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education

**MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA**

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS PAPER 2

September 2018

MARKING GUIDELINES

MARKS: 150

This marking guideline consist of 20 pages

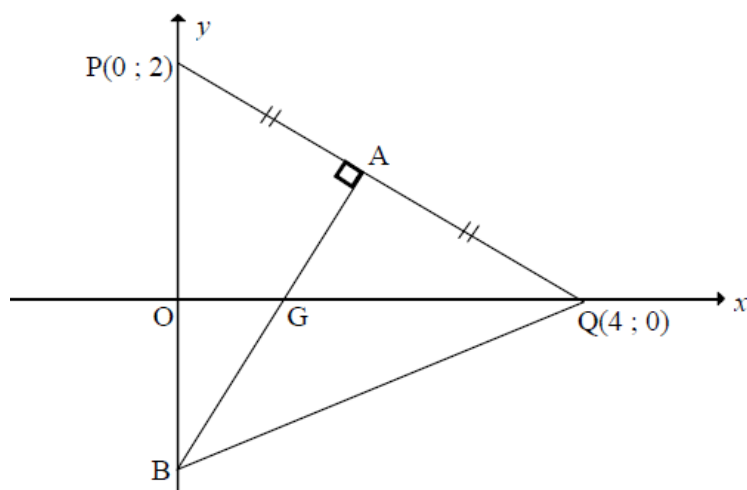
Question 1

1.1	$\begin{aligned}\text{IQR} &= Q_3 - Q_1 \\ &= 20 - 11 \\ &= 9\end{aligned}$	$\checkmark Q_3 - Q_1$ or $20 - 11$ $\checkmark 9$ (2)
1.2		$\checkmark$ min and max $\checkmark$ median $\checkmark$ upper and lower quartile (3)
1.3	Skewed to right or positively skewed	$\checkmark$ answer (1)
		[6]

Question 2

2.1.1	By using a calculator : $a = 29,22$ (29.21542...) $b = 0,89$ (0,886530...) $\therefore$ equation of line of least squares is $y = 29.22 + 0.89x$	✓ a ✓ b ✓ equation (3)
2.1.2	$y = 29.22 + (0.89)(32)$ $= 57,7$ OR $57,58$ Therefore the employee who undergoes 32 hours of training should produce about 58 units.	✓ substitution ✓ answer as integer (2)
2.1.3	Moderate OR not very strong Because the value of $r = 0.66$	✓ moderate ✓ reason (2)
2.2.1	$\bar{x} = \frac{45 + 70 + 44 + 56 + 60 + 48 + 75 + 60 + 63 + 38}{10} = \frac{559}{10}$ $= 55.9$	✓ sum ✓ answer (2)
2.2.2	$SD = 11.36$ $\bar{x} + SD$ $= 55.9 + 11.36$ $= 67.26$ $\therefore$ 2 employees	✓✓ SD ✓ 67.26 ✓ 2 employees (4)
		[13]

Question 3



3.1	$m_{PQ} = \frac{2-0}{0-4}$ $= -\frac{1}{2}$	✓ answer (1)
3.2	$A\left(\frac{0+4}{2}; \frac{2+0}{2}\right)$ $A(2; 1)$	✓ x_A ✓ y_A (2)
3.3	$m_{AB} . m_{PQ} = -1$ $m_{AB} . \left(\frac{-1}{2}\right) = -1$ $m_{AB} = 2$ Equation of AB is $y = 2x + c$ $\therefore 1 = 2(2) + c$ $c = -3$ Equation of AB is $y = 2x - 3$ OR	✓ $m_{AB} = 2$ ✓ substitution of (2;1) and m ✓ equation of AB (3)

	$m_{AB} \cdot m_{PQ} = -1$ $m_{AB} \cdot \left(\frac{-1}{2}\right) = -1$ $m_{AB} = 2$ $y - 1 = 2(x - 2)$ $y - 1 = 2x - 4$ Equation of AB is $y = 2x - 3$	✓ Substitution of (2;1) ✓ equation of AB (3)
3.4	B is the point (0 ; -3) $BQ = \sqrt{(0 - 4)^2 + (-3 - 0)^2}$ $BQ = 5$ OR $BQ^2 = 4^2 + 3^2$ (Pyth) $BQ = 5$	✓ substitution ✓ answer (2) ✓ substitution ✓ answer (2)
3.5	If PBQR is a rhombus then A is the midpoint of BR. Let the coordinates of R be (x ; y) $\frac{x + 0}{2} = 2$ AND $\frac{y - 3}{2} = 1$ $x = 4$ $y = 5$ R(4 ; 5) OR RQ PB so $x_R = 4$ RQ = PB = 5, so $y_R = 5$ ∴ R(4 ; 5)	 ✓✓ x coordinate ✓✓ y coordinate (4) ✓✓ x coordinate ✓✓ y coordinate (4)

3.6	$\tan \hat{A}GQ = 2$ $\therefore \hat{A}GQ = 63,43^\circ$ $m_{BQ} = \frac{-3-0}{0-4}$ $m_{BQ} = \frac{3}{4}$ $\therefore \tan \beta = \frac{3}{4}$ $\therefore \beta = 36,87^\circ$ $\therefore \hat{G}QB = 36,87^\circ$ (vertically opp angles) $63,43^\circ = 36,87^\circ + \hat{A}BQ$ (ext $\angle$ of Δ) $\therefore \hat{A}BQ = 26,56^\circ$ OR $\tan \hat{A}GQ = 2$ $\therefore \hat{A}GQ = 63,43^\circ$ $\therefore \hat{B}GQ = 180^\circ - 63,43^\circ$ ($\angle$ s on straight line) $= 116,57^\circ$ $m_{BQ} = \frac{-3-0}{0-4}$ $m_{BQ} = \frac{3}{4}$ $\therefore \tan \beta = \frac{3}{4}$ $\therefore \beta = 36,87^\circ$ $180 - 116,57^\circ - 36,87^\circ = \hat{A}BQ$ ($\angle$ in Δ) $\therefore \hat{A}BQ = 26,56^\circ$	$\checkmark 63,43^\circ$ $\checkmark m_{BQ}$ $\checkmark 36,87^\circ$ $\checkmark$ exterior angle of triangle $\checkmark$ answer (5) $\checkmark 116,57^\circ$ $\checkmark m_{BQ}$ $\checkmark 36,87^\circ$ $\checkmark$ angles in triangle $\checkmark$ answer (5)
3.7.1	$\hat{P}AB = 90^\circ$ $\therefore PB$ is the diameter (converse : $\angle$ at center = $2 \times \angle$ at circum)	$\checkmark BQ$ is diameter

	Midpoint PB $\left(0; -\frac{1}{2}\right)$ $x^2 + \left(y + \frac{1}{2}\right)^2 = r^2$ subst A(2 ; 1) $\therefore (2)^2 + \left(1 + \frac{1}{2}\right)^2 = r^2$ $r = \frac{5}{2}$ $x^2 + \left(y + \frac{1}{2}\right)^2 = \frac{25}{4}$ OR subst B(0 ; -3) $2^2 + \left(-3 + \frac{1}{2}\right)^2 = r^2$ $r^2 = \frac{25}{4}$ $x^2 + \left(y + \frac{1}{2}\right)^2 = \frac{25}{4}$ subst B(0 ; 2) $\left(2 + \frac{1}{2}\right)^2 = r^2$ $r^2 = \frac{25}{4}$ $x^2 + \left(y + \frac{1}{2}\right)^2 = \frac{25}{4}$	✓✓ midpoint ✓ substitution ✓ $r = \frac{5}{2}$ ✓ equation LHS ✓ equation RHS (6)
3.7.2	$y = 2$ ($r \perp$ tangent)	✓✓ $y = 2$ (2)
		[25]

Question 4

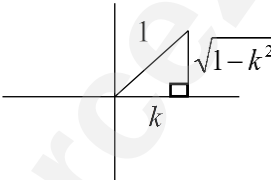
4.1.1	$x^2 + y^2 + 8x + 4y - 38 = 0$ $x^2 + 8x + 16 + y^2 + 4y + 4 = 38 + 16 + 4$ $(x + 4)^2 + (y + 2)^2 = 58$ B(-4 ; -2)	✓ $(x + 4)^2 + (y + 2)^2$ ✓ $r^2 = 58$ ✓ x_B ✓ y_B (4)
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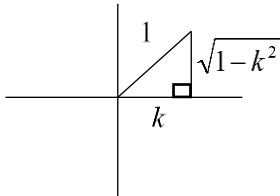
	OR $x_B = \frac{8}{-2} = -4$ $y_B = \frac{4}{-2} = -2$ B(-4; -2)	
4.1.2	radius = $\sqrt{58}$ OR $r = \sqrt{a^2 + b^2 - c}$ $r = \sqrt{16 + 4 + 38}$ $r = \sqrt{58}$	✓ radius (1)
4.2.1	Centre of second circle is (4 ; 6) Distance between of AB: $\sqrt{(4+4)^2 + (6+2)^2}$ $= \sqrt{128} \text{ OR } 11.31$	✓ centre ✓ substitution ✓ answer (3)
4.2.2	Sum of radii = $\sqrt{58} + \sqrt{26} = 12,71$ Distance between centres is 11,31. sum of the radii > distance between the centres ∴ the circles must overlap and hence the circles must intersect.	✓✓ sum of radii ✓ conclusion (3)
4.3	AB ⊥ CD (line from centre ⊥ chord) $m_{AB} = \frac{6+2}{4+4}$ $= \frac{8}{8}$ $= 1$ ∴ $m_{CD} = -1$ OR	✓ S/R ✓ subst ✓ answer (3) ✓ S/R ✓ subst

	$AB \perp CD$ (line from centre $\perp$ chord) $m_{AB} = \frac{-2 - 6}{-4 - 4}$ $= \frac{-8}{-8}$ $= 1$ $\therefore m_{CD} = -1$	✓ answer (3)
		[14]

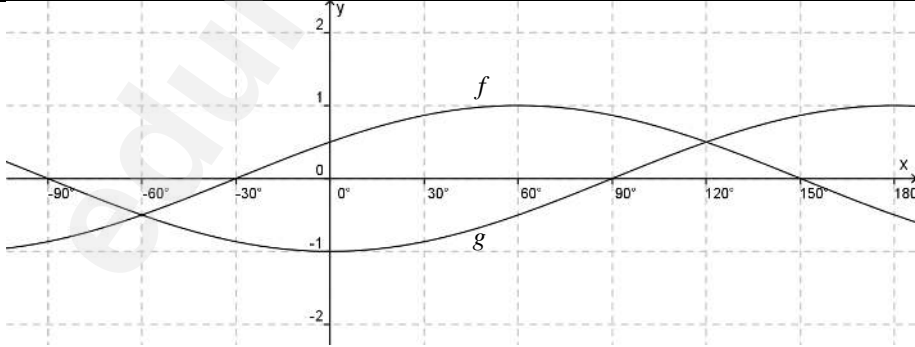
Question 5

5.1.1	$0,76604 \approx 0,77$	✓✓ answer (2)
5.1.2	$\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$ $= \frac{1 - \frac{\sin^2 \theta}{\cos^2 \theta}}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}} \quad \text{OR} \quad \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + \sin^2 \theta}$ $= \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + \sin^2 \theta} \times \frac{\cos^2 \theta}{\cos^2 \theta}$ $= \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + \sin^2 \theta} \times \frac{\cos^2 \theta}{\cos^2 \theta}$ $= \cos^2 \theta - \sin^2 \theta$ $= 2\cos^2 \theta - 1$ $= \cos 2\theta$	$\checkmark \frac{\sin^2 \theta}{\cos^2 \theta}$ $\checkmark \text{ simplify}$ $\checkmark \cos^2 \theta + \sin^2 \theta = 1$ $\checkmark \cos^2 \theta - \sin^2 \theta$ $\checkmark 2\cos^2 \theta - 1$ (5)
5.1.3	$\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \frac{1}{2}$ $\therefore \cos 2\theta = \frac{1}{2}$ $\therefore \text{ref } \angle = 60^\circ$ $2\theta = 60^\circ + 360.k; \quad k \in \mathbb{Z} \quad \text{or} \quad 2\theta = 300^\circ + 360k; \quad k \in \mathbb{Z}$ $\theta = 30^\circ + 180k; \quad k \in \mathbb{Z} \quad \theta = 150^\circ + 180k, \quad k \in \mathbb{Z}$	$\checkmark \cos 2\theta = \frac{1}{2}$ $\checkmark 60^\circ$ $\checkmark 300^\circ$ $\checkmark 30^\circ \text{ and } 150^\circ$ $\checkmark +180k; \quad k \in \mathbb{Z}$ (5)

	OR $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \frac{1}{2}$ $\therefore \cos 2\theta = \frac{1}{2}$ $\therefore \text{ref } \angle = 60^\circ$ $2\theta = \pm 60^\circ + 360k; k \in \mathbb{Z}$ $\theta = \pm 30^\circ + 180k; k \in \mathbb{Z}$	✓ $\cos 2\theta = \frac{1}{2}$ ✓ 60° ✓ -60° ✓ 30° and -30° ✓ $+180k; k \in \mathbb{Z}$ (5)
5.2.1	$\sin 245^\circ$ $= \sin(180^\circ + 65^\circ)$ $= -\sin 65^\circ$ $= -\cos 25^\circ$ $= -k$	✓ $-\sin 65^\circ$ ✓ $-\cos 25^\circ$ ✓ $-k$ (3)
5.2.2	$\sin 25^\circ$ $= \sqrt{1 - k^2}$  OR $\sin^2 25^\circ + \cos^2 25^\circ = 1$ $\sin^2 25^\circ = 1 - k^2$ $\sin 25^\circ = \sqrt{1 - k^2}$	✓ sketch ✓ answer (2)
5.2.3	$\cos 50^\circ$ $= 2\cos^2 25^\circ - 1$ $= 2k^2 - 1$ OR $\cos^2 25^\circ - \sin^2 25^\circ$ $= (k)^2 - (\sqrt{1 - k^2})^2$ $= k^2 - 1 + k^2$ $= 2k^2 - 1$ OR $1 - \sin^2 25^\circ$	✓ identity ✓ answer (2) ✓ identity ✓ answer (2) ✓ identity ✓ answer (2)

	$1 - 2(\sqrt{1 - k^2})^2$ $= 1 - 2(1 - k^2)$ $= 1 - 2 + 2k^2$ $= 2k^2 - 1$	
5.2.3	$\sin 25^\circ$ $= \sqrt{1 - k^2}$ 	✓ Sketch ✓ answer (2)
5.3.1	$\sqrt{3} \cos \beta + \sin \beta$ $= 2 \cdot \frac{\sqrt{3}}{2} \cdot \cos \beta + \frac{2}{2} \cdot \sin \beta$ $= 2 \left(\frac{\sqrt{3}}{2} \cdot \cos \beta + \frac{1}{2} \cdot \sin \beta \right)$ $= 2(\sin 60^\circ \cdot \cos \beta + \cos 60^\circ \cdot \sin \beta)$ $= 2 \sin(60^\circ + \beta)$	✓ $\times \frac{2}{2}$ ✓ $2 \left(\frac{\sqrt{3}}{2} \cdot \cos \beta + \frac{1}{2} \cdot \sin \beta \right)$ ✓ $\sin 60^\circ$ ✓ $\cos 60^\circ$ ✓ $2 \sin(60^\circ + \beta)$ (5)
5.3.2	Max - value = $2 - 5$ $= -3$	✓ ✓ answer (2)
		[26]

Question 6

6.1		$f(x)$ ✓ shape ✓ x-intercepts ✓ y-intercepts ✓ turning point (4)
6.2	$g(2x) = -\cos 2x$ $\therefore \text{period} = 180^\circ$	✓ $g(2x) = -\cos 2x$ ✓ answer (2)

6.3	$0^\circ < x < 60^\circ$ OR $x \in (0^\circ; 60^\circ)$	✓ notation ✓ end points (2)
		[8]

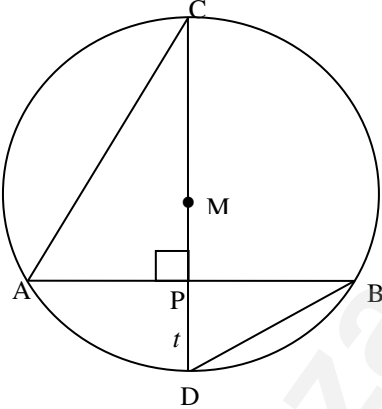
Question 7

7.1	$\frac{18}{PA} = \cos 23^\circ$ $PA = \frac{18}{\cos 23^\circ}$ $PA = 19.55\text{m}$ OR $\frac{AP}{\sin C} = \frac{CP}{\sin A}$ $\frac{AP}{\sin 90^\circ} = \frac{18}{\sin 67^\circ}$ $AP = \frac{18 \sin 90^\circ}{\sin 67^\circ}$ $AP = 19.55\text{m}$	✓ Ratio ✓ answer (2) ✓ Ratio ✓ answer (2)
7.2	$AB^2 = (22.65)^2 + (19.55)^2 - (2)(22.62)(19.55) \cdot \cos 42^\circ$ $AB^2 = 237.084...$ $AB = 15.40\text{m}$	✓ Use of cosine rule ✓ substitution ✓ 237.0847

		✓ answer (4)
		[6]

Question 8

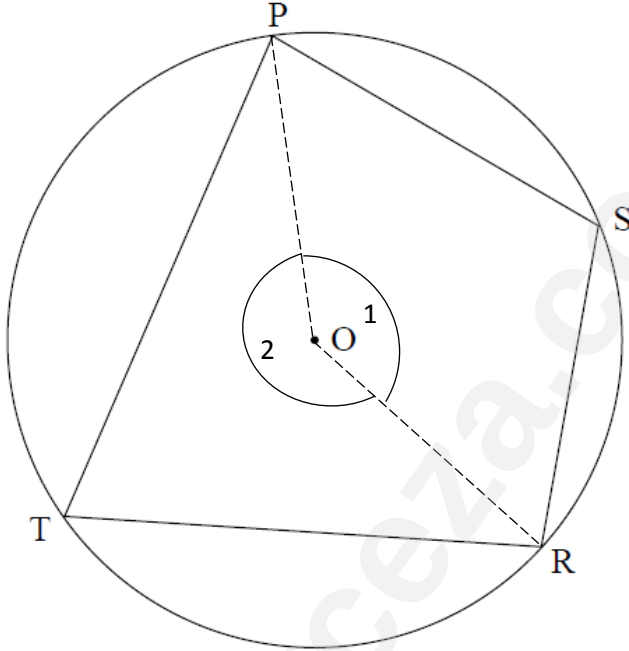
8.1		
8.1.1	$\hat{D}_1 = \hat{B}_1 = 40^\circ$ (tangent - chord theorem) $\therefore \hat{D}_2 = \hat{B}_1 = 40^\circ$ ($\angle$'s opp=sides)	✓S ✓R ✓S/R (3)
8.1.2	$\hat{C} = 180^\circ - (40^\circ + 40^\circ)$ ($\angle$ s in Δ) $= 100^\circ$	✓S ✓S (2)
8.1.3	$\hat{A} = 180^\circ - 100^\circ$ (Opposite angles of a cyclic quad) $= 80^\circ$	✓R ✓S (2)
8.1.4	$\hat{O}_1 = 2\hat{A}$ ($\angle$ at centre = $2 \times \angle$ at circun) $\hat{O}_1 = 160^\circ$ OR	✓R ✓S (2) ✓R

	$\hat{D}_2 = \hat{B}_1 = 40^\circ$ (From 9.1) $\hat{D}_3 = 90^\circ - 40^\circ - 40^\circ$ (tan $\perp$ rad) $\hat{D}_3 = 10^\circ$ $\therefore \hat{O}_1 = 180^\circ - 10^\circ - 10^\circ$ (sum of angles in Δ) $\hat{O}_1 = 160^\circ$	✓S (2)
8.2		
8.2.1	Line from centre $\perp$ to chord	✓R (1)
8.2.2	$\Delta CAP \parallel \Delta BDP$ $\frac{AP}{DP} = \frac{CP}{BP}$ (sides in proportion) $\frac{2t}{t} = \frac{15}{2t}$ $4t^2 = 15t$ $4t^2 - 15t = 0$ $2t(2t - 7.5) = 0$ $\therefore t \neq 0 \quad \text{or} \quad t = \frac{15}{4}$	✓S ✓S ✓S ✓answer (4)
		✓S

	OR $15 + t = 2r$ $\therefore r = \frac{15 + t}{2}$ <i>In ΔMPB:</i> $\left(\frac{15 + t}{2}\right)^2 = (2t)^2 + \left(\frac{15 + t}{2} - t\right)^2$ $\frac{225 + 30t + t^2}{4} = 4t^2 + \left(\frac{15 - t}{2}\right)^2$ $\frac{225 + 30t + t^2}{4} = 4t^2 + \frac{225 - 30t + t^2}{4}$ $225 + 30t + t^2 = 16t^2 + 225 - 30t + t^2$ $16t^2 - 60t = 0$ $4t(4t - 15) = 0$ $t = \frac{15}{4}$ OR $15 + t = 2r$ $t = 2r - 15$ <i>In ΔMPB:</i> $\left(\frac{15 + 2r - 15}{2}\right)^2 = (2(2r - 15))^2 + \left(\frac{15 + 2r - 15}{2} - (2r - 15)\right)^2$ $r^2 = 4(4r^2 - 60r + 225) + (r - 2r + 15)^2$ $r^2 = 16r^2 - 240r + 900 + (-r + 15)^2$ $r^2 = 16r^2 - 240r + 900 + r^2 - 30r + 225$ $0 = 16r^2 - 270r + 1125$ $0 = (2r - 15)(8r - 75)$ $\therefore r \neq \frac{15}{2} \text{ or } r = \frac{75}{8}$ $\therefore r = \frac{15}{4}$	✓S ✓S ✓answer (4) ✓S ✓S ✓S ✓answer (4)
8.2.2 (b)	$\text{Radius} = \frac{15 + \frac{15}{4}}{2}$ $= 9.375 \text{ OR } 9\frac{3}{8}$	✓method ✓answer (2)

		[16]
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Question 9

9.1	 Join PO and OR Let $\hat{O}_1 = 2x$ $\hat{T} = x$ ($\angle$ at circ centre = $2 \angle$ at circumference) $\hat{O}_2 = 360^\circ - 2x$ ($\angle$s round a point) $\hat{S} = 180 - x$ ($\angle$ at circ centre = $2 \angle$ at circumference) $\hat{T} + \hat{S} = 180^\circ - x + x$ $\hat{T} + \hat{S} = 180^\circ$ $\therefore \hat{PTR} + \hat{PSR} = 180^\circ$	✓ construction ✓ S ✓ R ✓ S ✓ S ✓ S (5)
9.2		

9.2.1	$\hat{R}_1 = x$ ($\angle$'s opp= radii) $\hat{O}_1 = 180^\circ - 2x$ ($\angle$ sum in ΔQRT) $\hat{P}_1 = 90^\circ - x$ ($\angle$ circle centre = twice at circum)	$\checkmark$ S /R $\checkmark$ S $\checkmark$ S $\checkmark$ R (4)
9.2.2	$PQ = QR$ (given) $\hat{QRP} = 90^\circ - x$ ($\angle$ opp=sides in Δ) $\hat{PQR} = 2x$ ($\angle$ sum in ΔPQR) $x + \hat{Q}_2 = 2x$ $\hat{Q}_2 = x$ TQ bisects $\hat{PQR}$	$\checkmark$ S $\checkmark$ S $\checkmark$ S (3)
9.2.3	$\hat{PQR} = 2x$ $\hat{S} = 180^\circ - 2x$ (opp $\angle$'s of cyclic quad are suppl) $\hat{O}_1 = 180^\circ - 2x$ (FROM 10.2.1) $STOR$ is a cyclic quad (converse – of ext $\angle$ of cyclic quad or ext $\angle$ of quad = int opp $\angle$) OR $\hat{PQR} = 2x$ $\hat{S} = 180^\circ - 2x$ (opp $\angle$'s of cyclic quad are suppl) $\hat{O}_1 = 180^\circ - 2x$ (from 10.2.1) $\hat{O}_2 = 2x$ ($\angle$ s on str. line) $\hat{S} + \hat{O}_2 = 180^\circ - 2x + 2x$ $\therefore \hat{S} + \hat{O}_2 = 180^\circ$ $STOR$ is a cyclic quad (convse : opp $\angle$'s are supplementary)	$\checkmark$ S $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R (5) $\checkmark$ S $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R (5)
		[17]

Question 10

10.1	$\hat{Q}_3 = \hat{R}_1 = x$ (ext angle of cyclic quad ...) and (RA bisects $\hat{R}$) $\hat{R}_2 = \hat{Q}_2 = x$ (angles in the same segment) $\hat{R}_1 = \hat{R}_2$ $\therefore \hat{Q}_2 = \hat{Q}_3$ OR $\hat{Q}_2 + \hat{Q}_3 = \hat{R}_1 + \hat{R}_2$ (ext $\angle$ of cyclic quad) But $\hat{Q}_2 = \hat{R}_2$ ($\angle$ s in same segment) $\hat{R}_1 = \hat{R}_2$ $\therefore \hat{Q}_3 = \hat{Q}_2$	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ S (5)
10.2	$\hat{Q}_3 = \hat{B} = x$ ($\angle$ s opp=sides, AQ=AB) $\hat{Q}_3 = \hat{R}_1 = \hat{B} = x$ (from 1.1) $\therefore TR = TB$ (sides opp = $\angle$ s)	$\checkmark$ S/R $\checkmark$ R (2)
10.3	$\hat{P} = \hat{A}_1$ ($\angle$ s in same segment) $\hat{A}_1 = \hat{Q}_3 + \hat{B}$ (ext $\angle$ of $\triangle ABC$) $\hat{Q}_3 + \hat{B} = 2\hat{Q}_3$ ($\hat{Q}_3 = \hat{B}$ $\angle$ s opp = sides) $2\hat{Q}_3 = 2\hat{R}_1$ (from 11.2) $\therefore 2\hat{R}_1 = \hat{PRT}$ $\therefore \hat{P} = \hat{TRP}$	$\checkmark$ S $\checkmark$ R $\checkmark$ S/R (3)

	OR $\hat{TRP} = 2x$ $\hat{A}_1 = \hat{Q}_3 + B = 2x$ (ext $\angle$ of $\triangle ABC$) $\hat{P} = \hat{A}_1 = 2x$ ($\angle$s in same segment) $= \hat{TRP}$	✓ S ✓ R ✓ S/R (3)
		[10]

Question 11

11.1	In $\triangle BPE$ and $\triangle BDA$ 1. $\hat{B}_1 = \hat{B}_1$ common 2. $\hat{P}_2 = \hat{D} = 90^\circ$ $\angle$ in semi-circle 3. $\hat{B}_2\hat{A}D = \hat{E}_3$ $\angle$ in Δ^s $\therefore \triangle BPE \parallel \triangle BDA$ (equiangular OR $\angle\angle\angle$)	✓ S ✓ S ✓ R ✓ R (4)
11.2	$\frac{BP}{BD} = \frac{BE}{AB}$ From Q11.1 $AB = \frac{BD \cdot BE}{BP}$ $AB^2 = \frac{BD^2 \cdot BE^2}{BP^2}$ In $\triangle BPE$; $BE^2 = BP^2 + PE^2$ (pyth) $AB^2 = \frac{BD^2 \cdot (BP^2 + PE^2)}{BP^2}$ $AB^2 = \frac{BD^2 \cdot BP^2}{BP^2} + \frac{BD^2 \cdot PE^2}{BP^2}$ $AB^2 = BD^2 + \frac{BD^2 \cdot PE^2}{BP^2}$	✓ S/ratio ✓ S ✓ S ✓ $BE^2 = BP^2 + PE^2$ ✓ substitution ✓ simplification (6)
		[10]
	TOTAL 150	