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NATIONAL SENIOR CERTIFICATE EXAMINATION
SUPPLEMENTARY EXAMINATION – MARCH 2017

**MATHEMATICS: PAPER I
MARKING GUIDELINES**

Time: 3 hours

150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

- (a) (1) $4 - \sqrt{2-x} = 3x$
 $-\sqrt{2-x} = 3x - 4$
 $2 - x = 9x^2 - 24x + 16$
 $9x^2 - 23x + 14 = 0$
 $(9x - 14)(x - 1) = 0$
 $x = \frac{14}{9} \text{ or } x = 1$
 Check: $x = \frac{14}{9}$: $4 - \sqrt{2-x} = \frac{10}{3}$
 $3x = \frac{14}{3}$
 $\therefore x = \frac{14}{9}$ is not a solution (6)
- (2) $2(5)^{9-x} = 1\,250$
 $5^{9-x} = 625$
 $5^{9-x} = 5^4$
 $9 - x = 4$
 $x = 5$ (3)
- (b) (1) For A and B: Let $y = 0$
 A $(-1; 0)$ B $(3; 0)$
 For C: let $x = 0$
 $y = 3(1)(-3) = -9$
 $\therefore C(0; -9)$
 For D: $x_D = \frac{-1+3}{2} = 1$ for writing as coordinates
 $\therefore y = 3(1+1)(1-3)$
 $\therefore y = -12$
 $\therefore D(1; -12)$ (6)
- (2) $x \leq -1$ or $x \geq 3$ (2)

(c) $x = 6$ or $x = 3$
 $(x-6)(x-3) = 0$
 $x^2 - 9x + 18 = 0$
 $t = -9$

OR

$x = -6$ or $x = -3$
 $(x+6)(x+3) = 0$
 $x^2 + 9x + 18 = 0$
 $t = 9$

Alternate: Product of roots = 18
 Numbers are 6 and 3 or -6 and -3
 Sum of roots = -t
 $\therefore -t = 6 + 3 = 9$ OR $-t = -6 - 3 = -9$
 $\therefore t = 9$ or $t = -9$

Alternate: Let the smaller root be a.
 The other root is $a + 3$.
 The equation is:

$$(x-a)[x-(a+3)] = 0$$

$$x^2 - (a+a+3)x + a(a+3) = 0$$

$$\therefore t = -2a - 3$$

$$a(a+3) = 18$$

$$a^2 + 3a - 18 = 0$$

$$(a+6)(a-3) = 0$$

$$a = -6 \text{ OR } a = 3$$

$$\therefore t = 9 \text{ or } t = -9$$

(5)

(d) $y = 2 - 3x$
 $(2-3x)^2 = 2x^2 - 1$
 $4 - 12x + 9x^2 - 2x^2 + 1 = 0$
 $7x^2 - 12x + 5 = 0$
 $(7x-5)(x-1) = 0$

$x = 1$ or $x = \frac{5}{7}$
 $\therefore y = -1$ or $y = -\frac{1}{7}$

(7)

[29]

QUESTION 2

(a)
$$P = x \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

$$2\,500\,000 = x \left[\frac{1 - \left(1 + \frac{6}{400}\right)^{-(5 \times 4)}}{\frac{6}{400}} \right]$$

$$x \approx \text{R}145\,614,34 \quad (4)$$

(b)
$$F = x \left[\frac{(1+i)^n - 1}{i} \right]$$

$$F = 145\,614,34 \left[\frac{\left(1 + \frac{6}{400}\right)^{(4 \times 4)} - 1}{\frac{6}{400}} \right]$$

$$F = 2\,611\,210,19$$

NB: Discrepancies in rounding

$$A = P \left(1 + \frac{r}{100t} \right)^{nt}$$

$$A = 2\,500\,000 \left(1 + \frac{6}{400} \right)^{16}$$

$$A = 3\,172\,463,87$$

$$\text{Balance of loan} = \text{R}561\,253,68$$

ALTERNATE:

$$\text{Balance outstanding} = 145\,614,34 \left[\frac{1 - \left(1 + \frac{6}{400}\right)^{-4}}{\frac{6}{400}} \right] = \text{R}561\,253,67 \quad (5)$$

- (c) In four years, the farmer paid: $145\,614,34 \times 4 \times 4 = \text{R}2\,329\,829,44$
 Balance of loan after four years = $\text{R}561\,253,68$
 Paid towards original loan: $2\,500\,000 - 561\,253,68 = \text{R}1\,938\,746,32$
 Interest charges were: $2\,329\,829,44 - 1\,938\,746,32 = \text{R}391\,083,12$ (3)

[12]

QUESTION 3

(a) (1) $r = \frac{1}{3}$

$$45 + 15 + 5 + \dots = \sum_{n=1}^{\infty} 45 \left(\frac{1}{3} \right)^{n-1} \quad (3)$$

(2) $-1 < \frac{1}{3} < 1$ (1)

(3) $S_{\infty} = \frac{45}{1 - \frac{1}{3}} = 67,5$ (2)

(b) (1) $S_n = \frac{n}{2}(7n + 19)$

$$4\,878 = \frac{n}{2}(7n + 19)$$

$$9\,756 = 7n^2 + 19n$$

$$0 = 7n^2 + 19n - 9\,756$$

$$n = \frac{-19 \pm \sqrt{19^2 - 4(7)(-9756)}}{14}$$

$$n = \frac{-19 \pm \sqrt{273\,529}}{14}$$

$$\therefore n = 36 \quad (5)$$

(2) $T_7 = S_7 - S_6$

$$S_6 = \frac{6}{2}(7(6) + 19)$$

$$S_6 = 183$$

$$S_7 = \frac{7}{2}(7(7) + 19)$$

$$S_7 = 238$$

$$T_7 = 55$$

Alternate

$$a = T_1$$

$$S_1 = 13 \quad S_2 = 33$$

$$\therefore T_2 = 20 \quad d = 7$$

$$\therefore T_7 = 13 + 6(7) = 55$$

Alternate

$$d = 2\left(\frac{7}{2}\right) = 7$$

$$a = S_1 = \frac{1}{2}(7 + 19) = 13$$

$$T_7 = a + (n-1)d$$

$$= 13 + 6(7)$$

$$= 55$$

(4)

(c) (1) Volume of each cylinder $= \pi r^2 h$

$$\text{Ratio of consecutive volumes} = \frac{\pi \left(\frac{9}{10}r\right)^2 \left(\frac{9}{10}h\right)}{\pi r^2 h}$$

$$= \left(\frac{9}{10}\right)^3$$

Which is a constant
 $\therefore$ The sequence is geometric.

(3)

(2) Volume $= S_{12}$

$$= \frac{a(1-r^{12})}{1-r}$$

$$= \frac{\pi(18)^2(64) \left(1 - \left[\left(\frac{9}{10}\right)^3\right]^{12}\right)}{1 - \left(\frac{9}{10}\right)^3}$$

$$= 234\,968,5 \text{ cm}^3$$

$$\approx 235 \text{ litres}$$

(4)
[22]**QUESTION 4**

Quadratic pattern $\therefore T_n = an^2 + bn + c$

$$a + b + c = -1$$

$$4a + 2b + c = 4$$

$$9a + 3b + c = 11$$

$$\therefore 3a + b = 5 \text{ and } 5a + b = 7$$

$$3a + b = 5 \quad 5a + b = 7$$

$$b = 5 - 3a$$

$$\text{Sub into: } 5a + b = 7$$

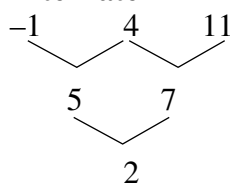
$$\therefore 5a + 5 - 3a = 7$$

$$2a = 2$$

$$a = 1$$

$$\therefore b = 2$$

$$\therefore c = -4$$

Alternate

First difference

Second difference

$$\therefore 2a = 2, \therefore a = 1$$

$$\therefore T_n = n^2 + bn + c$$

$$3a + b = 5 \text{ and Sub. } a = 1$$

$$3(1) + b = 5, \quad b = 2$$

$$a + b + c = -1 \quad \text{Sub. } a = 1 \text{ and } b = 2$$

$$1 + 2 + c = -1$$

$$\therefore c = -4$$

[6]

QUESTION 5

$$(a) \quad f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Working

$$f(x) = \frac{1}{2}x^2$$

$$f(x+h) = \frac{1}{2}(x+h)^2$$

$$f(x+h) = \frac{1}{2}(x^2 + 2xh + h^2)$$

$$f(x+h) = \frac{1}{2}x^2 + xh + \frac{1}{2}h^2$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\left[\frac{1}{2}x^2 + xh + \frac{1}{2}h^2 \right] - \left(\frac{1}{2}x^2 \right)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{xh + \frac{1}{2}h^2}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{h \left(x + \frac{1}{2}h \right)}{h}$$

$$f'(x) = x$$

(5)

$$(b) \quad y = \frac{2x^3}{x} - \frac{x^{\frac{1}{2}}}{x}$$

$$y = 2x^2 - x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 4x + \frac{1}{2}x^{-\frac{3}{2}}$$

$$\frac{dy}{dx} = 4x + \frac{1}{2x^{\frac{3}{2}}}$$

(5)

[10]**79 marks**

SECTION B

QUESTION 6

Alternate

$$\begin{aligned}
 \text{(a)} \quad f(x) &= 2\left(x^2 - 10x + \frac{47}{2}\right) \\
 f(x) &= 2\left[x^2 - 10x + (-5)^2 - (-5)^2 + \frac{47}{2}\right] \\
 f(x) &= 2\left[(x-5)^2 - 25 + \frac{47}{2}\right] \\
 f(x) &= 2(x-5)^2 - 3
 \end{aligned}$$

$$\begin{aligned}
 -\frac{b}{2a} &= \frac{20}{4} = 5 & \therefore \frac{b}{2a} &= -5 \\
 -\frac{\Delta}{4a} &= \frac{(-20)^2 - 4(2)(47)}{8} \\
 \frac{\Delta}{4a} &= -3
 \end{aligned}$$

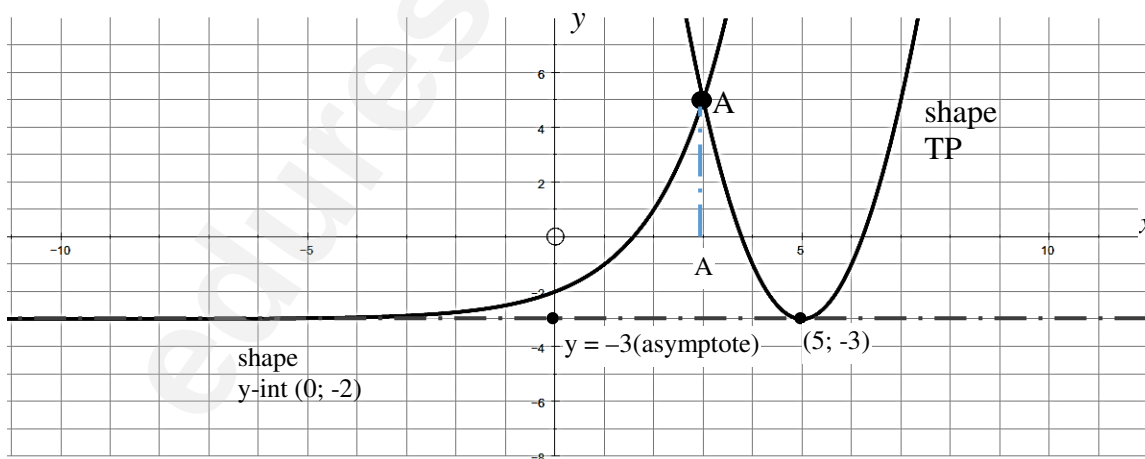
$$\begin{aligned}
 \therefore f(x) &= a\left(x + \frac{b}{2a}\right)^2 + \frac{\Delta}{4a} \\
 &= 2(x-5)^2 - 3
 \end{aligned}$$

(4)

$$\begin{aligned}
 \text{(b)} \quad h(x) &= b^x - q \quad \text{sub.: } (0; -2) \\
 -2 &= b^0 + q \\
 q &= -3 \\
 h(x) &= b^x - 3 \quad \text{sub.: } (1; -1) \\
 -1 &= b^1 - 3 \\
 b &= 2 \\
 h(x) &= 2^x - 3
 \end{aligned}$$

(3)

(c)



(5)

(d) Shown

(1)

$$\begin{aligned}
 \text{(e)} \quad x &= 2^y - 3 \\
 y &= \log_2(x + 3)
 \end{aligned}$$

(3)

(f) Domain of h^{-1} = Range of $h = (-3, \infty)$

(2)

[18]

QUESTION 7

$$\begin{aligned}\therefore f(x) &= \frac{-2(x+1)+4}{x+1} \\ &= \frac{4}{x+1} - 2\end{aligned}$$

Asymptotes are $x = -1$ and $y = -2$

Alternate 1

$$\begin{aligned}\frac{a}{x+1} + q &= \frac{-2x+2}{x+1} \\ a + q(x+1) &= -2x+2 \\ qx + (a+q) &= -2x+2 \\ q &= -2, \quad a+q=2, \quad a=4 \\ f(x) &= \frac{4}{x+1} - 2 \\ \therefore \text{Asymptotes are: } x &= -1 \text{ and } y = -2\end{aligned}$$

Alternate 2

$$\begin{array}{r} -2 \\ x+1 \overline{) -2x+2} \\ \underline{-2x-2} \\ 4 \end{array}$$

$$\begin{aligned}\therefore -2x+2 &= -2(x+1)+4 \\ \frac{-2x+2}{x+1} &= -2 + \frac{4}{x+1} \\ \therefore \text{Asymptotes are: } x &= -1 \text{ and } y = -2\end{aligned}$$

Alternate 3

$$\begin{aligned}\frac{-2x+2}{x+1} &= \frac{-2(x+1)+q}{x+1} \\ -2+q &= 2 \\ \therefore q &= 4 \\ \therefore \frac{-2(x+1)+4}{x+1} \\ &= \frac{-2+4}{x+1} \\ \therefore x &= -1; \quad y = -2 \quad \} \text{Asymptotes}\end{aligned}$$

[6]**QUESTION 8**

$$\begin{aligned}f(x) &= 2x + \frac{1}{2x} \\ f(x) &= 2x + \frac{1}{2}x^{-1} \\ f'(x) &= 2 - \frac{1}{2x^2} \\ f'(-1) &= 2 - \frac{1}{2} \\ &= \frac{3}{2}\end{aligned}$$

Equation of tangent is:

$$\begin{aligned}y &= \frac{3}{2}x + c \\ \frac{-5}{2} &= \frac{3}{2}(-1) + c \\ c &= -1 \\ \therefore Eq: y &= \frac{3}{2}x - 1\end{aligned}$$

QUESTION 9

$$\text{Eq. AB : } y = mx + 9 \quad \text{sub (12;0)}$$

$$0 = 12m + 9$$

$$m = \frac{-3}{4}$$

$$\therefore \text{ length CD} = -\frac{3}{4}x + 9$$

$$\begin{aligned} \text{Area } \Delta \text{COD} &= \frac{1}{2}xy = \frac{1}{2}x\left(-\frac{3}{4}x + 9\right) \\ &= -\frac{3}{8}x^2 + \frac{9}{2}x \end{aligned}$$

$$\frac{dA}{dx} = -\frac{3}{4}x + \frac{9}{2}$$

$$0 = -\frac{3}{4}x + \frac{9}{2}$$

$$x = 6$$

Alternate

$$x = \frac{-b}{2a}$$

$$= \left(\frac{-9}{2}\right)\left(\frac{1}{2}\right)\left(\frac{-8}{3}\right)$$

$$= 6$$

[8]**QUESTION 10**

$$(a) \quad f(x) = ax^3 + bx^2 - 3$$

$$f'(x) = 3ax^2 + 2bx$$

$$f''(x) = 6ax + 2b$$

$$f''(-2) = 6a(-2) + 2b$$

$$0 = -12a + 2b$$

$$b = 6a$$

$$\text{Sub: } (-3; 6) \text{ in } f(x)$$

$$f(-3) = a(-3)^3 + b(-3)^2 - 3 = 6$$

$$-27a + 9b - 9 = 0$$

$$-27a + 9(6a) - 9 = 0$$

$$27a = 9$$

$$a = \frac{1}{3} \quad \therefore b = 6\left(\frac{1}{3}\right) = 2$$

(7)

$$(b) \quad (1) \quad f(x) = \frac{1}{3}x^3 + 2x^2 - 3$$

$$f'(x) = x^2 + 4x$$

$$\frac{f'(x)}{x} = x + 4 \quad \text{and } x \neq 0$$

$$\frac{f'(x)}{x} \geq 0, \quad \text{when } x \geq -4 \quad x \neq 0$$

$$\text{OR } x \in [-4; 0) \text{ or } (0; \infty)$$

(4)

- (2) f is concave up when
 $f''(x) > 0$
 $f''(x) = 6ax + 2b = 2x + 4$
 $2x + 4 > 0$
 $x > -2$

(3)
[14]

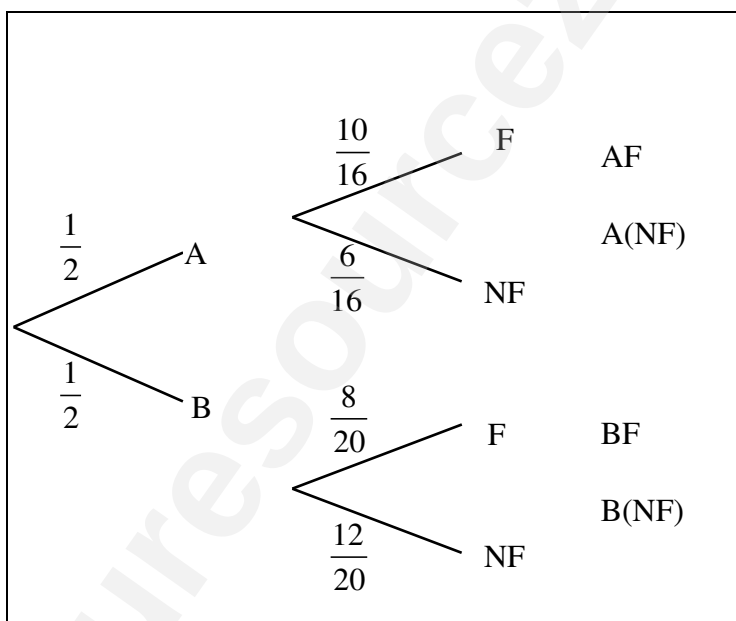
QUESTION 11

- (a) (1) Number of arrangements is $5! = 120$ (2)

- (2) $\frac{6!}{2!} = 360$ (2)

- (3) $\frac{120}{360} = \frac{1}{3}$ (1)

- (b) (1)



$$\begin{aligned}
 &P(\text{picking up a fiction book}) \\
 &= P(\{\text{Shelf A and fiction book}\} \text{ or } \{\text{Shelf B and fiction book}\}) \\
 &= P(AF) + P(BF) \\
 &= \frac{1}{2} \left(\frac{10}{16} \right) + \frac{1}{2} \left(\frac{8}{20} \right) \\
 &= \frac{41}{80}
 \end{aligned}$$

(4)

- (2) $1 - \left(\frac{1}{2} \right) \left(\frac{6}{16} \right) = \frac{13}{16}$ **Alternate:** $\left(\frac{1}{2} \right) \left(\frac{10}{16} \right) + \left(\frac{1}{2} \right) \left(\frac{8}{20} \right) + \frac{1}{2} \left(\frac{12}{20} \right) = \frac{13}{16}$ (3)

[12]

QUESTION 12

The full sphere touches all eight pieces. Let the side of the cube be a . The radius of the sphere is x . The diagonal of the cube is: $x + x + 2x = 4x$.

The diagonal of each face of the cube has length $\sqrt{2a^2}$ (by Pythag.)

$\therefore$ The diagonal of the cube has length:

$$\sqrt{\left(\sqrt{2a^2}\right)^2 + a^2} = \sqrt{3a^2} \text{ (by Pythag.)}$$

But: $\sqrt{3a^2} = 4x$

$$\sqrt{3}a = 4x$$

$$a = \frac{4}{\sqrt{3}}x$$

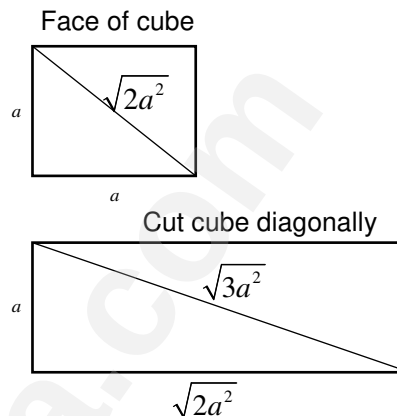
$$\text{Volume liquid} = a^3 - 2\left(\frac{4}{3}\pi x^3\right) = 116,8$$

$$\left(\frac{4}{\sqrt{3}}x\right)^3 - 2\left(\frac{4}{3}\pi x^3\right) = 116,8$$

$$\left(\frac{4}{\sqrt{3}}x\right)^3 \cdot x^3 - \frac{8}{3}\pi x^3 = 116,8$$

$$x^3 = \frac{116,8}{\left(\frac{4}{\sqrt{3}}\right)^3 - \frac{8}{3}\pi}$$

$$x = 3,095 \approx 3,1 \text{ cm}$$



ALTERNATE: Let the side of the cube be a ...

$$\sqrt{3a^2} = 4x$$

$$x = \frac{\sqrt{3}a}{4}$$

$$\text{Volume of liquid} = a^3 - 2\left(\frac{4\pi}{3}x^3\right)$$

$$116,8 = a^3 - \left(\frac{8\pi}{3}\right)\left(\frac{\sqrt{3}}{4}\right)^3 a^3$$

$$a^3 = \frac{116,8}{1 - \left(\frac{8\pi}{3}\right)\left(\frac{\sqrt{3}}{4}\right)^3}$$

$$\therefore x = \frac{\sqrt{3}a}{4} = \frac{\sqrt{3}}{4} \times \sqrt[3]{\frac{116,8}{1 - \left(\frac{8\pi}{3}\right)\left(\frac{\sqrt{3}}{4}\right)^3}} \approx 3,1 \text{ cm}$$

Use of volume of cube
Indication of volume of 2 spheres
Subtracting volumes

[7]

71 marks