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WISKUNDE
Graad 12



EKSAMEN V2
September 2014
Totaal: 150

MEMO

VRAAG 1

1.1	$\frac{1522}{15} = 101,47$	✓ 1522 ✓ 101,47	(2)
1.2	19,07	✓ 19,07 ✓ 2 desimale plekke	(2)
1.3	Onderste kwartiel = 89 Boonste kwartiel = 113	✓ 89 ✓ 113	(2)
1.4		✓ M by 100 ✓ Min=58 EN Max=145 ✓ Q₁=89 EN Q₃=113	(3)
1.5	$(\bar{x} - 1\sigma; \bar{x} + 1\sigma) = (82,4; 120,54)$ $\therefore 2dae$	✓✓ interval ✓ antwoord	(3)
			[12]

VRAAG 2

VRAAG 3

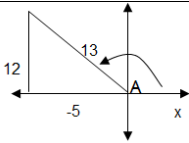
3.1	$M\left(\frac{2-3}{2}; \frac{-1+0}{2}\right)$ $= \left(-\frac{1}{2}; -\frac{1}{2}\right)$	✓ subst ✓ antwoord vir albei	(2)
3.2	$Mid BD\left(\frac{-1+0}{2}; \frac{-3+2}{2}\right)$ $= \left(-\frac{1}{2}; -\frac{1}{2}\right)$ Middelpunt van AC en BD is dieselfde Halveer AC en BD mekaar	✓ subst ✓ antwoord ✓ gevolgtrekking	(3)
3.3	$M_{AD} = \frac{2-0}{0+3} = \frac{2}{3}$ $M_{CD} = \frac{-1-2}{2-0} = \frac{-3}{2}$ $M_{AD} \times M_{CD} = \frac{2}{3} \times \frac{-3}{2} = -1$ $\therefore AD \perp CD$ $\therefore \hat{ADC} = 90^\circ$	✓ M_{AD} ✓ M_{CD} ✓ $= -1$ ✓ $\hat{ADC} = 90^\circ$	(4)
3.4	Diagonale halveer mekaar (3.2) DWS ABCD is parallelogram Vanaf 1.3: Omdat $\hat{ADC} = 90^\circ$ DWS ABCD reghoek $M_{AC} \times M_{BD}$ $= \frac{1}{-5} \times \frac{5}{1} = -1$ $AC \perp BD$ Diagonale halveer mekaar loodreg DWS ABCD is 'n vierkant	✓ halveer mekaar ✓ parallelogram ✓ $\hat{ADC} = 90^\circ$ ✓ reghoek ✓ diagonal halveer mekaar loodreg ✓ afleiding	(6)
3.5	$\tan \theta = \frac{2+1}{0-2} = \frac{-3}{2}$ $\therefore \theta = -56,30993247... + 180^\circ$ $= 123,7^\circ$	✓ gradient DC ✓ $\frac{-3}{2}$ ✓ antwoord	(3)

3.6	$OC^2 = (2-0)^2 + (-1-0)^2$ $OC^2 = 5$ $\therefore OC = 2,236067977$ $OC > 2$ Buite sirkel of $OC^2 = (2-0)^2 + (-1-0)^2$ $OC^2 = 5$ $\therefore OC^2 > 4$ Buite sirkel of $x^2 + y^2 = 4$ $(2)^2 + (-1)^2 = 5 > 4$ Buite sirkel	✓ OC^2 ✓ antwoord	(2)
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VRAAG 4

4.1	Middelpunt $AB \left(\frac{-8+0}{2}; \frac{1+5}{2} \right)$ $= (-4;3)$	✓ subst	(1)
4.2	$M_{AD} = \frac{5+1}{0-3} = \frac{-2}{1}$ $y = mx + c$ $\therefore 5 = -2(0) + c$ $\therefore c = 5$ $y = -2x + 5$	✓ subst ✓ M_{AD} ✓ subst (0;5) ✓ antwoord	(4)
4.3	$AM^2 = (5-2)^2 + (0+4)^2$ $= 4+16$ $AM = \sqrt{20}$	✓ subst ✓ vereenvoudig ✓ antwoord	(3)
4.4	$(x+4)^2 + (y-3)^2 = (\sqrt{20})^2$ $= (x+4)^2 + (y-3)^2 = 20$ $= x^2 + y^2 + 8x - 6y + 5 = 0$	✓ $(x+4)^2$ ✓ $(y-3)^2$ ✓ = 20 ✓ antwoord	(4)
4.5	$AT = TK = 6$ $CD \perp AK$ ACKD vlieër want diagonal CD halveer diagonal AK reghoekig.	✓ $AT=TK$ ✓ CD loodreg op AK ✓ vlieër ✓ rede	(4)
		✓	[16]

VRAAG 5

5.1	$\frac{\cos(-\theta) \cdot \tan(180^\circ - \theta) \cdot \cos(90^\circ - \theta)}{\sin(180^\circ - \theta) \cdot \sin(540^\circ + \theta)}$ $= \frac{\cos(\theta) \cdot \tan(180^\circ - \theta) \cdot \cos(90^\circ - \theta) \cdot \cos(90^\circ - \theta)}{\sin(180^\circ - \theta) \cdot \sin(180^\circ + \theta)}$ $= \frac{\cos \theta \cdot (-\tan \theta \cdot \sin \theta)}{\sin \theta \cdot (-\sin \theta)}$ $= \frac{\cos \theta \frac{\sin \theta}{\cos \theta}}{\sin \theta}$ $= 1$	✓	(7)
5.2.1	 $y = 12$ $\sin A = \frac{12}{13}$		(3)
5.2.2	$\sin 2A$ $= 2 \sin A \cdot \cos A$ $= 2 \left(\frac{12}{13} \right) \left(\frac{-5}{13} \right)$ $= \frac{-120}{169}$		(3)
5.3	$\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x}$ $= \frac{1 - (1 - 2\sin^2 x) - \sin x}{2\sin x \cos x - \cos x}$ $= \frac{2\sin^2 x - \sin x}{2\sin x \cos x - \cos x}$ $= \frac{\sin x (2\sin x - 1)}{\cos x (2\sin x - 1)}$ $= \frac{\sin x}{\cos x}$ $= \tan x$		(6)
5.3.2	$\sin 2x - \cos x = 0$ $= \cos x (2 \sin x - 1) = 0$ $\cos x = 0 \quad \sin x = \frac{1}{2}$ $x = 90^\circ + 360^\circ k \quad \text{of} \quad x = 30^\circ + 360^\circ k \quad k \in \mathbb{Z}$ $x = 270^\circ + 360^\circ k \quad x = 150^\circ + 360^\circ k$		(8)

5.4	$\sin 38^\circ = \sqrt{1-a^2}$ $\therefore \cos 52^\circ = \sin 38^\circ = \sqrt{1-a^2}$ $\frac{\sin 38^\circ + \cos 52^\circ}{\cos 52^\circ \cdot \cos 14^\circ + \sin 52^\circ \cdot \sin 14^\circ}$ $= \frac{\sin 38^\circ + \cos 52^\circ}{\cos(52^\circ - 14^\circ)}$ $= \frac{\sin 38^\circ + \cos 52^\circ}{\cos 38^\circ}$ $= \frac{\sqrt{1-a^2} + \sqrt{1-a^2}}{a}$ $= \frac{2\sqrt{1-a^2}}{a}$		(5)

VRAAG 6

6.1	$a = 2$ $b = \frac{1}{2}$		(2)
6.2	360°		(1)
6.3	-1		(1)
6.4	3 waardes van x		(2)
			[6]

VRAAG 7

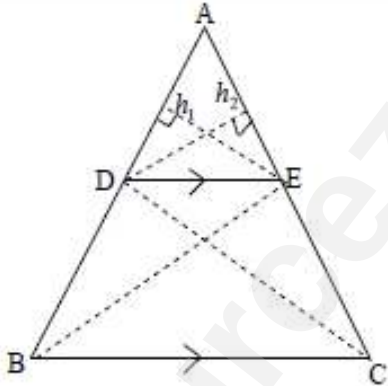
7.1	$\hat{C}\hat{A}\hat{D} = 180^\circ - (\alpha + \beta)$ $\therefore \frac{\sin[180^\circ - (\alpha + \beta)]}{x} = \frac{\sin \beta}{AC}$ $\therefore AC = \frac{x \sin \beta}{\sin(\alpha + \beta)}$ En $\sin \theta = \frac{AB}{AC}$ $\therefore AB = AC \sin \theta$ $= \frac{x \sin \theta \sin \beta}{\sin(\alpha + \beta)}$		
7.2	$AB = \frac{40 \sin 15^\circ \sin 70^\circ}{\sin 120^\circ}$ $= 11,25m$		
7.3	$AC = \frac{40 \sin 70^\circ}{\sin 120^\circ}$ $= 43,4m$ $AREA \triangle ACD = \frac{1}{2} AC \cdot CD \cdot \sin \alpha$ $= \frac{1}{2} (43,4)(40) \sin 50^\circ$ $= 664,93m^2$		

VRAAG 8

8.1	supplementêr	✓ antwoord	(1)
8.2.1	Hoek tussen raaklyn en radius	✓ antwoord	(1)
8.2.2	$\hat{S}_3 + \hat{S}_4 = 90^\circ \text{ (raaklyn } \perp \text{ radius)}$ $\hat{N}_1 + \hat{N}_2 = 90^\circ \text{ (raaklyn } \perp \text{ radius)}$ $\hat{S}_3 + \hat{S}_4 + \hat{N}_1 + \hat{N}_2 = 90^\circ + 90^\circ$ $= 180^\circ$ $\therefore RNOS \text{ koordevierhoek (teensthoekesuppl)}$	✓ Stelling ✓✓ Stelling & Rede ✓ Rede	(4)
8.2.3	$\hat{S}_1 = x$ $\hat{S}_1 = \hat{N}_2 = x \text{ (Raaklyn koordstelling)}$ $\hat{N}_2 = \hat{S}_3 = x \text{ (basis hoeke gelykbenige } \Delta)$ $\hat{S}_1 = \hat{R}_2 = x \text{ (hoeke in dieselfde segment)}$ $\hat{N}_2 = \hat{R}_1 = x \text{ (hoeke in dieselfde segment)}$	✓✓ Stelling, Rede ✓✓ Stelling, Rede ✓✓ Stelling, Rede ✓✓ Stelling, Rede	(8)

8.2.4	$\hat{O} + \hat{Q}_2 + \hat{N}_2 + \hat{S}_3 = 180^\circ$ (Som van hoeke $\triangle OSN$) Maar $\hat{S}_3 = \hat{N}_2$ (Teenst hoeke gelykesye) $\therefore \hat{O}_1 + \hat{O}_2 + 2\hat{S}_3 = 180^\circ$ $\hat{O}_1 + \hat{O}_2 + \hat{O}_3 = 180^\circ$ (hoekeopreguitlyn) $\therefore \hat{O}_1 + \hat{O}_2 + \hat{O}_3 = \hat{O}_1 + \hat{O}_2 + 2\hat{S}_3$ $2\hat{S}_3 = \hat{O}_3$ $\therefore \hat{S}_3 = \frac{1}{2}\hat{O}_3$		(4)
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VRAAG 9

9.1	Given: $\triangle ABC$ with D on AB and E on AC such that $DE \parallel BC$  RTP: $\frac{AD}{DB} = \frac{AE}{EC}$ Construction: Join DC and BE ✓ Proof: $\frac{\text{area } \triangle ADE}{\text{area } \triangle BDE} = \frac{\frac{1}{2}AD \cdot h_1}{\frac{1}{2}DB \cdot h_2} = \frac{AD}{DB} \checkmark \dots \text{areas of } \triangle\text{'s with the same height and common vertex are in the same ratio as their bases} \checkmark$ $\frac{\text{area } \triangle ADE}{\text{area } \triangle DEC} = \frac{\frac{1}{2}AE \cdot h_2}{\frac{1}{2}EC \cdot h_2} = \frac{AE}{EC} \checkmark$ but area $\triangle DBE = \text{area } \triangle DEC \checkmark \dots$ Same base, same parallel lines ✓ Thus $\frac{AD}{DB} = \frac{AE}{EC}$ (6)	construction Statement Reason NOTE: If area of $\triangle$ found, then reason not necessary Statement Statement Reason	
9.2.1	In $\triangle PQM$, $GH \parallel QT$ $\frac{QH}{HM} = \frac{GP}{GM} \checkmark \text{ (line parallel to one side of a } \triangle \text{ OR Prop Th: } GH \parallel PQ)$ $= \frac{1}{2} \checkmark$ (3)		

9.2.2	$QH = k; HM = 2k \therefore RM = 3k \checkmark$ $MR = QM = 3k \dots M \text{ is the midpoint of } QR$ $\frac{RG}{RT} = \frac{RH}{RQ} \checkmark$ (line parallel to one side of a Δ OR Prop Th: $GH \parallel PQ$) $= \frac{5k}{6k} \checkmark$ $= \frac{5}{6} \checkmark$ (5)		
9.3.1	Let $Z_2 = x = \alpha \checkmark \dots$ Tan chord theorem $\checkmark$ Then $\hat{ABX} = 90^\circ - \alpha \dots$ sum of $\angle$'s of $\Delta ABP \checkmark$ But $\hat{Z}_1 = \hat{ABP}$ $= 90 - \alpha \dots \angle$'s opposite equal sides: $AZ = AB \checkmark$ $\hat{Z}_1 + Z_2 = \alpha + 90^\circ - \alpha \dots$ adj. $\angle$'s on a straight line $\checkmark$ $= 90^\circ$ Thus $\hat{Z}_3 = 90^\circ$ (5)		
9.3.2	In ΔAYZ and ΔAZX $\hat{Z}_2 = \hat{X} \dots$ Tan chord theorem $\checkmark$ $\hat{A}_2 = \hat{A}_2 \dots$ common $\checkmark$ $\hat{AYZ} = \hat{AZX}$ (remaining angles) $\checkmark$ $\therefore \Delta AYZ \parallel \Delta AZX \quad \angle, \angle, \angle \quad \checkmark$ (3)		
9.3.3	$\therefore \frac{AZ}{AY} = \frac{AX}{AZ} \quad \Delta s \parallel \text{ sides in proportion } \checkmark$ $\therefore AZ^2 = AY \cdot AX$ (1)		