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KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS

COMMON TEST

MARCH 2022

TIME: 2 hours

N.B. This question paper consists of 6 pages,
2 diagram sheets and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 8 questions.
2. Answer **ALL** questions.
3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

Given the quadratic sequence: 5; x ; y ; 29; ... and its second constant difference is equal to 4.

1.1 Calculate the values of x and y . (4)

1.2 If $x = 9$ and $y = 17$, determine the n^{th} term of the quadratic sequence. (4)

1.3 Calculate the 50th term of the sequence. (2)

[10]**QUESTION 2**

The 2nd term of an arithmetic sequence is 8 and the 7th term is eleven times the value of the first term. Determine the first three terms of the sequence. [7]

QUESTION 3

3.1 Given:

$$\sum_{k=1}^n \frac{1}{p^{k-1}}$$

3.1.1 Write down the values of the first three terms of the series in terms of p . (1)

3.1.2 Determine the values of p for which the series is converging. (4)

3.2 The sum of the first n terms of a sequence is given by $4 - 4\left(\frac{1}{2}\right)^n$.

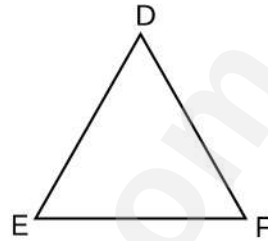
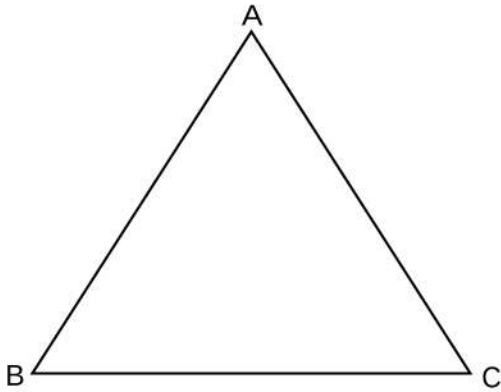
Calculate the first three terms. (5)

[10]

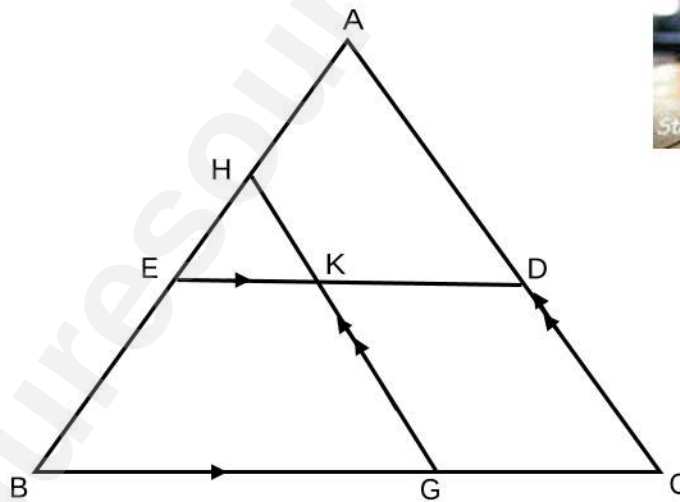
QUESTION 4

4.1 Given $\triangle ABC$ and $\triangle DEF$ with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.

Prove the theorem which states that $\frac{AB}{DE} = \frac{AC}{DF}$ (7)



4.2 In the figure, $\triangle ABC$ has $HG \parallel AC$ and $ED \parallel BC$. ED and HG intersect at K .
 $\frac{AD}{DC} = \frac{3}{2}$ and $BG = 2GC$. $AB = 15$ units.



Determine with reasons the value of:

4.2.1 AE (3)

4.2.2 AH (3)

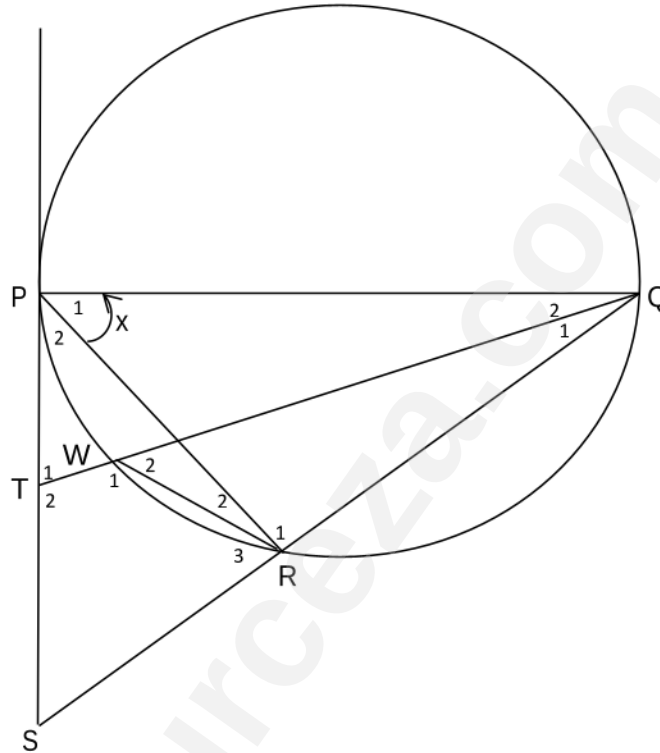
4.2.3 $\frac{GK}{KH}$ (3)

4.2.4 $\frac{\text{Area of } \triangle HEK}{\text{Area of } \triangle HBG}$ (3)

QUESTION 5

In the figure: PQ is the diameter of the circle. SP is the tangent to the circle at P.

QT intersects the circle at W and with T on the line SP. QS intersects the circle at R. $\hat{P}_1 = x$.



- 5.1 Give a reason why $\hat{PRQ} = 90^\circ$. (1)
- 5.2 Prove $\hat{S} = x$. (3)
- 5.3 Prove that SRWT is a cyclic quadrilateral. (3)
- 5.4 Prove that $\triangle QWR \parallel \triangle QST$. (3)
- 5.5 If $QW = 5$ cm, $TW = 1$ cm, $QR = 4$ cm and $WR = 2$ cm, calculate the lengths of:
- 5.5.1 TS (3)
- 5.5.2 SR (3)

[16]

QUESTION 6

6.1 Given $\cos 20^\circ = p$ and $\sin 14^\circ = q$

Without using a calculator, calculate the value of the following in terms of p , q or p and q .

6.1.1 $\sin 20^\circ$ (2)

6.1.2 $\cos 6^\circ$ (6)

6.2 Simplify into a single trigonometric ratio.

$$\sqrt{\frac{\frac{1}{2} \sin 2x}{\tan(540^\circ + x) \left(\frac{1}{\cos^2 x} - \tan^2 x \right)}} \quad (6)$$

[14]

QUESTION 7

7.1.1 Prove the following identity:

$$\cos 4x = 8 \cos^4 x - 8 \cos^2 x + 1 \quad (4)$$

7.1.2 Hence, determine, without the use of a calculator, the general solution of

$$16 \cos^4 x - 16 \cos^2 x + 2 = 1 \quad (5)$$

7.1.3 Write down the minimum value of the expression $16 \cos^4 x - 16 \cos^2 x + 2$. (2)

7.2 Calculate, without the use of a calculator, the value of:

$$\frac{2 \sin^2 22.5^\circ - 1}{4 \sin 22.5^\circ \cos 22.5^\circ} \quad (5)$$

[16]

QUESTION 8

8.1 Sketch the graphs of $f(x) = \sin(2x)$ and $g(x) = 2 \cos x$ for the domain $x \in [-90^\circ; 180^\circ]$. (Use the axes provided) (6)

8.2 Use your graphs to determine the solution $\frac{g(x)}{f(x)} \geq 1$. (2)

[8]

INFORMATION SHEET: MATHEMATICS
INLIGTING BLADSY

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum f \cdot x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

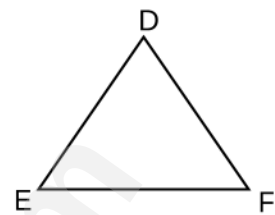
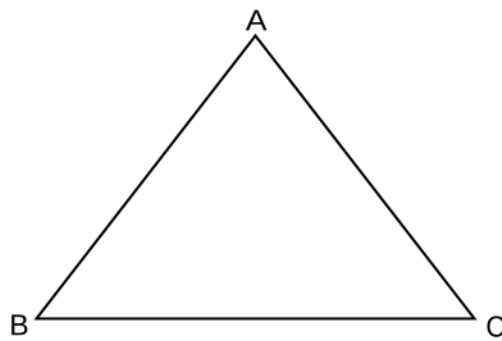
$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

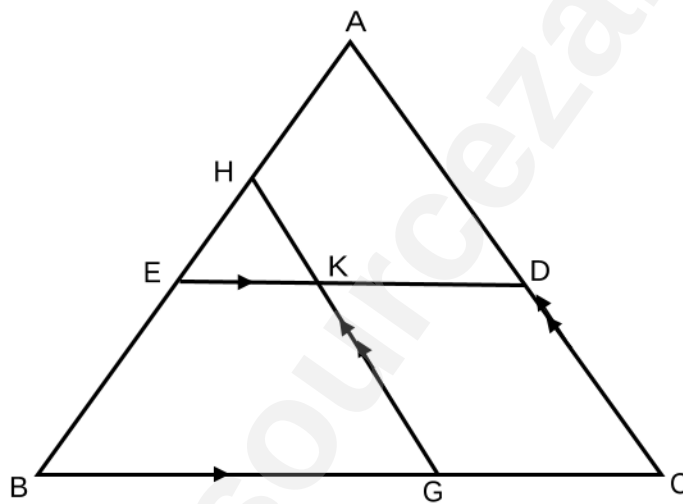
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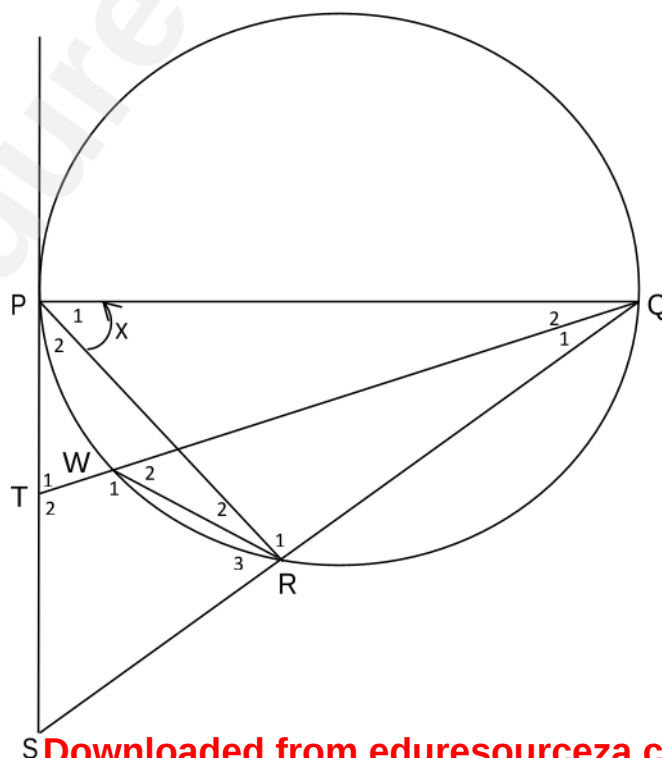
DIAGRAM SHEET
QUESTION 4.1



QUESTION 4.2



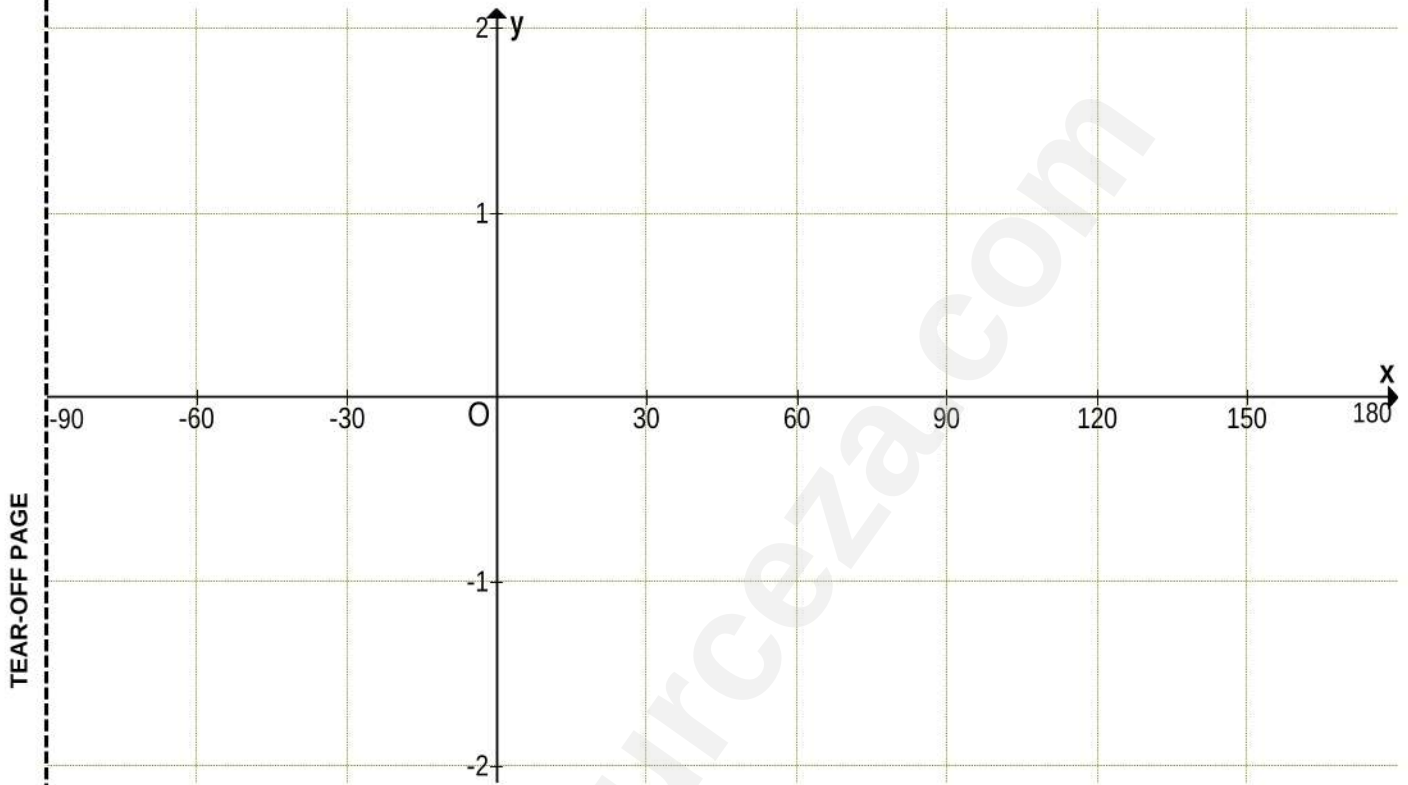
QUESTION 5



NAME: _____

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QUESTION 8





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MATHEMATICS MARKING GUIDELINE COMMON TEST MARCH 2022

MARKS: 100

This memorandum consists of 2 pages.

QUESTION 1

1.1	$y - 2x + 5 = 4$ $y - 2x = -1 \dots (1)$ $29 - 2y + x = 4$ $-2y + x = -25 \dots (2)$ $(1) \times 2: 2y - 4x = -2 \dots (3)$ $(3) + (2):$ $-3x = -27 \quad \therefore x = 9$ $\quad \quad \quad \therefore y = 17$	A✓ setting up equation 1st line A✓ setting up equation 1st line CA✓ x – value CA✓ y – value	(4)
1.2	$2a = 4 \quad \therefore a = 2$ $3a + b = 4 \quad \therefore b = -2$ $a + b + c = 5 \quad \therefore c = 5$ $T_n = 2n^2 - 2n + 5$ OR $2a = 4 \quad \therefore a = 2$ $3a + b = 5 \quad \therefore b = -2$ $\therefore c = T_0 = 5$ $T_n = 2n^2 - 2n + 5$ OR $T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)}{2}d_2$ OR $T_n = \frac{(n-1)}{2}[2a + (n-2)d] + T_1$	A✓ a – value CA✓ b – value CA✓ c – value CA✓ n^{th} term OR A✓ a – value CA✓ b – value CA✓ c – value CA✓ n^{th} term OR OR	(4)
1.3	$T_{50} = 2(50)^2 - 2(50) + 5 = 4905$	CA✓ substitution (from 1.2) CA✓ answer	(2)
			[10]

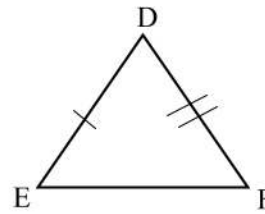
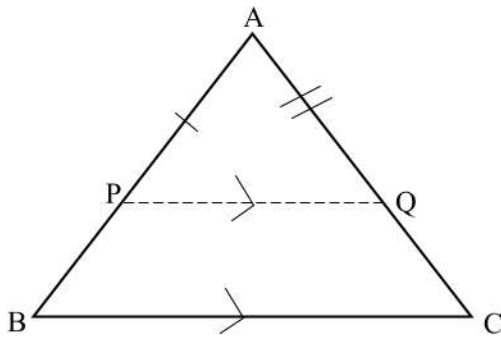
QUESTION 2

2	$a + d = 8 \rightarrow (1)$ $a + 6d = 11a \rightarrow (2)$ $d = 8 - a \rightarrow (3)$ $a + 6(8 - a) = 11a$ $a + 48 - 6a = 11a$ $48 = 16a$ $a = 3$ $d = 5$ $3 ; 8 ; 13 ; \dots$	A✓equation (1) A✓equation (2) CA✓making d/a the subject CA✓correct substitution of d/a CA✓ a/d value CA✓ d/a value CA✓sequence	[7]
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QUESTION 3

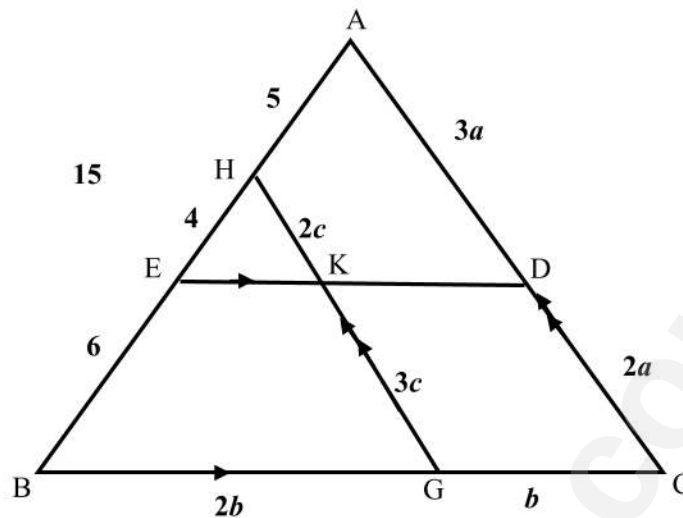
3.1.1	$1 ; \frac{1}{p} ; \frac{1}{p^2}$	A✓All three terms	(1)
3.1.2	$-1 < r < 1$ $-1 < \frac{1}{p} < 1$ $p < -1 \text{ or } p > 1$	A✓condition for convergence CA✓ r value in terms of p CACA✓✓answers	(4)
3.2	$S_n = 4 - 4\left(\frac{1}{2}\right)^n$ $T_1 = S_1 = 4 - 4\left(\frac{1}{2}\right)^1 = 2$ $T_1 + T_2 = S_2 = 4 - 4\left(\frac{1}{2}\right)^2 = 3$ $T_1 + T_2 + T_3 = S_3 = 4 - 4\left(\frac{1}{2}\right)^3 = 3\frac{1}{2}$ $\therefore T_2 = 1$ $\therefore T_3 = \frac{1}{2}$	A✓first term value A✓Sum of first two terms A✓Sum of first 3 terms CA✓second term value CA✓third term value	(5)
			[10]

QUESTION 4



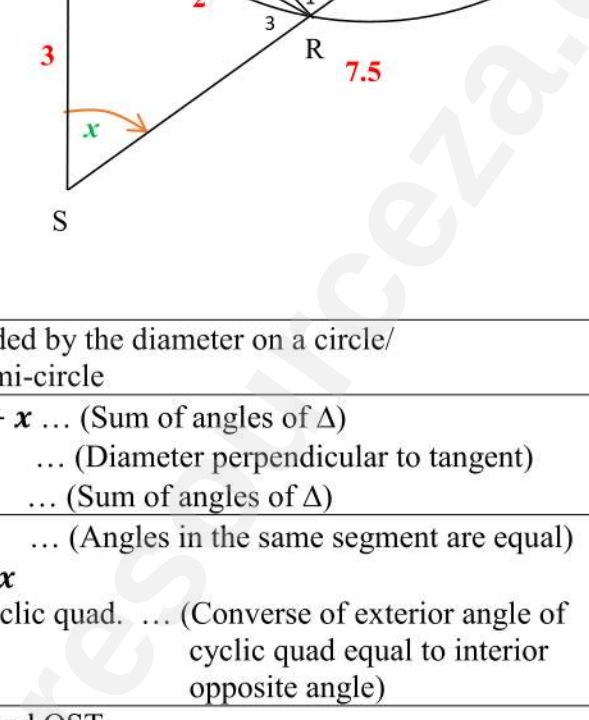
4.1	On AB mark off a point P such that $AP = DE$ and on AC mark off a point Q such that $AQ = DF$. Join PQ. In $\triangle ABC$ and $\triangle DEF$ $AP = DE$ (Construction) $AQ = DF$ (Construction) $\hat{A} = \hat{D}$ (Given) $\therefore \triangle APQ \equiv \triangle DEF$ (SAS) Now $\hat{APQ} = \hat{DEF}$ But $\hat{DEF} = \hat{B}$ (Given) $\therefore \hat{APQ} = \hat{B}$ $PQ \parallel BC$ (Corresponding angles =) $\frac{AB}{AP} = \frac{AC}{AQ}$ (Prop. Thm. $PQ \parallel BC$) $\frac{AB}{DE} = \frac{AC}{DF}$ (Construction $AP = DE$ and $AQ = DF$)	✓S Construction ✓S/R ✓S ✓S ✓S/R ✓S/R ✓R	[7]
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4.2



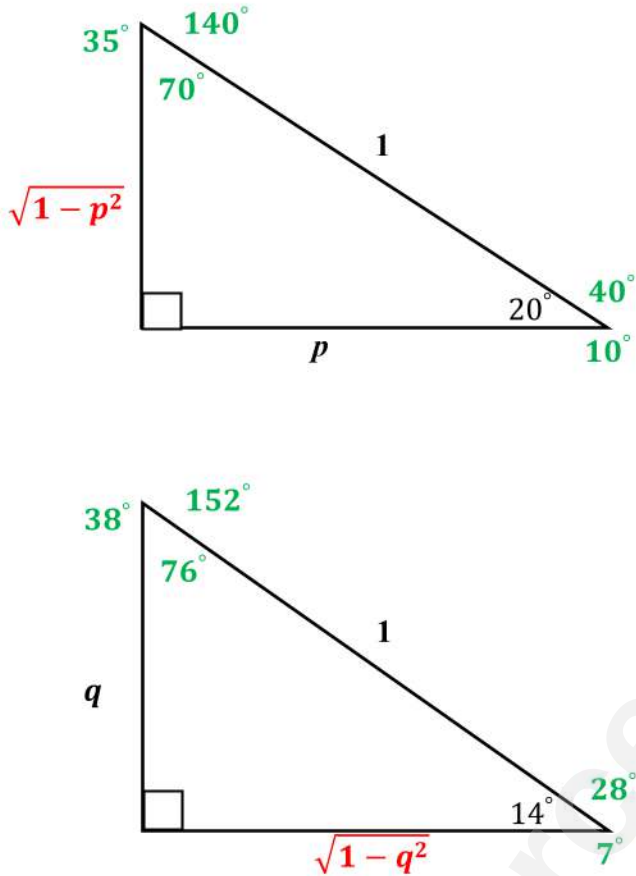
4.2.1	$\frac{AE}{15} = \frac{AD}{AC} = \frac{3}{5} \dots$ (Prop. Thm. $ED \parallel BC$) $AE = 9$ units	$\checkmark$ S $\checkmark$ R $\checkmark$ Answer	(3)
4.2.2	$\frac{AH}{15} = \frac{CG}{CB} = \frac{1}{3} \dots$ (Prop. Thm. $HG \parallel AC$) $AH = 5$ units	$\checkmark$ S $\checkmark$ R $\checkmark$ Answer	(3)
4.2.3	$HE = 4$ units and $BH = 6$ units $\frac{GK}{KH} = \frac{BE}{EH} = \frac{6}{4} = \frac{3}{2} \dots$ (Prop. Thm. $EK \parallel BG$)	$CA \checkmark HE = 4$ units $CA \checkmark S A \checkmark R$	(3)
4.2.4	$\frac{\Delta HEK}{\Delta HEG} = \frac{2}{5} \dots$ (Δ s with equal altitudes) $\frac{\Delta HEG}{\Delta HBG} = \frac{4}{10} = \frac{2}{5} \dots$ (Δ s with equal altitudes) $\frac{\text{Area of } \Delta HEK}{\text{Area of } \Delta HBG} = \frac{\Delta HEK}{\Delta HEG} \times \frac{\Delta HEG}{\Delta HBG} = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$ OR $\frac{\Delta HEK}{\Delta HBG} = \frac{\frac{1}{2} \cdot (4)(2c) \sin \widehat{E\hat{H}K}}{\frac{1}{2} \cdot (10)(5c) \sin \widehat{E\hat{H}K}}$ $\frac{\Delta HEK}{\Delta HBG} = \frac{4}{25}$ OR $\Delta HEK = \frac{2}{5} (\Delta HEG) \dots$ (Δ s with equal altitudes) $= \frac{2}{5} \left(\frac{2}{5} \Delta HBG \right) \dots$ (Δ s with equal altitudes) $\frac{\text{Area of } \Delta HEK}{\text{Area of } \Delta HBG} = \frac{4}{25}$	$\checkmark$ S/R $\checkmark$ S/R $\checkmark$ Answer OR $\checkmark$ Area of numerator $\checkmark$ Area of Denominator $\checkmark$ Answer OR $\checkmark$ S/R $\checkmark$ S/R $\checkmark$ Answer	(3) (3) (3)
			[19]

NSC – Marking Guideline



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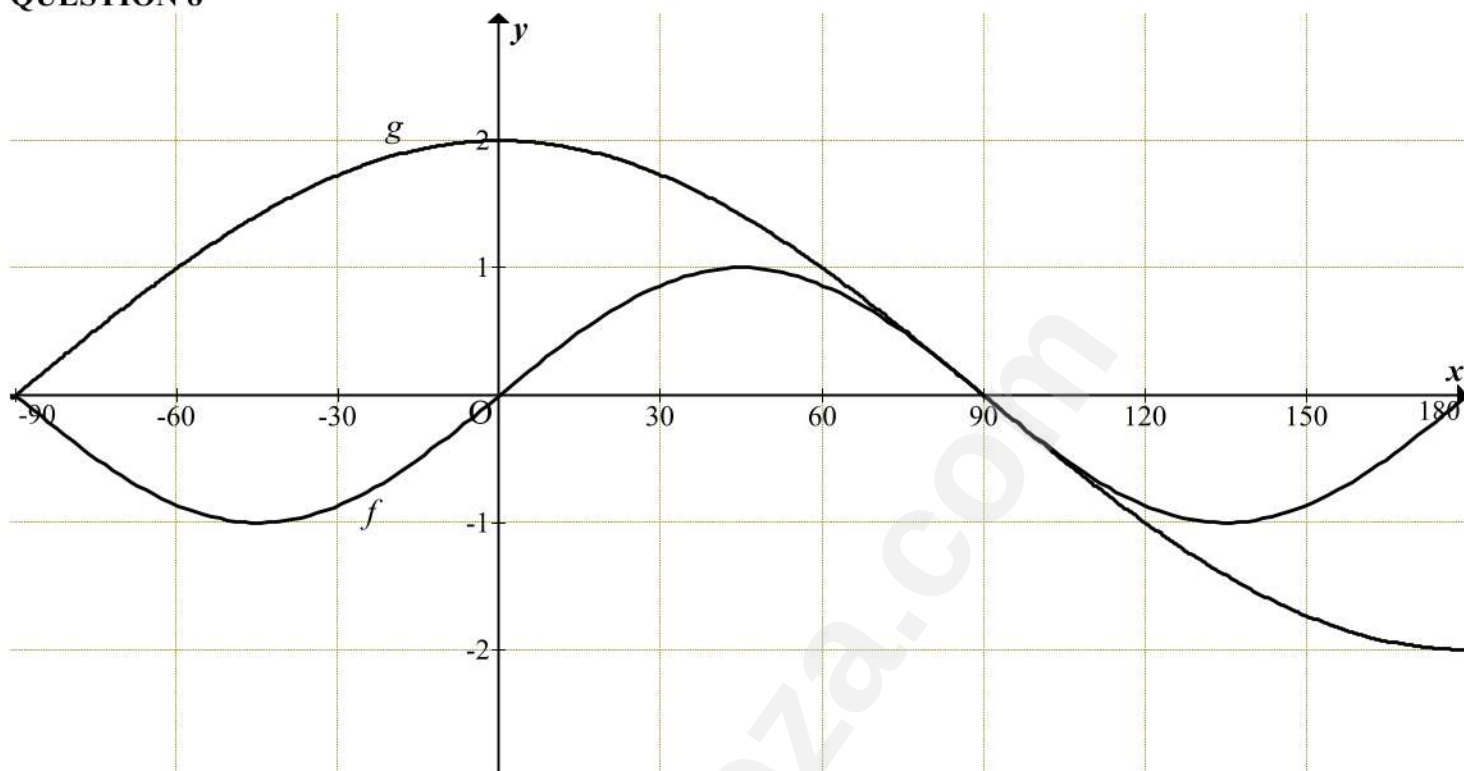
QUESTION 6

6.1.1	 $\sin 20^\circ = \sqrt{1 - p^2}$	M✓ A✓ Answer	(2)
6.1.2	$\begin{aligned}\cos 6^\circ &= \cos(20^\circ - 14^\circ) \\ &= \cos 20^\circ \cos 14^\circ + \sin 20^\circ \sin 14^\circ \\ &= p \cdot \sqrt{1 - q^2} + \sqrt{1 - p^2} \cdot q\end{aligned}$	A✓ Writing as difference A✓ Expansion A✓ ($\cos 20^\circ$) value CA ✓ $\sqrt{1 - q^2}$ CA ✓ $\sqrt{1 - p^2}$ A✓ ($\sin 14^\circ$) value	(6)
6.2	$\begin{aligned}&\sqrt{\frac{\frac{1}{2} \sin 2x}{\tan(540^\circ + x) \left(\frac{1}{\cos^2 x} - \tan^2 x \right)}} \\ &= \sqrt{\frac{\frac{1}{2} (2 \sin x \cos x)}{\tan x \cdot \left(\frac{1 - \sin^2 x}{\cos^2 x} \right)}} \\ &= \sqrt{\frac{\frac{1}{2} (2 \sin x \cos x)}{\frac{\sin x}{\cos x} \cdot \frac{\cos^2 x}{\cos^2 x}}} \\ &= \sqrt{\sin x \cos x \times \frac{\cos x}{\sin x}} \\ &= \sqrt{\cos^2 x} \\ &= \cos x\end{aligned}$	A✓ expansion of $\sin 2x$ A✓ reduction to $\tan x$ A✓ $1 - \sin^2 x = \cos^2 x$ A✓ $\tan x = \frac{\sin x}{\cos x}$ CA✓ simplifying $\sqrt{\cos^2 x}$ CA✓ answer	(6)
			[14]

QUESTION 7

7.1.1	$\cos 4x$ $= \cos(2 \cdot 2x)$ $= 2 \cos^2 2x - 1$ $= 2(2 \cos^2 x - 1)^2 - 1$ $= 2(4 \cos^4 x - 4 \cos^2 x + 1) - 1$ $= 8 \cos^4 x - 8 \cos^2 x + 1$	A✓ $\cos 4x = \cos (2 \cdot 2x)$ A✓ expansion of $\cos 4x$ A✓ expansion of $\cos 2x$ A✓ squaring bracket	(4)
7.1.2	$2 \cos 4x = 1$ $\cos 4x = \frac{1}{2}$ $4x = 60^\circ + 360k \text{ or } 4x = 300^\circ / -60^\circ + 360k$ $x = 15^\circ + 90k \text{ or } x = 75^\circ / -15^\circ + 90k ; k \in \mathbb{Z}$	A✓ setting up equation CA✓ both solutions A✓ $90k$ A✓ $k \in \mathbb{Z}$ CA✓ $x = 15^\circ, 75^\circ / -15^\circ$	(5)
7.1.3	$16 \cos^4 x - 16 \cos^2 x + 2$ $= 2 \cos 4x$ $\therefore \text{Minimum Value} = -2$	A✓ A✓	(2)
7.2	$\frac{2 \sin^2 22.5^\circ - 1}{4 \sin 22.5^\circ \cos 22.5^\circ}$ $= \frac{-(1 - 2 \sin^2 22.5^\circ)}{2 \cdot 2 \sin 22.5^\circ \cos 22.5^\circ}$ $= \frac{-\cos 45^\circ}{2 \cdot \sin 45^\circ}$ $= \frac{-1}{2} \times \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$ $= -\frac{1}{2} \times 1 = -\frac{1}{2}$ 	A✓ re-writing numerator and denominator A✓ Numerator A✓ Denominator A✓ Special angle values/ co - ratios A✓ Answer	(5)
			[16]

QUESTION 8



8.1	f : x -int., Maximum and Minimum and Shape g : y -int., Maximum and Minimum and Shape	AAA✓✓✓ AAA✓✓✓	(6)
8.2	$\frac{g(x)}{f(x)} \geq 1$ $g(x) \geq f(x); f(x) > 0$ $x \in (0^\circ; 180^\circ); x \neq 90^\circ$	AA✓✓ Answer	(2)
			[8]

Total Marks: 100