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Department:
Education
PROVINCE OF KWAZULU-NATAL

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

**MATHEMATICS
COMMON TEST
MARCH 2019**

MARKS: 100

TIME: 2 hours

This question paper consists of 9 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 9 questions.
2. Answers **ALL** questions.
3. Clearly show **ALL** calculations, diagrams, graphs, etc cetera that you have used in determining yours answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

Given the quadratic sequence: 2 ; 5 ; 10 ; 17 ; ...

1.1 Write down the next **two** terms of the quadratic sequence. (2)

1.2 Calculate the n^{th} term of the quadratic sequence. (4)

[6]

QUESTION 2

2.1 Given the combined constant and arithmetic sequences:

5 ; 4 ; 5 ; 7 ; 5 ; 10 ; ...

Determine the position of the term 1051 in the combined sequence. (3)

2.2 The series $3 + 8 + 13 + \dots$ consists of n terms. The sum of the last three terms is 699.

2.2.1 Determine the sum to n terms in terms of n . (2)

2.2.2 If the last three terms are excluded from the series, then determine in terms of n the sum of the remaining terms. (2)

2.2.3 Hence, or otherwise, determine the value of n . (3)

[10]

QUESTION 3

3.1 The sixth term of a geometric sequence is 80 more than the fifth term.

3.1.1 Show that $a = \frac{80}{r^5 - r^4}$. (2)

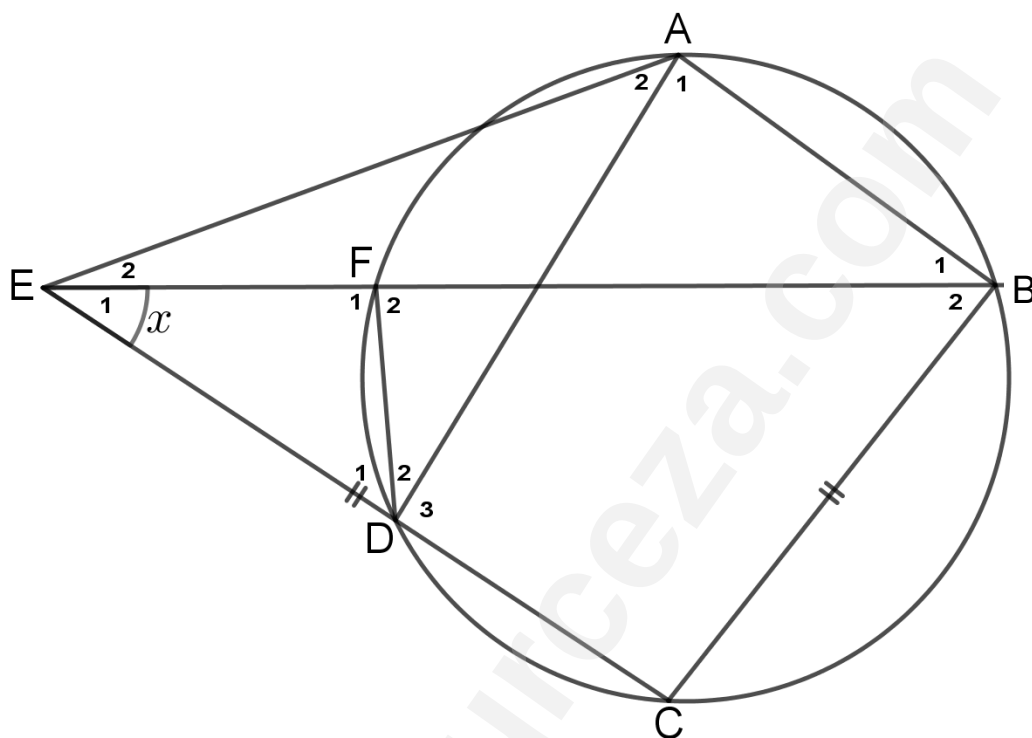
3.1.2 If it is further given that sum of the fifth and sixth terms is 240, determine the value of the common ratio. (5)

3.2 Write the geometric series $9 + 3 + 1 ; \dots$ to 130 terms in sigma notation. (2)

[9]

QUESTION 4

In the diagram below, $BC = CE$; $\hat{E}_1 = x$ and $\hat{D}_1 = \hat{D}_2$.



- 4.1 Name, with reasons, TWO other angles each equal to x and show that $FD = FE$. (4)
- 4.2 Prove that BF bisects $\hat{CBA}$. (4)
- 4.3 Hence, or otherwise, prove that $\hat{A}_1 = \hat{CBA}$. (4)

[12]

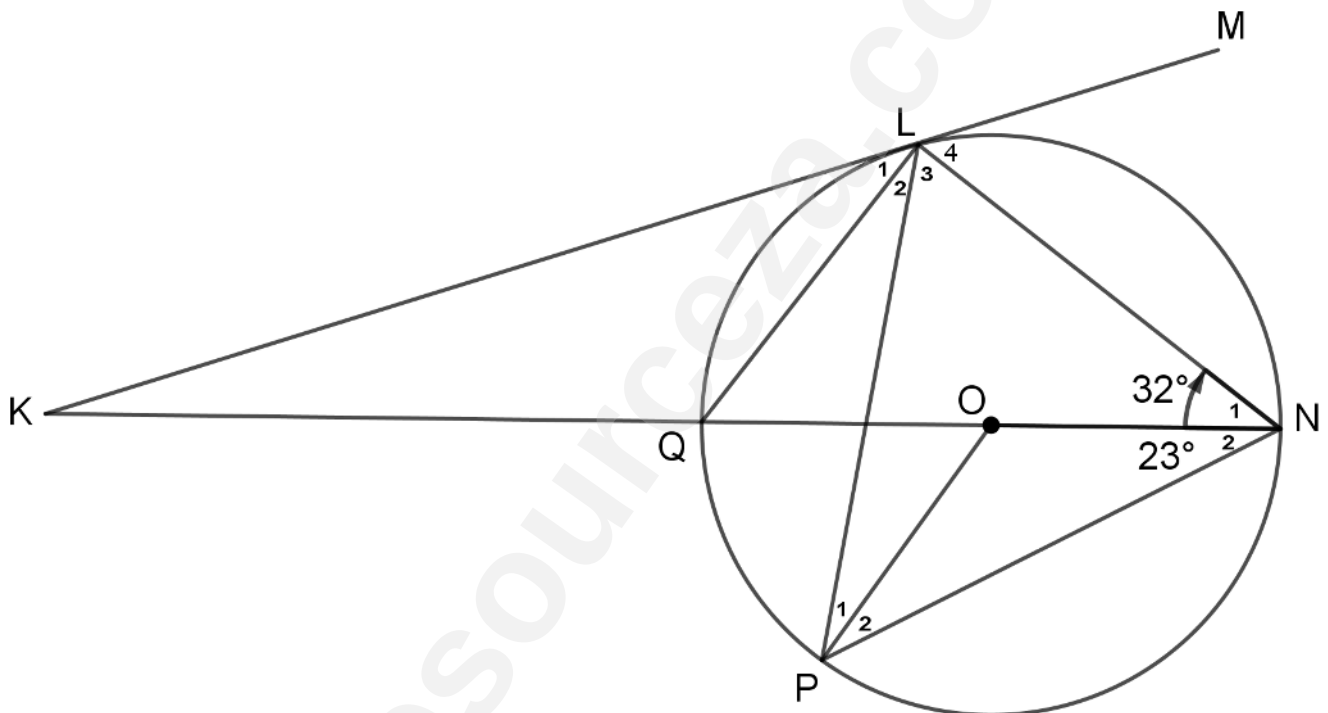
QUESTION 5

5.1 The angle at the point of contact between a tangent to a circle and a chord is ----- .

(1)

5.2 In the sketch below, circle centre O has a tangent KLM.
Diameter NQ produced meet the tangent in K.

$$\hat{N}_1 = 32^\circ \text{ and } \hat{N}_2 = 23^\circ.$$



Calculate, with reasons, the size of:

5.2.1 $\hat{P}_2$ (1)

5.2.2 $\hat{P}\hat{O}Q$ (2)

5.2.3 $\hat{L}_2$ (2)

5.2.4 $\hat{N}\hat{L}Q$ (1)

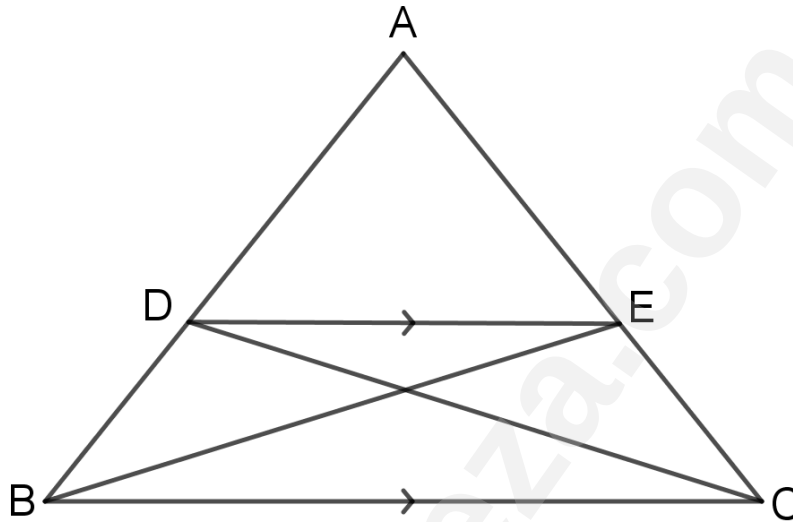
5.2.5 $\hat{L}_3$ (2)

5.2.6 $\hat{P}\hat{L}K$ (2)

[11]

QUESTION 6

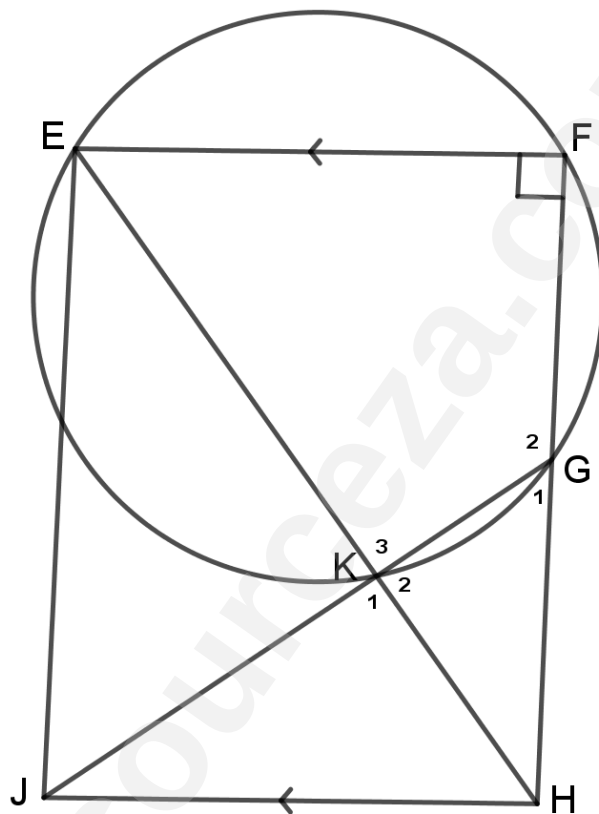
In the diagram below, $\triangle ABC$ has $DE \parallel BC$. Prove the theorem that states $\frac{AD}{DB} = \frac{AE}{EC}$.

**[7]**

QUESTION 7

In the diagram below, EFGK is a cyclic quadrilateral with $\hat{F} = 90^\circ$.

EK and FG are produced to meet at H. HJ is drawn parallel to FE. GK produced meets HJ at J.



7.1 Prove that:

7.1.1 $\hat{JHF} = 90^\circ$ (2)

7.1.2 $\hat{K}_2 = 90^\circ$ (2)

7.1.3 $\triangle HKG \parallel \triangle JHG$ (3)

7.2 Calculate JG and KG if HG = 5cm and JH = 10cm. (4)

[11]

QUESTION 8 (ANSWER THIS QUESTION WITHOUT THE USE OF CALCULATOR)

8.1 Show

$$\frac{\sin(90^\circ + x) \cos x \tan(-x)}{\cos(180^\circ + x)} = \sin x \quad (4)$$

8.2 If $\sin 36^\circ = m$, and $\cos 24^\circ = n$, determine the following in terms of m and /or n :

$$8.2.1 \quad \cos 36^\circ \quad (3)$$

$$8.2.2 \quad \sin 12^\circ \quad (4)$$

8.3 Simplify:

$$\frac{2 \cos 285^\circ \cos 15^\circ}{\cos(45^\circ - x) \cos x - \sin(45^\circ - x) \sin x} \quad (5)$$

8.4 Calculate the value of

$$(\sin 3x - \cos 3x)^2 \text{ if } \sin 6x = -\frac{2}{5} \quad (4)$$

[20]

QUESTION 9

Given $f(x) = \sin x + 1$ and $g(x) = \cos 2x$

9.1 Show that $f(x) = g(x)$ can be written as $(2 \sin x + 1) \sin x = 0$. (2)

9.2 Hence or otherwise determine the general solution of $\sin x + 1 = \cos 2x$. (6)

9.3 Write down the range of $g(x) - 1$. (1)

9.4 Consider the following geometric series

$$1 + 2 \cos 2x + 4 \cos^2 2x + \dots$$

Determine the values of x for the interval $0^\circ \leq x \leq 90^\circ$ for which the series will converge. (5)

[14]

TOTAL MARKS: [100]

INFORMATION SHEET: MATHEMATICS**INLIGTINGSBLAD: WISKUNDE**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

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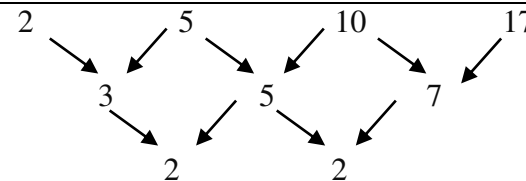
**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MARKS: 100

This marking guideline consists of 10 pages.

QUESTION 1

1.1	26 ; 37	AA✓✓ correct values	2
1.2	 $2a = 2$ $a = 1$ $3a + b = 3$ $b = 0$ $a + b + c = 2$ $c = 1$ $T_n = n^2 + 1$ OR $2a = 2$ $a = 1$ $3a + b = 3$ $b = 0$ $T_0 = c = 1$ $T_n = n^2 + 1$ OR $T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)}{2}d_2$ $= 2 + (n-1)(3) + \frac{(n-1)(n-2)}{2}(2)$ $= 2 + 3n - 3 + n^2 - 3n + 2$ $= n^2 + 1$ OR $T_n = \frac{n-1}{2}[2a + (n-2)d] + T_1$ $= \frac{n-1}{2}[2(3) + (n-2)(2)] + 2$ $= (n-1)[3 + n - 2] + 2$ $= (n-1)(n+1) + 2$ $= n^2 - 1 + 2 = n^2 + 1$	AA✓✓ correct values A✓ a value CA✓ b value CA✓ c value CA✓ general term OR A✓ a value CA✓ b value A✓ c value CA✓ general term OR A✓ formula A✓ substituting first and second difference values CA✓ simplifying CA✓ general term OR A✓ formula A✓ correct substitution into formula CA✓ simplifying CA✓ general term	2 4 4 4 4
			4
			[6]

QUESTION 2

2.1	$3n + 1 = 1051$ $3n = 1050$ $n = 350$ 1051 is in the 700 th position	A✓ equating n th term to 1051 CA✓ value of n CA✓ conclusion	3
2.2.1	$a = 3, d = 5$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{n}{2}[2(3) + (n-1)(5)]$ $= \frac{n}{2}[5n + 1]$	A✓ $d = 5$ CA✓ substitution of a & d into sum formula	2
2.2.2	$S_{n-3} = \frac{n-3}{2}[5(n-3) + 1]$ $= \frac{n-3}{2}[5n - 14]$	A✓ $n - 3$ CA✓ substituting into sum formula	2
2.2.3	$S_n - S_{n-3} = 699$ $\frac{n}{2}[5n + 1] - \frac{n-3}{2}[5n - 14] = 699$ $n(5n + 1) - (n-3)(5n - 14) = 1398$ $5n^2 + n - 5n^2 + 29n - 42 = 1398$ $30n = 1440$ $n = 48$ OR $T_n = 5n - 2$ $T_{n-1} = 5(n-1) - 2$ $T_{n-2} = 5(n-2) - 2$ $15n - 21 = 699$ $n = 45$	CA✓ forming equation CA✓ simplification CA✓ answer (n must be a natural number) OR CA✓ forming equations CA✓ simplification CA✓ answer (n must be a natural number)	3 3
			[10]

QUESTION 3

3.1.1	$T_6 = 80 + T_5$ $ar^5 = 80 + ar^4$ $ar^5 - ar^4 = 80$ $a(r^5 - r^4) = 80$ $a = \frac{80}{r^5 - r^4}$	A✓ forming equation A✓ factorizing	2
3.1.2	$T_5 + T_6 = 240$ $ar^4 + ar^5 = 240$ $a(r^4 + r^5) = 240$ $a = \frac{240}{r^4 + r^5}$ $\frac{80}{r^5 - r^4} = \frac{240}{r^5 + r^4}$ $80r^5 + 80r^4 = 240r^5 - 240r^4$ $320r^4 = 160r^5$ $r = 2$ OR Let the fifth term be = x and sixth term = y, then $x + y = 240 \rightarrow (1)$ $-x + y = 80 \rightarrow (2)$ $(1) + (2)$ $2y = 320$ $y = 160; x = 80$ $r = \frac{160}{80} = 2$	A✓ forming equation CA✓ factorizing CA✓ equating CA✓ simplifying CA✓ r – value OR A✓ forming equation (1) A✓ forming equation (2) CA✓ y – value CA✓ x – value CA✓ r – value	5
3.2	$\sum_{k=1}^{130} 9 \left(\frac{1}{3}\right)^{k-1}$	A✓ upper and lower limit values A✓ k^{th} term	2
			[9]

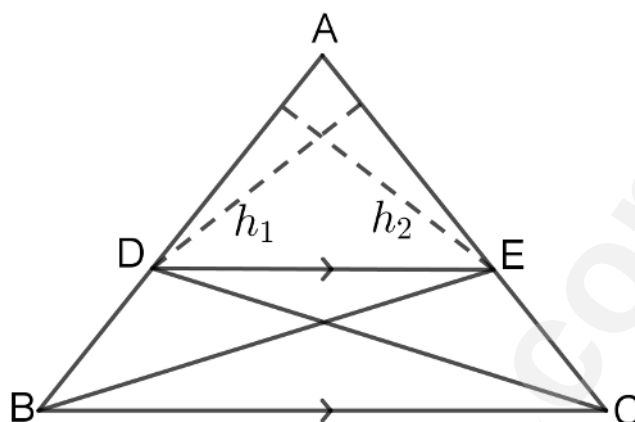
QUESTION 4

4.1	$\hat{B}_2 = \hat{E}_1 = x \dots\dots\dots (BC = CE)$ $\hat{D}_1 = \hat{B}_2 = x \dots$ (Ext. $\angle$ of cyclic quad = interior opposite angles) $\therefore \hat{D}_1 = \hat{E}_1$ $\therefore FD = FE$	$A \checkmark$ S/R $A \checkmark$ S $A \checkmark$ R $A \checkmark$ S	(4)
4.2	$\hat{D}_2 = \hat{D}_1 = x \dots\dots\dots$ given $\hat{D}_2 = \hat{B}_1 = x \dots$ subtended by arc AF $\therefore \hat{B}_1 = \hat{B}_2$ $\therefore EB$ bisects $C\hat{B}A$	$A \checkmark$ S $A \checkmark$ S $A \checkmark$ R $A \checkmark$ S	(4)
4.3	$\hat{A}_1 = \hat{F}_2 \dots\dots\dots$ subtended by arc $\hat{F}_2 = 2x \dots\dots$ ext. $\angle$ of $\triangle DFE$ $\therefore \hat{A}_1 = 2x = C\hat{B}A$	$A \checkmark$ S $A \checkmark$ R $A \checkmark$ S $A \checkmark$ R	(4)
			[12]

QUESTION 5

5.1	 equal to the angle subtended by the chord in the opposite circle segment.	A✓S	(1)
5.2.1	$\hat{P}_2 = 23^\circ$ (ON = OP; radii)	A✓ S/R	(1)
5.2.2	$P\hat{O}Q = 2\hat{N}_2 = 46^\circ$ ($\angle$ at centre) / (Ext $\angle$ of Δ)	A✓ S A✓R	(2)
5.2.3	$\hat{L}_2 = \hat{N}_2 = 23^\circ$ (subt. by arc PQ)	A✓ S A✓R	(2)
5.2.4	$N\hat{L}Q = 90^\circ$ (subt. by diameter NQ)	A✓S/R	(1)
5.2.5	$\hat{L}_3 = 90^\circ - 23^\circ$ $= 67^\circ$ OR $P\hat{O}N = 134^\circ$ (angles of triangle) $P\hat{O}N = 2\hat{L}$ (angle at centre theorem) $= 67^\circ$	CA✓ S CA✓ answer OR CA✓ S CA✓ answer	(2) (2)
5.2.6	$P\hat{L}K = L\hat{N}P$ (tan-chord theorem) $= 32^\circ + 23^\circ$ $= 55^\circ$	A✓ S/R A✓ answer	 (2)
			[11]

QUESTION 6

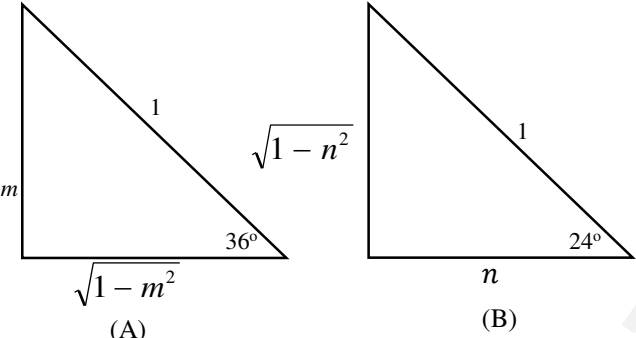


Construction: Draw line $h_1 \perp AC$ and $h_2 \perp AB$ Proof: $\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle BDE} = \frac{\frac{1}{2} \cdot AD \cdot h_2}{\frac{1}{2} \cdot DB \cdot h_2} \quad \text{same height } h_2$ and $\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle CED} = \frac{\frac{1}{2} \cdot AE \cdot h_1}{\frac{1}{2} \cdot EC \cdot h_1} \quad \text{same height } h_1$ but $\text{area of } \triangle BDE = \text{area of } \triangle CED$ $\therefore (\text{same base } DE; \text{ the same height; } DE \parallel BC)$ $\therefore \frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle BDE} = \frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle CED}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$	A✓ construction A✓ S A✓ R A✓ S/R AA✓ S/✓ R A✓ S	[7]
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QUESTION 7

7.1			
7.1.1	$J\hat{H}F + \hat{F} = 180^\circ \dots$ Co-interior angles; JH // EF $\therefore J\hat{H}F = 90^\circ \dots \hat{F} = 90^\circ$ (given)	A✓ S/R A✓ S	(2)
7.1.2	$\hat{K}_2 = \hat{F} = 90^\circ \dots$ ext. $\angle$ of cyclic quad	AA✓✓ S/R	(2)
7.1.3	In $\triangle HKG$ and $\triangle JHG$ $\widehat{G}_1 = \widehat{G}_1 \dots$ common $\hat{K}_2 = J\hat{H}G = 90^\circ \dots$ proved $\therefore K\hat{H}G = H\hat{J}G \dots$ remaining $\angle$ $\therefore \triangle HKG \sim \triangle JHG \dots (\angle\angle\angle)$	A✓ S/R A✓ S/R A✓ R	(3)
7.2	$JG^2 = HJ^2 + HG^2 \dots$ Pythagoras $= 10^2 + 5^2$ $= 125\text{cm}^2$ $JG = \sqrt{125\text{cm}^2}$ $= 5\sqrt{5}\text{cm}$ $\frac{KG}{HG} = \frac{HG}{JG}$ $KG = \frac{HG^2}{JG}$ $= \frac{5^2}{5\sqrt{5}}$ $= \frac{5}{\sqrt{5}}$ $= \frac{5\sqrt{5}}{5}$ $= \sqrt{5}\text{ cm}$ $= 2,24\text{ cm}$	A✓ $5\sqrt{5}$ A✓ $\triangle HKG \sim \triangle JHG$ CA✓ substitution CA✓ answer in any form	(4) [11]

QUESTION 8

8.1	$\frac{(\cos x)(\cos x)(-\tan x)}{-\cos x}$ $= \frac{(\cos x)(\cos x) \left(\frac{\sin x}{\cos x} \right)}{\cos x}$ $= \sin x$	A✓ $-\tan x$ A✓ $\cos x$ A✓ $\frac{\sin x}{\cos x}$ A✓ $\cos x$	(4)
8.2 8.2.1	 (A) $\cos 36^\circ = \sqrt{1 - m^2}$ (B)	A✓ A ✓ diagrams CA✓ $\sqrt{1 - m^2}$ answer	(3)
8.2.2	$\sin 12^\circ = \sin (36^\circ - 24^\circ)$ $= \sin 36^\circ \cos 24^\circ - \cos 36^\circ \sin 24^\circ$ $= m n - (\sqrt{1 - m^2})(\sqrt{1 - n^2})$ OR $\sin 12^\circ = \sqrt{\frac{1 - \cos 24^\circ}{2}}$ $= \sqrt{\frac{1 - n}{2}}$	A✓ compound angle A✓ expansion CACA✓✓ answer OR A✓ A✓ CACA✓✓ answer	(4) (4)
8.3	$\frac{2 \cos 75^\circ \cos 15^\circ}{\cos[45^\circ - x + x]}$ $= \frac{2 \sin 15^\circ \cos 15^\circ}{\cos 45^\circ}$ $= \frac{\sin 30^\circ}{\cos 45^\circ}$ $= \frac{1}{2} \div \frac{1}{\sqrt{2}}$ $= \frac{\sqrt{2}}{2}$	A✓ $\cos 75^\circ$ A✓ $\cos[45^\circ]$ A✓ $\sin 15^\circ$ A✓ $\frac{\sin 30^\circ}{\cos 45^\circ}$ CA✓ answer in any form	(5)
8.4	$\sin^2 3x - 2 \sin 3x \cos 3x + \cos^2 3x$ $= 1 - \sin 2(3x)$ $= 1 - \sin 6x$ $= 1 - \left(\frac{-2}{5} \right)$ $= 1 \frac{2}{5}$	A✓ expansion A✓ $1 - \sin 2(3x)$ A✓ $1 - \left(\frac{-2}{5} \right)$ A✓ answer in any form	(4)
			[20]

QUESTION 9

9.1	$\sin x + 1 = 1 - 2\sin^2 x$ $2\sin^2 x + \sin x = 0$ $\sin x (2\sin x + 1) = 0$	A✓equating A✓standard form	(2)
9.2	$\sin x (2\sin x + 1) = 0$ $\sin x = 0$ or $2\sin x = -1$ $x = 0^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ $x = 180^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ $\sin x = -\frac{1}{2}$ $x = 210^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ $x = 330^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$	A✓ $\sin x = 0$ A✓ $\sin x = -\frac{1}{2}$ A✓ $0^\circ + k \cdot 360^\circ$ A✓ $k \in \mathbb{Z}$ CA✓ $210^\circ + k \cdot 360^\circ$ CA✓ $330^\circ + k \cdot 360^\circ$	(6)
9.3	$-2 \leq y \leq 0$	A✓answer	(1)
9.4	$r = 2 \cos 2x$ $-1 < 2 \cos 2x < 1$ $\frac{-1}{2} < \cos 2x < \frac{1}{2}$; If $\cos 2x = \frac{1}{2}$ $\therefore 2x = 60^\circ$ then $x = 30^\circ$ $\therefore 30^\circ < x < 90^\circ$	A✓ $r = 2 \cos 2x$ A✓ $-1 < r < 1$ CA✓ $\frac{-1}{2} < \cos 2x < \frac{1}{2}$ CA✓ $x = 30^\circ$ CA✓answer $30^\circ < x < 90^\circ$	(5)
			[14]

TOTAL MARKS: [100]