



Be-Humble!

Cambridge International AS & A Level

CANDIDATE
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MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1

October/November 2025

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 1 Find the set of values of the constant k for which the quadratic equation

$$3kx^2 + (k+8)x + 3 = 0$$

has two distinct real roots.

[4]

$$b^2 - 4ac > 0$$

$$(k+8)^2 - 4(3k)(3) > 0$$

$$k^2 + 16k + 64 - 36k > 0$$

$$k^2 - 20k + 64 = 0$$

$$k^2 - 16k - 4k + 64 = 0$$

$$k(k-16) - 4(k-16) = 0$$

$$(k-16)(k-4) = 0$$

Critical values: $k=16, k=4$

$$k > 16 \text{ and } k < 4$$



- 2 A geometric progression has first term a and common ratio $\cos \theta$, where $0 < \theta < \frac{1}{2}\pi$. It is given that the second term is 8 and the fifth term is $\frac{1}{8}$.

(a) Find the value of θ . Give your answer correct to 3 significant figures.

[3]

$$a \cos \theta = 8$$

$$\frac{8}{a} = \cos \theta$$

$$a \cos^4 \theta = \frac{1}{8}$$

$$a \left(\frac{4096}{a^4} \right) = \frac{1}{8}$$

$$\frac{4096}{a^3} = \frac{1}{8}$$

$$a^3 = 32768$$

$$a = 32$$

$$\cos \theta = \frac{8}{32}$$

$$\cos \theta = \frac{1}{4}$$

$$\theta = \cos^{-1}\left(\frac{1}{4}\right)$$

$$\theta = 1.32$$

(b) Find the exact value of the sum to infinity.

[2]

$$S_{\infty} = \frac{a}{1-r}$$

$$S_{\infty} = \frac{32}{1-\frac{1}{4}}$$

$$S_{\infty} = \frac{128}{3}$$



3 In the expansion of

$$(px+3)^5 - \left(x^3 + \frac{p}{x}\right)^4,$$

the coefficient of x^4 is 216.

Find the value of the positive constant p .

[5]

$$(px+3)^5 = {}^5C_r (px)^r (3)^{5-r}$$

$$T_5 = {}^5C_4 (px)^4 (3)$$

$$T_5 = 5 \times p^4 \times x^4 \times 3$$

$$T_5 = 15p^4 x^4$$

$$\left(x^3 + \frac{p}{x}\right)^4 = {}^4C_r (x^3)^r \left(\frac{p}{x}\right)^{4-r}$$

$$T_3 = {}^4C_2 (x^3)^2 \left(\frac{p}{x}\right)^2$$

$$T_3 = 6 \times x^6 \times \frac{p^2}{x^2}$$

$$T_3 = 6p^2 x^4$$

$$15p^4 x^4 - 6p^2 x^4 = 216x^4$$

$$15p^4 - 6p^2 - 216 = 0$$

$$5a^2 - 2a - 72 = 0$$

$$5a^2 - 20a + 18a - 72 = 0$$

$$5a(a-4) + 18(a-4) = 0$$

$$(5a+18)(a-4) = 0$$

$$a = -\frac{18}{5} \text{ or } a = 4$$

$$p^2 = -\frac{18}{5} \text{ or } p^2 = 4$$

$$\text{Not Possible or } p = \pm 2$$

Since p is a positive constant

$$p = 2$$



- 4 (a) Express $1 - 6x - x^2$ in the form $a - (x + b)^2$, where a and b are constants.

[3]

$$-(x^2 + 6x - 1)$$

$$10 - (x^2 + 6x + 9)$$

$$10 - (x + 3)^2$$

- (b) The graph of $y = x^2$ is transformed to the graph of $y = 1 - 6x - x^2$ by a reflection followed by a translation of $\begin{pmatrix} m \\ n \end{pmatrix}$. Give details of the reflection and determine the values of m and n .

[3]

Reflection in x -axis.

Translation with $T = \begin{pmatrix} -3 \\ 10 \end{pmatrix}$, where $m = -3$ and $n = 10$.

working: $-x^2 \Rightarrow -(x-m)^2 + n$

$$-(x^2 - 2mx + m^2) + n$$

$$-x^2 + 2mx - m^2 + n$$

$$2m = -6$$

$$m = -3$$

$$-9 + n = 1$$

$$n = 10$$



5 (a) Show that $\tan^4 \theta - 1 \equiv \frac{1 - 2\cos^2 \theta}{\cos^4 \theta}$.

[3]

$$\tan^4 \theta - 1 \equiv \frac{\sin^4 \theta}{\cos^4 \theta} - 1$$

$$\tan^4 \theta - 1 \equiv \frac{\sin^4 \theta - \cos^4 \theta}{\cos^4 \theta}$$

$$\tan^4 \theta - 1 \equiv \frac{(1 - \cos^2 \theta)^2 - \cos^4 \theta}{\cos^4 \theta}$$

$$\tan^4 \theta - 1 \equiv \frac{1 - 2\cos^2 \theta + \cancel{\cos^4 \theta} - \cancel{\cos^4 \theta}}{\cos^4 \theta}$$

$$\tan^4 \theta - 1 \equiv \frac{1 - 2\cos^2 \theta}{\cos^4 \theta} \rightarrow \text{Shown!!}$$

(b) Hence solve the equation $\cos^2 \theta (\tan^4 \theta - 1) = 7$ for $0^\circ < \theta < 180^\circ$.

[4]

$$\cos^2 \theta \left(\frac{1 - 2\cos^2 \theta}{\cos^4 \theta} \right) = 7$$

$$\cos^2 \theta - 2\cos^4 \theta = 7\cos^4 \theta$$

$$9\cos^4 \theta - \cos^2 \theta = 0$$

$$(3\cos^2 \theta - \cos \theta)(3\cos^2 \theta + \cos \theta) = 0$$

$$3\cos^2 \theta = \cos \theta \quad \text{OR} \quad 3\cos^2 \theta = -\cos \theta$$

$$3\cos \theta = 1 \quad \text{OR} \quad 3\cos \theta = -1$$

$$\cos \theta = \frac{1}{3} \quad \text{OR} \quad \cos \theta = -\frac{1}{3}$$

$$\alpha = 70.528$$

$$\theta = 70.5 \text{ and } 109.5$$



6 Functions f and g are defined by

$$f(x) = (x+3)^2 - 12 \quad \text{for } x \geq 0,$$

$$g(x) = 2x - 5 \quad \text{for } x \in \mathbb{R}.$$

(a) State the range of f .

[1]

$$f(x) = (0+3)^2 - 12 = -3$$

$$f \geq -3$$

(b) Find an expression for $f^{-1}(x)$.

[2]

$$y = (x+3)^2 - 12$$

$$y+12 = (x+3)^2$$

$$\pm \sqrt{y+12} = x+3$$

$$-3 + \sqrt{y+12} = x$$

$$f^{-1}(x) = -3 + \sqrt{x+12}$$

(c) Solve the equation $gf(x) = 69$.

[4]

$$2[(x+3)^2 - 12] - 5 = 69$$

$$(x+3)^2 - 12 = 37$$

$$(x+3)^2 = 49$$

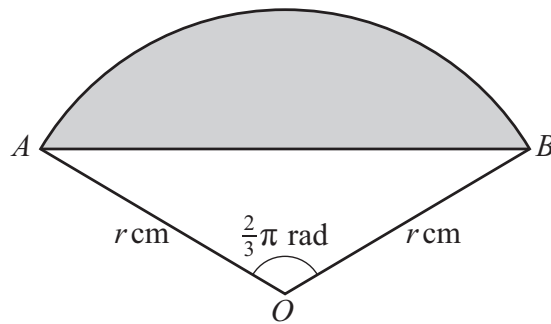
$$x+3 = 7 \quad \text{OR} \quad x+3 = -7$$

$$x = 4 \quad \text{OR} \quad x = -10$$

$$\text{Since } x \geq 0$$

$$x = 4$$





The diagram shows a sector of a circle with centre O and radius r cm. The shaded region is bounded by the chord AB and the arc AB . The size of angle AOB is $\frac{2}{3}\pi$ radians.

- (a) Show that the area of the shaded region is approximately $0.614r^2$ cm². [2]

$$\text{Shaded Area} = \frac{1}{2} r^2 \times \frac{2}{3} \pi - \frac{1}{2} r^2 \times \sin \frac{2}{3} \pi$$

$$\text{Shaded Area} = r^2 \left(\frac{1}{3} \pi - \frac{\sqrt{3}}{4} \right)$$

$$\text{Shaded Area} = 0.614 r^2 \text{ cm}^2 \rightarrow \text{shown!!}$$



It is given that the radius of the circle is increasing at a rate of 0.4 cm s^{-1} .

- (b) (i) Find the rate of increase of the area of the shaded region at the instant when $r = 20$.
Give your answer correct to 2 significant figures. [3]

$$\frac{dA}{dr} = 1.228r$$

$$\frac{dA}{dr} = 1.228(20) = 24.56$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

$$\frac{dA}{dt} = 24.56 \times 0.4$$

$$\frac{dA}{dt} = 9.8$$

- (ii) Find the rate of increase of the length of the arc AB . Give your answer correct to 2 significant figures. [3]

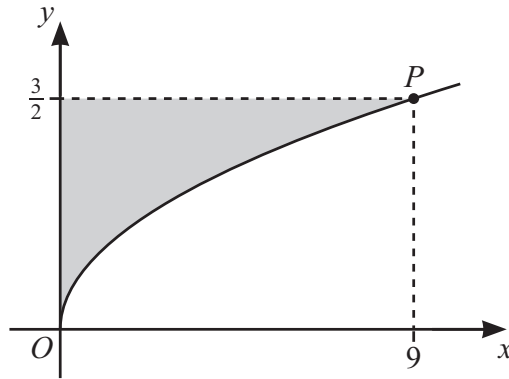
$$\frac{ds}{dr} = \frac{2}{3}\pi$$

$$\frac{ds}{dt} = \frac{ds}{dr} \times \frac{dr}{dt}$$

$$\frac{ds}{dr} = \frac{2}{3}\pi \times 0.4$$

$$\frac{ds}{dt} = 0.84$$





The diagram shows the curve with equation $y = \frac{1}{2}\sqrt{x}$ and the point P with coordinates $(9, \frac{3}{2})$. The shaded region is bounded by the curve and the lines $x = 0$ and $y = \frac{3}{2}$.

- (a) Find the area of the shaded region. [3]

Area of Rectangle: $9 \times \frac{3}{2} = \frac{27}{2}$

Area under curve: $\frac{1}{2} \int_0^9 x^{\frac{1}{2}}$

Area under curve: $\frac{1}{2} \times \frac{2}{3} x^{\frac{3}{2}}$

Area under curve: $\frac{1}{3} x^{\frac{3}{2}}$

Area under curve: $\frac{1}{3} (9)^{\frac{3}{2}}$

Area under curve: 9

Shaded Area: $\frac{27}{2} - 9$

Shaded Area: $\frac{9}{2}$



- (b) The shaded region is rotated through 360° about the y -axis.

Find the exact volume of the solid produced.

[4]

Volume: $y = \frac{1}{2} \sqrt{x}$

Volume: $2y = \sqrt{x}$

Volume: $4y^2 = x$

Volume: $\pi \int_0^{\frac{3}{2}} x^2 dy$

Volume: $\pi \int_0^{\frac{3}{2}} 16y^4 dy$

Volume: $\left[\frac{16}{5} \pi y^5 \right]_0^{\frac{3}{2}}$

Volume: $\frac{243}{10} \pi$



- 9 An arithmetic progression has first term 2 and common difference d . The sum of the first n terms is denoted by S_n .

(a) It is given that $(S_2 - 1)$, S_4 , S_9 are the first three terms of a second arithmetic progression.

Find the value of d .

[4]

$$S_2 = 2 + d + 2$$

$$S_2 - 1 = 3 + d$$

$$S_4 = 2[4 + 3d] = 8 + 6d$$

$$S_9 = \frac{9}{2}[4 + 8d] = 18 + 36d$$

$$8 + 6d - 3 - d = 18 + 36d - 8 - 6d$$

$$5 + 5d = 10 + 30d$$

$$-5 = 25d$$

$$d = -\frac{1}{5}$$



- (b) Hence find the difference between the values of the 15th terms of the two arithmetic progressions. [4]

First progression: $a=2$, $d=-\frac{1}{5}$

$$T_{15} = 2 + 14(-\frac{1}{5})$$

$$T_{15} = -\frac{4}{5}$$

Second progression: $a = 3 - \frac{1}{5} = \frac{14}{5}$

$$\therefore \text{common difference: } 8 - \frac{6}{5} - 3 + \frac{1}{5} = 4$$

$$T_{15} = \frac{14}{5} + 14(4)$$

$$T_{15} = \frac{294}{5}$$

$$\text{Difference of 15th terms} = \frac{294}{5} + \frac{4}{5}$$

$$\text{Difference of 15th terms} = \frac{298}{5} = 59.6$$



- 10 A circle has equation $x^2 + y^2 + 4y - 21 = 0$ and a straight line has equation $2x + y - 8 = 0$. The line intersects the circle at two points.

(a) Find the coordinates of these two points of intersection.

[4]

$$\text{line: } y = 8 - 2x$$

$$x^2 + (8 - 2x)^2 + 4(8 - 2x) - 21 = 0$$

$$x^2 + 64 - 32x + 4x^2 + 32 - 8x - 21 = 0$$

$$5x^2 - 40x + 75 = 0$$

$$x^2 - 8x + 15 = 0$$

$$x^2 - 3x - 5x + 15 = 0$$

$$x(x - 3) - 5(x - 3) = 0$$

$$(x - 5)(x - 3) = 0$$

$$x = 5 \text{ OR } x = 3$$

$$y = 8 - 2x$$

$$y = 8 - 2x$$

$$y = 8 - 2(5)$$

$$y = 8 - 2(3)$$

$$y = -2$$

$$y = 2$$

$$(5, -2)$$

$$(3, 2)$$

Two points of intersection: $(5, -2)$ and $(3, 2)$



- (b) The circle has centre C and the two points of intersection are denoted by A and B .

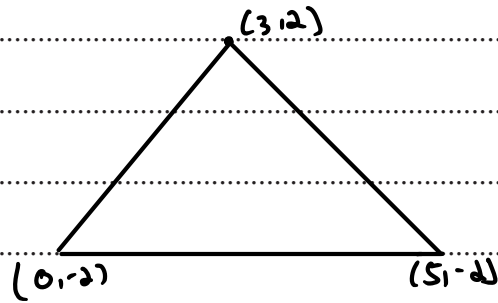
Find the area of the triangle ABC .

[3]

$$A = (5, -2)$$

$$B = (3, 2)$$

$$C = (0, -2)$$



$$\text{Area} = \frac{1}{2} \times 5 \times 4$$

$$\text{Area} = 10$$



- 11 A curve passes through the point $P(4, 3)$ and is such that

$$\frac{dy}{dx} = \frac{8}{x^2} - \frac{10}{(2x-3)^2}.$$

- (a) Find the equation of the normal to the curve at P . Give your answer in the form $y = mx + c$. [3]

$$m = \frac{8}{(4)^2} - \frac{10}{(5)^2}$$

$$m = \frac{1}{10} \rightarrow \text{Tangent at } P$$

$$m \text{ of normal} = -10$$

$$\text{Equation: } y - 3 = -10(x - 4)$$

$$\text{Equation: } y = -10x + 43$$

- (b) Find the rate of change of the gradient of the curve when $x = 4$. [3]

$$\frac{d^2x}{dy^2} = -16x^{-3} + 40(2x-3)^{-3}$$

$$\frac{d^2x}{dy^2} = -16(4)^{-3} + 40(5)^{-3}$$

$$\frac{d^2x}{dy^2} = \frac{7}{100}$$



(c) Given that the curve also passes through the point $(-1, q)$, find the value of q .

[5]

$$y = \int 8x^2 - 10(2x-3)^2$$

$$y = \frac{-8}{x} + \frac{5}{(2x-3)} + c$$

$$3 = \frac{-8}{-1} + \frac{5}{-2-3} + c$$

$$3 = -2 + 1 + c$$

$$3 = -1 + c$$

$$c = 4$$

$$y = \frac{-8}{x} + \frac{5}{(2x-3)} + 4$$

$$q = \frac{-8}{-1} + \frac{5}{-2-3} + 4$$

$$q = 8 - 1 + 4$$

$$q = 11$$



Cambridge International AS & A Level

MATHEMATICS**9709/11**

Paper 1 Pure Mathematics 1

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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This document consists of **19** printed pages.

PUBLISHED

Question	Answer	Marks	Guidance
1	Use of $b^2 - 4ac$	*M1	
	Obtain $k^2 - 20k + 64$	A1	
	Attempt solution of quadratic equation or inequality	DM1	Solve quadratic using suitable method. E.g., factorisation, quadratic formula, completing the square.
	Obtain $k < 4$, $k > 16$ or clear equivalent	B1	B0 for use of $\leq$ and/or $\geq$.
		4	

Question	Answer	Marks	Guidance
2(a)	State or imply that second and fifth terms are $a\cos\theta$ and $a\cos^4\theta$ or $r^3 = \frac{1}{64}$	B1	Condone poor notation.
	Attempt solution of equation of form $\cos^3\theta = k$ as far as $\cos\theta = \dots$, where $0 < k < 1$	M1	Must come from division.
	Obtain $\cos\theta = 0.25$, and hence $\theta = 1.32$ only	A1	Allow greater accuracy 1.318116... A0 for 75.5° .
		3	
2(b)	Attempt to find a using <i>their</i> θ and substitute in correct formula for sum to infinity	M1	Using exact or approximate value for r . $-1 < r < 1$
	Obtain $a = 32$ and use exact value of r to obtain $\frac{128}{3}$	A1	Or exact equivalent.
		2	

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Question	Answer	Marks	Guidance
3	x^4 term from $(px+3)^5$ is $5p^4x^4 \times 3$	B1	Or coefficient is $15p^4$. Not $5C1 \times 3$ or factorial form or $15(px)^4$. Can be in a list.
	Identify term involving $x^6 \times \frac{p^2}{x^2}$ as relevant term from $\left(x^3 + \frac{p}{x}\right)^4$	M1	
	Obtain $6p^2x^4$ or $-6p^2x^4$	A1	Or coefficient is $6p^2$ or $-6p^2$. Not $4C2$ or factorial form.
	Attempt solution for p^2 of equation that is quadratic in p^2	M1	Solve quadratic in p^2 using suitable method. E.g. factorisation, quadratic formula, completing the square. E.g. $3(5p^2 + 18)(p^2 - 4)$
	Obtain $15p^4 - 6p^2 - 216 = 0$ or equivalent and hence $p = 2$	B1	No other solutions.
		5	

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Question	Answer	Marks	Guidance
4(a)	$\{10\} - (x \{+\} \{3\})^2$	B1	
		B1	
		B1	
		3	
4(b)	State reflection in x -axis	B1	
	Obtain $m = -3$	B1 FT	Following <i>their</i> value of b .
	Obtain $n = 10$	B1 FT	Following <i>their</i> value of a .
		3	

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Question	Answer	Marks	Guidance
5(a)	Use $\tan^4 \theta = \frac{\sin^4 \theta}{\cos^4 \theta}$ or $\tan^4 \theta - 1 = \left(\frac{\sin^2 \theta}{\cos^2 \theta} + 1 \right) + \left(\frac{\sin^2 \theta}{\cos^2 \theta} - 1 \right)$	B1	Candidates can work from LHS to RHS or RHS to LHS for full marks. If working on both sides simultaneously, maximum of B1 M1 only if a common correct expression is reached.
	Attempt to express $\tan^4 \theta - 1$ as a single fraction in terms of $\cos \theta$ only	M1	Using $\frac{(1-c^2)^2 - c^4}{c^4}$ or $\frac{(s^2+c^2)(s^2-c^2)}{c^4}$ etc.
	Confirm result $\frac{1-2\cos^2 \theta}{\cos^4 \theta}$	A1	AG – necessary detail needed. WWW
	Alternative Method for Question 5(a)		
	$\frac{1-2\cos^2 \theta}{\cos^4 \theta} + \frac{\cos^4 \theta}{\cos^4 \theta} - 1$	B1	
	$\frac{(1-\cos^2 \theta)^2}{\cos^4 \theta} - 1$	M1	
	Confirm result $\tan^4 \theta - 1$	A1	AG – necessary detail needed.
	Alternative Method 2 for Question 5(a)		
	$\tan^4 \theta = \sec^4 \theta - 2\sec^2 \theta + 1$	B1	
	$\tan^4 \theta - 1 = \frac{1}{\cos^4 \theta} - \frac{2}{\cos^2 \theta}$	M1	
	Confirm result $\frac{1-2\cos^2 \theta}{\cos^4 \theta}$	A1	AG – necessary detail needed.
		3	

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Question	Answer	Marks	Guidance
5(b)	Use identity to form equation using $\cos\theta$ only	*M1	Expect $\frac{1-2\cos^2\theta}{\cos^2\theta} = 7$ OE Allow unsimplified.
	Obtain $9\cos^2\theta = 1$ or similarly simplified equivalent	A1	Condone $\cos^2\theta (1 - 9\cos^2\theta) = 0$ Could be implied by one correct solution
	Solve equation of form $\cos^2\theta = k$ to obtain at least one solution	DM1	Provided $0 < k < 1$.
	Obtain 70.5 and 109.5	A1	Or greater accuracy 70.528..., 109.471..., and no others between 0 and 180. SC B1 for 1.23° and 1.91° only. A0 if $\cos^2\theta (1 - 9\cos^2\theta) = 0$ is used.
		4	

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Question	Answer	Marks	Guidance
6(a)	State $f(x) \geq -3$ or $y \geq -3$ or clear equivalent	B1	Must be $\geq$ rather than $>$. Allow ≥ -3 but not $x \geq -3$.
		1	
6(b)	Attempt to arrange to $x = \dots$ in terms of y or equivalent	M1	Or in terms of x already. Sign errors only.
	Obtain $-3 + \sqrt{x+12}$	A1	Now in terms of x . A0 for $\pm$ in final answer. Condone $y = \dots$
		2	
6(c)	Attempt expression for $gf(x)$	*M1	Expect $2((x+3)^2 - 12) - 5$. Sign errors only.
	Obtain $2(x+3)^2 - 29 = 69$	A1	OE E.g. $2x^2 + 12x - 80 [= 0]$.
	Attempt solution of quadratic equation to find at least one value of x	DM1	No method needed.
	Obtain $x = 4$ only	A1	
	Alternative Method for Question 6(c)		
	Attempt solution of $g(x) = 69$ or evaluation of $g^{-1}(69)$	*M1	Incorrect order of f^{-1} and then g^{-1} scores *M0.
	Obtain 37	A1	
	Attempt solution of $f(x) = \textit{their 37}$ or evaluation of $f^{-1}(\textit{their 37})$	DM1	
	Obtain $x = 4$ only	A1	
		4	

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Question	Answer	Marks	Guidance
7(a)	Obtain correct $\frac{1}{2}r^2 \times \frac{2}{3}\pi - \frac{1}{2}r^2 \sin \frac{2}{3}\pi$	M1	
	$0.614r^2$	A1	AG Greater accuracy is $0.61418\dots r^2$.
		2	
7(b)(i)	Obtain $\left[\frac{dA}{dr}\right] = 1.228r$	B1	Or greater accuracy
	Use $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$, or equivalent, with $r = 20$	M1	'their' $\frac{dA}{dr} \times 0.4$ with $r = 20$
	Obtain 9.8	A1	AWRT
		3	
7(b)(ii)	State or imply $\frac{2}{3}\pi r$ for length of arc AB	B1	Could be implied by $\frac{dl}{d\theta} = \frac{2\pi}{3}$.
	Differentiate and apply correct use of chain rule	M1	
	Obtain 0.84	A1	AWRT
		3	

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Question	Answer	Marks	Guidance
8(a)	Integrate to obtain expression $\frac{1}{2}x^{\frac{3}{2}} \div \frac{3}{2}$	B1	OE
	Evaluate integral of the form $kx^{\frac{3}{2}}$ between limits 0 and 9 and subtract result from $\frac{27}{2}$	M1	
	Obtain $\frac{1}{3}x^{\frac{3}{2}}$ giving area under curve is 9 and shaded area is $\frac{9}{2}$	A1	
	Alternative Method for Question 8(a)		
	Obtain $x = 4y^2$	B1	
	Integrate to obtain expression of form ky^3 and evaluate between limits 0 and $\frac{3}{2}$	M1	
	Obtain $\frac{4}{3}y^3$ and hence shaded area is $\frac{9}{2}$	A1	
		3	
8(b)	Attempt to express x^2 in terms of y	M1	Condone ky^4 . Attempting $\int y^2 dx$ scores 0/4.
	Obtain or imply volume is $[\pi] \int 16y^4 [dy]$	A1	
	Integrate to obtain ky^5 and evaluate between limits 0 and $\frac{3}{2}$	M1	
	Obtain $\frac{16}{5}y^5$ and hence $\frac{243}{10}\pi$ or exact equivalent	A1	Condone omission of π except for last mark.
		4	

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Question	Answer	Marks	Guidance
9(a)	Attempt to express each of $S_2 - 1$, S_4 , S_9 in terms of d (with at least one correct)	*M1	Allow unsimplified.
	Obtain $S_2 - 1 = 3 + d$, $S_4 = 8 + 6d$, $S_9 = 18 + 36d$	A1	Must be simplified but can be implied by later work.
	Attempt equation in d by linking the three values	DM1	
	Obtain correct $(3 + d) + (18 + 36d) = 2(8 + 6d)$ or equivalent, and hence $d = -\frac{1}{5}$	A1	
		4	
9(b)	$2 + 14 \times \text{their } d$	M1	
	Second progression: obtain either first term = $\frac{14}{5}$ or common difference = 4	B1	Must be from correct d in part (a).
	Second progression: attempt to find 15th term	M1	Using $a = 3 + \text{their } d$ from part (a).
	Obtain 58.8, and hence difference = 59.6	A1	
		4	

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Question	Answer	Marks	Guidance
10(a)	Attempt to substitute for x or y to produce quadratic equation in one variable	*M1	Allow unsimplified.
	Obtain $5x^2 - 40x + 75 = 0$ or $5y^2 - 20 = 0$	A1	Or similarly simplified equivalent.
	Find two values of one variable	B1	
	Obtain coordinates $(3, 2)$ and $(5, -2)$	B1	
		4	
10(b)	Obtain centre $(0, -2)$ [and radius 5]	B1	
	Attempt area of ABC using mid-point of AB or $\frac{1}{2} \times \text{base} \times \text{height}$	M1	OE Use of correct formula for area of a triangle.
	Obtain $\frac{1}{2}\sqrt{20} \times \sqrt{20}$ or $\frac{1}{2} \times 5 \times 4$ (or equivalent) and hence 10	A1	
		3	

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Question	Answer	Marks	Guidance
11(a)	Substitute 4 to obtain gradient of curve is $\frac{1}{10}$	B1	
	Attempt equation of normal using (4, 3) and $-\frac{1}{m}$ for their gradient	M1	
	$y = -10x + 43$	A1	
		3	
11(b)	$\{-16x^{-3}\}\{+40(2x-3)^{-3}\}$	B1 B1	OE
	Substitute 4 to obtain $\frac{7}{100}$	B1	
		3	
11(c)	$[y =]\{-8x^{-1}\}\{+5(2x-3)^{-1}\}[+c]$	B1 B1	
	Substitute $x = 4, y = 3$ in an integrated expression to find value of c	M1	
	Obtain $3 = -2 + 1 + c$ and hence $y = -8x^{-1} + 5(2x-3)^{-1} + 4$	A1	OE For finding $c = 4$.
	Substitute $x = -1$ to obtain $q = 11$	A1	Not $y = 11$.
		5	



Cambridge International AS & A Level

CANDIDATE
NAME



CENTRE
NUMBER

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NUMBER

MATHEMATICS

9709/12

Paper 1 Pure Mathematics 1

October/November 2025

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 1 (a) Express $9x^2 - 36x + 8$ in the form $p(x+q)^2 + r$, where p , q and r are constants. [2]

$$9(x^2 - 4x) + 8$$

$$9(x-2)^2 + r$$

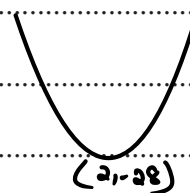
$$36 + r = 8$$

$$r = -28$$

$$9(x-2)^2 - 28$$

- (b) Hence find the set of values of the constant k for which the equation $9x^2 - 36x + 8 = k$ has no real roots. [1]

$$k < -28$$



- (c) Find the exact roots of the equation $9x^2 - 36x + 8 = -15$. [2]

$$9x^2 - 36x + 23 = 0$$

$$\frac{36 \pm \sqrt{1296 - 828}}{18}$$

$$18$$

$$\frac{36 \pm 6\sqrt{13}}{18}$$

$$18$$

$$2 \pm \frac{\sqrt{13}}{3}$$



- 2 Find the term independent of x in the expansion of $\left(2x^2 - \frac{3}{x}\right)^6$. [3]

$$T_{r+1} = {}^6C_r (2x^2)^r \left(-\frac{3}{x}\right)^{6-r}$$

$$T_{2+1} = {}^6C_2 \times 4x^4 \times \frac{81}{x^4}$$

$$T_3 = 4860.$$



- 3 (a) The graph of $y = f(x)$ is transformed to the graph of $y = f(3x) + 2$.

Describe fully the two transformations which have been combined to give the resulting graph. [3]

Translation with $T = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$
Horizontal stretch with factor $\frac{1}{3}$

- (b) A different graph has equation $y = g(x)$. This graph is stretched by scale factor 3 in the y -direction and then reflected in the y -axis.

Write down the equation of the transformed graph in terms of the function g . [2]

Stretch: $y = 3g(x)$
Stretch and Reflection: $y = -3g(x)$



- 4 The equation of a curve is such that $\frac{dy}{dx} = kx^3 + \frac{2}{x^2}$, where k is a constant. The curve passes through the point $S(2, 20)$ and the gradient of the curve at S is $\frac{65}{2}$.

(a) Find the value of k .

[1]

$$\frac{65}{2} = 8k + \frac{1}{2}$$

$$65 = 16k + 1$$

$$64 = 16k$$

$$k = 4$$

- (b) The coordinates of a point T on the curve are $(1, t)$.

Find the value of t .

[5]

$$\frac{dy}{dx} = 4x^3 + 2x^{-2}$$

$$y = x^4 - \frac{2}{x} + c$$

$$20 = 16 - 1 + c$$

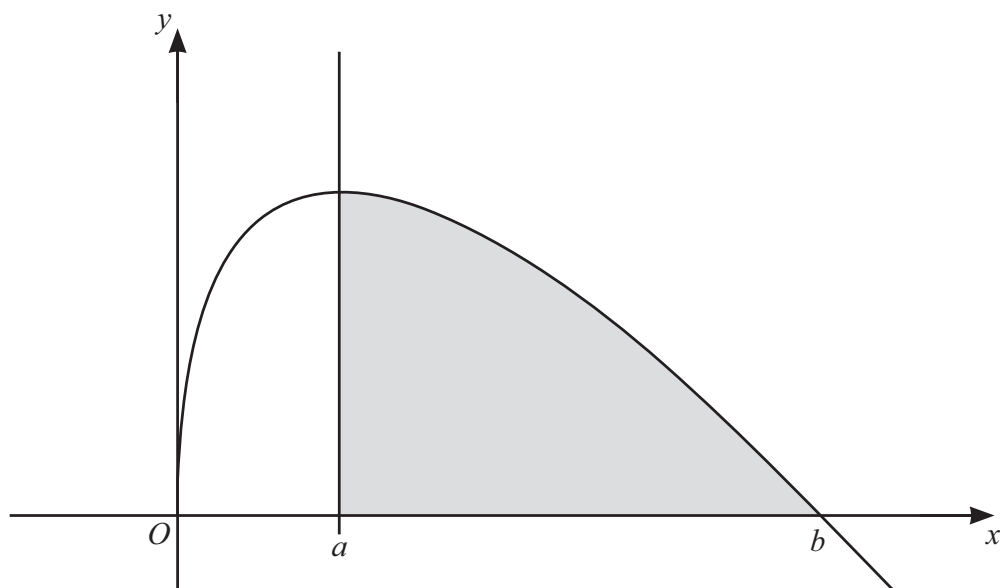
$$c = 5$$

$$y = x^4 - \frac{2}{x} + 5$$

$$t = 1 - 2 + 5$$

$$t = 4$$





The equation of a curve is $y = 4x^{\frac{1}{2}} - x$. The curve has a maximum point when $x = a$ and crosses the x -axis at the point with coordinates $(b, 0)$, where $b > 0$. The shaded region is bounded by the curve, the line $x = a$ and the x -axis (see diagram).

(a) Find the value of a .

[3]

$$y = 4x^{1/2} - x$$

$$\frac{dy}{dx} = 2x^{-1/2} - 1$$

$$\frac{2}{\sqrt{x}} - 1 = 0$$

$$\frac{2}{\sqrt{x}} = 1$$

$$2 = \sqrt{x}$$

$$x = 4$$



(b) Find the exact area of the shaded region.

[5]

$$4b^{1/2} - b = 0$$

$$4u - u^2 = 0$$

$$u^2 - 4u = 0$$

$$u(u-4) = 0$$

$$u = 4$$

$$b^{1/2} = 4$$

$$b = 16$$

$$\text{Area} = \int_4^{16} 4x^{1/2} - x$$

$$\text{Area} = \left[\frac{8}{3} x^{3/2} - \frac{x^2}{2} \right]_4^{16}$$

$$\text{Area} = \left[\frac{512}{3} - 128 \right] - \left[\frac{64}{3} - 8 \right]$$

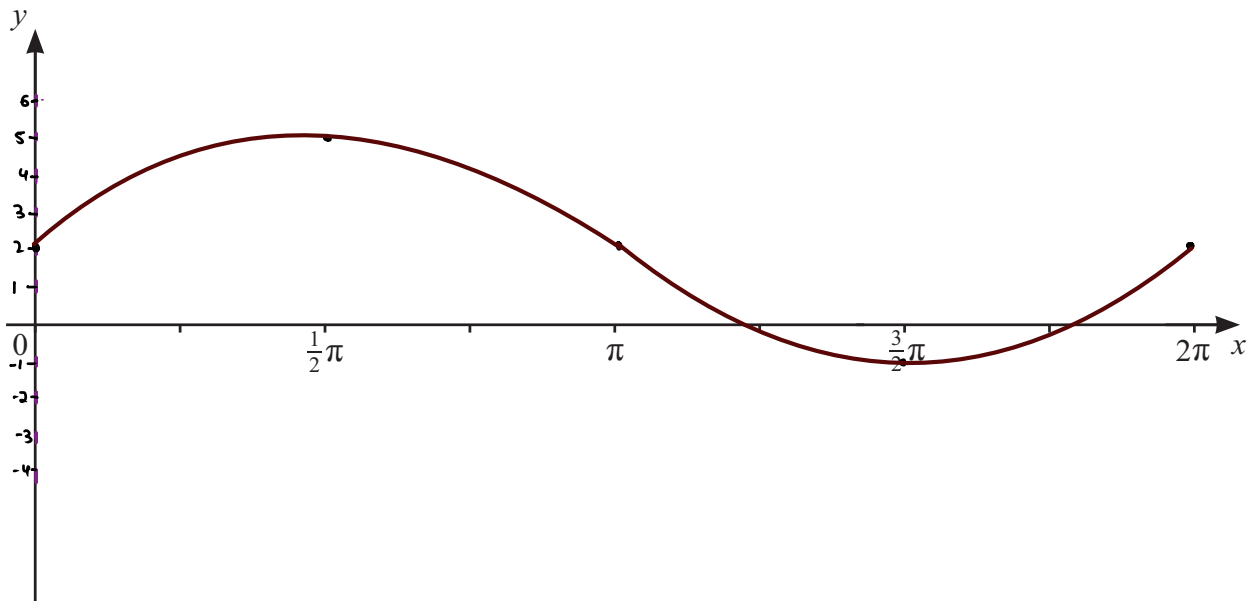
$$\text{Area} = \frac{128}{3} - \frac{40}{3}$$

$$\text{Area} = \frac{88}{3}$$



6 (a) Sketch the graph of $y = 3 \sin x + 2$ for $0 \leq x \leq 2\pi$.

[2]



(b) Determine the number of solutions in the interval $0 \leq x \leq 2\pi$ of each of the following equations.

(i) $3 \sin x + 2 = x$

[1]

1 solution.

(ii) $3 \sin x + 2 = 5 - x$

[1]

3 solutions.



(c) Solve the equation $3 \sin x + 2 = 5 \cos^2 x - 1$ for $0 \leq x \leq 2\pi$.

[5]

$$3 \sin x + 2 = 5 - 5 \sin^2 x - 1$$

$$5 \sin^2 x + 3 \sin x - 2 = 0$$

$$5s^2 + 3s - 2 = 0$$

$$5s(s+1) - 2(s+1) = 0$$

$$(5s-2)(s+1) = 0$$

$$\sin x = \frac{2}{5}, \quad \sin x = -1$$

$$\alpha = 0.411516, \quad \alpha = \frac{3}{2}\pi$$

$$\theta = 0.412, 2.73, \frac{3}{2}\pi$$



- 7 The coordinates of the points P and Q are $(1, 1)$ and $(7, 11)$ respectively. The line segment PQ forms a diameter of a circle.

(a) Find the equation of the circle.

[4]

$$\text{Midpoint of } PQ = \text{Centre} = (4, 6)$$

$$\text{Radius} = \sqrt{3^2 + 5^2} = \sqrt{34}$$

$$\text{Equation of circle} = (x-4)^2 + (y-6)^2 = 34$$

(b) Find the equation of the tangent to the circle at the point Q .

[3]

$$m \text{ of } PQ = \frac{11-6}{7-4} = \frac{5}{3}$$

$$m \text{ of Tangent} = -\frac{3}{5}$$

$$y - 11 = -\frac{3}{5}(x - 7)$$

$$y - 11 = -\frac{3}{5}x + \frac{21}{5}$$

$$y = -\frac{3}{5}x + \frac{76}{5}$$



- (c) The other point on the circle with x -coordinate 7 is R .

Find the coordinates of the point of intersection of the tangent at Q with the tangent at R .

[4]

$$(y-6)^2 = 25$$

$$y-6 = -5$$

$$y = +1$$

$$R = (7, +1)$$

$$\text{m of CR} = \frac{+1-6}{7-4} = \frac{-5}{3}$$

$$\text{m of Tangent at R} = \frac{3}{5}$$

$$y-1 = \frac{3}{5}(x-7)$$

$$y-1 = \frac{3}{5}x - \frac{21}{5}$$

$$y = \frac{3}{5}x - \frac{16}{5}$$

$$\frac{3}{5}x - \frac{16}{5} = -\frac{3}{5}x + \frac{76}{5}$$

$$3x - 16 = -3x + 76$$

$$6x = 92$$

$$x = \frac{46}{3}$$

$$y = \frac{3}{5}\left(\frac{46}{3}\right) - \frac{16}{5}$$

$$y = 6$$

Co-ordinates of point of intersection: $\left(\frac{46}{3}, 6\right)$



- 8 The first three terms of a geometric progression are a , b and c respectively, where a , b and c are positive constants. The first three terms of an arithmetic progression are a , b and $-3c$ respectively.

(a) Show that $a^2 - 10ac + 9c^2 = 0$.

[3]

$$\frac{b}{a} = \frac{c}{b}$$

$$b - a = -3c - b$$

$$b^2 = ca$$

$$2b = a - 3c$$

$$4b^2 = 4ca$$

$$4b^2 = a^2 - 6ca + 9c^2$$

$$4ca = a^2 - 6ca + 9c^2$$

$$a^2 - 10ca + 9c^2 = 0 \rightarrow \text{Shown!!}$$

It is now given that $a = 9$ and c takes the smaller of its two possible values.

- (b) (i) Find the sum to infinity of the geometric progression.

[5]

$$9c^2 - 90c + 81 = 0$$

$$c^2 - 10c + 9 = 0$$

$$c^2 - 9c - c + 9 = 0$$

$$c(c-9) - 1(c-9) = 0$$

$$(c-1)(c-9) = 0$$

$$c = 1 \text{ or } c = 9$$

Since c takes the smaller value,

$$c = 1$$

Geometric: $a = 9$,

$$b^2 = ac$$

$$b^2 = 9$$

$$b = 3$$

$$c = 1$$





$$r = \frac{3}{9} = \frac{1}{3}$$

$$S_{\infty} = \frac{a}{1-r} = \frac{9}{1-\frac{1}{3}}$$

$$S_{\infty} = \frac{27}{2}$$

(ii) Find the sum of the first 20 terms of the arithmetic progression.

[3]

$$a = 9$$

$$d = b - a = 3 - 9 = -6$$

$$S_{20} = 10[18 + 19(-6)]$$

$$S_{20} = 10[18 - 114]$$

$$S_{20} = 10 \times -96$$

$$S_{20} = -960$$



- 9 The function f is defined by $f(x) = \frac{4}{(3x-6)^2} + \frac{1}{(3x-6)^3}$ for $x > 2$.

- (a) Find an expression for $f'(x)$ and hence determine whether f is an increasing function, a decreasing function or neither. [4]

$$f(x) = 4(3x-6)^{-2} + (3x-6)^{-3}$$

$$f'(x) = -24(3x-6)^{-3} - 3(3x-6)^{-4}$$

$$f'(x) = \frac{-24}{(3x-6)^3} - \frac{3}{(3x-6)^4}$$

The function is an decreasing function.

- (b) State whether f^{-1} exists. Give a reason for your answer. [1]

Yes, because it passes the horizontal line test.



The function g is defined by $g(x) = 4x - 3$ for $x > a$.

(c) Find the range of g in terms of the constant a .

[1]

$$g(x) > 4a - 3$$

(d) Find the set of values of a for which the composite function fg exists.

[2]

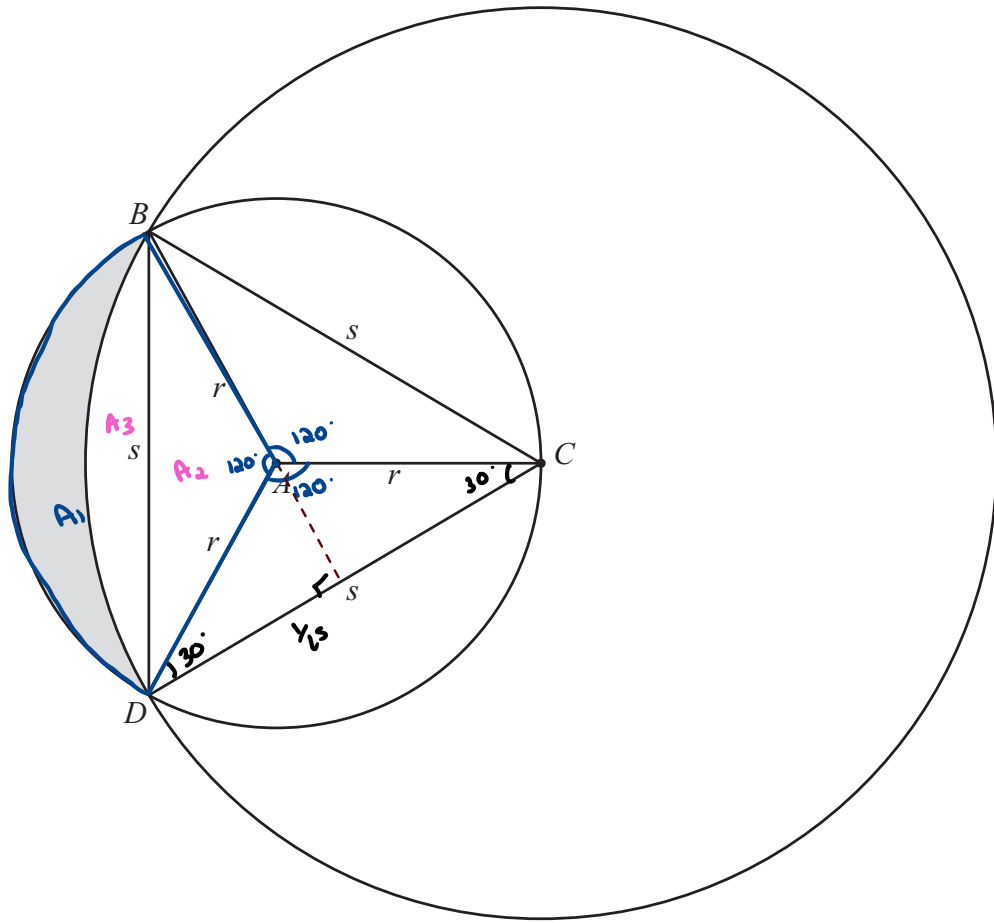
Fg Rst g is within the domain of f.

$$4a - 3 > 2$$

$$4a > 5$$

$$a > 5/4$$





The diagram shows a circle with centre A and radius r passing through points B , C and D . A larger circle of radius s has centre C and passes through B and D . The length BD is also s .

- (a) Show that $s = \sqrt{3}r$. [2]

$$\begin{aligned} \cos 30^\circ &= \frac{0.5s}{r} \\ \frac{\sqrt{3}}{2} &= \frac{1}{2} \times \frac{s}{r} \\ s &= \sqrt{3}r \rightarrow \text{Proved!!} \end{aligned}$$



- (b) Find an expression for the area of the shaded region. Give your answer in the form $(a + b\pi)r^2$, where a and b are constants to be found. [7]

$$\text{Area 1} = \frac{1}{2}r^2 \times \frac{2}{3}\pi = \frac{1}{3}r^2\pi$$

$$\text{Area 2: } \frac{1}{2}r^2 \times \sin \frac{2}{3}\pi = \frac{\sqrt{3}}{4}r^2$$

$$\text{Area 3: } \frac{1}{2}s^2 \times \frac{1}{3}\pi - \frac{1}{2}s^2 \times \sin \frac{1}{3}\pi$$

$$\text{Area 3: } \frac{1}{2} \times 3r^2 \times \frac{1}{3}\pi - \frac{\sqrt{3}}{4} \times 3r^2$$

$$\text{Area 3: } \frac{1}{2}r^2\pi - \frac{3\sqrt{3}}{4}r^2$$

$$\text{Shaded Area: Area 1} - \text{Area 2} - \text{Area 3}$$

$$\text{Shaded Area: } \frac{1}{3}r^2\pi - \frac{\sqrt{3}}{4}r^2 - \frac{1}{2}r^2\pi + \frac{3\sqrt{3}}{4}r^2$$

$$\text{Shaded Area: } r^2 \left(\frac{\sqrt{3}}{2} - \frac{1}{6}\pi \right)$$



Cambridge International AS & A Level

MATHEMATICS**9709/12**

Paper 1 Pure Mathematics 1

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **25** printed pages.

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Question	Answer	Marks	Guidance
1(a)	$\{9\} \{(x-2)^2\} \{-28\}$	B2,1,0	B2 for all 3 correct values. B1 for any two correct. Allow $p=9$, $q=-2$, $r=-28$. If contradictory, give preference to the expression. Condone correct values in an expression of the correct form, even if missing the square on the bracket. Allow B1 for $9\{(x-2)^2+k\}$.
		2	
1(b)	$k < -28$	B1FT	$k < \text{their } r$.
		1	
1(c)	$9(x - \text{their } 2)^2 - \text{their } 28 = -15 \Rightarrow (x-2)^2 = \frac{13}{9} \text{ or } x = \frac{36 \pm \sqrt{36^2 - 4 \times 9 \times 23}}{2 \times 9}$	M1	Solving as far as ' $(x+q)^2 =$ '. Allow sign errors only in the rearrangement, or rearranging and using the quadratic formula (see quadratics guidance).
	$x = 2 \pm \frac{\sqrt{13}}{3} \text{ or } \frac{6 \pm \sqrt{13}}{3} \text{ or } 2 \pm \sqrt{\frac{13}{9}} \text{ or } \frac{36 \pm \sqrt{468}}{18} \text{ oe}$	A1	ISW if decimals given after the exact answers.
		2	

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Question	Answer	Marks	Guidance
2	$\binom{6}{2} \times (2x^2)^2 \times \left(-\frac{3}{x}\right)^{6-2}$ or $\binom{6}{2} \times 2^6 \times (x^2)^2 \times \left(-\frac{3}{2x}\right)^{6-2}$	B1	$\binom{6}{2}$ may be replaced by $\binom{6}{4}$ or 15. May be seen in a list. Condone missing brackets only if recovered later.
	Identifying the relevant term of the form: $k \times (2x^2)^2 \times \left(-\frac{3}{x}\right)^{6-2}$ or $k \times 2^6 \times (x^2)^2 \times \left(-\frac{3}{2x}\right)^{6-2}$	M1	May still be part of a list if, e.g., underlining or arrow. k can be any constant or any $\left(\frac{p}{q}\right)$. May be implied by correct final answer. If constants evaluated, then condone errors for this mark.
	= 4860	A1	Accept $4860x^0$ SC B2 for -4860
		3	

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Question	Answer	Marks	Guidance
3(a)	Translation $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$, or Translation 2 ‘parallel to the y -axis’ or ‘in the y -direction’ or ‘vertically’.	B1	Condone ‘shift’ and vector written as co-ordinates. Allow any mention of the y -axis but not just ‘up’.
	{Stretch} {factor $\frac{1}{3}$ } {parallel to x -axis or in x -direction or horizontally}	B2,1,0	B2 for 3 correct components. B1 for 2. Allow ‘the y -axis is invariant’.
		3	
3(b)	$[y=]3g(-x)$	B2,1,0	B1 for 3 outside the function and nowhere else, or – inside the function and nowhere else. So $-3g(x)$, $3(-gx)$, $g(-3x)$, $g\left(\frac{x}{-3}\right)$, $3g(x)+2$, $-3g(-x)$ and $3g(-3x)$ all score B1. But $3g(3x)$, $-g(-x)$ score B0. SC B1 for $3f(-x)$.
		2	

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Question	Answer	Marks	Guidance
4(a)	$[k =]4$	B1	
		1	
4(b)	$\left\{\frac{k}{4}x^4\right\} + \left\{\frac{2x^{-2+1}}{-2+1}\right\} [+c]$	B2,1,0FT	FT <i>their k</i> from 4(a) or the letter <i>k</i> . B2 for both correct components and no other <i>x</i> -terms. B1 for one correct.
	$20 = \frac{\text{their } k}{4} \times 2^4 - \frac{2}{2} + c$	M1	Substituting $x = 2$ and $y = 20$ into <i>their</i> integrated expression (defined by having at least one correct power and including a '+c'). Allow one slip.
	$c = 5$	A1	
	$[t =]4$	A1	
		5	

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Question	Answer	Marks	Guidance
5(a)	$\left[\frac{dy}{dx}\right] 2x^{-\frac{1}{2}} - 1$	B1	
	$2x^{-\frac{1}{2}} - 1 = 0 \Rightarrow x^{\frac{1}{2}} = 2$	M1	Setting <i>their</i> $\frac{dy}{dx}$ of the form $kx^{-\frac{1}{2}} - 1$ equal to 0 and solving as far as $x^{\frac{1}{2}} = d$, condoning sign errors only. Can be implied by correct final answer but not if clearly following wrong working.
	$[a=]4$	A1	
	Alternative Method for Question 5(a):		
	$y = 4\sqrt{x} - (\sqrt{x})^2$	B1	Recognising the quadratic in $\sqrt{x}$.
	Max at $\sqrt{x} = \frac{-b}{2a} = \frac{-4}{-2} = 2$ or $-(\sqrt{x} - 2)^2 + 4 \Rightarrow x^{\frac{1}{2}} = 2$	M1	Condone sign errors only. Allow $z = \sqrt{x}$, $-(z - 2)^2 + 4 \Rightarrow \text{max at } 2$.
	$[a=]4$	A1	
		3	

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Question	Answer	Marks	Guidance
5(b)	$[x = \text{or } b =] 16$	B1	SOI It may be found in 5(a) , but must be seen in 5(b) . Condone extra ‘solution’ $x = 0$.
	$\left[\int (4x^{\frac{1}{2}} - x) dx = \right] \left\{ \frac{4}{\frac{3}{2}} x^{\frac{3}{2}} \right\} + \left\{ -\frac{1}{2} x^2 \right\}$	B2, 1, 0	B2 for both correct components and no other x terms. B1 for one correct term. Allow any correct unsimplified form.
	$[\text{Area} =] \left(\frac{8}{3} \times 16^{\frac{3}{2}} - \frac{1}{2} \times 16^2 \right) - \left(\frac{8}{3} \times 4^{\frac{3}{2}} - \frac{1}{2} \times 4^2 \right)$	M1	Substituting <i>their a</i> (from part (a)) and <i>their b</i> (from an attempt to solve $4x^{\frac{1}{2}} - x = 0$) into an integrated expression (defined by having at least one correct power) and subtracting. If correct limits and integration, then minimum acceptable working is $\frac{128}{3} - \frac{40}{3}$. If incorrect limits or integration, then full substitution of every term must be seen. Note: needs $0 < a < b$, otherwise M0, but allow limits applied either way round. Allow missing brackets if recovered.
	$\left\{ = \frac{128}{3} - \frac{40}{3} \right\} = \frac{88}{3} \text{ or } 29\frac{1}{3}$	DB1	Must be exact. Allow $-\frac{88}{3}$ if it becomes $\frac{88}{3}$. Do not ISW if a further area is added or subtracted. Dependent upon B1B2 scored earlier.
		5	

Question	Answer	Marks	Guidance
6(a)		B2, 1, 0	B2 Fully correct curve, including 2, 5 and -1 clearly labelled or implied on the y -axis and the curve horizontal only at $(\frac{1}{2}\pi, 5)$ and $(\frac{3}{2}\pi, -1)$ (approximately). Condone the x -axis intercepts and the stationary points not being in exactly the correct positions and y -axis not to scale. B1 for one full cycle from 0 to 2π starting on the positive y -axis and initially increasing. Note: Starting at the origin or finishing at $(2\pi, 0)$ scores B0.
		2	
6(b)(i)	1	B1	Ignore any working out seen or graphs drawn.
		1	
6(b)(ii)	3	B1	
		1	

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Question	Answer	Marks	Guidance
6(c)	$3\sin x + 2 = 5(1 - \sin^2 x) - 1$	M1*	For replacing $\cos^2 x$ with $1 - \sin^2 x$.
	$5\sin^2 x + 3\sin x - 2 \quad [=0]$	DM1	For simplifying to a 3-term quadratic. The terms don't all need to be on the same side. Condone sign errors only. Candidates who miss out the '−1' can score the M1DM1 only.
	$\sin x = 0.4$ and -1	A1	
	$x = 0.41, 2.73, \frac{3}{2}\pi$ AWRT or $0.13\pi, 0.87\pi, 1.5\pi$ or $\frac{59}{450}\pi, \frac{391}{450}\pi, \frac{3}{2}\pi$	A2, 1, 0	A2 for three correct values and no others in the range $0 \leq x \leq 2\pi$. A1 for 2 correct values. Ignore $^\circ$ symbol if seen with answers. Ignore answers outside the domain. Allow 4.71 or equivalent fractions instead of $\frac{3}{2}\pi$ SC A1 for $23.6^\circ, 156.4^\circ$ AWRT and 270° or $\frac{3}{2}\pi$ only. SC A1 for 0.4, 2.7 and 4.7 or $\frac{3}{2}\pi$ only.
		5	

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Question	Answer	Marks	Guidance
7(a)	[Centre is] (4, 6)	B1	SOI
	$\sqrt{3^2 + 5^2}$ or $3^2 + 5^2$ or $\sqrt{6^2 + 10^2}$ or $6^2 + 10^2$ oe	B1	SOI Finding PQ or PQ^2 or $\frac{1}{2}PQ$ or $(\frac{1}{2}PQ)^2$. Allow 5.83 or 5.83^2 or 11.7 or 11.7^2 AWR. T. Allow unsimplified, e.g. $\sqrt{(4-1)^2 + (6-1)^2}$.
	$(x - \text{their } 4)^2 + (y - \text{their } 6)^2 = (\text{their radius})^2$	M1	Using what they think that the centre and radius are, correctly, in the equation of a circle. Do not allow P or Q as the centre or the diameter being used as the radius.
	$(x-4)^2 + (y-6)^2 = 34$ or $x^2 + y^2 - 8x - 12y + 18 = 0$	A1	May be done by other equivalent method. Do not allow $(\sqrt{34})^2$ or 5.83^2 for the final answer.
		4	

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Question	Answer	Marks	Guidance
7(b)	[Gradient of PQ =] $\frac{11-1}{7-1}$ or $\frac{5}{3}$ oe	B1	Allow unsimplified.
	[Gradient of tangent =] $-\frac{3}{5}$	M1	Use of $m_1m_2 = -1$ with <i>their</i> gradient PQ .
	$\frac{y-11}{x-7} = -\frac{3}{5}$ oe e.g. $y = -\frac{3}{5}x + \frac{76}{5}$	A1	ISW Condone stopping at $c = \frac{76}{5}$ if $y = -\frac{3}{5}x + c$ stated earlier.
	Alternative Method for Question 7(b):		
	$2(x-4) + 2(y-6)\frac{dy}{dx} = 0$	B1	Or rearranging to form ' $y =$ ' and differentiating using the chain rule.
	Substitute $x = 7, y = 11 \Rightarrow 6 + 10\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{3}{5}$	M1	Substituting $x = 7$ and $y = 11$ into a correct differential equation and rearranging to give $\frac{dy}{dx}$ allowing sign errors only.
	$\frac{y-11}{x-7} = -\frac{3}{5}$ oe e.g. $y = -\frac{3}{5}x + \frac{76}{5}$	A1	
		3	

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Question	Answer	Marks	Guidance
7(c)	y -coordinate of R is 1	B1	
	[Equation of tangent at R is] $\frac{y-1}{x-7} = \frac{3}{5}$ oe eg $y = \frac{3}{5}x - \frac{16}{5}$	B1	
	$\frac{3}{5}(x-7)+1 = -\frac{3}{5}(x-7)+11 \Rightarrow x = \dots$ or $y = \dots$	M1	Using <i>their</i> stated line and the equation from part (b) to reach a value of x or y . Do not allow if the gradients are the same.
	$x = \frac{46}{3}$ oe and $y = 6$	A1	
	Alternate Method for Question 7(c):		
	y -coordinate of R is 1	B1	Can be implied by correct y -coordinate of point of intersection.
	y -coordinate of point of intersection is 6	B1	
	Then either		
	$6-11 = -\frac{3}{5}(x-7)$	M1	Using $y = 6$ in <i>their</i> equation of tangent at R .
	Or		
	$(x-4)^2 = 34 + (x-7)^2 + 25$	(M1)	Using the right-angled triangle formed by the centre, Q and the point of intersection of the tangents.
	$x = \frac{46}{3} \left[\frac{46}{3}, 6 \right]$ oe	A1	Allow 15.3 AWRT.
		4	

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Question	Answer	Marks	Guidance
8(a)	$-3c - b = b - a$ or $2b = a - 3c$ or $\frac{b}{a} = \frac{c}{b}$ or $b^2 = ac$ oe	B1	For one correct equation connecting a , b and c only. Allow $a + 2(b - a) = -3c$.
	$4ac = (a - 3c)^2$ or $2\sqrt{ac} = a - 3c$ oe	M1	Forming an equation in a and c only using both the AP and GP. Condone sign errors only.
	$\Rightarrow a^2 - 6ac + 9c^2 = 4ac \Rightarrow a^2 - 10ac + 9c^2 = 0$	A1	AG Convincing proof needed, e.g. $(a - 3c)^2$ expanded.
		3	

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Question	Answer	Marks	Guidance
8(b)(i)	$81 - 90c + 9c^2 = 0 \Rightarrow [9](c-9)(c-1) = 0$ or $\frac{90 \pm \sqrt{5184}}{18} \left[= \frac{90 \pm 72}{18} \right]$	M1	Factorising or other accepted method for solving. See quadratics guidance but condone missing the factor 9. Allow presence of a .
	$c = 1$	B1	Condone extra ‘solution’ $c = 9$. B0 if any error seen leading to $c = 1$ but all further marks are still available in this part and (b)(ii) .
	$r^2 = \frac{\text{their } 1}{9} \Rightarrow r = \left[\frac{1}{3} \right],$ or $b^2 = \text{their } 1 \times 9 \Rightarrow b = 3 \Rightarrow r = \left[\frac{1}{3} \right]$	M1*	Correctly finding a value for r from <i>their</i> c . May be implied by $r = \frac{1}{3}$ or $r = \pm \frac{1}{3}$ or $r = -\frac{1}{3}$.
	$S_{\infty} = \frac{9}{1 - \frac{1}{3}}$	DM1	Use of correct sum to infinity formula with 9 and <i>their</i> $ r < 1$. Ignore any extra working with a negative value of r .
	$S_{\infty} = \frac{27}{2}$ oe	A1	Do not condone extra ‘answer’. This mark is not dependent upon the first M1.
		5	

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Question	Answer	Marks	Guidance
8(b)(ii)	$d = -6$	B1	SOI
	$S_{20} = \frac{20}{2}(2 \times 9 + (20 - 1) \times (-6))$	M1	Use of a correct sum formula with $n = 20$, $a = 9$ and <i>their</i> non-zero d stated in this part (can be implied by quoting the correct formula then substituting). But allow use of ± 6 without stating either the formula or ' $d =$ '.
	$S_{20} = -960$	A1	
		3	

Question	Answer	Marks	Guidance
9(a)	$\frac{4 \times 3 \times (-2)}{(3x - 6)^3} + \frac{1 \times 3 \times (-3)}{(3x - 6)^4}$ or $4 \times 3 \times (-2)(3x - 6)^{-3} + 1 \times 3 \times (-3)(3x - 6)^{-4}$	B1B1B1	B1 for the correct powers, B1 for $\times 3$ in at least one term, B1 for all correct which can be unsimplified.
	Decreasing.	B1*	This mark is only available if $f'(x)$ is of the form $(-p)(3x - 6)^{-3} + (-q)(3x - 6)^{-4}$, where p and q are positive coefficients.
		4	
9(b)	f^{-1} exists because f is a decreasing function Or f^{-1} exists because it is one-to-one, or passes the horizontal line test.	DB1	
		1	

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Question	Answer	Marks	Guidance
9(c)	$g(x) > 4a - 3$	B1	Allow ' $y >$ ' or ' $g >$ ' only. Accept $4a - 3 < y < \infty$ or $(4a - 3, \infty)$. Condone $(4a - 3, \infty]$. Accept $g(x) > g(a)$, but not if they make an error 'simplifying' it.
		1	
9(d)	Either $4a - 3 > 2$ allow with x or a or $\geq$	M1	Do not allow $=$ or $\neq$, unless they reach a correct inequality later.
	$a > \frac{5}{4}$ or $a \geq \frac{5}{4}$ oe	A1	Accept $\frac{5}{4} < a < \infty$, $\frac{5}{4} \leq a < \infty$, $\left(\frac{5}{4}, \infty\right)$ or $\left[\frac{5}{4}, \infty\right)$.
	Or $3 \times (4a - 3) - 6 > 0$ allow with x or a or $\geq$	M1	Do not allow $=$ or $\neq$, unless they reach a correct inequality later.
	$a > \frac{5}{4}$ or $a \geq \frac{5}{4}$ oe	A1	Accept $\frac{5}{4} < a < \infty$, $\frac{5}{4} \leq a < \infty$, $\left(\frac{5}{4}, \infty\right)$ or $\left[\frac{5}{4}, \infty\right)$.
		2	

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Question	Answer	Marks	Guidance
10(a)	$s^2 = r^2 + r^2 - 2r^2 \cos \frac{2\pi}{3}$ or $\frac{r}{\sin \frac{\pi}{6}} = \frac{s}{\sin \frac{2\pi}{3}}$ or $s = 2r \times \cos \frac{\pi}{6}$ or $\left(\frac{r}{2}\right)^2 + \left(\frac{s}{2}\right)^2 = r^2$ or $\left(\frac{3r}{2}\right)^2 + \left(\frac{s}{2}\right)^2 = s^2$ oe	M1	Forming a correct equation. Allow use of degrees.
	$\Rightarrow$ E.g. $s = 2r \times \frac{\sqrt{3}}{2} \Rightarrow s = \sqrt{3}r$	A1	AG Convincing proof required. E.g., for those using trig, the exact value of the trig ratio substituted before the final answer is needed.
		2	

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Question	Answer	Marks	Guidance
10(b)	[Angle BAD, BAC or CAD =] $\frac{2\pi}{3}$ or 120°	B1	SOI, unless clearly associated with the wrong angle. Condone 2.09 for this mark only.
	[Sector BAD =] $\frac{1}{2}r^2 \times \frac{2\pi}{3}$ oe	B1	
	[Sector BCD =] $\frac{1}{2}s^2 \times \frac{\pi}{3}$ oe	B1*	
	$= \frac{3}{6}\pi r^2$ or $\frac{1}{2}\pi r^2$	DB1	Sector BCD simplified in terms of r .
	Either [Area triangle ABD, ABC or ACD =] $\frac{1}{2}r^2 \times \sin \frac{2\pi}{3}$ or $\frac{r}{2} \times \frac{s}{2}$ oe	B1*	May be multiplied by 2 or by 3. Note: $\frac{1}{2}r^2 \times \sin \frac{\pi}{3} = \frac{\sqrt{3}}{4}r^2$ is B0DB0.
	$= \frac{\sqrt{3}}{4}r^2$	DB1	May be multiplied by 2 or by 3.
	Or [Area triangle CBD =] $\frac{1}{2}s^2 \times \sin \frac{\pi}{3}$ or $\frac{3r}{2} \times \frac{s}{2}$ oe	(B1*	Note: $\frac{1}{2}s^2 \times \sin \frac{2\pi}{3} = \frac{3\sqrt{3}}{4}r^2$ is B0DB0.
	$= \frac{3\sqrt{3}}{4}r^2$	DB1)	
	$\left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right)r^2$ or e.g. $\left(\frac{12\sqrt{3} - 4\pi}{24}\right)r^2$ oe in the required exact form	B1	Fully correct solution. Accept $a = \frac{\sqrt{3}}{2}$, $b = -\frac{\pi}{6}$.
		7	



Cambridge International AS & A Level

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MATHEMATICS

9709/13

Paper 1 Pure Mathematics 1

October/November 2025

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages.

- 1 (a) Expand $\left(2 - \frac{1}{2}x\right)^6$ in ascending powers of x up to and including the term in x^3 . [3]

$${}^6C_0 (2)^6 x \left(-\frac{1}{2}x\right)^0 + {}^6C_1 (2)^5 \left(-\frac{1}{2}x\right)^1 + {}^6C_2 (2)^4 \left(-\frac{1}{2}x\right)^2 + {}^6C_3 (2)^3 \left(-\frac{1}{2}x\right)^3$$

$$1 \times 64 \times 1 + 6 \times 32 \times -\frac{1}{2}x + 15 \times 16 \times \frac{1}{4}x^2 + 20 \times 8 \times -\frac{1}{8}x^3$$

$$\left(2 - \frac{1}{2}x\right)^6 = 64 - 96x + 60x^2 - 20x^3$$

- (b) Hence find the coefficient of x^3 in the expansion of $(3 - x + 2x^3)\left(2 - \frac{1}{2}x\right)^6$. [2]

$$(3 - x + 2x^3)(64 - 96x + 60x^2 - 20x^3)$$

$$\text{Coefficient of } x^3 = -60 - 60 + 128$$

$$\text{coefficient of } x^3 = 8$$



- 2 (a) Solve the equation $\tan^{-1}(5x-3) = -\frac{1}{4}\pi$.

[2]

$$5x-3 = \tan\left(-\frac{1}{4}\pi\right)$$

$$5x-3 = -1$$

$$5x = 2$$

$$x = \frac{2}{5}$$

- (b) Solve the equation $5\cos^2\theta = 4\sin\theta + 4$ for $\frac{1}{2}\pi \leq \theta \leq 2\pi$.

[4]

$$5(1 - \sin^2\theta) = 4\sin\theta + 4$$

$$5 - 5\sin^2\theta = 4\sin\theta + 4$$

$$5\sin^2\theta + 4\sin\theta - 1 = 0$$

$$5s^2 + 4s - 1 = 0$$

$$5s(s+1) - 1(s+1) = 0$$

$$(5s-1)(s+1) = 0$$

$$\sin\theta = \frac{1}{5}, \sin\theta = -1$$

$$\theta = 0.201358, \frac{1}{2}\pi$$

$$\theta = 2.94 \text{ and } \frac{3}{2}\pi$$

$$\theta = 2.94 \text{ and } 4.71$$



- 3 The equation of a curve is $y = f(x)$, where $f(x) = \frac{1}{2}x^{\frac{2}{3}}(x-2)^2$. The following points lie on the curve. Non-exact values of the y -coordinates are given correct to 6 decimal places.

$A(8, 72), B(8.001, k), C(8.01, 72.300388), D(8.1, 75.038882)$

- (a) Find the value of k . Give your answer correct to 6 decimal places. [1]

$$k = \frac{1}{2} (8.001)^{\frac{2}{3}} (8.001 - 2)^2$$

$$k = 72.030004$$

The table below shows the gradients of the chords AB and AC , given correct to 4 decimal places.

Chord	AB	AC	AD
Gradient of chord	30.0039	30.0388	

- (b) Find the gradient of the chord AD . Give your answer correct to 4 decimal places. [1]

$$m_{AD} = \frac{75.038882 - 72}{8.1 - 8}$$

$$m_{AD} = 30.3888$$

- (c) State what the values in the table suggest about the value of $f'(8)$. [1]

It suggest that $f'(8) = 30$.



- 4 The first, second and third terms of a progression are 20, k and $k-5$ respectively.

(a) Given that the progression is arithmetic, find the 30th term.

[2]

$$k - 20 = k - 5 - k$$

$$k = 15$$

$$a = 20$$

$$d = -5$$

$$T_{30} = 20 + 29(-5)$$

$$T_{30} = -125$$

(b) Given instead that the progression is geometric, find the sum to infinity.

[4]

$$\frac{k}{20} = \frac{k-5}{k}$$

$$k^2 = 20k - 100$$

$$k^2 - 20k + 100 = 0$$

$$(k-10)^2 = 0$$

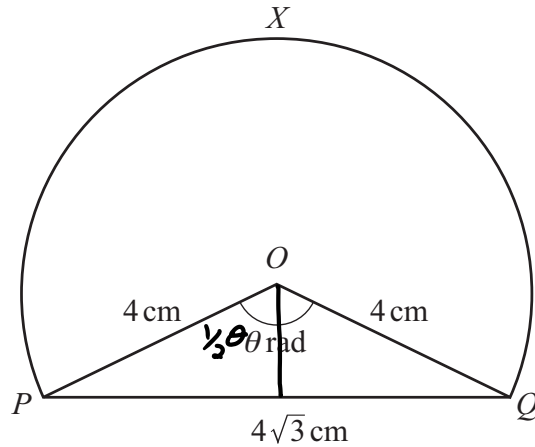
$$k = 10$$

$$a = 20, \quad r = \frac{10}{20} = \frac{1}{2}$$

$$S_{\infty} = \frac{a}{1-r} = \frac{20}{1-\frac{1}{2}}$$

$$S_{\infty} = 40$$





The diagram shows part of a circle with centre O and radius 4 cm . The chord PQ is of length $4\sqrt{3}\text{ cm}$ and angle $POQ = \theta$ radians. The point X lies on the circle.

- (a) Find the exact value of θ . [2]

$$\sin \frac{1}{2}\theta = \frac{2\sqrt{3}}{4}$$

$$\sin \frac{1}{2}\theta = \frac{\sqrt{3}}{2}$$

$$\frac{1}{2}\theta = \frac{1}{3}\pi$$

$$\theta = \frac{2}{3}\pi$$

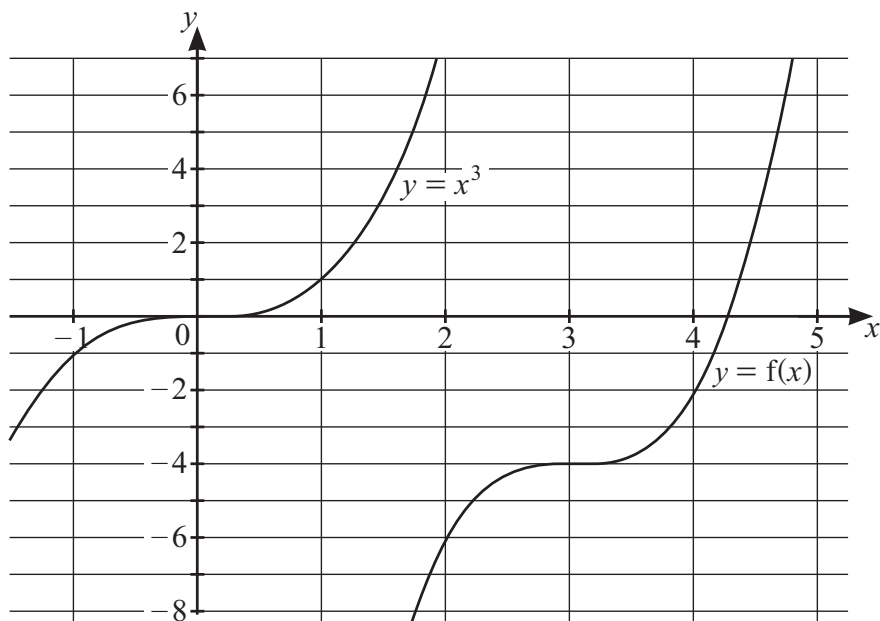
- (b) Find the exact area of the segment PXQ . [3]

Total Area = Area of Sector + Area of Triangle

$$\text{Total Area} = \left(\frac{1}{2} \times 4^2 \times \frac{4}{3}\pi \right) + \left(\frac{1}{2} \times 4^2 \times \sin \frac{2\pi}{3} \right)$$

$$\text{Total Area} = \frac{32}{3}\pi + 4\sqrt{3}$$





The diagram shows the graphs of $y = x^3$ and $y = f(x)$. The graph of $y = x^3$ is transformed to the graph of $y = f(x)$ by a sequence of transformations.

- (a) Describe fully a suitable sequence of transformations. Make clear the order in which the transformations are applied. [5]

Translation with $T = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$
vertical stretch of factor 2

- (b) You are given that $f(x) = a(x+b)^3 + c$.

State the values of the constants a , b and c .

[3]

$$f(x) = 2[(x-3)^3 - 2]$$

$$f(x) = 2(x-3)^3 - 4$$

$$a = 2$$

$$b = -3$$

$$c = -2$$



- 7 The function g is defined by $g(x) = \frac{2}{ax-3} + \frac{1}{2}$ for $x > \frac{3}{a}$, where a is a positive constant.

Find $g^{-1}(x)$ and hence verify that if $a = 6$ then $g^{-1}(x) \equiv g(x)$.

[4]

$$y = \frac{4 + ax - 3}{2ax - 6}$$

$$2axy - 6y = 1 + ax$$

$$2axy - ax = 1 + 6y$$

$$x(2ay - a) = 1 + 6y$$

$$x = \frac{1 + 6y}{2ay - a}$$

$$g^{-1}(x) = \frac{1 + 6x}{2ax - a}$$

$$\text{If } a = 6, \quad g^{-1}(x) = \frac{1 + 6x}{12x - 6}$$

$$g(x) = \frac{4 + ax - 3}{2ax - 6} = \frac{1 + 6x}{12x - 6}$$

Since both expressions are same,

$$g(x) \equiv g^{-1}(x).$$



- 8 The points (6, 1) and (−2, 7) lie at the opposite ends of a diameter of a circle.

(a) Find the equation of the circle.

[3]

$$\text{Centre} = \left(\frac{6-2}{2}, \frac{1+7}{2} \right) = (2, 4)$$

$$\text{Radius} = \sqrt{(3)^2 + (-4)^2} = 5$$

$$\text{Equation: } (x-2)^2 + (y-4)^2 = 25$$

- (b) There are two tangents to the circle which have gradient $-\frac{1}{2}$.

Find the exact values of the x -coordinates of the points at which these tangents touch the circle.

[5]

$$y = \sqrt{25 - (x-2)^2} + 4$$

$$y = (21 + 4x - x^2)^{1/2} + 4$$

$$\frac{dy}{dx} = \frac{1}{2}(4-2x)(21+4x-x^2)^{-1/2}$$

$$(2-x) \frac{1}{\sqrt{21+4x-x^2}} = -\frac{1}{2}$$

$$2x-4 = \sqrt{21+4x-x^2}$$

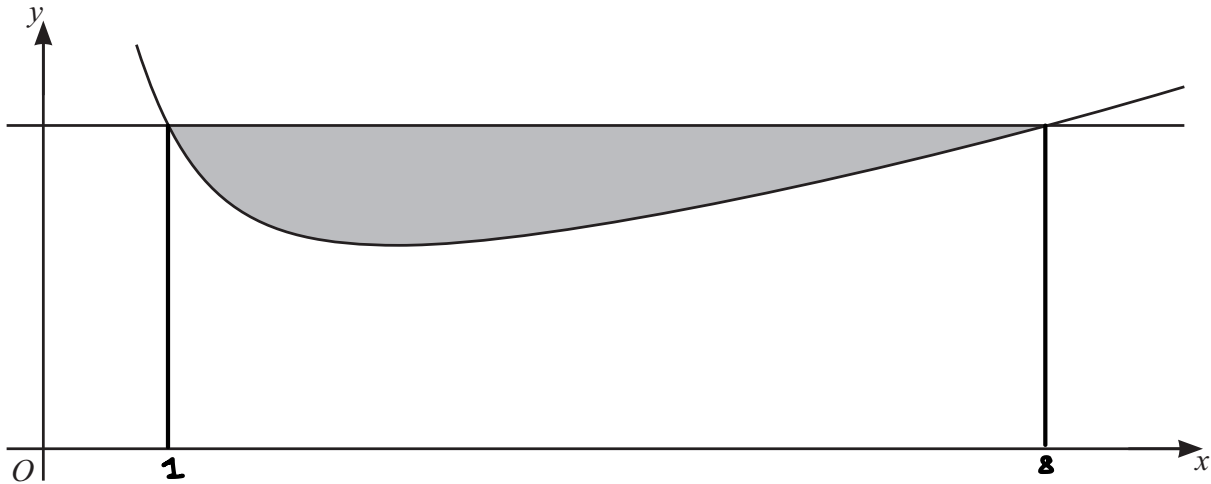
$$4x^2 - 16x + 16 = 21 + 4x - x^2$$

$$5x^2 - 20x - 5 = 0$$

$$x = \frac{20 \pm \sqrt{400 + 100}}{10}$$

$$x = 2 + \sqrt{5} \text{ and } x = 2 - \sqrt{5}$$





The diagram shows part of the curve with equation $y = \frac{1}{2}x + \frac{4}{x}$ and the line $y = 4.5$.

Find the exact volume of the solid formed when the shaded region is rotated through 360° about the x -axis. [8]

Points of Intersection: $\frac{9}{2} = \frac{x}{2} + \frac{4}{x}$

$$\frac{9}{2} = \frac{x^2 + 8}{2x}$$

$$9x = x^2 + 8$$

$$x^2 - 9x + 8 = 0$$

$$x^2 - 8x - x + 8 = 0$$

$$x(x-8) - 1(x-8) = 0$$

$$(x-1)(x-8) = 0$$

$$x = 1 \text{ and } x = 8.$$

Volume = Volume of line - Volume of line.

$$\text{Volume} = \pi \int_1^8 (4.5)^2 dx - \pi \int_1^8 \left(\frac{x}{2} + \frac{4}{x} \right)^2 dx$$

$$\text{Volume} = \pi [20.25x]_1^8 - \pi \int_1^8 \left(\frac{x^2}{4} + 4 + \frac{16}{x^2} \right) dx$$

$$\text{Volume} = \pi [20.25x]_1^8 - \pi \left[\frac{x^3}{12} + 4x - \frac{16}{x} \right]_1^8$$

$$\text{Volume} = 162\pi - \frac{81}{4}\pi - \frac{128}{3}\pi - 32\pi + 2\pi + \frac{1}{12}\pi + 4\pi - 16\pi$$

$$\text{Volume} = \frac{343}{6}\pi$$





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10 A function f is defined by $f(x) = px^2 + 4x + q$ for $x \in \mathbb{R}$, where p and q are constants.

(a) It is given that $p = 2$ and $q = 10$.

(i) Express $f(x)$ in the form $a(x+b)^2 + c$, where a , b and c are constants. [3]

$$f(x) = 2x^2 + 4x + 10$$

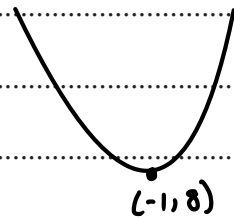
$$f(x) = 2(x^2 + 2x) + 10$$

$$f(x) = 2(x^2 + 2x + 1) + 8$$

$$f(x) = 2(x^2 + 1)^2 + 8$$

(ii) State the range of f . [1]

$$f \geq 8$$



- (b) It is given instead that $q = -5$ and the roots of $f(x) = 0$ are $5m$ and $-9m$, where m is a constant.

Find the values of p and m .

[5]

$$px^2 + 4x - 5 = 0$$

$$5m \rightarrow 25m^2p + 20m - 5 = 0$$

$$-9m \rightarrow 81m^2p - 36m - 5 = 0$$

$$25m^2p + 20m = 81m^2p - 36m$$

$$56m = 56m^2p$$

$$mp = 1$$

$$p = \frac{1}{m}$$

$$25m + 20m = 5$$

$$45m = 5$$

$$m = \frac{1}{9}$$

$$p = 1 \div \frac{1}{9}$$

$$p = 9$$



11 The equation of a curve is $y = \frac{8}{3x-8} - \frac{6}{x-1}$.

- (a) Find the coordinates of the point at which the tangent to the curve at the point (3, 5) intersects the line $y = -8x$. [6]

$$y = 8(3x-8)^{-1} - 6(x-1)^{-1}$$

$$\frac{dy}{dx} = -24(3x-8)^{-2} + 6(x-1)^{-2}$$

$$\frac{dy}{dx} \text{ at } x=3 \rightarrow -\frac{45}{2}$$

$$\text{Tangent: } y - 5 = -\frac{45}{2}(x-3)$$

$$y = -\frac{45}{2}x + \frac{145}{2}$$

$$2y = -45x + 145$$

$$-45x + 145 = -16x$$

$$145 = 29x$$

$$x = 5$$

$$y = -8(5)$$

$$y = -40$$

Co-ordinates of point of intersection: (5, -40)



- (b) (i) Find the x -coordinates of each of the stationary points of the curve.

[3]

$$\frac{dy}{dx} = \frac{-24}{(3x-8)^2} + \frac{6}{(x-1)^2} = 0$$

$$-24(x-1)^2 + 6(3x-8)^2 = 0$$

$$-24x^2 + 48x - 24 + 54x^2 - 288x + 384 = 0$$

$$30x^2 - 240x + 360 = 0$$

$$x^2 - 8x + 12 = 0$$

$$x^2 - 6x - 2x + 12 = 0$$

$$x(x-6) - 2(x-6) = 0$$

$$(x-2)(x-6) = 0$$

$$x = 2 \text{ OR } x = 6.$$

- (ii) Find $\frac{d^2y}{dx^2}$ and hence determine the nature of each of the stationary points.

[4]

$$\frac{dy}{dx} = -24(3x-8)^{-2} + 6(x-1)^{-2}$$

$$\frac{d^2y}{dx^2} = 144(3x-8)^{-3} - 12(x-1)^{-3}$$

$$\text{Nature: } 144(3 \times 2 - 8)^{-3} - 12(2 - 1)^{-3} \text{ OR } 144(3 \times 6 - 8)^{-3} - 12(6 - 1)^{-3}$$

$$\frac{d^2y}{dx^2} = -30$$

$$\frac{d^2y}{dx^2} = \frac{6}{125}$$

Negative shows maximum.

Positive shows minimum.

Nature: when $x=2$, it is a maximum point.

when $x=6$, it is a minimum point.



Cambridge International AS & A Level

MATHEMATICS**9709/13**

Paper 1 Pure Mathematics 1

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

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This document consists of **26** printed pages.

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Question	Answer	Marks	Guidance
1(a)	$\left[\begin{pmatrix} 6 \\ 0 \end{pmatrix} \right] 2^6 \pm \begin{pmatrix} 6 \\ 1 \end{pmatrix} 2^5 \left(\frac{x}{2} \right) \pm \begin{pmatrix} 6 \\ 2 \end{pmatrix} 2^4 \left(\frac{x}{2} \right)^2 \pm \begin{pmatrix} 6 \\ 3 \end{pmatrix} 2^3 \left(\frac{x}{2} \right)^3$ or $2^6 \left(1 \pm 6 \left(\frac{x}{4} \right) \pm \frac{6.5}{2!} \left(\frac{x}{4} \right)^2 \pm \frac{6.5.4}{3!} \left(\frac{x}{4} \right)^3 \right)$	M1	OE A correct unsimplified expansion of at least four terms. Condone $\pm$ sign errors for this mark. Condone omission of brackets if recovered. This mark can be implied by sight of $64 + 96x + 60x^2 + 20x^3$.
	$64 - 96x + 60x^2 - 20x^3$	A1	Three correct simplified terms.
		A1	Four correct simplified terms. Accept these terms given in a list.
		3	
1(b)	$(3 \times \text{their}(-20)) + (-1 \times \text{their} 60) + (2 \times \text{their} 64)$	M1	Selecting the correct products using <i>their</i> expansion.
	8	A1	Accept $8x^3$.
		2	

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Question	Answer	Marks	Guidance
2(a)	$5x - 3 = \tan\left(-\frac{1}{4}\pi\right)$ or $-\tan\left(\frac{1}{4}\pi\right)[=-1]$	M1	Condone $5x - 3 = \tan\left(\frac{1}{4}\pi\right)$ and $5x - 3 = -0.0137\dots$
	$x = \frac{2}{5}$ or 0.4	A1	
		2	
2(b)	$5 - 5\sin^2\theta = 4\sin\theta + 4$	*M1	Replacing $\cos^2\theta$ with $1 - \sin^2\theta$.
	$5\sin^2\theta + 4\sin\theta - 1 [= 0]$ $\quad [\Rightarrow (5\sin\theta - 1)(\sin\theta + 1) = 0]$	DM1	Collecting terms to form a three-term quadratic. Condone only $\pm$ errors.
	$\sin\theta = \frac{1}{5}$ and -1	A1	
	$\theta = 2.94, 4.71$	A1	AWRT Accept $\left(\frac{3}{2}\pi\right)$ in place of 4.71 and 0.936π or $\left(\frac{337}{360}\pi\right)$ in place of 2.94. Condone repetition of $\frac{3}{2}\pi$ but no other values within the range. Ignore answers outside the range $\frac{\pi}{2} \leq \theta \leq 2\pi$.
		4	

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Question	Answer	Marks	Guidance
3(a)	72.030004	B1	CAO. Not AWRT.
		1	
3(b)	30.3888	B1	CAO. Not AWRT.
		1	
3(c)	30[.0]	B1	CAO. 30 may be accompanied by ‘around’, ‘approximately’ etc.
		1	

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Question	Answer	Marks	Guidance
4(a)	$20 + (30 - 1) \times (their - 5)$	M1	Correct expression for u_{30} using <i>their</i> calculated value of d .
	-125	A1	
		2	
4(b)	$r = \frac{k}{20}$ and $r = \frac{k-5}{k}$	B1	OE For two correct expressions for r . Can be implied by $\frac{k}{20} = \frac{k-5}{k}$ OE.
	$k^2 - 20k + 100 [= 0]$	M1	Forming a 3-term quadratic. Condone only $\pm$ errors.
	$[r =] \frac{1}{2}$	A1	May be seen in the S_{∞} formula.
	$\left[S_{\infty} = \frac{20}{1 - \frac{1}{2}} \right] 40$	A1	
	Alternative Method for Question 4(b)		
	$20r = k$ and $20r^2 = k - 5$	B1	Stating two correct relationships between k and r . This can be implied by $20r^2 = 20r - 5$ OE.
	$20r^2 - 20r + 5 [= 0]$	M1	Forming a three-term quadratic in r . Condone only $\pm$ errors.
	$[r =] \frac{1}{2}$	A1	May be seen in the S_{∞} formula.
	$\left[S_{\infty} = \frac{20}{1 - \frac{1}{2}} \right] 40$	A1	
		4	

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Question	Answer	Marks	Guidance
5(a)	$\sin\left(\frac{1}{2}\theta\right) = \frac{2\sqrt{3}}{4}$ or $\cos\left(\frac{1}{2}\theta\right) = \frac{2}{4}$ or $\cos\theta = \frac{4^2 + 4^2 - (4\sqrt{3})^2}{2 \times 4 \times 4} \left[= -\frac{1}{2} \right]$	B1	Use of a correct expression for either $\frac{\theta}{2}$ or θ .
	$[\theta] = \frac{2\pi}{3}$	B1	Correct answer can imply B1B1. Condone inconsistent use of θ .
		2	
5(b)	$\frac{1}{2} \times 4^2 \times \sin \frac{2\pi}{3}$ or $\frac{1}{2} \times 4^2 \times \frac{4\pi}{3}$ or $\frac{1}{2} \times 4^2 \times \frac{2\pi}{3}$	*M1	Use of a correct formula with <i>their</i> θ or $2\pi - \text{their } \theta, \theta > 0$. Triangle and/or minor sector may be seen in the formula for finding the minor segment.
	$\frac{1}{2} \times 4^2 \times \frac{4\pi}{3} + \frac{1}{2} \times 4^2 \times \sin \frac{2\pi}{3}$ Or $16\pi - \left(\frac{1}{2} \times 4^2 \times \frac{2\pi}{3} - \frac{1}{2} \times 4^2 \times \sin \frac{2\pi}{3} \right)$	DM1	Addition of relevant sector area with $2\pi - \text{their } \theta$, where $\theta > 0$, and triangle area with <i>their</i> θ , where $\theta > 0$. Subtraction of triangle area, with <i>their</i> $\theta, \theta > 0$, from relevant sector area, with <i>their</i> $\theta, \theta > 0$, and subtract from circle area.
	$\frac{32\pi}{3} + 4\sqrt{3}$	A1	OE. Exact equivalents only. SC B2 following M0DM0 for 40.4 AWRT.
		3	

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Question	Answer	Marks	Guidance
6(a)	{Stretch} {[scale] factor 2} {[parallel to] in/on y[-axis] or x-axis invariant}	B2,1,0	B2 for all three {} elements correct. B1 for two {} elements correct.
	{Translation} $\begin{pmatrix} \{3\} \\ \{-4\} \end{pmatrix}$ or {[+]3 in/on [the] x[direction]} and {-4 in/on [the] y [direction]}	B2,1,0	B2 for all three {} elements correct. B1 for two {} elements correct. Do not condone use of ‘Right’ and ‘Down’ in place of x and y directions. Note: The vectors can be seen as $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$.
	Correct transformations in the correct order without any extra transformations. Down and right can be condoned for this mark if the candidate’s intention is clear.	B1	Combined translations (condone transformations accompanied by a vector) must come after the stretch or, if separate, the $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$ must come after the stretch.
	Alternative Method for Question 6(a)		
	{Translation} $\begin{pmatrix} \{3\} \\ \{-2\} \end{pmatrix}$ or {[+]3 in/on [the] x [direction]} and {-2 in/on [the] y [direction]}	B2,1,0	B2 for all three {} elements correct. B1 for two {} elements correct. Do not condone use of ‘Right’ and ‘Down’ in place of x and y directions. Note: The vectors can be seen as $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$.
	{Stretch} {[scale]factor 2} {[parallel to] in/on y-[axis] or x axis invariant OE}	B2,1,0	
	Numerically correct transformations in the correct order without any extra transformations. Down and right can be condoned for this mark if the candidate’s intention is clear.	B1	Combined translations (condone transformations accompanied by a vector) must come before the stretch or, if separate, the $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ must come before the stretch.
		5	

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Question	Answer	Marks	Guidance
6(b)	$a = 2$	B1	Individual values are considered the final answer and NOT those in $a(x+b)^3 + c$, unless individual values are not stated.
	$b = -3$	B1	
	$c = -4$	B1	
		3	

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Question	Answer	Marks	Guidance
7	$\left[y = \frac{2}{ax-3} + \frac{1}{2} \Rightarrow \right] \left(y - \frac{1}{2} \right)(ax-3) = 2 \text{ or } ax-3 = \frac{2}{y - \frac{1}{2}}$	*M1	OE For setting 'y =' and rearranging. Condone only $\pm$ errors.
	$ax = \frac{4}{2y-1} + 3$	DM1	For making ax the subject. Condone only $\pm$ errors.
	$[y = \text{ or } g^{-1}(x) =] \frac{4}{a(2x-1)} + \frac{3}{a} \text{ or } \frac{2}{a\left(x - \frac{1}{2}\right)} + \frac{3}{a}$	A1	OE Note: x and y may be interchanged from the start.
	Clear replacement of a with 6 in g^{-1} and, if necessary, followed by simplification to arrive at exactly the same expressions.	A1	This mark is dependent on getting the correct inverse.
	Alternative Method for Question 7		
	$\left[y = \frac{ax+1}{2(ax-3)} \Rightarrow \right] 2y(ax-3) = ax+1$	*M1	OE For setting 'y =' and rearranging. Condone only $\pm$ errors.
	$ax(2y-1) = 6y+1$	DM1	For making $ax(2y-1)$ the subject, condone only $\pm$ errors.
	$[y = \text{ or } g^{-1}(x) =] \frac{6x+1}{a(2x-1)} \text{ or } \frac{3x + \frac{1}{2}}{a\left(x - \frac{1}{2}\right)}$	A1	OE Note: x and y may be interchanged from the start.
	Clear replacement of a with 6 in g^{-1} and, if necessary, followed by simplification to arrive at exactly the same expressions.	A1	This mark is dependent on getting the correct inverse.
		4	

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Question	Answer	Marks	Guidance
8(a)	Centre is (2, 4)	B1	
	[(Radius) ² =] $(6-2)^2 + (1-4)^2$ or $\left(\frac{1}{2}\sqrt{(6-(-2))^2 + (1-7)^2}\right)^2 [= 25]$	M1	Use of a correct method to find (radius) ² or radius. This can be implied by $r = 5$.
	$(x-2)^2 + (y-4)^2 = (5)^2$	A1	OE
	Alternative Method for Question 8(a)		
	Either $\frac{y-1}{x-6}$ or $\frac{y-7}{x+2}$	B1	Correct expression using either (6,1) or (-2,7) and (x,y).
	$\frac{y-1}{x-6} \times \frac{y-7}{x+2} = -1$	M1	Product of <i>their</i> gradients = -1.
	$x^2 + y^2 - 4x - 8y - 5 = 0$	A1	OE Simplification to get the correct five terms.
		3	

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Question	Answer	Marks	Guidance
8(b)	[Gradient of normal through centre =] 2	B1	SOI
	$y - 4 = (\text{their gradient})(x - 2)$ Or $4 = (\text{their } 2)(2) + c \Rightarrow c = [0]$	*M1	Correct form of a line equation with <i>their</i> <u>normal</u> gradient i.e. not $-\frac{1}{2}$
	$(x - 2)^2 + (2x - 4)^2 = 25$	DM1	OE Using <i>their</i> $2x$ and <i>their</i> circle to form an equation in x .
	$[5(x - 2)^2 = 25 \Rightarrow](x - 2)^2 = 5$	A1	OE Accept $x^2 - 4x - 1 [= 0]$.
	$[x =] 2 \pm \sqrt{5}$	A1	OE Exact values only.
	Alternative Method for Question 8(b)		
	$2x + 2y \frac{dy}{dx} - 4 - 8 \frac{dy}{dx} = 0$	B1	OE Implicit differentiation of the correct circle equation.
	$2x + 2y \left(-\frac{1}{2}\right) - 4 - 8 \left(-\frac{1}{2}\right) = 0$	*M1	Replacing $\frac{dy}{dx}$ with $-\frac{1}{2}$ in their differential which must contain two terms in $\frac{dy}{dx}$. Expect $y = 2x$.
	$(x - 2)^2 + (2x - 4)^2 = 25$	DM1	OE Using <i>their</i> $2x$ and <i>their</i> circle to form an equation in x .
	$[5(x - 2)^2 = 25 \Rightarrow](x - 2)^2 = 5$	A1	OE Accept $x^2 - 4x - 1 [= 0]$.
	$[x =] 2 \pm \sqrt{5}$	A1	OE Exact values only.

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Question	Answer	Marks	Guidance
8(b)	Alternative Method 2 for Question 8(b)		
	$y = \sqrt{25 - (x - 2)^2} + 4$ or $(-x^2 + 4x + 21)^{\frac{1}{2}} + 4$	B1	This mark can be implied by a correct differential.
	$\frac{dy}{dx} = \frac{1}{2}(25 - (x - 2)^2)^{-\frac{1}{2}} \times (4 - 2x)$	*M1	Differentiating <i>their</i> expression for y . Use of the chain rule must be clear.
	$-\frac{1}{2} = \frac{1}{2}(25 - (x - 2)^2)^{-\frac{1}{2}} \times (4 - 2x) \quad [\Rightarrow 25 - (x - 2)^2 = (2x - 4)^2]$	DM1	Equating <i>their</i> $\frac{dy}{dx}$ to $-\frac{1}{2}$.
	$5x^2 - 20x - 5 = 0$	A1	OE
	$[x =] 2 \pm \sqrt{5}$	A1	OE Exact values only.
	Alternative Method 3 for Question 8(b)		
	$(x - 2)^2 + ((-\frac{1}{2}x + c) - 4)^2 = 25$	B1	Replacing y with $-\frac{1}{2}x + c$ in a correct circle equation.
	$\frac{5}{4}x^2 - cx + (c - 4)^2 - 21 = 0$ or $\frac{5}{4}x^2 - cx + (c^2 - 8c - 5) = 0$	M1	Forming a three-term quadratic in x . Condone only $\pm$ errors.
	Using " $b^2 - 4ac$ " = 0 with <i>their</i> values from the quadratic in x	DM1	Expect $c^2 - 4 \times \frac{5}{4} \times \{(c - 4)^2 - 21\} = 0$.
	$[c =] \frac{40 \pm \sqrt{1600 + 400}}{8} \left[= \frac{10 \pm 5\sqrt{5}}{2} \right]$	A1	OE
	$[x =] 2 \pm \sqrt{5}$	A1	OE Exact values only.
		5	

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Question	Answer	Marks	Guidance
9	$\frac{1}{2}x + \frac{4}{x} = 4.5 \Rightarrow x^2 - 9x + 8 = 0$	M1	Equating the curve and the line to form a 3-term quadratic.
	$[x=]1, 8$	A1	SOI
	$\pi \times 4.5^2 \times (\text{their } 8 - \text{their } 1) \left[= \frac{567}{4} \pi \text{ or } 445.3 \right]$	A1 FT	FT <i>their</i> limits from the quadratic, both limits must be positive.
	$[\pi] \int \left(\frac{1}{2}x + \frac{4}{x} \right)^2 [dx]$	*M1	For use of $\int y^2 dx$.
	$[\pi] \int \left(\frac{1}{4}x^2 + 4 + \frac{16}{x^2} \right) dx = [\pi] \left(\frac{1}{12}x^3 + 4x - \frac{16}{x} \right)$	A1	OE First A1 for two correct terms, second A1 for all three correct. May be unsimplified. May include +C.
		A1	
	$[\pi] \left(\frac{218}{3} + \frac{143}{12} \right) = [\pi] \frac{1015}{12}$	DM1	Clear use of their <i>limits</i> in <i>their</i> integrated expression. At least one correct power required. Both limits must be positive, or if correct limits are used, then $\pm \left(\frac{218}{3} + \frac{143}{12} \right)$ is the minimum required.
	Volume of solid = $\frac{343}{6} \pi$	A1	Condone a negative answer changed to a positive. A0 for omission of π . SC B1: following DM0. A0 for 179.594...

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Question	Answer	Marks	Guidance
	Alternative Method for Question 9		
	$\frac{1}{2}x + \frac{4}{x} = 4.5 \Rightarrow x^2 - 9x + 8 = 0$	M1	Equating the curve and the line to form a 3-term quadratic
	$[x=]1, 8$	A1	SOI
	$[\text{Volume} =] [\pi] \int \left(4.5^2 - \left(\frac{1}{2}x + \frac{4}{x} \right)^2 \right) [dx]$	*M1	For use of $\int (y_1^2 - y_2^2)$. Condone $\int (y_1 - y_2)^2$ for this *M1.
	$\pm [\pi] \left(\left\{ 4.5^2 x - 4x \right\} \left\{ -\frac{1}{12}x^3 \right\} \left\{ +\frac{16}{x} \right\} \right)$	A1	OE
		A1	A1 for each element of the integral. May be unsimplified.
		A1	May include +C. 'Correct' terms from $\int (y_1 - y_2)^2$ score A0A0A0 XP.
	$[\pi] \left(\frac{268}{3} - \frac{193}{6} \right)$	DM1	Clear use of <i>their</i> limits in their integrated expression. At least one correct power is required. Both limits must be positive, or if correct limits are used, then $\pm \left(\frac{268}{3} - \frac{193}{6} \right)$ is the minimum required.
	Volume of solid = $\frac{343}{6}\pi$	A1	Condone a negative answer changed to a positive. A0 for omission of π . SC B1: following DM0. A0 for 179.594...
		8	

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Question	Answer	Marks	Guidance
10(a)(i)	$\{2\}\{(x+1)^2\}\{+8\}$	B1	B1 for each {} element.
		B1	
		B1	
		3	
10(a)(ii)	$\left[\text{Range or } f[(x)] \text{ or } y \right] \geq 8$	B1 FT	OE. FT <i>their</i> value of c . $x \geq 8$ scores B0.
		1	

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Question	Answer	Marks	Guidance
10(b)	$p(5m)^2 + 4(5m) - 5 = 0$ and $p(-9m)^2 + 4(-9m) - 5 = 0$	*M1	Allow one error for this mark if the intention is clear.
	$81\left(\frac{5-20m}{(5m)^2}\right)m^2 + 4(-9m) - 5 = 0$ or $25\left(\frac{36m+5}{(-9m)^2}\right)m^2 + 4(5m) - 5 = 0$ or $\left(\frac{5-20m}{(5m)^2}\right) = \left(\frac{36m+5}{(-9m)^2}\right)$	*DM1	Eliminate p by substitution or equating two expressions for p .
	$2520m = 280$	DM1	OE Simplify as far as a linear equation or a quadratic where $m = 0$ is ignored or dismissed.
	$m = \frac{1}{9}, p = 9$	A1, A1	
	Alternative Method for Question 10(b)		
	$p(5m)^2 + 4(5m) - 5 = 0$ and $p(-9m)^2 + 4(-9m) - 5 = 0$	*M1	Allow one error for this mark if the intention is clear.
	$p(5m)^2 + 4(5m) - 5 = p(-9m)^2 + 4(-9m) - 5$	*DM1	Equating <i>their</i> two expressions.
	$\left[56pm^2 - 56m = 0 \Rightarrow m = \frac{1}{p} \Rightarrow\right] 5 = \frac{45}{p} \text{ or } 45m = 5$	DM1	Simplify as far as a quadratic where $m = 0$ is ignored or dismissed and substitute for m or p .
	$p = 9, m = \frac{1}{9}$	A1, A1	

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Question	Answer	Marks	Guidance
10(b)	Alternative Method 2 for Question 10(b)		
	$\frac{-4 + \sqrt{4^2 - 4(p)(-5)}}{2p} = 5m$ and $\frac{-4 - \sqrt{4^2 - 4(p)(-5)}}{2p} = -9m$	*M1	Equating $5m$ and $-9m$ to the two solutions from the quadratic formula. Allow one error for this mark if the intention is clear.
	$\frac{-4 + \sqrt{4^2 - 4(p)(-5)}}{(5 \times 2)p} = \frac{-4 - \sqrt{4^2 - 4(p)(-5)}}{(-9 \times 2)p}$	*DM1	Attempt to solve simultaneously by equating <i>their</i> expressions for m , or dividing one by the other and equating to $-\frac{5}{9}$.
	$16 + 20p = 196$	DM1	OE Simplify as far as a linear expression – no square root.
	$p = 9, m = \frac{1}{9}$	A1, A1	
	Alternative Method 3 for Question 10(b)		
	$[(x - 5m)(x + 9m) \Rightarrow] x^2 + 4mx - 45m^2 [= 0]$ Or $\alpha + \beta = -4m$ and $\alpha\beta = -45m^2$	*M1	Recognising the relationship between $5m$ and $-9m$ and the quadratic equation.
	$4m = \frac{4}{p}, -45m^2 = -\frac{5}{p}$	*DM1	Using $x^2 + \frac{4}{p}x - \frac{5}{p} [= 0]$. Comparing coefficients or using $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$.
	$m = \frac{1}{p} \Rightarrow -45m^2 = -5m$ or $p = \frac{1}{m} \Rightarrow 5p^2 = 45p$	DM1	Forming an equation in one unknown.
	$m = \frac{1}{9}, p = 9$	A1, A1	
		5	

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Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = \left\{ \frac{-8 \times 3}{(3x-8)^2} \right\} \left\{ + \frac{6}{(x-1)^2} \right\}$	B1	B1 for each {} element.
		B1	
	$\left\{ \frac{-8 \times 3}{(3.3-8)^2} \right\} \left\{ + \frac{6}{(3-1)^2} \right\} \left[= \frac{-45}{2} \right]$	*M1	Using <i>their</i> differentiated expression, which must contain $(3x-8)^{-2}$ and $(x-1)^{-2}$, and $x=3$. This may be seen in <i>their</i> line equation.
	$y-5 = \left(\text{their} \frac{-45}{2} \right) (x-3)$ or $5 = \left(\text{their} \frac{-45}{2} \right) (3) + c \Rightarrow c = \left[\frac{145}{2} \right]$	*DM1	Correct form of a line equation with their gradient, but not the negative reciprocal, and (3, 5).
	$\left[-8x - 5 = \frac{-45}{2} (x-3) \Rightarrow \right] 29x = 145$	DM1	Replacing y with $-8x$ and collecting terms.
	$x=5, y=-40$	A1	Accept (5, -40).
		6	

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Question	Answer	Marks	Guidance
11(b)(i)	$\frac{-24}{(3x-8)^2} + \frac{6}{(x-1)^2} = 0 \Rightarrow 24(x-1)^2 = 6(3x-8)^2$ $[\Rightarrow 4(x-1)^2 = (3x-8)^2]$	*M1	Equate <i>their</i> $\frac{dy}{dx}$, which must contain $(3x-8)^{-2}$ and $(x-1)^{-2}$ to 0, and clear of fractions. Only condone $\pm$ errors.
	$5x^2 - 40x + 60 [= 0]$ or $\pm 2(x-1) = 3x-8$	DM1	OE Forming a three-term quadratic. Condone only $\pm$ errors. Or taking square roots. For this method, the $\pm$ must be present.
	2, 6	A1	Both values.
		3	
11(b)(ii)	$\frac{d^2y}{dx^2} = \left\{ \frac{-24 \times -2 \times 3}{(3x-8)^3} \right\} + \left\{ \frac{6 \times -2}{(x-1)^3} \right\}$	B1FT B1FT	Correct differentials of the elements of <i>their</i> $\frac{dy}{dx}$, which must contain $(3x-8)^{-2}$ and $(x-1)^{-2}$.
	At $x=2$, $\frac{d^2y}{dx^2} = \frac{\text{their}144}{(6-8)^3} - \frac{\text{their}12}{(2-1)^3}$ and at $x=6$, $\frac{d^2y}{dx^2} = \frac{\text{their}144}{(18-8)^3} - \frac{\text{their}12}{(6-1)^3}$	M1	Replacing x with <i>their</i> 2 and <i>their</i> 6 in <i>their</i> $\frac{d^2y}{dx^2}$, which must contain $(3x-8)^{-3}$ and $(x-1)^{-3}$. Correct final answers.
	At $x=2$, $\frac{d^2y}{dx^2} = -30 < 0$, therefore max[imum] At $x=6$, $\frac{d^2y}{dx^2} = \frac{48}{10^3} \left[\frac{6}{125} \right] > 0$, therefore min[imum]	A1	WWW Correct values, or working, and > 0 and < 0 are required.
		4	



Cambridge International AS & A Level

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MATHEMATICS

9709/15

Paper 1 Pure Mathematics 1

October/November 2025

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 1 A circle has centre (2, 6) and radius 10.

(a) State the equation of the circle.

[2]

$$\text{Equation: } (x-2)^2 + (y-6)^2 = 100$$

- (b) The circle passes through the point (8, k).

Find the two possible values of k.

[3]

$$(8-2)^2 + (k-6)^2 = 100$$

$$36 + (k-6)^2 = 100$$

$$(k-6)^2 = 64$$

$$k-6 = 8 \quad \text{OR} \quad k-6 = -8$$

$$k = 14 \quad \text{OR} \quad k = -2$$



2 A geometric progression has first term $3 + 4\sqrt{2}$ and second term $5 - \sqrt{2}$.

- (a) Find the common ratio of the geometric progression. Give your answer in the form $\sqrt{2} + p$, where p is an integer to be found. [3]

$$r = \frac{5 - \sqrt{2}}{3 + 4\sqrt{2}}$$

$$r = \frac{5 - \sqrt{2}}{3 + 4\sqrt{2}} \times \frac{3 - 4\sqrt{2}}{3 - 4\sqrt{2}}$$

$$r = \frac{23 - 23\sqrt{2}}{9 - 32}$$

$$r = \frac{23(1 - \sqrt{2})}{-23}$$

$$r = \sqrt{2} - 1$$

- (b) Find the sum to infinity of the geometric progression. [2]

$$S_{\infty} = \frac{3 + 4\sqrt{2}}{1 - \sqrt{2} + 1}$$

$$S_{\infty} = \frac{3 + 4\sqrt{2}}{2 - \sqrt{2}}$$

$$S_{\infty} = \frac{3 + 4\sqrt{2}}{2 - \sqrt{2}} \times \frac{2 + \sqrt{2}}{2 + \sqrt{2}}$$

$$S_{\infty} = \frac{14 + 11\sqrt{2}}{2}$$

$$S_{\infty} = 7 + \frac{11}{2}\sqrt{2}$$



- 3 (a) Express $4x^2 + 10x + 6$ in the form $a(x+b)^2 + c$, where a , b and c are rational constants to be determined. [2]

$$4\left(x^2 + \frac{5}{2}x\right) + 6$$

$$4\left(x + \frac{5}{4}\right)^2 + a$$

$$\frac{25}{4} + a = 6$$

$$a = -\frac{1}{4}$$

$$4\left(x + \frac{5}{4}\right)^2 - \frac{1}{4}$$

- (b) The curve with equation $y = 4x^2 + 10x + 6$ and the line $y = k$ have exactly one point of intersection.

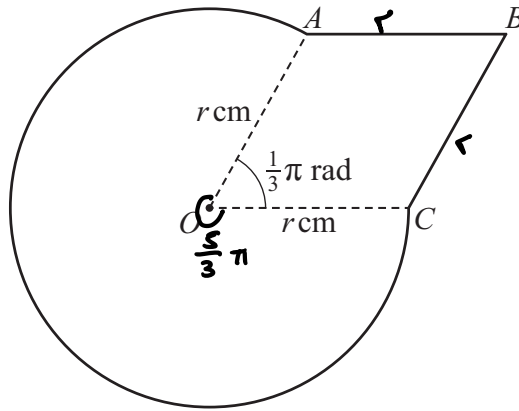
Using your answer to part (a) or otherwise, state the value of the constant k .

[1]

$$y = -\frac{1}{4}$$

$$k = -\frac{1}{4}$$





The diagram shows the design for a company's new logo. The sector of the circle, centre O , has radius r cm. The acute angle $AOC = \frac{1}{3}\pi$ radians. The quadrilateral $OABC$ is a rhombus.

- (a) Find an expression for the perimeter of the design. Give your answer in terms of π and r . [2]

$$\text{Perimeter} = \text{SAC} + AB + BC$$

$$\text{Perimeter} = \frac{5}{3}\pi r + r + r$$

$$\text{Perimeter} = r \left(\frac{5}{3}\pi + 2 \right)$$

- (b) It is now given that the perimeter of the design is 200 cm.

Find the area of the design. Give your answer to 3 significant figures. [5]

$$r \left(\frac{5}{3}\pi + 2 \right) = 200$$

$$r = 27.64$$

$$\text{Area of Sector} = \frac{1}{2} \left(\frac{200}{\frac{5}{3}\pi + 2} \right)^2 \times \frac{5}{3}\pi$$

$$\text{Area of Sector} = 2000.013704$$

$$\text{Area of Rhombus} = \frac{1}{2} \times \left(\frac{200}{\frac{5}{3}\pi + 2} \right)^2 \times \sin\left(\frac{1}{3}\pi\right)$$

$$\text{Area of Rhombus} = 661.5992$$

$$\text{Area of Design} = \text{Area of Sector} + \text{Area of Rhombus}$$

$$\text{Area of Design} = 2000.013704 + 661.5992$$

$$\text{Area of Design} = 2660 \text{ cm}^2$$



5 Solve the equation

$$x^3 - 28 + \frac{27}{x^3} = 0.$$

[3]

$$\text{let } x^3 = a$$

$$a - 28 + \frac{27}{a} = 0$$

$$a^2 - 28a + 27 = 0$$

$$a^2 - 27a - a + 27 = 0$$

$$a(a - 27) - 1(a - 27) = 0$$

$$(a - 1)(a - 27) = 0$$

$$a = 1 \text{ or } a = 27$$

$$x^3 = 1 \text{ or } x^3 = 27$$

$$x = 1 \text{ or } x = 3$$



6 (a) Show that the equation

$$6 \sin \theta + \frac{1}{\tan \theta} = \frac{4}{\sin \theta}$$

can be written in the form

$$6 \cos^2 \theta - \cos \theta - 2 = 0. \quad [3]$$

$$\frac{6 \sin \theta + \frac{1}{\tan \theta}}{\sin \theta}$$

$$\frac{6 \sin^2 \theta + \cos \theta}{\sin \theta} = \frac{4}{\sin \theta}$$

$$6[1 - \cos^2 \theta] + \cos \theta - 4 = 0$$

$$6 - 6 \cos^2 \theta + \cos \theta - 4 = 0$$

$$6 \cos^2 \theta - \cos \theta - 2 = 0 \rightarrow \text{Shown!!}$$



(b) Hence, solve the equation

$$6 \sin \theta + \frac{1}{\tan \theta} = \frac{4}{\sin \theta}$$

for $0^\circ \leq \theta \leq 360^\circ$.

[4]

$$6 \cos^2 \theta - \cos \theta - 2 = 0$$

$$6c^2 - 4c + 3c - 2 = 0$$

$$2c(3c - 2) + 1(3c - 2) = 0$$

$$(2c + 1)(3c - 2) = 0$$

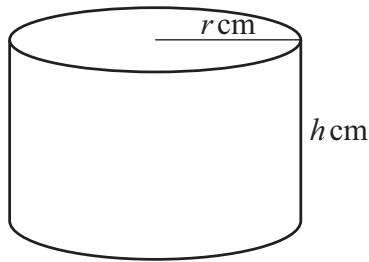
$$\cos \theta = -\frac{1}{2} \quad \text{OR} \quad \cos \theta = \frac{2}{3}$$

$$\alpha = 60^\circ \quad \text{OR} \quad \alpha = 48.19$$

$$\theta = 120, 240. \quad \text{OR} \quad \theta = 48.2, 311.8$$

Hence $\theta = 48.2, 120, 240, 311.8$





A manufacturer wishes to design an open cylindrical tank, as shown in the diagram. The tank will have a base but no top. The outside of the tank will have a fixed surface area of $600\pi \text{ cm}^2$. The radius $r \text{ cm}$ and height $h \text{ cm}$ of the tank can vary.

- (a) Show that the volume, $V \text{ cm}^3$, of the tank is given by

$$V = \frac{\pi r(600 - r^2)}{2}.$$

[3]

$$\pi r^2 + 2\pi r h = 600\pi$$

$$r^2 + 2rh = 600$$

$$2rh = 600 - r^2$$

$$h = \frac{600 - r^2}{2r}$$

$$V = \pi r^2 \left(\frac{600 - r^2}{2r} \right)$$

$$V = \frac{\pi r (600 - r^2)}{2r} \rightarrow \text{Shown!!}$$



(b) Find the exact value of r which corresponds to the maximum value of V .

[3]

$$V = \frac{\pi}{2} [600r - r^3]$$

$$\frac{dV}{dr} = \frac{\pi}{2} [600 - 3r^2]$$

$$\frac{\pi}{2} [600 - 3r^2] = 0$$

$$600 - 3r^2 = 0$$

$$200 = r^2$$

$$r = \sqrt{200}$$

$$r = 10\sqrt{2}$$

(c) Hence, find the maximum value of V .

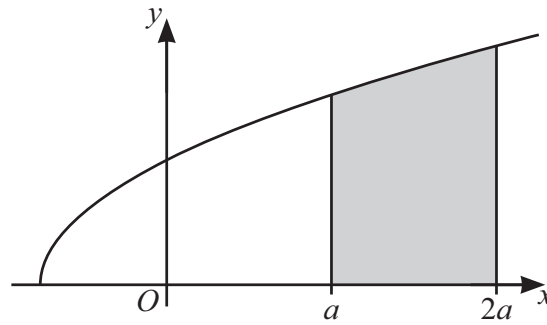
[2]

$$V = \frac{\pi}{2} [600 \times 10\sqrt{2} - (10\sqrt{2})^3]$$

$$V = 8885.765$$

$$V = 8890$$





The diagram shows the curve with equation $y = \sqrt{6x+5}$. The shaded region is bounded by the curve, the x -axis and the lines $x = a$ and $x = 2a$, where a is a positive constant.

The shaded region is rotated through 360° about the x -axis to form a solid. The solid has volume, V , such that $V \geq 46\pi$.

- (a) Show that $9a^2 + 5a - 46 \geq 0$. [4]

$$\pi \int_a^{2a} (6x+5) dx \geq 46\pi$$

$$[3x^2 + 5x]_a^{2a} \geq 46$$

$$12a^2 + 10a - 3a^2 - 5a - 46 \geq 0$$

$$9a^2 + 5a - 46 \geq 0 \rightarrow \text{Shown!!}$$

- (b) Find the range of possible values of a . [3]

$$9a^2 + 5a - 46 = 0$$

$$9a^2 + 23a - 18a - 46 = 0$$

$$a(9a+23) - 2(9a+23) = 0$$

$$(a-2)(9a+23) = 0$$

$$a=2 \text{ or } a = -23/9$$

$$a \geq 2$$



- 9 In the expansion of $(p + qx)^4$, the coefficient of x is equal to the coefficient of x^2 . The constants p and q are both positive.

(a) Find the ratio $p : q$. Give your answer in its simplest form.

[3]

$$T_2 = {}^4C_2 (p)^2 (qx)^2 = 6p^2 q^2 x^2$$

$$T_3 = {}^4C_3 (p)^3 (qx)^1 = 4p^3 q x$$

$$6p^2 q^2 = 4p^3 q$$

$$q = \frac{2}{3} p$$

$$p : \frac{2}{3} p$$

$$p : q = 3 : 2$$

- (b) It is given that the coefficient of x^3 is 486.

Find the values of p and q .

[4]

$$T_1 = {}^4C_1 (p)^1 (qx)^3$$

$$4p q^3 x = 486 x^3$$

$$4p \left(\frac{2}{3}p\right)^3 = 486$$

$$\frac{32}{27} p^4 = 486$$

$$p^4 = \frac{6561}{16}$$

$$p = \pm \frac{9}{2}$$

Since p is positive

$$p = \frac{9}{2}$$



10 The function f is defined by

$$f(x) = 3 + \frac{7}{x-2}$$

for $x > 2$.

(a) It is given that $f(a) = 4$.

Find the value of a .

[2]

$$3 + \frac{7}{a-2} = 4$$

$$\frac{7}{a-2} = 1$$

$$a-2 = 7$$

$$a = 9$$

(b) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} .

[4]

$$y = 3 + \frac{7}{x-2}$$

$$y-3 = \frac{7}{x-2}$$

$$x-2 = \frac{7}{y-3}$$

$$x = \frac{7}{y-3} + 2$$

$$f^{-1}(x) = \frac{7}{x-3} + 2$$

$$\text{Domain of } f^{-1} = 3 + \frac{7}{x-2}$$

$$x > 3$$



(c) The function g is defined by

$$g(x) = \frac{1+4x}{2x-3}$$

for $x > \frac{3}{2}$.

Show that $fg(x) \equiv kx$, where k is a constant to be determined.

[3]

$$fg(x) = 3 + \frac{7}{\left(\frac{1+4x}{2x-3}\right) - 2}$$

$$fg(x) = 3 + \frac{7}{\frac{1+4x-4x+6}{2x-3}}$$

$$fg(x) = 3 + \left(7 \div \frac{7}{2x-3}\right)$$

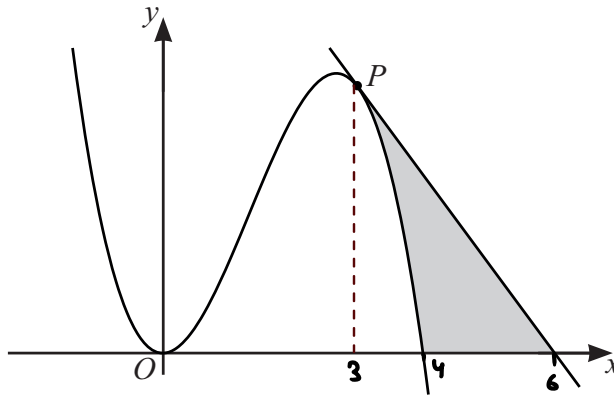
$$fg(x) = 3 + \left(7 \times \frac{2x-3}{7}\right)$$

$$fg(x) = 3 + 2x - 3$$

$$fg(x) = 2x$$

$$k = 2$$





The diagram shows the curve with equation $y = 4x^2 - x^3$ and the tangent to the curve at the point P . The point P has x -coordinate 3.

- (a) Find the equation of the tangent to the curve at the point P . Give your answer in the form $y = mx + c$. [5]

$$y = 4(3)^2 - (3)^3$$

$$y = 9$$

$$\text{Point } P = (3, 9)$$

$$\frac{dy}{dx} = 8x - 3x^2$$

$$m \text{ of } P = 8(3) - 3(3)^2$$

$$m \text{ of } P = -3$$

$$\text{Equation} = y - 9 = -3(x - 3)$$

$$\text{Equation} = y - 9 = -3x + 9$$

$$\text{Equation } y = -3x + 18$$



- (b) The shaded region is bounded by the curve, the x -axis and the tangent to the curve at P .

Find the exact area of the shaded region.

[6]

Point where curve touches x -axis: $\rightarrow 4x^2 - x^3 = 0 \rightarrow -x^2(x-4) = 0 \rightarrow x = 4$

Point where tangent touches x -axis: $\rightarrow -3x + 18 = 0 \rightarrow x = 6$

Area of Triangle = $\frac{1}{2} \times 9 \times 3 = \frac{27}{2}$

Area under curve: $\int_3^4 4x^2 - x^3$

Area under curve: $\left[\frac{4}{3}x^3 - \frac{x^4}{4} \right]_3^4$

Area under curve: $\frac{64}{3} - \frac{63}{4}$

Area under curve: $\frac{67}{12}$

Shaded Area = Area of Triangle - Area under curve

Shaded Area = $\frac{27}{2} - \frac{67}{12}$

Shaded Area = $\frac{95}{12}$

The graph of $y = 4x^2 - x^3$ is transformed by a stretch of scale factor $\frac{1}{3}$ in the x -direction. The point Q is the image of P under this transformation. The transformed shaded region is bounded by the transformed curve, the x -axis and the tangent to the transformed curve at Q .

- (c) (i) Find the equation of the transformed curve in the form $y = mx^2 + nx^3$, where m and n are integers to be found. [1]

$y = 4(3x)^2 - (3x)^3$

$y = 36x^2 - 27x^3$

- (c) (ii) State the coordinates of Q and the area of the transformed shaded region. [2]

$Q: (1, 9)$

Shaded Area = $\frac{1}{3} \times \frac{95}{12}$

Shaded Area = $\frac{95}{36}$



Cambridge International AS & A Level

MATHEMATICS**9709/15**

Paper 1 Pure Mathematics 1

October/November 2025**MARK SCHEME**Maximum Mark: 75

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Question	Answer	Marks	Guidance
1(a)	$\{(x-2)^2\}\{+(y-6)^2\} = \{100 \text{ or } 10^2\}$ or $\{x^2 - 4x\}\{+y^2 - 12y\}\{-60 = 0\}$ oe	B2,1,0	B2 All 3 components correct, B1 Any 2 components correct.
		2	
1(b)	$(8-2)^2 + (k-6)^2 = 100$ or $k^2 - 12k - 28 = 0$	M1*	Substituting the point into <i>their</i> equation to obtain an equation in k .
	$k-6 = \pm 8$ or $(k-14)(k+2) = 0$ oe	DM1	Simplify to the point where k can be found (allow one sign error).
	$k = 14, -2$	B1	SC B2 only for both answers without working. Allow ‘ $y =$ ’.
		3	

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Question	Answer	Marks	Guidance
2(a)	$r = \frac{5 - \sqrt{2}}{3 + 4\sqrt{2}}$	B1	Correct quotient SOI.
	$= \frac{(5 - \sqrt{2})(3 - 4\sqrt{2})}{9 - 32}$	M1	Rationalise <i>their</i> denominator (must see clear process), or equate <i>their</i> r to $\sqrt{2} + p$ and solve.
	$= \frac{23 - 23\sqrt{2}}{-23} = \sqrt{2} - 1$ [$p = -1$]	B1	
		3	
2(b)	$S_{\infty} = \frac{3 + 4\sqrt{2}}{1 - (\sqrt{2} - 1)}$	M1	Uses S_{∞} formula. FT $a = 3 + 4\sqrt{2}$ and <i>their</i> r from 2(a) , provided $-1 < r < 1$ and r is evaluated. Do not condone missing brackets unless corrected in the calculation.
	$\left[= \frac{14 + 11\sqrt{2}}{2} \right]$ or $\left[7 + \frac{11\sqrt{2}}{2} \right]$	A1	Allow answers from a correct expression which round to 14.8 to 3sf.
		2	

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Question	Answer	Marks	Guidance
3(a)	$\{4\} \left\{ \left(x + \frac{5}{4} \right)^2 \right\} \left\{ -\frac{1}{4} \right\}$	B2,1,0	B2 All 3 components correct, B1 Any 2 components correct. Allow $a = 4$, $b = \frac{5}{4}$, $c = -\frac{1}{4}$ unless contradicted by the algebraic expression. Allow decimals.
		2	
3(b)	$k = -\frac{1}{4}$ or -0.25 (allow $-\frac{1}{4}$ or $y = -\frac{1}{4}$)	B1 FT	Follow through <i>their c</i> from 3(a) . Don't allow $x = -\frac{1}{4}$. If solved through restarting answer must be WWW.
		1	

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Question	Answer	Marks	Guidance
4(a)	Arc Length = $\left(2\pi - \frac{1}{3}\pi\right) \times r = \frac{5}{3}\pi r$	M1	Finding the reflex angle AND using arc length formula or $2\pi r - r\left(\frac{1}{3}\pi\right)$.
	Perimeter = $\frac{5}{3}\pi r + 2r$ oe e.g. $2\pi r - \frac{1}{3}\pi r + 2r$	A1	Answer must be exact.
		2	
4(b)	$r\left(\frac{5\pi}{3} + 2\right) = 200$	M1	Equating <i>their</i> expression to 200 and attempting to solve (<i>their</i> expression must be linear).
	$r = 27.6[4]$ allow $r = \frac{200}{\frac{5\pi}{3} + 2}$ oe	A1	CAO
	Area of Rhombus = $2\left(\frac{1}{2}\right)(27.64)^2 \sin\left(\frac{1}{3}\pi\right) (= 661.6)$ Or Area of segment = $\frac{1}{2}(27.64)^2\left(\frac{\pi}{3}\right) - \frac{1}{2}(27.64)^2 \sin\left(\frac{1}{3}\pi\right)$	M1	Using $2(0.5ab \sin C)$ with $a = b = \text{their } r$ and $C = \frac{1}{3}\pi$ to get the area of the rhombus (or suitable other method), with <i>their</i> value of r which may be substituted later. Expect segment area = $400 - 330.8 = 69.2$. Must be an attempt to find and use r .
	Area of Sector = $\left(\frac{1}{2}\right)(27.64)^2\left(\frac{5}{3}\pi\right) (= 2000)$ oe Or Total Area = Circle area – Segment area + Triangle area	M1	Using sector area formula with <i>their</i> r and $\frac{5}{3}\pi$, or complete method using the segment. Must be an attempt to find and use r .
	Total Area = $2000 + 661.6 = 2660$ (3sf) Or Total area = $2400 - 69.2 + 330.8 = 2660$ (3sf)	A1	CAO WWW AWRT 2660 (3sf).
		5	

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Question	Answer	Marks	Guidance
5	$[x^6 - 28x^3 + 27 = 0] \rightarrow u^2 - 28u + 27 [= 0]$	M1*	Multiply by x^3 and recognise as a quadratic in x^3 .
	$[(x^3 - 1)(x^3 - 27) = 0] \rightarrow (u - 27)(u - 1) [= 0]$	DM1	Solve quadratic using suitable method. E.g. factorisation, quadratic formula, completing the square.
	$x = 1, 3$	B1	WWW
		3	
6(a)	$6\sin\theta + \frac{\cos\theta}{\sin\theta} = \frac{4}{\sin\theta} \Rightarrow 6\sin^2\theta + \cos\theta = 4$	M1	Using identity for $\tan\theta$ and multiplying through by $\sin\theta$.
	$6(1 - \cos^2\theta) + \cos\theta = 4$	M1	Uses $\sin^2\theta + \cos^2\theta = 1$.
	$6\cos^2\theta - \cos\theta - 2 = 0$	A1	AG Expands out and simplifies to give the answer.
		3	
6(b)	$(3\cos\theta - 2)(2\cos\theta + 1) = 0$	M1	Suitable method for solving quadratic.
	$\cos\theta = \frac{2}{3}, \cos\theta = -\frac{1}{2}$	B1	Obtaining both values of $\cos\theta$.
	$\theta = 48.2, 120, 240, 311.8$	B1	Any 2 values of θ correct.
		B1	All 4 values of θ correct and no other values in the given range. SC B1 All four answers in radians: $0.841, 2\pi/3, 4\pi/3, 5.44$.
		4	

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Question	Answer	Marks	Guidance
7(a)	$2\pi rh + \pi r^2 = 600\pi$ oe $\left[h = \frac{600\pi - \pi r^2}{2\pi r} = \frac{600 - r^2}{2r} \right]$	M1*	Uses given info and correct formulae to form equation in h and r .
	$V = \frac{(600 - r^2)}{2r} \times \pi r^2$ oe	DM1	Sub <i>their</i> expression for h into $V = \pi r^2 h$.
	$= \frac{\pi r(600 - r^2)}{2}$	A1	AG CAO WWW
		3	
7(b)	$V = \frac{1}{2}\pi(600r - r^3) \frac{dV}{dr} = \frac{1}{2}\pi(600 - 3r^2)$ or $\left(300\pi - \frac{3\pi r^2}{2} \right)$ Alternative: $h = r$ at max V , so $2\pi r(r) + \pi r^2 = 600\pi$ oe	M1*	Differentiate given expression for V . Condone missing $\frac{1}{2}\pi$ (must be of form $a - br^2$).
	$600 - 3r^2 = 0$ Alternative: Solving $2\pi r(r) + \pi r^2 = 600\pi$	DM1	Equate their derivative to zero and attempt to solve as far as an equation of the form ' $r =$ '.
	$r = 10\sqrt{2}$	A1	CAO (accept $\sqrt{200}$).
		3	
7(c)	$V = \frac{\pi}{2} \left(600(10\sqrt{2}) - (10\sqrt{2})^3 \right)$	M1	Sub <i>their</i> value of r from 7(b) into the given expression for V , providing <i>their</i> value of $r > 0$ and gives $V > 0$.
	8890(3sf) Accept exact answer $2000\pi\sqrt{2}$	A1	AWRT 8890 (3sf).
		2	

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Question	Answer	Marks	Guidance
8(a)	$V = \pi \int_a^{2a} (6x + 5) [dx]$	B1	CAO SOI Must include π and the correct limits.
	$= [\pi] \left\{ 3x^2 + 5x \right\}_a^{2a}$ or $= [\pi] \left\{ \frac{(6x + 5)^2}{12} \right\}_a^{2a}$	M1*	Correct integral of $6x + 5$ (can be awarded if π is missing and the limits are missing or incorrect).
	$= [\pi] (12a^2 + 10a - 3a^2 - 5a) [\geq 46\pi]$	DM1	Correct substitution of correct limits.
	$9a^2 + 5a - 46 \geq 0$	A1	AG Can only be awarded if the argument leading to the statement is complete and clear.
		4	
8(b)	Solve inequality (or equation)	M1	Solve quadratic by suitable method.
	Obtain $-\frac{23}{9}, 2$	B1	Condone absence of $-\frac{23}{9}$. Condone use of x .
	Final answer $a \geq 2$	B1	WWW Don't allow $x \geq 2$.
		3	

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Question	Answer	Marks	Guidance
9(a)	$4 \times p^3 \times q$ or $6 \times p^2 \times q^2$	M1	Attempt to find either term in x or x^2 – must use binomial coefficients. May see x terms.
	$\frac{p}{q} = \frac{3}{2}$ oe	DM1	Equate coefficients and simplify to an equation equivalent to $2p = 3q$ (no x 's should be present).
	$p : q = 3 : 2$	A1	Must be in this form only.
		3	
9(b)	$4p \times q^3 = 486$	B1	Set coefficient of the term in x^3 equal to 486. May include x^3 on both sides of the equation.
	$4 \times \frac{3q}{2} \times q^3 = 486$	M1	Substitute <i>their</i> 9(a) equation, providing this is of the form $p = aq$ where a is positive and rational.
	$q^4 = 81 \Rightarrow q = 3, p = \frac{9}{2}$	A1	Solve for q (or p).
		A1	Solve for p and q with no negative answer. SC B1 for answers without working.
		4	

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Question	Answer	Marks	Guidance
10(a)	$4 = 3 + \frac{7}{a-2}$	M1	Forms an equation in a .
	$a = 9$	A1	Condone $x = 9$.
		2	
10(b)	Swap x and y and change the subject. This can be viewed as four stages: e.g. $x = 3 + \frac{7}{y-2}$, $x-3 = \frac{7}{y-2}$, $y-2 = \frac{7}{x-3}$, $y = 2 + \frac{7}{x-3}$ or $x = 3 + \frac{7}{y-2}$, $x(y-2) = 3y-6+7$, $xy-3y = 2x+1$, $y = \frac{2x+1}{x-3}$	M1	Any two stages.
		M1	The other two stages. Condone one sign error present in the complete method. One algebraic slip would result in only one M1.
	$f^{-1}(x) = 2 + \frac{7}{x-3}$	A1	CAO $\left(\text{Could be written as } \frac{2x+1}{x-3} \right)$ Need to see ' $f^{-1}(x) =$ ' or ' $f^{-1}: x \rightarrow$ ', not just ' $y =$ '.
	Domain: $x > 3$	B1	Allow equivalent statement but must involve x .
		4	

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Question	Answer	Marks	Guidance
10(c)	$fg(x) \equiv 3 + \frac{7}{\left(\frac{1+4x}{2x-3}\right) - 2}$ Alternative 1: $g(x) = f^{-1}(kx) \rightarrow \frac{1+4x}{2x-3} = 2 + \frac{7}{kx-3}$	B1	Substitute $g(x)$ into $f(x)$.
	$3 + \frac{14x-21}{1+4x-4x+6} \text{ or } 3 + \frac{7(2x-3)}{1+4x-2(2x-3)} \text{ oe}$ Alternative 1: Clearly amending fractions to a linear equation to solve Alternative 2: Solve $3 + \frac{7}{\left(\frac{1+4x}{2x-3}\right) - 2} = kx$ to a linear equation	M1	Simplify to expression with no embedded fractions (working may be minimal).
	$3 + \frac{14x-21}{7} \equiv 2x \text{ or } k=2$	A1	Cannot just state $k=2$. Must have supporting algebra. Trying several values to establish $k=2$ scores M0A0.
		3	

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Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = 8x - 3x^2$	B1	CAO
	$\frac{dy}{dx} = 24 - 27 = -3$ [when $x = 3$]	B1	
	$y = 9$ [when $x = 3$]	B1	SOI
	$y - 9 = -3(x - 3)$ or $y = -3x + c \rightarrow 9 = -9 + c \rightarrow c = 18$ oe	M1	Uses <i>their</i> y and <i>their</i> numerical $\frac{dy}{dx}$ to find equation of the tangent; condone one sign error.
	$y = -3x + 18$	A1	
		5	

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Question	Answer	Marks	Guidance
11(b)	Area between curve and x -axis = $\int_3^4 (4x^2 - x^3) [dx]$ and attempt to integrate	M1*	Must obtain $ax^3 + bx^4$ and indicate the limits 3 and 4.
	$= \left[\frac{4x^3}{3} - \frac{x^4}{4} \right]_3^4$	A1	SC B1 for use of wrong or no limits (only for correct integral).
	$\left(\frac{256}{3} - 64 \right) - \left(36 - \frac{81}{4} \right)$	DM1	Correct sub of correct limits (allow one slip). Minimum acceptable: $\frac{64}{3} - \frac{63}{4}$.
	$= \frac{67}{12}$	A1	SOI May be implied by a correct final answer if the two areas are combined. SC B1 if substitution of the limits is not seen.
	Shaded region = $(6-3) \times \frac{9}{2} - \text{their} \frac{67}{12}$ or $\left[\frac{-3x^3}{2} + 18x \right]_3^6 - \text{their} \frac{67}{12}$	DM1	Expect $\frac{27}{2} - \text{'their integral'}$, but must be 'area under <i>their</i> line' minus ' <i>their</i> area under the curve', where ' <i>their integral</i> ' is an attempt at the area under the curve between $x=3$ and $x=4$. May use the lengths from <i>their</i> tangent equation.
	$= \frac{95}{12}$ any equivalent exact answer	A1	Calculating area of triangle – (correct) area under the curve.

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Question	Answer	Marks	Guidance
11(b)	Alternative Method for Question 11(b):		
	Finds area between curve and tangent between $x=3$ and $x=4$ $\int_3^4 (-3x+18) - (4x^2 - x^3) dx$	M1*	Integrate at least two of the four terms correctly. Area under the line could be found from the trapezium area $\frac{(4-3)}{2}(9+6)$.
	$= \left[-\frac{3x^2}{2} + 18x - \frac{4x^3}{3} + \frac{x^4}{4} \right]_3^4$	A1	Integrating all four terms correctly. SC B1 for the correct integral $\pm \left(-\frac{4x^3}{3} + \frac{x^4}{4} \right)$.
	$= \left(-24 + 72 - \frac{256}{3} + 64 \right) - \left(-\frac{27}{2} + 54 - 36 + \frac{81}{4} \right)$	DM1	Correct sub of limits (allow one slip).
	$= \frac{23}{12}$	A1	SOI SC B1 if substitution of the limits is not seen.
	Shaded region = $(6-4) \times 6 \times \frac{1}{2} + \frac{23}{12}$	DM1	Calculating area of triangle between $x=4$ and $x=6$ + <i>their</i> area, providing limits of 3 and 4 are used to find the area between the curve and the tangent.
	$= \frac{95}{12}$	A1	Must be exact.
		6	

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Question	Answer	Marks	Guidance
11(c)(i)	$y = 36x^2 - 27x^3$ or state $m = 36, n = -27$	B1	CAO (must be expanded)
		1	
11(c)(ii)	$Q(1, 9)$	B1	CAO coordinates of Q .
	Area = $\frac{95}{36}$	B1 FT	$\frac{1}{3}$ of <i>their</i> area from 11(b) . Allow 2.64.
		2	