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DEPARTMENT: EDUCATION
MPUMALANGA PROVINCE

NATIONAL SENIOR CERTIFICATE MEMORANDUM

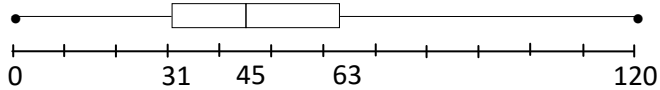
MATHEMATICS MEMORANDUM P2

SEPTEMBER 2016

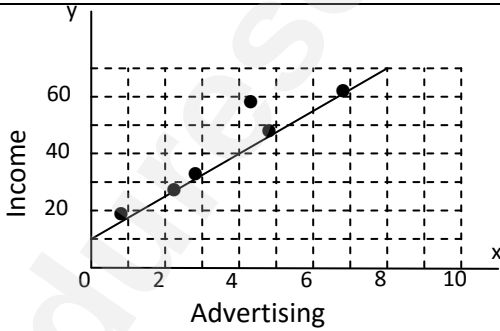
GRADE 12

This memo consists of 15 pages

QUESTION 1

1.1.1	3700	✓ answer (1)
1.1.2	700	✓✓ answer (2)
1.1.3	45 minutes	✓ answer (1)
1.1.4		✓ min / max ✓ $Q_1 + Q_3$ ✓ median (3)
1.2.1	$40 \leq p \leq 54$	✓ values ✓ notation (2)
1.2.2	$\frac{32 + 40 + p + 56 + 83}{5}$ $209 + p = 300$ $p = 91$	✓ equation ✓ answer (2)
		[11]

QUESTION 2

2.1		✓ 2 points plotted correctly ✓ 4 points plotted correctly ✓ 6 points plotted correctly (3)
2.2	$y = 7,69x + 10,49$	✓ $7,69x$ ✓ $10,49$ (2)
2.3	$r = 0,95$	✓ answer (1)
2.4	It is a very strong positive correlation.	✓ answer (1)
2.5	$y = 7,69(3,5) + 10,49 = 37,405$ Income = R 37 405	✓ substitution ✓ answer (2)
		[9]

QUESTION 3

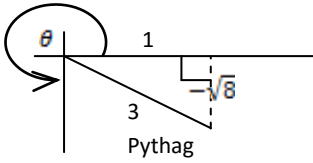
3.1.1	$\frac{x+2}{2} = 5$ $x+2=10$ $x=8$ $\therefore B(8;5)$	$\frac{3+y}{2} = 4$ $3+y=8$ $y=5$	✓ $x=8$ ✓ $y=5$ (2)
3.1.2	$m_{AM} = \frac{4-3}{5-2}$ $= \frac{1}{3}$ $\perp m = -3$ $y-4 = -3(x-5)$ $y-4 = -3x+15$ $y = -3x+19$		✓ gradient of AM ✓ perp gradient ✓ correct sub into equation ✓ answer (4)
3.1.3	$0 = -3x+19$ $x = \frac{19}{3}$ $C\left(\frac{19}{3}; 0\right)$ $m_{AC} = \frac{3-0}{2-\frac{19}{3}}$ $= \frac{-9}{13}$ $\tan \hat{ACX} = \frac{-9}{13}$ $\hat{ACX} = 180^\circ - 34,7^\circ$ $= 145,3^\circ$ $\alpha = 34,7^\circ$		✓ $y=0$ ✓ coordinates of C ✓ gradient of AC ✓ $145,3^\circ$ ✓ answer (5)

3.1.4	BC is a diameter converse of $\angle$ in a semi circle	✓S ✓R (2)
3.2.1	$D\left(\frac{-4+8}{2}; \frac{10+2}{2}\right)$ $D(2;6)$ $AD = \sqrt{(7-2)^2 + (11-6)^2}$ $= \sqrt{25+25}$ $= \sqrt{50}$ $= 5\sqrt{2}$	✓✓ coordinates of D ✓ sub in distance formula ✓ answer (4)
3.2.2	$AD = \frac{1}{2} DE$ $D(2;6)$ $E(x;-4)$ $5\sqrt{2} = \frac{1}{2} \sqrt{(x-2)^2 + (-4-6)^2}$ $10\sqrt{2} = \sqrt{x^2 - 4x + 4 + 100}$ $(10\sqrt{2})^2 = x^2 - 4x + 104$ $0 = x^2 - 4x - 96$ $0 = (x-12)(x+8)$ $x = 12$ or $x = -8$ NA	✓ sub in distance formula ✓ simplification ✓ standard form ✓ factors ✓ $x = -8$ only (5)
3.3	$m_{AB} = m_{AC}$ $\frac{y-4}{-3+7} = \frac{2-4}{5+7}$ $12(y-4) = -8$ $12y = 40$ $y = \frac{10}{3}$	✓ equating gradients ✓ simplification ✓ answer (3)
		[25]

QUESTION 4

4.1	$x^2 + 2x + 1 + y^2 - 4y + 4 = 44 + 1 + 4$ $(x+1)^2 + (y-2)^2 = 49$ $\therefore M(-1;2)$	✓ ✓ equation of circle ✓ answer (3)
4.2	Line AK : $y = -x + 5$ let $A(a;b)$ then: $A(a; -a + 5)$ $m_{AM} = 1$ $\frac{-a + 5 - 2}{a - (-1)} = 1$ $-a + 3 = a + 1$ $a = 1$ $b = 4$ $\therefore A(1;4)$	✓ standard form ✓ coordinates of A ✓ substitution (3)
4.3	$r^2 = AM^2 = (1+1)^2 + (4-2)^2 = 8$ $(x+1)^2 + (y-2)^2 = 8$	✓ sub and ✓ r value ✓ equation (3)
4.4	Distance = radius of bigger circle – radius of smaller circle $= 7 - \sqrt{8}$ $= 7 - 2\sqrt{2}$ or 4,17	✓ method ✓ answer (2)
4.5	$K(5;0)$ $AK = \sqrt{(5-1)^2 + (4-0)^2}$ $= \sqrt{32}$ $\text{Area } \triangle AMK = \frac{1}{2} AM \cdot AK$ $= \frac{1}{2} \sqrt{8} \sqrt{32}$ $= 8$	✓ $x = 5$ ✓ $y = 0$ ✓ length of AK ✓ sub in area formula ✓ answer (5)
		[16]

QUESTION 5

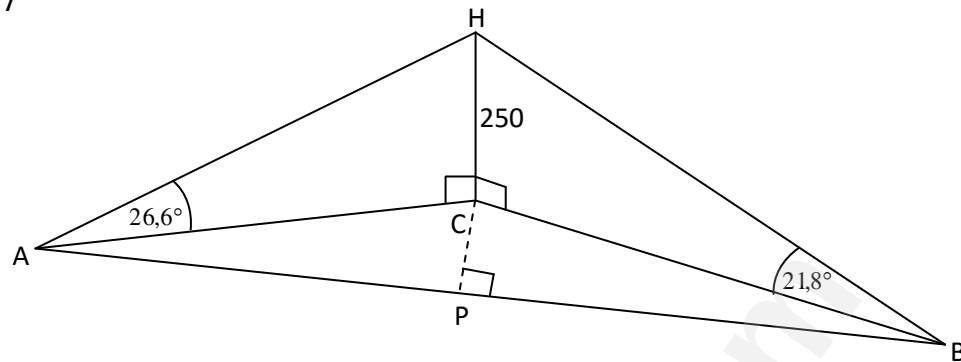
5.1	$\sqrt{8} \sin \theta + \cos \theta$ $= (\sqrt{8}) \left(-\frac{\sqrt{8}}{3} \right) + \frac{1}{3}$ $= -\frac{7}{3}$ 	✓ correct sketch in correct quadrant ✓ $r = 3$ ✓ $\frac{1}{3}$ ✓ $-\frac{\sqrt{8}}{3}$ ✓ answer (5)
5.2	$\frac{(-\cos 45^\circ)^2 \cdot (\tan x)(-\sin x)}{-\sin x}$ $= \left(-\frac{1}{\sqrt{2}} \right)^2 \cdot \tan x$ $= \frac{1}{2} \tan x$	✓ ✓ ✓ ✓ reduction formulae ✓ value of special angle ✓ answer (6)
		[11]

QUESTION 6

6.1.1	$2 \cos \theta = \sin(\theta + 30^\circ)$ $= \sin \theta \cos 30^\circ + \cos \theta \sin 30^\circ$ $= \frac{\sqrt{3}}{2} \sin \theta + \frac{1}{2} \cos \theta$ $\therefore 4 \cos \theta = \sqrt{3} \sin \theta + \cos \theta$ $\therefore 3 \cos \theta = \sqrt{3} \sin \theta$	✓ expansion ✓ substitution of special angles ✓ $4 \cos \theta = \sqrt{3} \sin \theta + \cos \theta$ (3)
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6.1.2	$3 \cos \theta = \sqrt{3} \sin \theta$ $\frac{\sin \theta}{\cos \theta} = \frac{3}{\sqrt{3}}$ $\tan \theta = \frac{3}{\sqrt{3}}$ $\theta = 60^\circ + 180.n$ $\theta = -120^\circ \text{ or } \theta = 60^\circ$	✓ division ✓ $\tan \theta = \frac{3}{\sqrt{3}}$ ✓✓ answers (4)
6.2.1		g : ✓ y-intercept and x- intercepts ✓ shape f : ✓ y-intercept and x-intercepts ✓ turning points ✓ shape (5)
6.2.2	$-30^\circ < \theta < 90^\circ$	✓✓ values ✓ notation (3)
6.2.3	$-120^\circ < \theta < 60^\circ$	✓✓ values ✓ notation (3)
		[18]

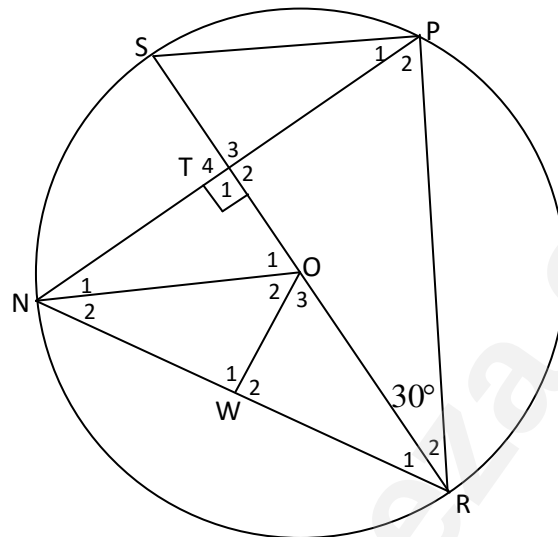
QUESTION 7



7.1	In $\triangle AHC$; $\hat{AHC} = 63,4^\circ$ $\frac{AC}{\sin 63,4^\circ} = \frac{250}{\sin 26,6^\circ}$ $AC = 624,43$ In $\triangle CHB$; $\hat{CHB} = 68,2^\circ$ $\frac{BC}{\sin 68,2^\circ} = \frac{250}{\sin 21,8^\circ}$ $BC = 625,04$ In $\triangle ABC$ $AB^2 = (624,43)^2 + (625,04)^2 - 2(624,43)(625,04)\cos 104,5^\circ$ $= 9760313165$ $AB = 987,94$	✓ method ✓ value of AC ✓ method ✓ value of BC ✓ sub in cos-rule ✓ answer (6)
7.2	In $\triangle ABC$: $\frac{\sin A}{625,04} = \frac{\sin 104,5^\circ}{987,94}$ $\sin A = 0,6317$ $\hat{A} = 39,2^\circ$	✓ method ✓ $\sin A = 0,6317$ ✓ answer (3)
		[9]

QUESTION 8

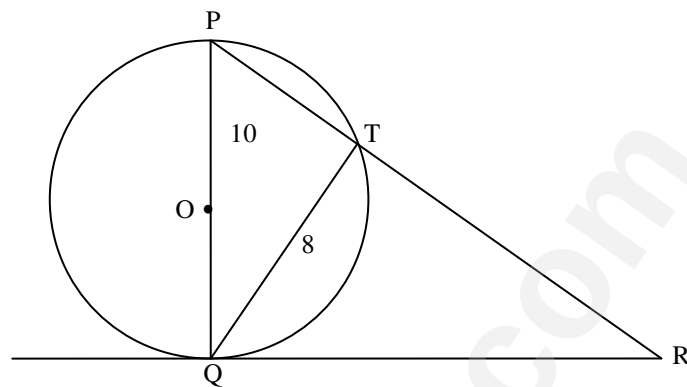
8.1



8.1.1	$\hat{P}_1 + \hat{P}_2 = 90^\circ$ $\hat{S} + \hat{P}_1 + \hat{P}_2 + \hat{R}_2 = 180^\circ$ $\hat{S} + 90^\circ + 30^\circ = 180^\circ$ $\therefore \hat{S} = 60^\circ$	$\angle$ in semi circle $\angle$'s in Δ	$\checkmark$ S $\checkmark$ R $\checkmark$ answer (3)
8.1.2	$\hat{S} = \hat{N}_1 + \hat{N}_2$ $\hat{N}_1 + \hat{N}_2 = 60^\circ$ $\hat{N}_1 + \hat{N}_2 + \hat{T}_1 + \hat{R}_1 = 180^\circ$ $60^\circ + 90^\circ + \hat{R}_1 = 180^\circ$ $\therefore \hat{R}_1 = 30^\circ$ OR $\hat{T}_4 + \hat{P}_1 + \hat{S} = 180^\circ$ $90^\circ + \hat{P}_1 + 60^\circ = 180^\circ$ $\hat{P}_1 = 30^\circ$ $\hat{P}_1 = \hat{R}_1$ $\therefore \hat{R}_1 = 30^\circ$	$\text{PR subt} = \angle$'s $\angle$'s in Δ $\angle$'s in Δ $\text{NR subtends} = \angle$'s	$\checkmark$ S $\checkmark$ R $\checkmark$ answer (3) $\checkmark \hat{P}_1 = 30^\circ$ $\checkmark$ S/ R $\checkmark$ answer (3)

8.1.3	$\hat{O}_1 = 2\hat{R}_1$ $\therefore \hat{O}_1 = 2(30^\circ) = 60^\circ$ $\hat{N}_1 + \hat{T}_1 + \hat{O}_1 = 180^\circ$ $\hat{N}_1 + 90^\circ + 60^\circ = 180^\circ$ $\hat{N}_1 = 30^\circ$ OR $\hat{O}_1 = 2(30^\circ) = 60^\circ$ $N\hat{O}_1R = 120^\circ$ $\hat{N}_2 = \hat{R}_1$ $N\hat{O}_1R + 2\hat{N}_2 = 180^\circ$ $120^\circ + 2\hat{N}_2 = 180^\circ$ $2\hat{N}_2 = 180^\circ - 120^\circ$ $\hat{N}_2 = 30^\circ$ But $\hat{N}_1 + \hat{N}_2 = 60^\circ$ $\therefore \hat{N}_1 = 30^\circ$	$\checkmark$ S $\checkmark$ R $\checkmark$ answer (3)
8.1.4	$NW = WR$ $\therefore \hat{W}_1 = 90^\circ$ $\hat{T}_1 = 90^\circ$ $\therefore \hat{W}_1 + \hat{T}_1 = 180^\circ$ $\therefore TNWO$ is a cyclic quad	given line from centre, midpoint chord $\checkmark$ S $\checkmark$ S/R $\checkmark$ S $\checkmark$ R (4)

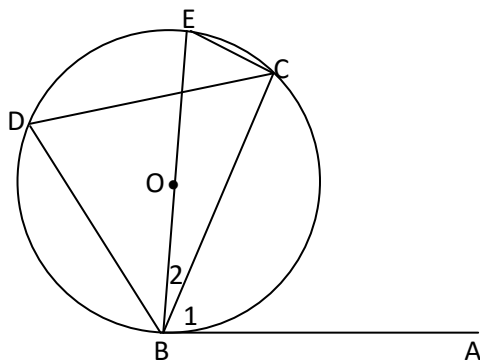
8.2



8.2.1	$\hat{P}TQ = 90^\circ$ $\angle$ in semi-circle $PQ^2 = QT^2 + PT^2$ Pythagoras theorem $10^2 = 8^2 + PT^2$ $PT = 6$	✓ S ✓ R ✓ S (3)
8.2.2	$\hat{P}TQ = 90^\circ$ rad $\perp$ tangent $PR^2 = PQ^2 + QR^2$ Pythagoras theorem $(x+6)^2 = 10^2 + QR^2$ $x^2 + 12x - 64 = QR^2$ $\sqrt{x^2 + 12x - 64} = QR$	✓ S/R ✓ substitution (2)
8.2.3	$QR^2 = QT^2 + TR^2$ Pythagoras theorem $QR^2 = 8^2 + x^2$ $(x+6)^2 - 10^2 = 8^2 + x^2$ $x^2 + 12x - 64 = 64 + x^2$ $12x = 128$ $x = \frac{32}{3}$	✓ S ✓ substituting QR^2 ✓ answer (3)
		[21]

QUESTION 9

9.1



Draw diameter OBE and EC

constr.

$$\hat{B}_1 + \hat{B}_2 = 90^\circ$$

radius $\perp$ tangent

$$\hat{C} = 90^\circ$$

$\angle$ in semi circle

$$\hat{E} + \hat{B}_2 = 90^\circ$$

$\angle$ s in Δ

$$\hat{E} + \hat{B}_2 = 90^\circ$$

$$\therefore \hat{B}_1 = \hat{E}$$

$$\text{but } \hat{D} = \hat{E}$$

subt BD

$$\therefore \hat{B}_1 = \hat{D}$$

✓ construction

✓ S/R

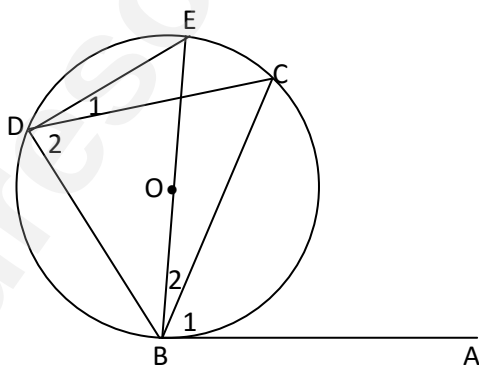
✓ S/R

✓ S

✓ S/R

(5)

OR



Draw diameter BOE and ED

constr.

$$\hat{B}_1 + \hat{B}_2 = 90^\circ$$

radius $\perp$ tangent

$$\hat{D}_1 + \hat{D}_2 = 90^\circ$$

$\angle$ in semi circle

$$\text{But } \hat{D}_1 = \hat{B}_2$$

EC subtends $= \angle$ s

$$\text{But } \hat{D} = \hat{E}$$

$$\therefore \hat{B}_2 = \hat{D}_2 \text{ or } \hat{ABC} = \hat{D}$$

✓ construction

✓ S/R

✓ S/R

✓ S/R

✓ S

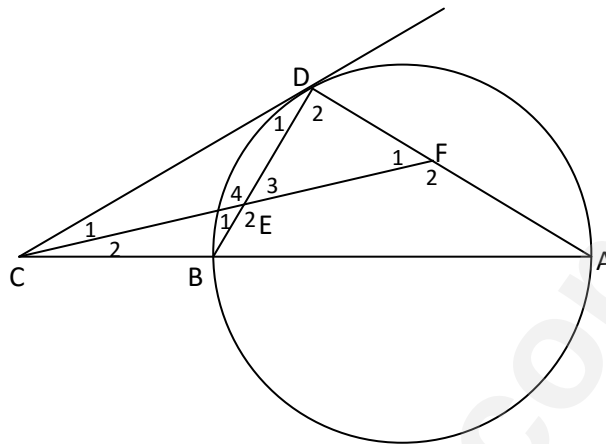
(5)

9.2

$$AB = 6$$

$$CD = 4$$

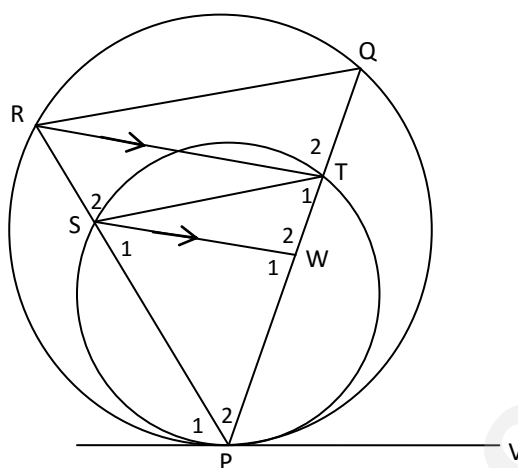
$$\hat{D}_2 = 90^\circ$$



9.2.1	In $\triangle DBC$ and $\triangle ADC$, $\hat{D}_1 = \hat{A}$ tan-chord $\hat{DCA} = \hat{CA}$ common $\therefore \triangle DBC \sim \triangle ADC$ $\angle, \angle, \angle$	✓ S/R ✓ S ✓ R (3)
9.2.2	$\frac{DC}{BC} = \frac{AC}{DC}$ $\triangle DBC \sim \triangle ADC$ $DC^2 = BC \cdot AC$ $4^2 = BC(AB + BC)$ $= BC(6 + BC)$ $16 = 6BC + BC^2$ $BC^2 + 6BC - 16 = 0$ $(BC + 8)(BC - 2) = 0$ $BC = -8$ or $BC = 2$ $\therefore BC = 2$	✓ correct ratio ✓ simplification ✓ substitution ✓ std form ✓ factors ✓ $BC = 2$ only (6)
9.2.3	$\triangle DEC \sim \triangle AFC$ given $\frac{DC}{EC} = \frac{AC}{FC}$ $\frac{4}{EC} = \frac{8}{FC}$ $\therefore EC = \frac{1}{2} FC$ $\therefore EC = EF$	✓ S ✓ correct ratio ✓ $AC = 8$ ✓ simplification (4)

9.2.4	$\frac{\Delta CDF}{\Delta CFA} = \frac{1}{2}$ $FA = 2FD \quad \text{given}$ $2\Delta CDF = \Delta CFA$ but $\Delta CDF + \Delta CFA = \Delta ADC$ $\therefore \Delta CDF + 2\Delta CDF = \Delta ADC$ $\therefore 3\Delta CDF = \Delta ADC$ and $\Delta CDE = \frac{1}{2} \Delta CDF$ $\frac{\Delta CDE}{\Delta ACD} = \frac{\frac{1}{2} \Delta CDF}{3\Delta CDF}$ $= \frac{1}{6}$	✓ S ✓ R ✓ S ✓ S ✓ answer (5)
		[23]

QUESTION 10



10.1	$\hat{P}_1 = \hat{T}_1$ tan chord theorem $\hat{P}_1 = \hat{Q}$ tan chord theorem $\hat{T}_1 = \hat{Q}$ $\therefore RS \parallel ST$ corresp $\angle$ s are equal	✓ S ✓ R ✓ S ✓ R (4)
10.2	$\frac{PS}{SR} = \frac{PW}{WT}$ lines $\parallel$ one side of Δ $\frac{PS}{SR} = \frac{PT}{TQ}$ $\therefore \frac{PW}{WT} = \frac{PT}{TQ}$	✓ S ✓ R ✓ S (3)
		[7]

TOTAL: 150