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NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2018

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours | 150 marks

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SECTION A

QUESTION 1

(a) $T_{100} = a + 99d$ ✓

$a + 99(7) = 512$ ✓

$a = -181$ ✓ (3)

(b) (1) $T_1 = 2(1) + 3 \therefore T_1 = 5$; $T_2 = 7$; $T_3 = 9$ ✓✓

$\therefore$ Constant first difference $= 2$ ✓ (3)

(2) $S_n = \frac{n}{2}[2(5) + (n-1)(2)]$ ✓✓

$S_n = \frac{n}{2}[8 + 2n]$

$S_n = 4n + n^2$ ✓ (3)

Alternate:

$S_n = \frac{n}{2}(a + l)$

$S_n = \frac{n}{2}(5 + 2n + 3)$ ✓ ✓

$S_n = n^2 + 4n$ ✓

(c) $2a = 4$ $\therefore a = 2$ ✓

$3a + b = 3$ $\therefore 3(2) + b = 3$ $\therefore b = -3$ ✓

$a + b + c = 4$ $\therefore 2 + (-3) + c = 4$ $\therefore c = 5$ ✓

$T_n = 2n^2 - 3n + 5$ ✓ (4)

[13]

QUESTION 2

(a) (1) $T_1 = 108 \times \left(\frac{2}{3}\right)^1$ $\therefore T_1 = 72$ ✓

$T_2 = 108 \times \left(\frac{2}{3}\right)^2$ $\therefore T_2 = 48$ ✓ (2)

(2) $T_3 = 108 \times \left(\frac{2}{3}\right)^3$ $\therefore T_3 = 32$ ✓

$T_4 = 108 \times \left(\frac{2}{3}\right)^4$ $\therefore T_4 = \frac{64}{3}$ ✓

$\therefore$ First 4 items add up to $\frac{520}{3}$ ✓

$\therefore x = 4$ ✓ (4)

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Alternative:

Geometric sequence with $a = 72$ and $r = \frac{2}{3}$ ✓

$S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$
✓

$\left(\frac{2}{3}\right)^n = \frac{16}{81}$ ✓

$\log_{\frac{2}{3}}\left(\frac{16}{81}\right) = n$

$n = 4$

$\therefore x = 4$ ✓

(b) Area $1 = 2\pi(21)^2$

Area $2 = 2\pi(3)^2$ ✓

Area $3 = 2\pi\left(\frac{3}{7}\right)^2$

Common ratio: $\frac{1}{49}$ ✓ indicating a convergent series

$S\infty = \frac{a}{1-r}$; $-1 < r < 1$ ✓

$S\infty = \frac{2\pi(21)^2}{1 - \frac{1}{49}}$ ✓

$S\infty = \frac{7\ 203}{8}\pi$ $\therefore S\infty \approx 2\ 828{,}6 \text{ cm}^2$ ✓ (5)

[11]

QUESTION 3

(a) (1) Working with: $\frac{1}{(x^2 - 3x - 4)(x+1)}$ ✓, undefined for:

$(x^2 - 3x - 4)(x+1) = 0$

$(x-4)(x+1)(x+1) = 0$ ✓

$x = 4$ ✓ or $x = -1$ ✓ (4)

(2) $x^2 - 3x - 4 \leq 0$ ✓

Critical values: 4 ; -1 ✓

$\therefore -1$ ✓ $\leq x \leq 4$ ✓ (4)

(b) (1) $x + 4 \geq 0$ ✓✓

$\therefore x \geq -4$ (2)

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(2) $\sqrt{x+4}-3=x$

$\left(\sqrt{x+4}\right)^2=(x+3)^2$ ✓

$x+4=x^2+6x+9$ ✓

$x^2+5x+5=0$ ✓

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

✓ ✓

$x\approx-1{,}4$ or $x\approx-3{,}6$ (n/v) ✓ (6)

[16]

QUESTION 4

(a) (1) Average Gradient $=\dfrac{[2(1+h)^3]-[2(1)^3]}{(1+h)-1}$ ✓

Average Gradient $=\dfrac{2(1+h)(1+2h+h^2)-2}{h}$

Average Gradient $=\dfrac{2(1+2h+h^2+h+2h^2+h^3)-2}{h}$

Average Gradient $=\dfrac{2(1+3h+3h^2+h^3)-2}{h}$

✓ ✓

Average Gradient $=\dfrac{(2+6h+6h^2+2h^3)-2}{h}$

Average Gradient $=\dfrac{h(6+6h+2h^2)}{h}$

Average Gradient $=6+6h+2h^2$ ✓ (4)

(2) $f'(1)=\lim\limits_{h\to0}\left(6+6h+2h^2\right)$ ✓

$f'(1)=6$ ✓ (2)

Alternate:

(2) $f'(x)=\lim\limits_{h\to0}\dfrac{2(x+h)^3-2x^3}{h}$

$f'(x)=\lim\limits_{h\to0}\dfrac{2(x+h)(x^2+2xh+h^2)-2x^3}{h}$

$f'(x)=\lim\limits_{h\to0}\dfrac{2(x^3+2x^2h+h^2x+x^2h+2xh^2+h^3)-2x^3}{h}$

$f'(x)=\lim\limits_{h\to0}\dfrac{6x^2h+6h^2x+2h^3}{h}$

$f'(x)=\lim\limits_{h\to0}\dfrac{h(6x^2+6hx+2h^2)}{h}$

$f'(x)=6x^2$ ✓

$f'(1)=6(1)^2 \quad \therefore f'(1)=6$ ✓

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Alternate:

$$f(x) = 2x^3$$
$$f'(x) = 6x^2 \checkmark$$
$$f'(1) = 6 \checkmark$$

(b) $y = 3x^{-2} \checkmark - 10x^{\frac{1}{5}} \checkmark$

$\frac{dy}{dx} = -6x^{-3} \checkmark \; -2x^{-\frac{4}{5}} \checkmark$ (4)

[10]

QUESTION 5

(a) $\overset{\checkmark}{A} = 300000\left(1+\frac{0,16}{12}\right)^{\overset{\checkmark}{60}}\overset{\checkmark}{(1+0,11)^{10}} - 500000\overset{\checkmark}{(1+0,11)^{2}}$

$A = 1269728,917 \checkmark$

Alternate:

$T_0 - T_5$: $A = 300\ 000\left(1+\frac{16}{100(12)}\right)^{5\times 12} \checkmark \checkmark$

$A = 664\ 142,0648$

$T_6 - T_{13}$: $A = 664\ 142,0648\left(1+\frac{11}{100}\right)^{8} \checkmark$

At the end of the 13th year: $1\ 530\ 540,473 - 500\ 000$

$T_{14} - T_{15}$: $A = 1\ 030\ 540,473\left(1+\frac{11}{100}\right)^{2} \checkmark$

At the end of the 15th year he has: R1 269 728,917 ✓ (5)

(b) $F = x\left[\frac{(1+n)^n - 1}{i}\right] \checkmark$

$$1\ 270\ 000 \checkmark = x\left[\frac{\left(1+\frac{8}{100(12)}\right)^{(15\times 12)} - 1}{\frac{8}{100(12)}}\right] \checkmark$$

$x = \text{R}3\ 670,114804 \checkmark$ (4)

[9]

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QUESTION 6

(a) Y-intercept: $y = 2(0)+5$ $\therefore$ y-intercept for both graphs: (0 ; 5)

For horizontal asymptote for f :substitute $(-1\,;\,y)$ in $g(x) = 2x+5$

$\therefore g(-1) = 2(-1)+5$ ✓ $\therefore g(-1) = 3$ ✓

$\therefore$ Horizontal asymptote of f: $y = 3$

$f(x) = \dfrac{a}{x+1} + 3$ ✓ ✓ substitute (0 ; 5)

$5 = \dfrac{a}{0+1} + 3$ ✓ $\therefore a = 2$

$a = 2$; ✓ $b = 1$ and $c = 3$ (6)

(b) (1) X-intercept of f : $0 = \dfrac{2}{x+1} + 3$ ✓ $\therefore x = -\dfrac{5}{3}$ ✓

X-intercept of g : $0 = 2x+5$ $\therefore x = -\dfrac{5}{2}$ ✓ (3)

(2) $-\dfrac{5}{3} \leq x < -1$ or $x \leq -\dfrac{5}{2}$ ✓ ✓ ✓ (3)

(c) (1) $g(x) = 2x+5$

$x = 2y+5$ ✓

$y = \dfrac{1}{2}x$ ✓ $-\dfrac{5}{2}$ ✓ (3)

(2) Point of intersection: $2x+5 = \dfrac{x-5}{2}$ ✓ $\therefore x = -5$ ✓

The values of x for which $g^{-1}(x) > g(x)$: $x < -5$ ✓ (3)

[18]

77 marks

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SECTION B

QUESTION 7

(a)

$$x = 5 \pm \sqrt{2} \quad \checkmark$$

$$\therefore \left[x - \left(5 + \sqrt{2}\right)\right]\left[x - \left(5 - \sqrt{2}\right)\right] = 0 \quad \checkmark$$

$$x^2 - 5x + \sqrt{2}x - 5x - \sqrt{2}x + 23 = 0 \quad \checkmark$$

$$x^2 - 10x + 23 = 0 \quad \checkmark$$

(4)

(b) For real and equal roots: Quadratic must be a perfect square $\therefore$ ✓

$$x^2 + ax + b = 0$$

$$\left(x + \sqrt{b}\right)^2 = 0$$

$$x^2 + 2\sqrt{b}x + b = 0$$

$$\therefore a = 2\sqrt{b} \quad \checkmark$$

$$\therefore \left(\sqrt{b}\right)^2 = \left(\frac{a}{2}\right)^2$$

$$\therefore b = \frac{a^2}{4} \quad \ldots \text{ eq. 1}$$

$$x^2 + bx + a = 0$$

$$\left(x + \sqrt{a}\right)^2 = 0$$

$$x^2 + 2\sqrt{a}x + a = 0$$

$$\therefore b = 2\sqrt{a} \quad \ldots \text{ eq. 2} \quad \checkmark$$

Substitute eq1 in eq 2:

$$\frac{a^2}{4} = 2\sqrt{a} \quad \checkmark$$

$$\therefore a^{\frac{3}{2}} = 2^3$$

$$\therefore \left(a^{\frac{3}{2}}\right)^{\frac{2}{3}} = \left(2^3\right)^{\frac{2}{3}} \quad \checkmark$$

$$\therefore a = 4 \checkmark \quad \text{and} \quad b = 4 \checkmark$$

(7)

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Alternate:

For real and equal roots, $\Delta = b^2 - 4ac = 0$ ✓

For $x^2 + ax + b = 0$: $0 = a^2 - 4b$

$\therefore b = \frac{a^2}{4}$ … eq1 ✓

For $x^2 + bx + a = 0$: $0 = b^2 - 4a$ … eq2 ✓

Substitute eq1 in eq 2:

$\left(\frac{a^2}{4}\right)^2 - 4a = 0$ ✓

$a^4 - 64a = 0$

$a(a^3 - 64) = 0$ ✓

$a = 0$ or $a = 4$

$\therefore a = 4$ ✓ only and $b = 4$ ✓

[11]

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QUESTION 8

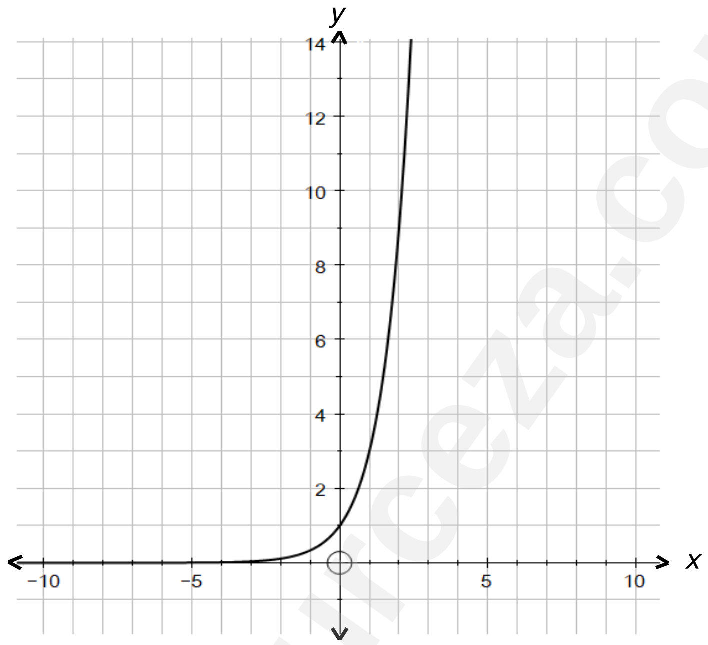
(a) $A = P(1+i)^n$

$y^2 = y(1+i)^x$ ✓

$y^2 = y\left(1+\frac{200}{100}\right)^x$ ✓

$y = (3)^x$ ✓ (3)

(b)



Shape	✓
Y-intercept	✓
Asymptote	✓

(3)

(c) (1) $750 = (3)^x$ ✓

$x = \log_3 750$ ✓

$x \approx 6{,}03$

It took approximately 6 years ✓ (3)

(2) Domain: $x > 6$ (accept: $x \geq 6$) ✓ (1)

[10]

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QUESTION 9

(a) For point of inflection: Let $g''(x)=0$ ✓

$g'(x)=3x^2-6x$ ✓

$g''(x)=6x-6$ ✓

$6x-6=0 \quad \therefore x=1$ ✓

$g(1)=-2$ ✓and $h(1)=-2$ ✓

Hence, g and h intersect at $x=1$, the point of inflection. (6)

Alternate:

For point of inflection: Let $g''(x)=0$ ✓

$g'(x)=3x^2-6x$ ✓

$g''(x)=6x-6$ ✓

$6x-6=0 \quad \therefore x=1$ ✓

Point of intersection: $x^3-3x^2=-\frac{2}{3}x-\frac{4}{3}$ ✓

$3x^3-9x^2+2x+4=0$

$x=\frac{3\pm\sqrt{21}}{3}$ or $x=1$ ✓

Therefore, the graph of h does intersect the graph of g at its point of inflection.

Alternate:

For point of inflection: Let $g''(x)=0$ ✓

$g'(x)=3x^2-6x$ ✓

$g''(x)=6x-6$ ✓

$6x-6=0 \quad \therefore x=1$ ✓

For co-ordinate of point of inflection:

Substitute $x=1$ in $f(1)=(1)^3-3(1)^2$ ✓

$f(1)=-2$

Substitute $(1;-2)$ in $y=-\frac{2}{3}x-\frac{4}{3}$

$\text{RHS}=-\frac{2}{3}(1)-\frac{4}{3}$ ✓

$\text{RHS}=-2$

$\text{RHS}=\text{LHS}$

Therefore, the graph of h does intersect the graph of g at its point of inflection.

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(b) (1) For stationary point of $y = g'(x)$

$y = 3x^2 - 6x$

$\frac{dy}{dx} = 6x - 6$

$6x - 6 = 0$ ✓

$\therefore x = 1$

Startionary point $(1;-3)$ ✓ ✓ Min. value function ✓ (4)

(2) (i) Concave down for: $x < 1$ ✓ (1)

(ii) $g'(1) = 3(1)^2 - 6(1)$ ✓

$g'(1) = -3$ ✓ (2)

(3) Decreasing gradient occurs for: $0 < x < 2$ ✓

Maximum decreasing gradient occurs at the point of inflection. ✓ (2)

(c) The graph of g decreases for the interval: $0 < x < 2$ ✓✓

We must shift the graph of g , 3 units to the left ✓

$\therefore k = 3$ ✓ (4)

[19]

QUESTION 10

(a) (1) Since $b > 2a$, then $b^2 > 4a^2$ ✓

Since $c < a$

Then $b^2 > 4ac$ ✓ (2)

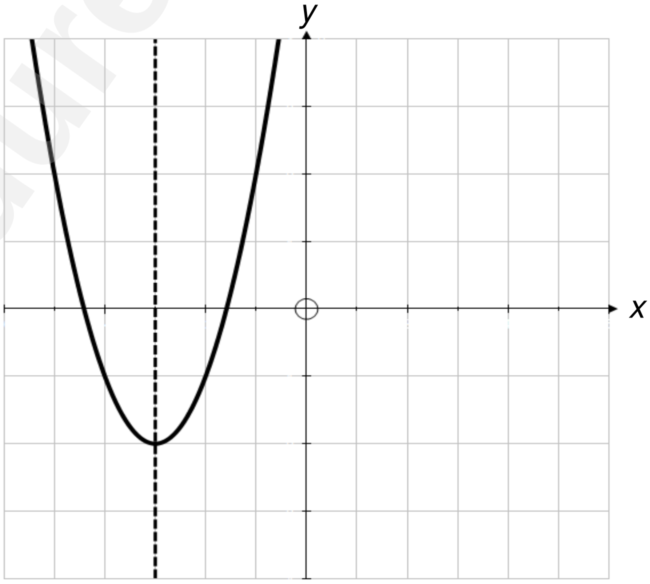
Alternate:

$b > 2a$ and $b > c$ ✓✓

$(b > 2a > a > c)$, hence

$b^2 > 4ac$

(2)



From the given constraints:

a, b and c are positive(+)

Therefore:

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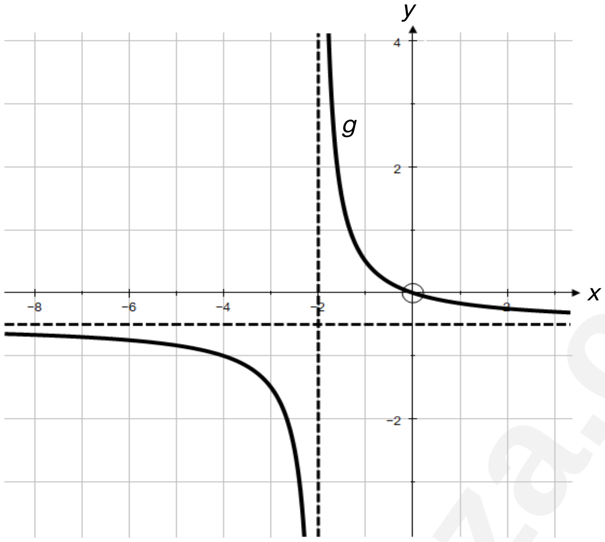
Shape: min. value function ✓
Y-int. : + ✓
Axis of Symm.: $x =$ negative value ✓
$b^2 - 4ac > 0$ $\therefore$ roots are real and unequal ✓ (4)

(b) (1)



Shape and Pt.$(0;0)$ ✓

Horizontal asymptote: $y = -\frac{1}{2}$ ✓

Vertical asymptote: $x = -2$ ✓ (3)

(2) $p \overset{✓}{\geq} -\frac{1}{2}$ ✓ (2)

[11]

QUESTION 11

(a) (1) P(both letters are C) $= \frac{2}{6} \overset{✓}{\times} \frac{1}{5}$

$= \frac{1}{15}$ ✓ (2)

(2) P(only one letter is C) $\overset{✓}{\left(\frac{2}{6} \times \frac{4}{5}\right)} \overset{✓}{+} \left(\frac{4}{6} \overset{✓}{\times} \frac{2}{5}\right)$

$= \frac{8}{15}$ (3)

(b) $\frac{6!}{2!}$ ✓ $= 360$ ✓ (2)

(c) $4! = 24$ ✓✓ (2)

[9]

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QUESTION 12

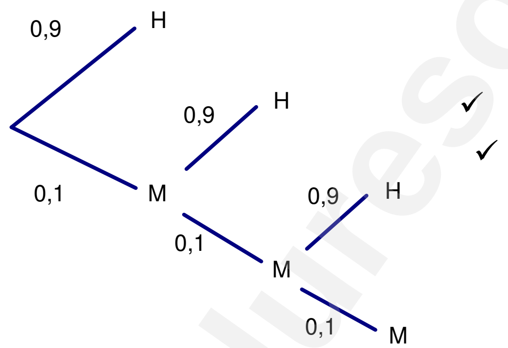
Let the number of missiles required for firing be n.
P(all will miss) $= (1-0,9)^n$ $\therefore$ P(all will miss) $= 0,1^n$ ✓✓
P(at least 1 will hit) $= 1-0,1^n$ ✓
We require: $1-0,1^n > 0,97$
When $n=1$, $1-0,1^1 = 0,9$
When $n=2$, $1-0,1^2 = 0,99$ ✓✓
When $n=3$, $1-0,1^3 = 0,999$
Therefore, at least 2 missiles should be fired.
Therefore, Lulu was correct. ✓

[6]

Alternate:

Let the number of missiles required for firing be n.
P(all will miss) $= (1-0,9)^n$ $\therefore$ P(all will miss) $= 0,1^n$ ✓✓
Let: $1-0,1^n = 0,97$ ✓
$0,03 = 0,1^n$ ✓
$\log_{0,1} 0,03 = n$ ✓
$n \approx 1,5$ ✓
Therefore, at least 2 missiles need to be fired to ensure at least a 0,97 chance of hitting the target.
Lulu was correct✓

Alternate:



First missile fired: P(a hit) = 0,9

Second missile fired: P(a hit) = 0,9 + MH

= 0,9 + (0,1 ✓ x 0,9 ✓)

= 0,99 ✓

Third missile fired: P(hit) = 0,9 + MH + MMH

= 0,9 + (0,1 x 0,9) + (0,1 x 0,1 x 0,9)

= 0,999   Hence Lulu was correct ✓

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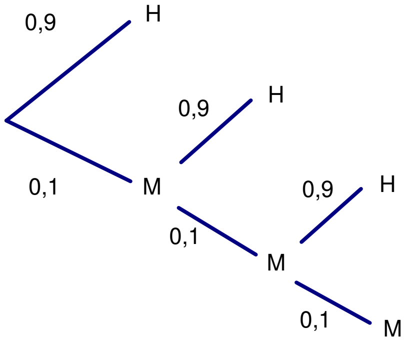
Alternate:



✓
✓

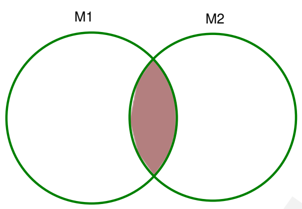
2 Missiles fired:

P(a hit) = 1 – P(no hit) ✓
= 1 - P(MM)
= 1 - (0,1 x 0,1) ✓
= 0,99 ✓

3 missiles fired:
P(a hit) = 1 – P(no hit)
= 1 - P(MMM)
= 1 - (0,1 x 0,1 x 0,1)
= 0,999

Hence Lulu was correct. ✓

Alternate:



✓

$P(M_1 \cup M_2) = P(M_1) + P(M_2) - P(M_1 \quad M_2)$ ✓
✓ $= 0{,}9 + 0{,}9 - (0{,}9 \times 0{,}9)$ ✓
$= 0{,}99$ ✓

Similarly, if 3 missiles are fired:
$P(M_1 \cup M_2 \cup M_3) = 0{,}999$

Hence Lulu was correct ✓

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QUESTION 13

$y=-\frac{3}{2}x+3$ ✓

Area ΔOMN $=\frac{1}{2}b.h$

Area ΔOMN $=\frac{1}{2}x\left(-\frac{3}{2}x+3\right)$

Area ΔOMN $=-\frac{3}{4}x^2+\frac{3}{2}x$ ✓

For max. value x_1 : $\frac{dA}{dx}=0$

$0=-\frac{3}{2}x_1+\frac{3}{2}$ ✓

$\therefore x_1=1$ ✓

$f(x)=rx^2+bx+c$ where $r=-\frac{3}{4}$

From: $f'(x)=-\frac{3}{2}x+3$ …. By inspection, $b=3$

$f(x)=-\frac{3}{4}x^2+\overset{✓}{3}x+c$

Stationary point $(x;5)$

X-Intercept if f ’(x) represents x-coordinate of the Stationary Point

$\therefore$ Stationary Point (2;5)

Substitute (2;5) in $f(x)=-\frac{3}{4}x^2+3x+c$

$5=-\frac{3}{4}(2)^2+3(2)+c$

$c=2$

For value of x_2 that give max. distance (S) between f and f':

$S=-\frac{3}{4}x^2+3x+2-\left(-\frac{3}{2}x+3\right)$ ✓

$S=-\frac{3}{4}x^2+\frac{9}{2}x-1$

$\frac{dS}{dx}=0$

$-\frac{3}{2}x_2+\frac{9}{2}=0$

$x_2=3$ ✓

They differ.

(7)

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Alternate:

$y = -\frac{3}{2}x + 3$

Area ΔOMN $= \frac{1}{2}b.h$

Area ΔOMN $= \frac{1}{2}x\left(-\frac{3}{2}x + 3\right)$

Area ΔOMN $= -\frac{3}{4}x^2 + \frac{3}{2}x$ ✓

For max. value x_1 : $\frac{dA}{dx} = 0$

$0 = -\frac{3}{2}x_1 + \frac{3}{2}$ ✓

$\therefore x_1 = 1$ ✓

$f(x) = rx^2 + bx + c$ where $r = -\frac{3}{4}$

From: $f'(x) = -\frac{3}{2}x + 3$ …. By inspection, $b = 3$ ✓

$f(x) = -\frac{3}{4}x^2 + 3x + c$

Stationary point $(x;5)$

X-Intercept if f '(x) represents x-coordinate of the Stationary Point

$\therefore$ Stationary Point (2;5)

Substitute (2;5) in $f(x) = -\frac{3}{4}x^2 + 3x + c$ ✓

$5 = -\frac{3}{4}(2)^2 + 3(2) + c$

$c = 2$

For value of x_2 that give max. distance (S) between f and f':

$S(x) = -\frac{3}{4}x^2 + 3x + 2 - \left(-\frac{3}{2}x + 3\right)$ ✓

$S(x) = -\frac{3}{4}x^2 + \frac{9}{2}x - 1$

$S(1) = 2{,}75$ and $S(2) = 5$, ✓

Hence maximum distance is not at $x = 1$

$\boxed{\textbf{73 marks}}$

Total: 150 marks

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