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basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE
NASIONALE SENIOR SERTIFIKAAT

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2012

MEMORANDUM

MARKS/*PUNTE*: 150

This memorandum consists of 23 pages.
Hierdie memorandum bestaan uit 23 bladsye.

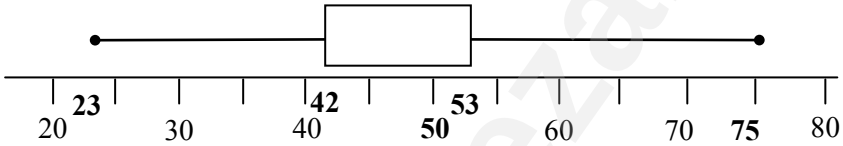
NOTE:

- If a candidate answered a question TWICE, mark only the first attempt.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed-out question.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

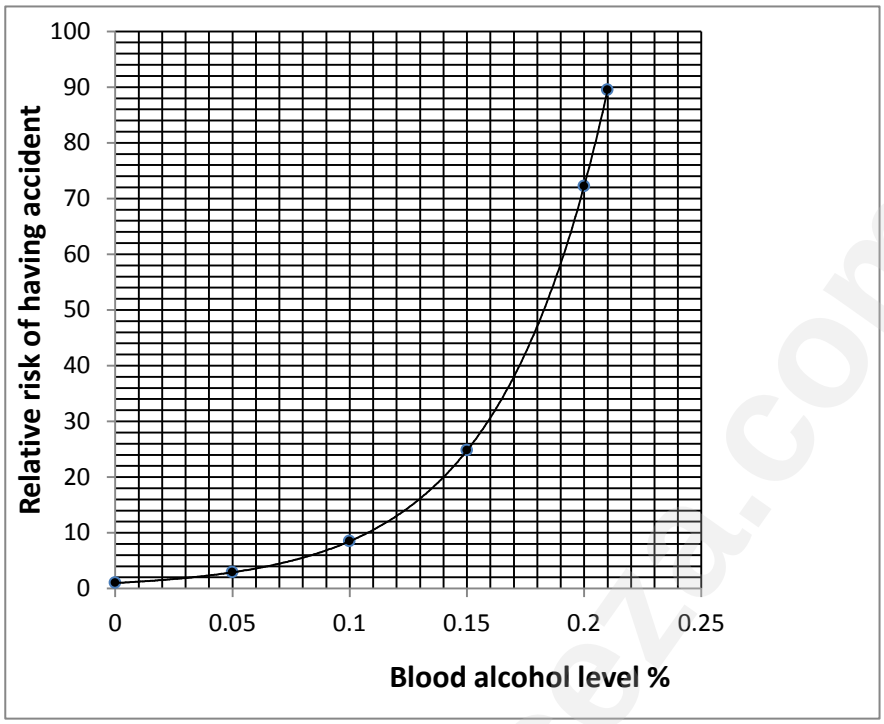
- *As 'n kandidaat 'n vraag TWEE keer beantwoord het, merk slegs die eerste poging.*
- *As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, merk die deurgehaalde antwoord.*
- *Volgehoue akkuraatheid is DEURGAANS in ALLE aspekte van die memorandum van toepassing.*
- *Aanvaarding van waardes/antwoorde om 'n problem op te los, is onaanvaarbaar.*

QUESTION/*VRAAG* 1



1.1	Interquartile range/*Interkwartielvariasiewydte* = 53 – 42 = 11 Answer only: Full marks No CA	✓critical values (42 ; 53) ✓11 (2)
1.2	25% of trees have a height in excess of 53 cm. *25% van bome het 'n hoogte van meer as 53 cm.*	✓✓25% (2)
1.3	Between Q_2(50) and Q_3 (53)/*Tussen Q_2 en Q_3* ***REASON / REDE*** The distance between these two quartiles is the smallest/*Die afstand tussen hierdie twee kwartiele is die kleinste.* **OR** The third quarter has smallest length / *Die derde kwart het die kortste lengte*	✓ Q_2 and Q_3 ✓ reason (2) **[6]**

QUESTION/*VRAAG* 2

2.1 and/en 2.2		6 points correctly plotted (3 marks) 4 points correctly plotted (2 marks) 2 points correctly plotted (1 mark) (3) ✓ exponential curve/ *eksponensiale kromme* (1)
2.3	The trend shows that as the blood alcohol levels increase, the risk of having an accident increases rapidly. *Die tendens(neiging) toon dat indien die bloed-alkoholvlakke toeneem, die risiko van 'n motorongeluk neem vinnig toe.*	✓reason (1)
2.4	Approximately 47% (Accept 44% - 51%)	✓✓ 47% (2) **[7]**

QUESTION/*VRAAG* 3

3.1	more than 15 minutes: 140 – 104 = 36 people Approximately 36 people (Accept 34 – 37) Answer only: Full marks	✓ 104 ✓ 36 (2)
3.2	At 8 minutes approximately 27 people and at 12 minutes approximately 62 people left the auditorium/*By 8 minute het ongeveer 27 mense en by 12 minute ongeveer 62 mense die ouditorium verlaat.* $\therefore 62 - 27 = 35$ Approximately 35 people left the auditorium between 8 and 12 minutes/*Ongeveer 35 mense het tussen 8 en 12 minute die ouditorium verlaat.*	✓ 27 and 62 ✓ 35 (Accept 33 – 36) (2)
3.3	Modal class/*modale klas*: $11 < x \leq 16$ **OR** $11 \leq x < 16$ Mark for critical values	✓ $11 < x \leq 16$ ✓ $11 \leq x < 16$ (1) **[5]**

QUESTION/*VRAAG* 4

	SCHOOL A	SCHOOL B	SCHOOL C
Mean	9,8	9,8	14,8
Standard deviation	2,3	3,1	2,3

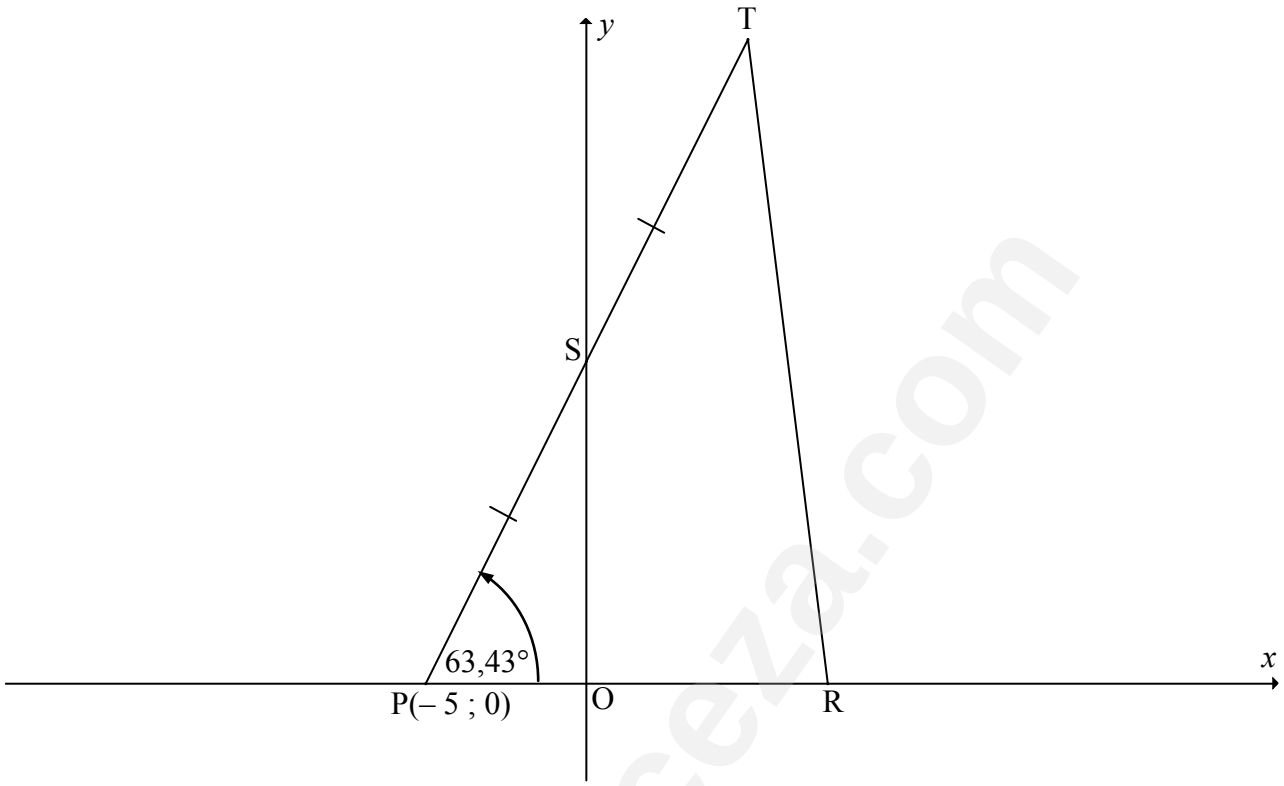
4.1	School B, because the standard deviation of B is the largest. *Skool B, want die standaardafwyking is die grootste.*	✓ School B ✓ reason (2)
4.2	There is no difference in the spread of the marks. *Daar is geen verskil in die verspreiding van punte nie.*	✓no difference / the same (1)
4.3	Add/increase each score in School A by 5 marks. *Vermeerder(tel by) elke punt in Skool A met 5 punte.*	✓ increase each mark *vermeerder elke punt* ✓ 5 marks (2)
4.4	The mean will decrease (by 10%) *Die gemiddelde sal verminder (met 10%).* The standard deviation will also decrease (by 10%) *Die standaardafwyking sal verminder (ook met 10%).*	✓ mean decreased /*gemiddeld verminder* ✓ SD decreased/ *SD verminder* (2) **[7]**

Explanation why values decrease by 10%:

$$mean = \frac{\sum 0{,}9x_i}{n} = 0{,}9\frac{\sum x_i}{n} = 0{,}9\bar{x}$$

$$SD = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} = \sqrt{\frac{\sum (0{,}9x_i - 0{,}9\bar{x})^2}{n}} = \sqrt{\frac{0{,}9^2\sum (x_i - \bar{x})^2}{n}} = 0{,}9\sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

QUESTION/*VRAAG* 5

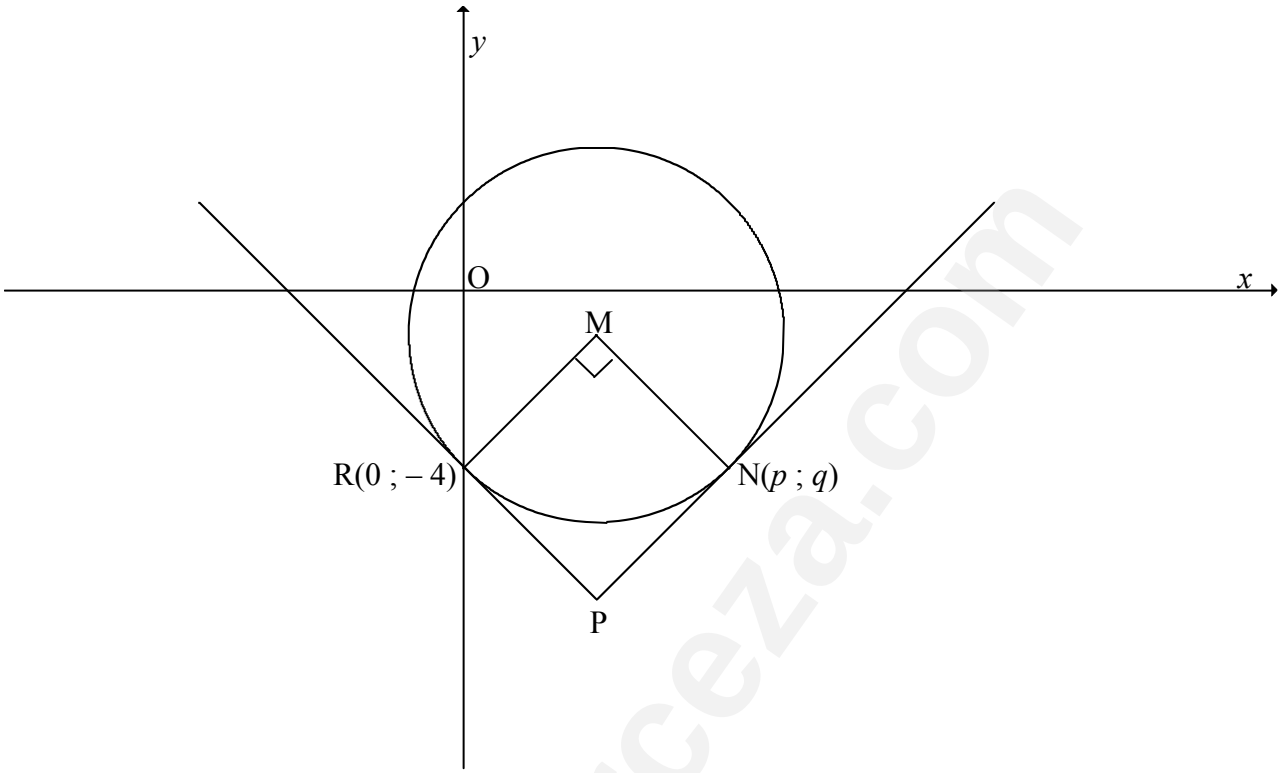


5.1.1	$m_{PT} = \tan 63{,}43°$ $= 2$	✓ tan 63,43° ✓ 2 (2) Answer only: full marks
5.1.2	Coordinates of P(– 5 ; 0) $y - y_1 = m(x - x_1)$ $y - 0 = 2(x + 5)$ $y = 2x + 10$ **OR** $y = mx + c$ $0 = (2)(-5) + c$ $c = 10$ $y = 2x + 10$ **OR** $m_{PT} = 2 = \tan 63{,}43°$ $\tan 63{,}43° = \frac{OS}{OP} = \frac{OS}{5} = 2$ $\therefore OS = 10$ $y = 2x + 10$	✓substitution of P(–5 ; 0) and $m = 2$ into equation ✓ equation (2) ✓substitution of P(–5 ; 0) and $m = 2$ into equation ✓ equation (2) ✓ $\frac{OS}{5} = 2$ ✓ equation (2)

5.1.3	$OS = 10$ units $PS^2 = (5)^2 + (10)^2$ $= 125$ $PS = \sqrt{125} = 5\sqrt{5}$ Accept PS = 11,18 **OR** P(–5 ; 0) ; OS = 10 units $PS^2 = (-5-0)^2 + (0-10)^2$ $= 25 + 100$ $= 125$ $PS = \sqrt{125} = 5\sqrt{5}$ Accept PS = 11,18 **OR** $\frac{PS}{5} = \frac{1}{\cos 63{,}43°}$ $\therefore PS = \frac{5}{\cos 63{,}43°}$ $PS = 11{,}18$ **OR** $\frac{PS}{10} = \frac{1}{\sin 63{,}43°}$ $\therefore PS = \frac{10}{\sin 63{,}43°}$ $PS = 11{,}18$	✓ OS = 10 ✓ substitution of correct distances into Pythagoras ✓ $\sqrt{125}$ (3) ✓ OS = 10 ✓ substitution of correct distances into Pythagoras ✓ $\sqrt{125}$ (3) ✓ ratio ✓ $PS = \frac{5}{\cos 63{,}43°}$ ✓11,18 (3) ✓ ratio ✓ $PS = \frac{10}{\sin 63{,}43°}$ ✓11,18 (3)
5.1.4	Let T be $(x ; y)$. Then $\frac{-5+x}{2} = 0$ and $\frac{0+y}{2} = 10$ $x = 5$ $y = 20$ T(5 ; 20) **OR** by inspection: T(5 ; 20)	✓ 5 ✓ 20 (2) ✓ 5 ✓ 20 (2)
5.2	$OR = \left(\frac{3}{2}\right)(5) = \frac{15}{2} = 7{,}5$ $R\left(\frac{15}{2}; 0\right)$ If only x-coordinate : 2 marks	✓$x = 7{,}5$ / $\frac{15}{2}$ ✓$y = 0$ (2)

5.3	$\text{Area } \Delta PTR = \frac{1}{2}(\text{base PR}) \times (\text{height})$ $= \frac{1}{2}(5 + \frac{15}{2}) \times 20$ $= 125 \text{ square units}$ **OR** $\text{Area } \Delta PTR = \frac{1}{2} PT.PR.\sin T\hat{P}R$ $= \frac{1}{2}\left(10\sqrt{5}\right)\left(\frac{25}{2}\right)\sin 63{,}43°$ $= 124{,}99 \text{ square units}$	✓ area formula ✓ $5 + \frac{15}{2} = 12{,}5$ ✓ 20 ✓ 125 (4) ✓ area formula ✓ $10\sqrt{5}$ ✓ $\frac{25}{2}$ ✓ 124,99 (4) **[15]**

QUESTION/*VRAAG* 6



6.1	$x^2 + y^2 - 6x + 2y - 8 = 0$ $x^2 - 6x + 9 + y^2 + 2y + 1 = 8 + 9 + 1$ $(x-3)^2 + (y+1)^2 = 18$ $\therefore$ M(3 ; – 1) If only $(x-3)^2 + (y+1)^2 = r^2$ $(r^2 \neq 18)$, then 2 marks **OR** $x_M = -\frac{1}{2}(\text{coefficient of } x)$ $x_M = -\frac{1}{2}(-6)$ $x_M = 3$ $y_M = -\frac{1}{2}(\text{coefficient of } y)$ $y_M = -\frac{1}{2}(2)$ $y_M = -1$ $\therefore$ M(3 ; – 1)	✓ $x^2 - 6x + 9$ ✓ $y^2 + 2y + 1$ ✓ $(x-3)^2$ ✓ $(y+1)^2$ (4) ✓ $x_M = -\frac{1}{2}(-6)$ ✓ $x_M = 3$ ✓ $y_M = -\frac{1}{2}(2)$ ✓ $y_M = -1$ (4)

6.2	$m_{RM} = \frac{-1-(-4)}{3-0}$ $= 1$ y-intercept is -4 $y = x - 4$	✓ substitution into gradient formula ✓ $m_{RM} = 1$ ✓ equation (3)
6.3	MR ⊥ RP (radius ⊥ tangent/*raaklyn*) $m_{MN} = m_{PR} = -1$ $\frac{q-(-1)}{p-3} = -1$ $-p+3 = q+1$ $q = 2-p$ **OR** MR ⊥ RP (radius ⊥ tangent/*raaklyn*) $m_{MN} = m_{PR} = -1$ $y-(-1) = -1(x-3)$ $y+1 = -x+3$ $y = -x+2$ $q = 2-p$	✓✓ $m_{MN} = -1$ ✓ substitution into gradient formula ✓ $-p+3 = q+1$ (4) ✓✓ $m_{MN} = -1$ ✓ $y = -x+2$ ✓ substitution into equation of line (4)
6.4	$(x-3)^2 + (y+1)^2 = 18$ $(p-3)^2 + (q+1)^2 = 18$ $(2-q-3)^2 + (q+1)^2 = 18$ $q^2+2q+1+q^2+2q+1-18 = 0$ $2q^2+4q-16 = 0$ $q^2+2q-8 = 0$ $(q+4)(q-2) = 0$ $q = -4$ *or* $q \neq 2$ $p = 6$ **OR** MRPN is a square/*vierkant* (rectangle with/*reghoek met* MN = MR) $\therefore$ $M\hat{P}N = 45°$ But MR has a slope/gradient of 1, so RN || x-axis $\therefore$ $q = -4$ and $p = 2-(-4) = 6$ **OR**	✓ method ✓✓ $q = -4$ ✓✓ $p = 6$ (5) ✓ method ✓✓ $q = -4$ ✓✓ $p = 6$ (5)

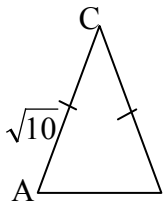
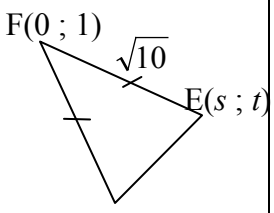
	$q = 2 - p$ $(p-3)^2 + (2-p+1)^2 = 18$ $(p-3)^2 = 9$ $\therefore \quad p-3 = 3 \quad (p > 0)$ $p = 6$ $\therefore q = -4$ **OR** Using symmetry: $q = -4$ (since $y_M = y_R$) $-4 = 2 - p$ $p = 6$ OR $p = 2 \times 3$ (since $x_M = 2x_N$)	✓ method ✓✓ $p = 6$ ✓✓ $q = -4$ (5) ✓method ✓✓ q ✓✓ p (5)
6.5	$r^2 = (6)^2 + (-4)^2$ $= 36+16 = 52$ $x^2 + y^2 = 52$ **OR** $p^2 + q^2 = (6)^2 + (-4)^2$ $= 36+16 = 52$ $x^2 + y^2 = p^2 + q^2$ $= 52$	✓ substitution ✓ equation (2) ✓ substitution ✓ equation (2)
6.6	area of circle M $= \pi r^2$ $= \pi(\sqrt{18})^2$ $= 18\pi$ square *units* $= 56{,}55$ *square units*	✓ $r = \sqrt{18}$ ✓ area of circle (2)
6.7	MRPN is a square (all angles equals 90°, adj sides equal) $N\hat{M}P = 45°$ (diagonals of a square bisect the angles/*hoeklyne van vierkant halveer hoeke)* $\frac{NP}{MP} = \sin N\hat{M}P$ $= \sin 45°$ $= \frac{1}{\sqrt{2}}$ *or* $\frac{\sqrt{2}}{2}$ **OR** MRPN is a square (all angles equals 90°, adj sides equal) $MP^2 = 18+18$ $= 36$ $MP = 6$ $\frac{NP}{MP} = \frac{\sqrt{18}}{6} = \frac{1}{\sqrt{2}}$ *or* $\frac{\sqrt{2}}{2}$	✓ $N\hat{M}P = 45°$ ✓✓ $\frac{NP}{MP} = \sin N\hat{M}P$ ✓ $\frac{1}{\sqrt{2}}$ (4) ✓ $MN^2 = 18$ ✓ $MP^2 = 36$ ✓ 6 ✓ $\frac{1}{\sqrt{2}}$ (4)

	OR	
	By inspection: P(3 ; – 7)	✓ P(3 ; – 7)
	$\frac{NP}{MP} = \frac{\sqrt{(6-3)^2+(4-7)^2}}{\sqrt{(3-3)^2+(-7+1)^2}}$ $= \frac{\sqrt{18}}{6} = \frac{1}{\sqrt{2}}$ *or* $\frac{\sqrt{2}}{2}$	✓$NP^2 = 18$ ✓MP = 6 ✓ $\frac{1}{\sqrt{2}}$ (4) **[24]**

QUESTION/*VRAAG* 7

7.1	$(x\ ;\ y) \rightarrow (y\ ;\ -x) \rightarrow (-y\ ;\ -x)$	✓ $(y\ ;\ -x)$ ✓ $(-y\ ;\ -x)$ (2)
7.2	$(x\ ;y) \rightarrow (-x\ ;y) \rightarrow (y\ ;x)$	✓ $(-x\ ;\ y)$ ✓ $(y\ ;\ x)$ (2)
7.3	Mo's claim is correct/*Mo se bewering is korrek.* If order was unimportant then the image of P would be the same in both cases. This is not so./*As volgorde onbelangrik is, sal die beeld van P in beide gevalle dieselfde wees. Wat nie so is nie.* **OR** Choose any point ≠ (0 ; 0) and show that their images both cases are not the same. For example: $(3\ ;\ 4) \rightarrow (4\ ;\ -3) \rightarrow (-4\ ;\ -3)$ $(3\ ;\ 4) \rightarrow (-3\ ;\ 4) \rightarrow (4\ ;\ 3)$ Mo is correct	✓ Mo ✓ reason (2) ✓ calculation of both images ✓ Mo (2) **[6]**

QUESTION/*VRAAG* 8

8.1	ΔABC is translated by 4 units to the left and 4 units up/*ΔABC word getransleer met 4 eenhede na links en 4 eenhede opwaarts.* Accept $(x;y) \rightarrow (x-4;y+4)$	✓ translation/*translasie* ✓ 4 left/*links* and 4 up/*opwaarts* (2)
8.2	$R'\ (3\ ;-4)$	✓ 3 ✓ – 4 (2)
8.3.1	Area $\Delta A'B'C'$ = 16 × Area of ΔABC Scale factor/*skaalfaktor* = 4	✓ 4 (1)
8.3.2	$AC = \sqrt{10}$ $A'C' = 4\sqrt{10}$	✓ $4\sqrt{10}$ (1)
8.4	EF = AC $\sqrt{(s-0)^2+(t-1)^2} = \sqrt{10}$ $\sqrt{s^2+(t-1)^2} = \sqrt{10}$ $s^2+(t-1)^2 = 10$ $s^2+t^2-2t+1-10=0$ $s^2+t^2-2t-9=0$   	✓✓✓ recognising that EF = AC ✓ equation in terms of *s* and *t* (4) **[10]**

QUESTION/*VRAAG* 9

9.1	Anti-clockwise / Anti-kloksgewys: $-\frac{16}{\sqrt{2}}\cos(-\theta)-\frac{16}{\sqrt{2}}\sin(-\theta)=8$ .....(1) $\frac{16}{\sqrt{2}}\cos(-\theta)-\frac{16}{\sqrt{2}}\sin(-\theta)=-8\sqrt{3}$ .....(2) (1) +(2): $-\frac{32}{\sqrt{2}}\sin(-\theta)=8-8\sqrt{3}$ $\sin(-\theta)=\frac{-8+8\sqrt{3}}{\frac{32}{\sqrt{2}}}$ $\sin\theta=\frac{-\sqrt{6}+\sqrt{2}}{4}=-0{,}258819...$ $\theta=180°+15°$ *or* $\theta=360°-15°$ $=195°$ **OR** $-\frac{16}{\sqrt{2}}\cos(-\theta)-\frac{16}{\sqrt{2}}\sin(-\theta)=8$ .....(1) $\frac{16}{\sqrt{2}}\cos(-\theta)-\frac{16}{\sqrt{2}}\sin(-\theta)=-8\sqrt{3}$ .....(2) (1) – (2): $-\frac{32}{\sqrt{2}}\cos(-\theta)=8+8\sqrt{3}$ $\cos\theta=\frac{8+8\sqrt{3}}{-\frac{32}{\sqrt{2}}}=\frac{-\sqrt{6}-\sqrt{2}}{4}=-0{,}96592...$ $\theta=180°+15°$ *or* $\theta=180°-15°$ $=195°$ **OR** Clockwise /Kloksgewys: $-\frac{16}{\sqrt{2}}\cos(\theta)+\frac{16}{\sqrt{2}}\sin(\theta)=8$ .....(1) $\frac{16}{\sqrt{2}}\cos(\theta)+\frac{16}{\sqrt{2}}\sin(\theta)=-8\sqrt{3}$ .....(2) (1) +(2): $\frac{32}{\sqrt{2}}\sin(\theta)=8-8\sqrt{3}$ $\sin(\theta)=\frac{8-8\sqrt{3}}{\frac{32}{\sqrt{2}}}$ $\sin\theta=\frac{-\sqrt{6}+\sqrt{2}}{4}=-0{,}258819...$ $\theta=180°+15°$ *or* $\theta=360°-15°$ $=195°$	✓ substitution into *x* image of rotation ✓ substitution into *y* image of rotation ✓ addition of equations ✓ value of sin θ ✓ 180° + 15° (5) ✓ substitution into *x* image of rotation ✓ substitution into *y* image of rotation ✓ subtraction of equations ✓ value of cos θ ✓ 180° + 15° (5) ✓ substitution into *x* image of rotation ✓ substitution into *y* image of rotation ✓ addition of equations ✓ value of sin θ ✓ 180° + 15° (5)

OR

$$-\frac{16}{\sqrt{2}}\cos(\theta)+\frac{16}{\sqrt{2}}\sin(\theta)=8 \quad .....(1)$$

$$\frac{16}{\sqrt{2}}\cos(\theta)+\frac{16}{\sqrt{2}}\sin(\theta)=-8\sqrt{3} \quad .....(2)$$

$$(1)-(2): \quad -\frac{32}{\sqrt{2}}\cos(\theta) = 8+8\sqrt{3}$$

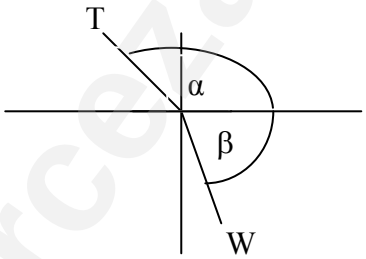
$$\cos\theta=\frac{8+8\sqrt{3}}{-\frac{32}{\sqrt{2}}}=\frac{-\sqrt{6}-\sqrt{2}}{4}=-0{,}96592...$$

$$\theta=180°+15° \quad or \quad \theta=180°-15°$$
$$=195°$$

✓ substitution into x image of rotation
✓ substitution into y image of rotation
✓ subtraction of equations
✓ value of cos θ
✓ 180° + 15°

(5)

OR



$$\tan\alpha=\frac{-\frac{16}{\sqrt{2}}}{\frac{16}{\sqrt{2}}}=-1$$
$$\alpha = 135°$$

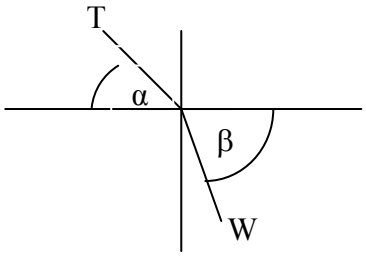
$$\tan\beta=\frac{-8\sqrt{3}}{8}=-\sqrt{3}$$
$$\beta=-60°$$

$$\therefore\theta = 135°+60°=195°$$

✓ $\tan\alpha=-1$
✓ 135°
✓ $\tan\beta=-\sqrt{3}$
✓ – 60°
✓ 195°

(5)

OR



$$\tan\alpha=1$$
$$\alpha=45°$$

$$\tan\beta=\sqrt{3}$$
$$\beta=60°$$

$$\therefore\theta = (180°-45°)+60°=195°$$

✓ $\tan\alpha=1$
✓ 45°
✓ $\tan\beta=\sqrt{3}$
✓ 60°
✓ 195°

(5)

9.2	195° in 1,3 secs $\therefore$ 1 revolution/*omwenteling* in $\frac{360}{195}\times 1{,}3$ secs = 2,4 secs/*sek* 1 minute = 60 sec: $\therefore \frac{60}{2{,}4} = 25$ revolutions/*omwentelings* $\therefore$ 25 rev/min or 25 *omw/min* Answer only: 1 mark OR $\text{speed/sec} = \frac{195°}{1{,}3} = 150°/\text{sec}$ $\text{speed/minute} = 150° \times 60 = 9000°/\text{min}$ $\text{no of revolutions} = \frac{9000}{360} = 25 \text{ rev/min}$	✓ $\frac{360}{195}$ ✓ $\frac{360}{195}\times 1{,}3$ ✓ 2,4 secs ✓ $\frac{60}{2{,}4}$ ✓ 25 rev/min (5) ✓ $\frac{195}{1{,}3}$ or 150 ✓ 150×60 ✓ 9000 ✓ $\frac{9000}{360}$ ✓ 25 rev/min (5) **[10]**

QUESTION/*VRAAG* 10

10.1	$OP^2 = 25+144 = 169$ $OP = 13$ $\cos\alpha = -\frac{5}{13}$ **OR** $r^2 = x^2 + y^2$ $25+144 = 169$ $r = 13$ $\cos\alpha = -\frac{5}{13}$ Answer only: full marks	✓ $OP^2 = 25+144$ ✓ $OP = 13$ ✓ answer (3) ✓ $r^2 = 25+144$ ✓ $r = 13$ ✓ answer (3)
10.2	$\tan(180° - \alpha)$ $= -\tan\alpha$ $= -\frac{12}{5}$	✓ $-\tan\alpha$ ✓ answer (2)
10.3	$\sin(30° - \alpha)$ $= \sin 30°\cos\alpha - \cos 30°\sin\alpha$ $= \left(\frac{1}{2}\right)\left(\frac{-5}{13}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{-12}{13}\right)$ $= \frac{-5+12\sqrt{3}}{26}$	✓ expansion ✓ $\left(\frac{1}{2}\right)\left(\frac{-5}{13}\right)$ ✓ $\left(\frac{\sqrt{3}}{2}\right)\left(\frac{-12}{13}\right)$ (3) **[8]**

QUESTION/*VRAAG* 11

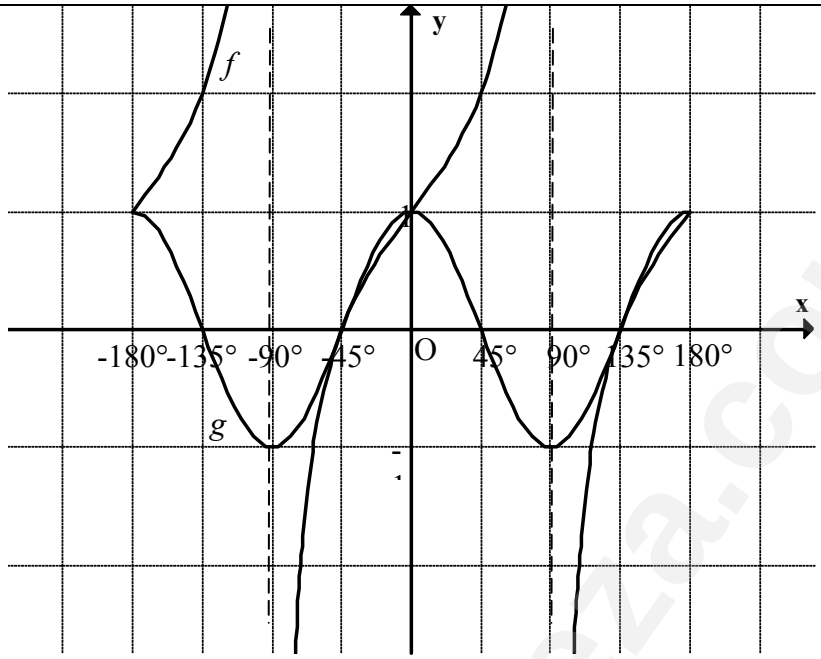
11.1	$LHS = \dfrac{\cos^2(90°+\theta)}{\cos(-\theta)+\sin(90°-\theta).\cos\theta}$ $= \dfrac{(-\sin\theta)^2}{\cos\theta+\cos\theta.\cos\theta}$ $= \dfrac{1-\cos^2\theta}{\cos\theta+\cos^2\theta}$ $= \dfrac{(1-\cos\theta)(1+\cos\theta)}{\cos\theta(1+\cos\theta)}$ $= \dfrac{1-\cos\theta}{\cos\theta}$ $= \dfrac{1}{\cos\theta}-1$ $= RHS$	✓ $\cos^2(90°+\theta) = \sin^2\theta$ ✓ $\sin(90°-\theta) = \cos\theta$ ✓ $\cos(-\theta) = \cos\theta$ ✓ $1-\cos^2\theta$ ✓ factors ✓ $\dfrac{1-\cos\theta}{\cos\theta}$ (6)
11.2	$\tan x.\sin x+\cos x.\tan x = 0$ $\tan x(\sin x+\cos x) = 0$ $\tan x = 0$ *or* $\sin x+\cos x = 0$ $\sin x = -\cos x$ $\tan x = -1$ $x = 0°+k.180°;\ k\in Z$ *or* $x = 135°+k.180°;\ k\in Z$ **OR** $\dfrac{\sin x}{\cos x}.\sin x+\cos x.\dfrac{\sin x}{\cos x} = 0$ $\dfrac{\sin^2 x}{\cos x}+\dfrac{\cos x.\sin x}{\cos x} = 0$ $\sin^2 x+\cos x.\sin x = 0$ $\sin x(\sin x+\cos x) = 0$ $\sin x+\cos x = 0$ $\sin x = 0$ *or* $\tan x = -1$ $x = 0°+k.180°;\ k\in Z$ *or* $x = 135°+k.180°;\ k\in Z$ $(\sin x+\cos x)^2 = 0$ $1+\sin 2x = 0$ $\sin 2x = -1$ ✓ $x = 135°+k.180°$ **OR** $\tan x.\sin x+\cos x.\tan x = 0$ $(\cos x\neq 0)$ $\tan^2 x+\tan x = 0$ $\tan x(\tan x+1) = 0$ $\tan x = 0$ *or* $\tan x+1 = 0$ $\tan x = -1$ $x = 0°+k.180°;\ k\in Z$ *or* $x = 135°+k.180°;\ k\in Z$	✓ factorising ✓ $\tan x = 0$ and $\sin x+\cos x = 0$ ✓ $\tan x = -1$ ✓ $x = 0°$ ✓ $x = 135°$ or $-45°$ ✓ $k.180°$ ✓ $k\in Z$ (7) ✓ factorising ✓ $\sin x = 0$ and ✓ $\tan x = -1$ ✓ $x = 0°$ ✓ $x = 135°$ or $-45°$ ✓ $k.180°$ ✓ $k\in Z$ (7) ✓ factorising ✓ $\tan x = 0$ and $\tan x+1 = 0$ ✓ $\tan x = -1$ ✓ $x = 0°$ ✓ $x = 135°$ or $-45°$ ✓ $k.180°$ ✓ $k\in Z$ (7)

11.3.1	$2\sin^2 3x-\sin^2 x-\cos^2 x$ $=2\sin^2 3x-(\sin^2 x+\cos^2 x)$ $=2\sin^2 3x-1$ $=-\cos 6x$ OR $2\sin^2(2x+x)-\sin^2 x-\cos^2 x$ $=2(\sin 2x.\cos x+\cos 2x.\sin x)^2-(\sin^2 x+\cos^2 x)$ $=2((2\sin x.\cos x)\cos x+(1-2\sin^2 x)\sin x)^2-1$ $=2(2\sin x.\cos^2 x+\sin x-2\sin^3 x)^2-1$ $=2(2\sin x(1-\sin^2 x)+\sin x-2\sin^3 x)^2-1$ $=2(2\sin x-2\sin^3 x+\sin x-2\sin^3 x)^2-1$ $=2(-4\sin^3 x+3\sin x)^2-1$ $=2(16\sin^6 x-24\sin^4 x+9\sin^2 x)-1$ $=32\sin^6 x-48\sin^4 x+18\sin^2 x-1$	✓ $-(\sin^2 x+\cos^2 x)$ ✓ 1 ✓ $2\sin^2 3x-1$ (3) ✓ $-(\sin^2 x+\cos^2 x)$ ✓1 ✓ answer (3)
11.3.2	Max value = 1	✓1 (1)
11.4.1 (a)	$p=\cos\alpha+\sin\alpha$ $q=\cos\alpha-\sin\alpha$ $\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$ $=(\cos\alpha+\sin\alpha)(\cos\alpha-\sin\alpha)$ $=pq$ **OR** $\sin\alpha=\frac{p-q}{2}$ $\cos 2\alpha=1-2\sin^2\alpha$ $=1-2(\frac{p-q}{2})^2$ **OR** $\cos\alpha=\frac{p+q}{2}$ $\cos 2\alpha=2\cos^2\alpha-1$ $=2(\frac{p+q}{2})^2-1$ **OR** $\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$ $=(\frac{p+q}{2})^2-(\frac{p-q}{2})^2$	✓ expansion ✓ factorise ✓ answer (3) ✓ $\frac{p-q}{2}$ ✓ expansion ✓ answer (3) ✓ $\frac{p+q}{2}$ ✓ expansion ✓ answer (3)

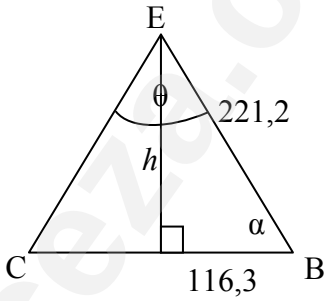
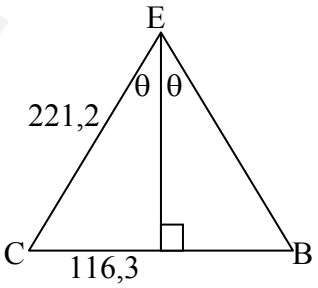
		✓ expansion ✓ $\frac{p-q}{2}$ ✓ $\frac{p+q}{2}$ (3)
11.4.1 (b)	$p+q=2\cos\alpha \quad \therefore \cos\alpha=\frac{p+q}{2}$ $p-q=2\sin\alpha \quad \therefore \sin\alpha=\frac{p-q}{2}$ $\tan\alpha=\frac{\sin\alpha}{\cos\alpha}$ $=\frac{2\sin\alpha}{2\cos\alpha}$ $=\frac{p-q}{p+q}$ **OR** $\cos\alpha+\sin\alpha=p$ $\cos\alpha-\sin\alpha=q$ $\Rightarrow 2\cos\alpha=p+q$ $\cos\alpha=\frac{p+q}{2}$ $y^2=2^2-(p+q)^2$ $y=\sqrt{4-(p+q)^2}$ $\therefore \tan\alpha=\frac{\sqrt{4-(p+q)^2}}{p+q}$ **OR** $\cos\alpha+\sin\alpha=p$ $\cos\alpha-\sin\alpha=q$ $\Rightarrow 2\sin\alpha=p-q$ $\sin\alpha=\frac{p-q}{2}$ $x^2=2^2-(p-q)^2$ $x=\sqrt{4-(p-q)^2}$ $\therefore \tan\alpha=\frac{p-q}{\sqrt{4-(p-q)^2}}$ OR	✓ $p+q$ ✓ $p-q$ ✓ identity ✓ answer (4) ✓ $2\cos\alpha=p+q$ ✓ sketch ✓ $y=\sqrt{4-(p+q)^2}$ ✓ answer (4) ✓ $2\sin\alpha=p-q$ ✓ sketch ✓ $x=\sqrt{4-(p-q)^2}$ ✓ answer (4)

	$\cos 2\alpha = 1 - 2\sin^2\alpha$ $\sin^2\alpha = \frac{1-pq}{2}$ $\cos 2\alpha = 2\cos^2\alpha - 1$ $\cos^2\alpha = \frac{pq+1}{2}$ $(\tan\alpha)^2 = \left(\frac{\sin\alpha}{\cos\alpha}\right)^2$ $\tan^2\alpha = \frac{1-pq}{1+pq}$ $\therefore \tan\alpha = \sqrt{\frac{1-pq}{1+pq}}$	✓ $\sin^2\alpha = \frac{1-pq}{2}$ ✓ $\cos^2\alpha = \frac{pq+1}{2}$ ✓ $(\tan\alpha)^2 = \left(\frac{\sin\alpha}{\cos\alpha}\right)^2$ ✓ $\tan\alpha = \sqrt{\frac{1-pq}{1+pq}}$ (4)
11.4.2	$\frac{p}{2q} - \frac{q}{2p}$ $= \frac{p^2-q^2}{2pq}$ $= \frac{(p+q)(p-q)}{2pq}$ $= \frac{(2\cos\alpha)(2\sin\alpha)}{2\cos 2\alpha}$ $= \frac{4\sin\alpha\cos\alpha}{2\cos 2\alpha}$ $= \frac{2\sin 2\alpha}{2\cos 2\alpha}$ $= \tan 2\alpha$	✓ $\frac{p^2-q^2}{2pq}$ ✓ factorising substituting ✓ from 11.4.1(a) ✓ and 11.4.1(b) ✓ $2\sin 2\alpha$ ✓ $\tan 2\alpha$ (6)
	OR $\frac{p}{2q} - \frac{q}{2p}$ $= \frac{\cos\alpha+\sin\alpha}{2(\cos\alpha-\sin\alpha)} - \frac{\cos\alpha-\sin\alpha}{2(\cos\alpha+\sin\alpha)}$ $= \frac{(\cos\alpha+\sin\alpha)^2 - (\cos\alpha-\sin\alpha)^2}{2(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)}$ $= \frac{\cos^2\alpha + 2\sin\alpha\cos\alpha + \sin^2\alpha - (\cos^2\alpha - 2\sin\alpha\cos\alpha + \sin^2\alpha)}{2(\cos^2\alpha - \sin^2\alpha)}$ $= \frac{4\sin\alpha\cos\alpha}{2\cos 2\alpha}$ $= \frac{2\sin 2\alpha}{2\cos 2\alpha}$ $= \tan 2\alpha$	✓ substitution ✓ single fraction ✓ $4\sin\alpha\cos\alpha$ ✓ $2\cos 2\alpha$ ✓ $2\sin 2\alpha$ ✓ $\tan 2\alpha$ (6) **[30]**

QUESTION/*VRAAG* 12

12.1		$y = \tan x + 1$ ✓ asymptotes and shape for whole domain ✓ y intercept ✓ x intercepts $y = \cos 2x$ ✓ x intercepts ✓ y intercept/TP ✓ minimum values (6)
12.2	Period of g is 180°.	✓ 180° (1)
12.3	Reflected about the x-axis and then translated by 10° to the left/ *refleksie om die x-as en dan 'n translasie van 10° links*. **OR** Translated by 10° to the left and then reflected about the x-axis/ *Translasie van 10° links en dan 'n refleksie om die x-as.*	✓ reflected about x –axis/*refleksie om x-as* ✓ 10° to the left *10° na links* (2) ✓ 10° to the left *10° na links* ✓ reflected about x –axis/*refleksie om x-as* (2)
12.4	f is always increasing $\therefore f'(x) > 0$ always $\therefore g(x) > 0$ $\therefore 0° < x < 45°$ or $135° < x \leq 180°$ **OR** $\therefore x \in (0°; 45°)$ *or* $(135° ; 180°]$	✓ critical values 0° and 45° ✓ inequality ✓ critical values 135° and 180° ✓ inequality (4) **[13]**

QUESTION/*VRAAG* 13

13.1	In ΔCEB : $BC^2 = EC^2 + EB^2 - 2(EC)(EB)\cos C\hat{E}B$ $(232,6)^2 = (221,2)^2 + (221,2)^2 - 2(221,2)(221,2)\cos C\hat{E}B$ $\cos C\hat{E}B = \dfrac{2(221,2)^2 - (232,6)^2}{2(221,2)^2}$ $= 0,447.....$ $C\hat{E}B = 63,44°$	✓ substitution into correct formula ✓ 0,447.... ✓ 63,44° (3)
	OR $\cos\alpha = \dfrac{116,3}{221,2}$ $\alpha = 58,28°$ $\theta = 180° - 2(58,28°) = 63,44°$  	✓ substitution into correct definition ✓ α = 58,28° ✓ 63,44° (3)
	OR $\sin\theta = \dfrac{116,3}{221,2}$ $\theta = 31,72°$ $A\hat{E}B = 2\theta = 2(31,72°) = 63,44°$  	✓ substitution into correct formula ✓ θ = 31,72° ✓ 63,44° (3)
13.2	$EF^2 = EB^2 - BF^2$ $= (221,2)^2 - (116,3)^2$ $= 35403,75$ $EF = 188,16\,m$ $\cos E\hat{F}G = \dfrac{GF}{EF}$ $= \dfrac{116,3}{188,16}$ $= 0,618....$ $E\hat{F}G = 51,82°$	✓ using Pythagoras correctly ✓ BF = 116,3 ✓ 188,16 ✓ using cos EFG ✓ $\dfrac{116,3}{188,16}$ ✓ 51,82° (6)
	OR	

$$\sin\alpha = \frac{h}{221,2}$$

$$h = 221,2\sin 58,28°$$

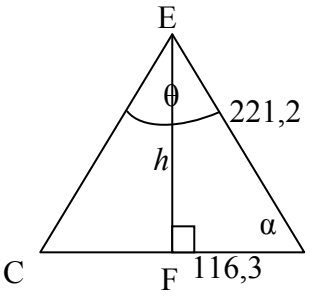
$$= 188,158...$$

In ΔEFG :

$$\cos E\hat{F}G = \frac{116,3}{188,158...}$$

$$= 0,61809...$$

$$E\hat{F}G = 51,82°$$



✓ $\sin\alpha = \frac{h}{221,2}$

✓ h subject

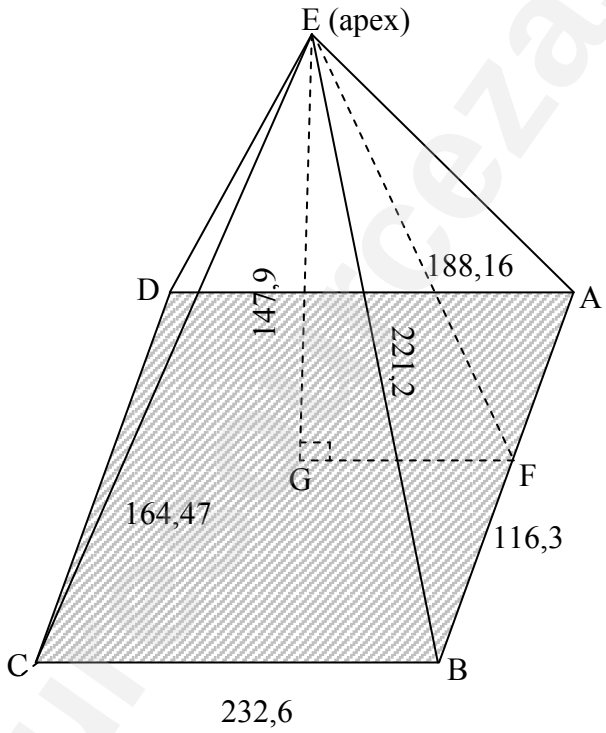
✓ $h = 188,158...$

✓ using $\cos E\hat{F}G$

✓ $\frac{116,3}{188,16}$

✓ $51,82°$

(6)



[9]

TOTAL/*TOTAAL*: 150