the particles A and B passes. Derive an equation that connects  $M$ ,  $m$  and  $\mu$  where  $\mu$  is a coefficient of friction between the plane NM and particle B using the least and the greatest possible values of the tensions in the string. Basically, you need to derive two such equations in total. Also draw a diagram to support your working. [10]



→ Max  $T$  when A is about to slide down the plane

$$A: T = 10M \left( \frac{1}{\sqrt{5}} \right)$$

$$B: T = 10m \left( \frac{2}{\sqrt{5}} \right) + \mu \left( 10m \cdot \frac{1}{\sqrt{5}} \right)$$

$$\Rightarrow \frac{10M}{\sqrt{5}} = \frac{20m}{\sqrt{5}} + \frac{10\mu m}{\sqrt{5}}$$

$$\Rightarrow M = m(2 + \mu)$$

→ Min  $T$  when A is about to slide up the plane

$$A: T = \frac{10M}{\sqrt{5}}$$

$$B: T + \frac{10\mu m}{\sqrt{5}} = \frac{20m}{\sqrt{5}}$$

$$\Rightarrow \frac{10M}{\sqrt{5}} + \frac{10\mu m}{\sqrt{5}} = \frac{20m}{\sqrt{5}}$$

$$\therefore M = m(2 - \mu)$$

LEMNISCATES ACADEMY