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September P2 2015

MEMORANDUM

Grade/Graad 12

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- As 'n kandidaat 'n vraag TWEEKEER beantwoord, merk slegs die EERSTE poging.
- As 'n kandidaat 'n poging om die vraag te beantwoord, doodgetrek het en nie dit oorgedoen het nie, merk die doodgetrekte poging.
- Volgehoue akkuraatheid word in ALLE aspekte van die nasienmemorandum toegepas.
- Aanvaarding van antwoorde/waardes om 'n probleem op te los, is ONaanvaarbaar.

VRAAG/QUESTION 1

1.1	$a = 16,16$ (16,1571639) $b = 0,88$ (0,8847043972) $\hat{y} = 16,16 + 0,88x$	answ only:full marks	✓ a ✓ b ✓ equation	(3)
1.2	$\hat{y} = 16,157... + 0,884...(73)$ $= 81\%$	answ only:full marks	✓ Subst into eq ✓ Answer (accept 80%)	(2)
1.3	$r = 0.92$ (0.9177373213)		✓ ✓ Answer (accept 0,91)	(2)
1.4	Yes./Ja The correlation between the two exams is very strong/ Die korrelasie tussen die twee eksamens is baie sterk..		✓ Yes ✓ correct justification	(2)
				[9]

QUESTION/ VRAAG 2

2.1	Modal Class: $19 \leq t < 23$	✓ ✓ answer	(1)
2.2	OGIVE - OGIEF Time (in minutes) Tyd (in minute)	✓ grounding at (11;0) ✓ plotting upper limits ✓ plotting cumulative frequency ✓ drawing a smooth curve ✓ geanker by (11;0) ✓ plot boonste limiete ✓ plot kumulatiewe frekwensie ✓ gladde kurwe	(4)
2.3	$Q_1 = 17,5$ (accept 17 – 18) $Q_3 = 25,5$ (accept 25 – 26) $IQR/IKO = 25,5 - 17,5$ $= 8$ (accept 7 – 9)	✓17,5 ✓25,5 ✓answer	(3)
2.4	Read off from graph: 18 occasions $\therefore 48 - 18 = 30$ times answ only:full marks	✓ accept 17 – 18 ✓ answer	(2)
			[10]

QUESTION/ VRAAG 3

3.1	$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{BC} = \frac{0 + 8}{-5 - 1}$ $m_{BC} = \frac{8}{-6} = -\frac{4}{3}$	✓ Substitution ✓ Answer	(2)
3.2	$E = \left(\frac{x_2 + x_1}{2} ; \frac{y_2 + y_1}{2} \right)$ $= \left(\frac{-5 + 1}{2} ; \frac{0 + 8}{2} \right)$ $= (-2; -4)$	✓ x value ✓ y value	(2)
3.3	$m_{DE} = \frac{3}{4} \quad [DE \perp BC]$ $y - (-4) = \frac{3}{4}(x - (-2)) \quad OR \quad -4 = \frac{3}{4}(-2) + c$ $y = \frac{3}{4}x - 2\frac{1}{2}$	✓ m_{DE} ✓ Substitute m_{DE} and $E(-2; -4)$ ✓ equation	(3)
3.4	$\tan \theta = m_{AD}$ $\tan \theta = -\frac{4}{3}$ $\theta = 180^\circ - 53,13^\circ$ $= 126,87^\circ$	✓ $\tan \theta = -\frac{4}{3}$ ✓ $53,13^\circ$ ✓ answer (obtuse)	(3)
3.5	$\hat{OFD} = 126,87^\circ - 90^\circ \quad [\text{ext } \angle \text{ of } \Delta]$ $= 36,87^\circ$	✓ method ✓ answer	(2)
3.6	$(5\sqrt{2})^2 = (-5 - x)^2 + (0 - 7)^2$ $50 = (-5 - x)^2 + (0 - 7)^2$ $50 = x^2 + 10x + 25 + 49$ $x^2 + 10x + 24 = 0$ $(x + 6)(x + 4) = 0$ $x \neq -6 \text{ or } x = -4$	✓ Subst into dist formula ✓ $AB^2 = 50$ ✓ standard form ✓ factors ✓ correct choice	(5)
3.7	$(x - a)^2 + (y - b)^2 = r^2$ $(x - (-2))^2 + (y - (-4))^2 = (-5 - (-2))^2 + (0 - (-4))^2$ $(x + 2)^2 + (y + 4)^2 = (-5 + 2)^2 + (0 + 4)^2$ $(x + 2)^2 + (y + 4)^2 = 25$ OR $(x + 2)^2 + (y + 4)^2 = r^2$ $(-5 + 2)^2 + (0 + 4)^2 = r^2$ $(x + 2)^2 + (y + 4)^2 = 25$ OR $(x + 2)^2 + (y + 4)^2 = r^2$ $(1 + 2)^2 + (-8 + 4)^2 = r^2$	✓ Subst LHS ✓ Subst RHS ✓ Answer ✓ Subst LHS ✓ Subst $(-5; 0)$ ✓ Answer ✓ Subst LHS ✓ Subst $(1; -8)$	(3)

Please turn over/Blaai asseblief om

	$(x+2)^2 + (y+4)^2 = 25$	✓ Answer	
			[20]

QUESTION/ VRAAG 4

4.1.1	$x^2 + y^2 + 4x - 4y - 12 = 0$ $(x+2)^2 + (y-2)^2 = 12 + 4 + 4 = 20$ $M(-2 ; 2)$ OR $M(\frac{4}{-2} ; \frac{-4}{-2})$ $M(-2 ; 2)$ answ only:full marks	✓ x value ✓ y value ✓ x value ✓ y value	(2)
4.1.2	$x^2 + 0^2 + 4x - 4(0) - 12 = 0$ $x^2 + 4x - 12 = 0$ $(x+6)(x-2) = 0$ $x \neq -6$ and $x = 2$, $C(2 ; 0)$ OR $\frac{x_C - 6}{2} = -2$ $x_C = 2$ $C(2 ; 0)$ answ only:full marks	✓ y- coordinate = 0 ✓ factors ✓ correct x value ✓ correct use of formula ✓ x value ✓ y = 0	(3)
4.2	$m_{MC} = -\frac{1}{2}$ $m_{\text{tangent}} = 2$ [tangent $\perp$ diameter] $y - y_1 = m(x - x_1)$ $y - 4 = 2(x - (-6))$ $y = 2x + 16$	✓ S ✓ R ✓ substituting A(-6 ; 4) ✓ equation	(4)
4.3	$B(x ; 0)$ $y = 2x + 16$ $\therefore 0 = 2x + 16$ $\therefore x = -8$ Area $\Delta ABC = \frac{1}{2} BC \times h$ $= \frac{1}{2} (10) \times 4$ $= 20$ sq units OR $B(x ; 0)$ $y = 2x + 16$ $\therefore 0 = 2x + 16$ $\therefore x = -8$ $AB = \sqrt{(-6 + 8)^2 + (4 - 0)^2}$ $AB = \sqrt{20}$	✓ y = 0 in equation ✓ x_B ✓ BC = 10 ✓ h = 4 ✓ answer ✓ y = 0 in equation ✓ x_B	(5)

	$\begin{aligned} \text{Area of } \Delta &= \frac{1}{2}(AB)(AC) \\ &= \frac{1}{2}(\sqrt{20})(2\sqrt{20}) \\ &= 20 \text{ unit}^2 \end{aligned}$	✓ $AB = \sqrt{20}$ ✓ $AC = 2\text{radius} = 2\sqrt{20}$ ✓ answer	
4.4	Eq of tangent parallel to AB through C: $y - y_1 = m(x - x_1)$ $y - 0 = 2(x - 2)$ $y = 2x - 4$ $\therefore -4 < k < 16$ OR $x^2 + (2x + k)^2 + 4x - 4(2x + k) - 12 = 0$ $5x^2 + 4xk - 4x + k^2 - 4k - 12 = 0$ $5x^2 + (4k - 4)x + (k^2 - 4k - 12) = 0$ $\therefore \Delta > 0$ $(4k - 4)^2 - 4(5)(k^2 - 4k - 12) > 0$ $-4k^2 + 64k + 256 > 0$ $k^2 - 16k - 64 > 0$ $(k - 16)(k + 4) > 0$ $\therefore -4 < k < 16$	✓ subst C(2 ; 0) into equation ✓ eq of tangent through C ✓ -4 ✓ 16 ✓ between ✓ standard form ✓ $\Delta > 0$ ✓ -4 ✓ 16 ✓ between	(5) [19]

QUESTION/ VRAAG 5

5.1.1	$\sin 149^\circ = \sin 31^\circ$ $= p$	✓ reduction ✓ answer	(2)
5.1.2	$\cos(-59^\circ) = \cos 59^\circ$ $= \sin 31^\circ$ $= p$	✓ co-ratio ✓ answer	(2)
5.1.3	$\cos 62^\circ = 1 - 2\sin^2 31^\circ$ $= 1 - 2p^2$	✓ double formula ✓ answer in terms of p	(2)
5.2	$(-\tan \theta)(\cos \theta)^2 + (-\cos \theta)(\sin \theta)$ $\left(-\frac{\sin \theta}{\cos \theta}\right)(\cos \theta)^2 - (\cos \theta)(\sin \theta)$ $-(\cos \theta)(\sin \theta) - (\cos \theta)(\sin \theta)$ $-2(\cos \theta)(\sin \theta)$ $-\sin 2\theta$	✓ $-\tan \theta$ ✓ $\cos \theta$ ✓ $-$ $\cos \theta$ ✓ identity ✓ simplify and add ✓ answer	(6)
5.3.1	$LHS = \frac{\sin 2x + \sin x}{\cos 2x + \cos x + 1}$ $= \frac{2 \sin x \cdot \cos x + \sin x}{2 \cos^2 x - 1 + \cos x + 1}$ $= \frac{\sin x (2 \cos x + 1)}{2 \cos^2 x + \cos x}$ $= \frac{\sin x (2 \cos x + 1)}{\cos x (2 \cos x + 1)}$ $= \frac{\sin x}{\cos x} = \tan x$ $= RHS$	✓ expansion of $\sin 2x$ ✓ correct expansion of $\cos 2x$ ✓ simplify numerator ✓ factorize numerator & denominator ✓ Identity	(5)
5.3.2	$\cos x (2 \cos x + 1) = 0$ or $\tan x = \infty$ $x = 240^\circ$ or 270°	✓ 240° ✓ 270°	(2)
			[19]

QUESTION/ VRAAG 6

6.1	$\sin(x + 30^\circ) = \cos 3x$ $= \sin(90^\circ - 3x)$ $x + 30^\circ = 90^\circ - 3x + k \cdot 360^\circ$ $4x = 60^\circ + k \cdot 360^\circ$ $x = 15^\circ + k \cdot 90^\circ$ or $x + 30^\circ = 180^\circ - (90^\circ - 3x) + k \cdot 360^\circ$ $-2x = 60^\circ + k \cdot 360^\circ$ $x = -30^\circ - k \cdot 180^\circ ; k \in \mathbb{Z}$ OR $\sin(x + 30^\circ) = \cos 3x$ $\cos[90^\circ - (x + 30^\circ)] = \cos 3x$ $90^\circ - (x + 30^\circ) = 3x + k \cdot 360^\circ$ $-4x = -60^\circ + k \cdot 360^\circ$ $x = 15^\circ - k \cdot 90^\circ$ or $90^\circ - (x + 30^\circ) = 360^\circ - 3x + k \cdot 360^\circ$ $2x = 300^\circ + k \cdot 360^\circ$ $x = 150^\circ + k \cdot 180^\circ ; k \in \mathbb{Z}$	✓ co-ratio ✓ equation ✓ general solution ✓ equation ✓ general solution ✓ $k \in \mathbb{Z}$ ✓ co-ratio ✓ equation ✓ general solution ✓ equation ✓ general solution ✓ $k \in \mathbb{Z}$	(6)
6.2.1		✓ max tp's ✓ min tp ✓ x intercepts	(3)
6.2.2	Period = 120°	✓ answer	(1)
6.2.3	$x \in [15^\circ ; 105^\circ]$ OR $15^\circ \leq x \leq 105^\circ$	✓ 15° ✓ 105° ✓ notation	(3)
			[13]

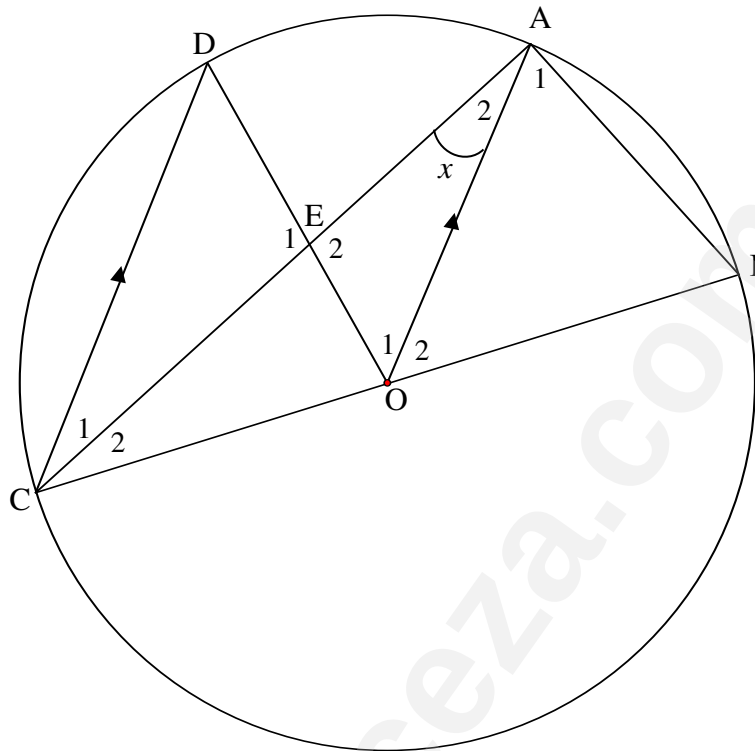
QUESTION 7

7.1	$\tan \alpha = \frac{AP}{AB}$ $AP = AB \tan \alpha$	✓ correct ratio ✓ AP ito AB and α	(2)
7.2	$\frac{AB}{\sin \beta} = \frac{20}{\sin(180^\circ - (\theta + \beta))}$ $\frac{AB}{\sin \beta} = \frac{20}{\sin(\theta + \beta)}$ $\therefore AB = \frac{20 \sin \beta}{\sin(\theta + \beta)}$ $\therefore AP = \frac{20 \cdot \sin \beta \cdot \tan \alpha}{\sin(\theta + \beta)}$	✓ correct subst into sine rule ✓ reduction ✓ AB as subject and subst into 7.1	(3)
7.3	$\theta = \beta$ [$\angle$ s opp equal sides] $AP = \frac{20 \cdot \sin \beta \cdot \tan \alpha}{\sin 2\beta}$ $AP = \frac{20 \cdot \sin \beta \cdot \tan \alpha}{2 \sin \beta \cdot \cos \beta}$ $AP = \frac{10 \cdot \tan \alpha}{\cos \beta}$	✓ replace θ ✓ expansion of $\sin 2\beta$ ✓ simplified answer	(3)
			[8]

QUESTION/ VRAAG 8

8.1	Line from centre to midpt of chord OR Line from centre bisects chord	✓ reason	(1)
8.2	$\hat{B} = 43^\circ$ [$\angle$ s in same segment]	✓S ✓R	(2)
8.3	$\hat{BAD} = 90^\circ$ [$\angle$ in semi-circle] $= \hat{E}_1$	✓S ✓R	(2)
8.4	$BD = 28$ units $\therefore \cos 43^\circ = \frac{AB}{28}$ $\therefore AB = 28 \cos 43^\circ = 20,48$ units	✓ correct ratio ✓ answer	(2)
			[7]

QUESTION/ VRAAG 9

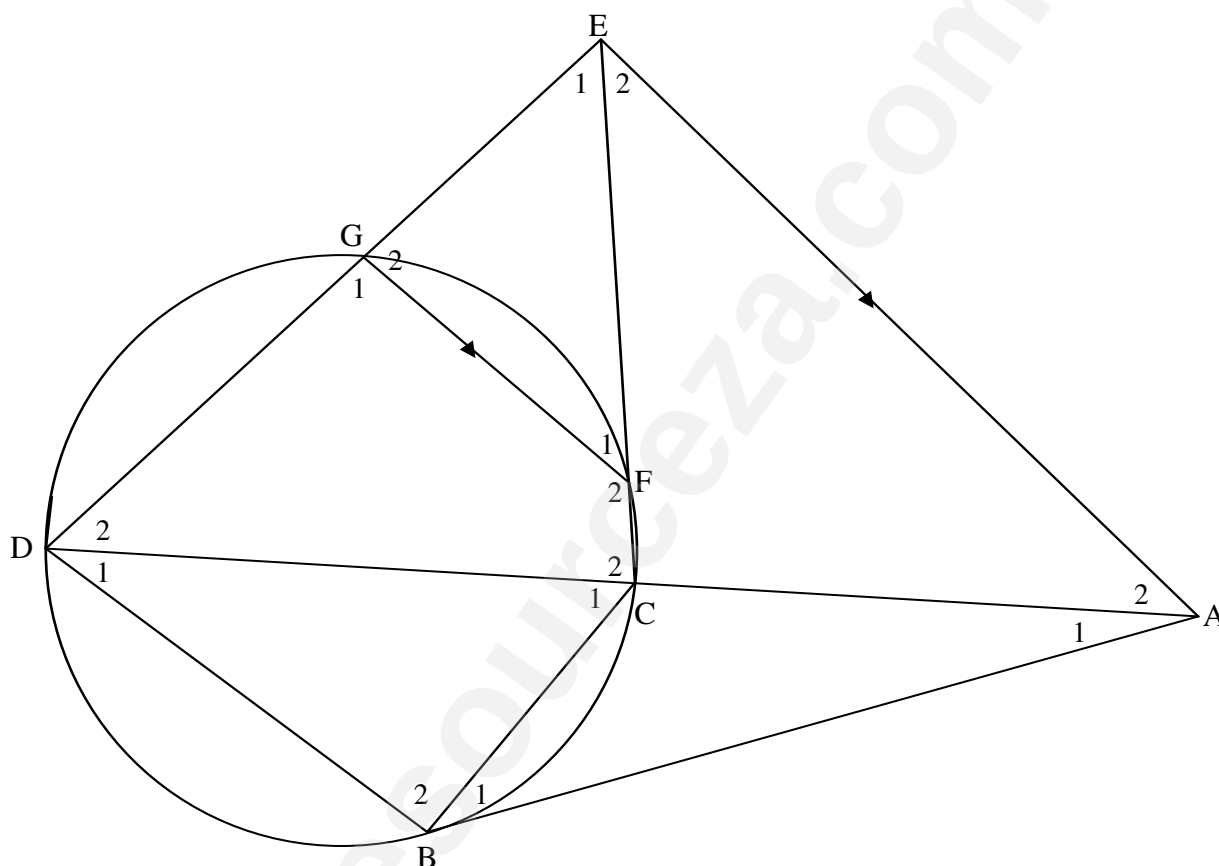


9.1.1	verwys $\angle$ e/alternate $\angle$ s; $CD \parallel OA$	✓ R	(1)
9.1.2	$CO = OA$ [radii] $\angle$ s opp equal sides/ $\angle$ e to gelyke sye	✓ R	(1)
9.2.1	$\hat{CAB} = 90^\circ$ [$\angle$ in semi-circle/halfsirkel] $\therefore \hat{A}_1 = 90^\circ - x$	✓ S ✓ R ✓ S	(3)
9.2.2	$\hat{O}_1 = 2x$ [$\angle$ at centre = $2 \times \angle$ at circum/midpts $\angle = 2 \times$ omtr $\angle$]	✓ S ✓ R	(2)
9.2.3	$\hat{O}_2 = 2x$ [ext $\angle$ of $\triangle ACO$] OR $\hat{O}_2 = 2x$ [$\angle$ at centre = $2 \times \angle$ at circum/midpts $\angle = 2 \times$ omtr $\angle$] OR $\hat{A}_1 = 90^\circ - x = \hat{B}$ [$\angle$ s opp equal sides/ $\angle$ e to gelyke sye] $\therefore \hat{O}_2 = 180^\circ - 2(90^\circ - x) = 2x$ [$\angle$ s of $\triangle$]	✓ S ✓ R ✓ S ✓ R ✓ R ✓ S	(2)
9.3	$\hat{EOB} = 4x$ [$\hat{O}_1 = 2x$ and $\hat{O}_2 = 2x$] $\hat{EOB} + \hat{EAB} = 180^\circ$ [opp $\angle$ s of cyc quad supp/ to $\angle$ e v kdvh suppl] $\therefore 4x + 90^\circ = 180^\circ$ $x = 22,5^\circ$	✓ R ✓ equation ✓ answer	(3)
			[12]

QUESTION/ VRAAG 10

10.1	proportional/eweredig	✓ S	(1)
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10.2

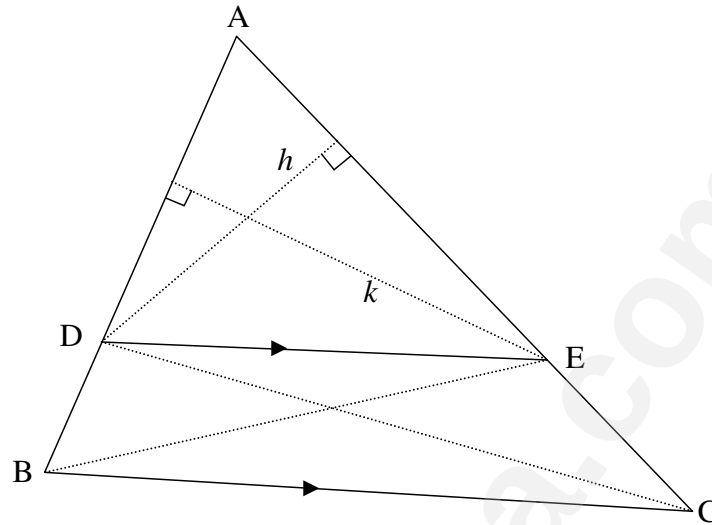


10.2.1	tangent-chord theorem / raaklyn-koordstelling	✓ R	(1)
10.2.2	In $\triangle ABC$ and $\triangle ADB$: $\hat{A}_1 = \hat{A}_1$ [common/gemeenskaplik] $\hat{B}_1 = \hat{D}_1$ [proven/bewys in 10.2.1] $\therefore \triangle ABC \parallel \triangle ADB$ [$\angle$; $\angle$; $\angle$] OR In $\triangle ABC$ and $\triangle ADB$: $\hat{A}_1 = \hat{A}_1$ [common/gemeenskaplik] $\hat{B}_1 = \hat{D}_1$ [proven/bewys in 10.2.1] $\hat{BCA} = \hat{B}_2$ [$\angle$s of $\Delta = 180^\circ$] $\therefore \triangle ABC \parallel \triangle ADB$	✓ S ✓ S ✓ R ✓ S ✓ S ✓ R	(3)

$\triangle AEC \parallel \triangle ADE$ [$\angle$; $\angle$; $\angle$] $\frac{AE}{DE} = \frac{AC}{AE}$ $AE^2 = AD \times AC$	✓ R ✓ S
$\triangle AEC$ and $\triangle ADE$: $\hat{A}_2$ [common/gemeenskaplik] $\hat{D}_2$ [proven/bewys in 10.2.3] $\hat{G}_1$ [$\angle$ s of $\triangle = 180^\circ$ OR ext $\angle$ of cyc quad DGFC/ buite $\angle$ v kdvh DGFC]	✓ S ✓ S ✓ R
$\triangle AEC \parallel \triangle ADE$ $\frac{AE}{DE} = \frac{AC}{AE}$ $AE^2 = AD \times AC$	✓ S
$\frac{AC}{AB} = \frac{AD}{AB}$ [$\triangle ABC \parallel \triangle ADB$] $AC = AD$ $AC = AE^2$ [from 10.2.4] $AC = AE$	✓ S ✓ S ✓ S

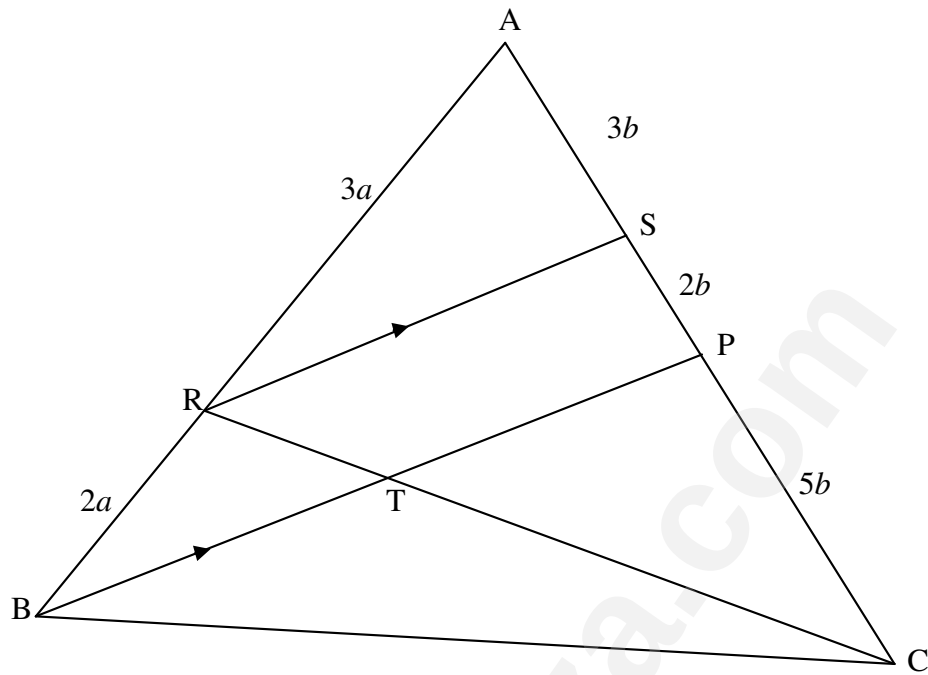
QUESTION/ VRAAG 11

11.1



	Construction: Connect DC and BE and draw the altitudes k and h Konstruksie: Verbind DC en BE en trek hoogtelyne k en h.	✓ constr/ konstr (on sketch or wording/ op skets of beskrywing)	
	$\frac{\text{Area } \triangle ADE}{\text{Area } \triangle BDE} = \frac{\frac{1}{2} \times AD \times k}{\frac{1}{2} \times BD \times k} = \frac{AD}{BD}$ $\frac{\text{Area } \triangle ADE}{\text{Area } \triangle DEC} = \frac{\frac{1}{2} \times AE \times h}{\frac{1}{2} \times EC \times h} = \frac{AE}{EC}$ but/maar: Area $\triangle BDE$ = Area $\triangle DEC$ [DE common base and DE $\parallel$ BC/ DE gemeensk basis en DE $\parallel$ BC] $\therefore \frac{\text{Area } \triangle ADE}{\text{Area } \triangle BDE} = \frac{\text{Area } \triangle ADE}{\text{Area } \triangle DEC}$ $\therefore \frac{AD}{BD} = \frac{AE}{EC}$	✓S ✓S ✓S ✓R ✓conclusion	(6)

11.2



11.2.1	$\frac{AS}{SP} = \frac{AR}{RB}$ $= \frac{3}{2}$ $\frac{AS}{SC} = \frac{3}{7}$ [line side of Δ OR RS BP] [AP = PC]	✓S ✓R ✓answer ✓S	(4)
11.2.2	$\frac{RT}{TC} = \frac{SP}{PC}$ $= \frac{2}{5}$ [line side of Δ OR RS TP]	✓S ✓R ✓answer	(3)
11.2.3	$\frac{\text{Area } \Delta TPC}{\text{Area } \Delta RSC} = \frac{\frac{1}{2} TC \cdot PC \cdot \sin \hat{C}P}{\frac{1}{2} RC \cdot SC \cdot \sin \hat{C}P}$ $= \frac{TC}{RC} \times \frac{PC}{SC}$ $= \frac{2}{7} \times \frac{5}{7}$ $= \frac{10}{49}$	✓Correct subst into area rule ✓ $\frac{2}{7}$ ✓ $\frac{5}{7}$ ✓answer	(4)
			[17]

Total/Totaal: [150]