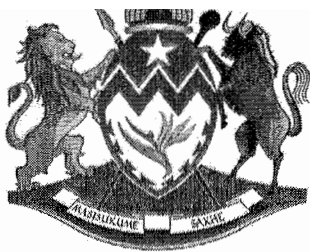




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Basic Education

KwaZulu-Natal Department of Basic Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS

COMMON TEST

MARCH 2016

NATIONAL
SENIOR CERTIFICATE

GRADE 12

Marks: 100

Time: 2 hours

N.B. This question paper consists of 7 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 8 questions.
2. Answer **ALL** questions.
3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

1.1 Given: $f(x) = \frac{-5}{x}$, determine $f'(x)$ from first principles. (5)

1.2 Determine:-

1.2.1 $\frac{dy}{dx}$ if $y = (x^2 + 3)\sqrt{x}$ (4)

1.2.2 $f'(x)$ if $f(x) = \frac{x^3 - 8}{2 - x}$ (4)

[13]

QUESTION 2

Given the equation of the curve $h(x) = x^3 + 3x^2 + 15x$

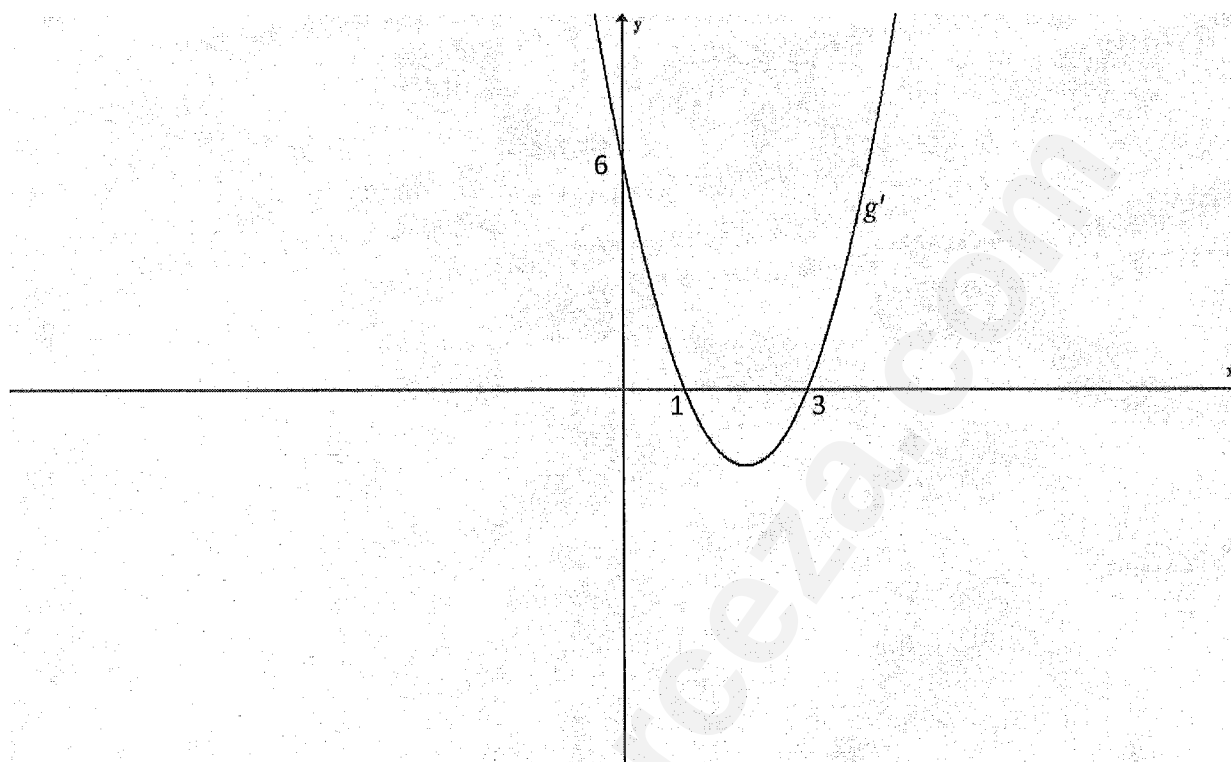
2.1 Determine the equation of the tangent to h at $x = -3$. (5)

2.2 Show that the function h is increasing for all real value(s) of x . (5)

[10]

QUESTION 3

The sketch below represents $g'(x) = ax^2 + bx + c$ with $g'(1) = g'(3) = 0$ and $g'(0) = 6$.

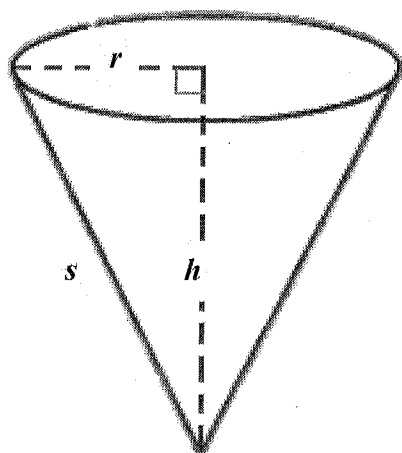


- 3.1 Write down the x -coordinates of the stationary points of g . (2)
- 3.2 Determine the value of x , for which the gradient of the curve is 6, except $x = 0$. (3)
- 3.3 For which value(s) of x is the graph of g strictly increasing? (3)
- 3.4 Calculate the x - coordinate of the point of inflection of g . (1)
- 3.5 If it is further given that $g(-1) = \frac{4}{3}$ and g is a cubic function.
 $g(x) = ax^3 + bx^2 + cx + d$, calculate the value of the y - intercept of g . (7)
- 3.6 Determine the value(s) of x for which g is concave up. (2)

[18]

QUESTION 4

A right circular cone with perpendicular height (h cm), radius (r cm) has a slant height(s) of 12 cm.



$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{4}{3} \pi r^3$$

$$A = \pi r^2 + \pi r s$$

$$A = 4\pi r^2$$

4.1 Express the radius of the cone in terms h . (2)

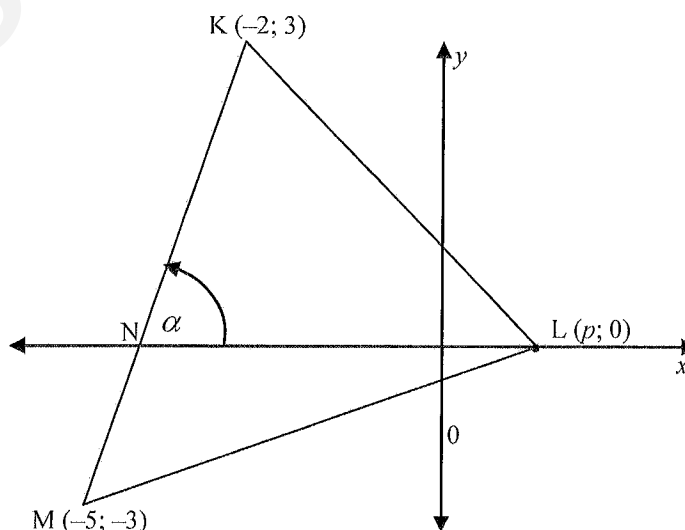
4.2 Show that the Volume of the cone is given by $V = 48\pi h - \frac{1}{3}\pi h^3$. (2)

4.3 Calculate the height of the cone for which the volume is at a maximum. (5)

[9]

QUESTION 5

In the sketch alongside, points $K(-2;3)$, $L(p;0)$ and $M(-5;-3)$ are vertices of $\triangle KLM$ in a Cartesian plane. KM cuts the x -axis at N and $\hat{KNL} = \alpha$.



5.1.1 Determine the equation of the line KM . (3)

5.1.2 Hence, calculate the co-ordinates of N (1)

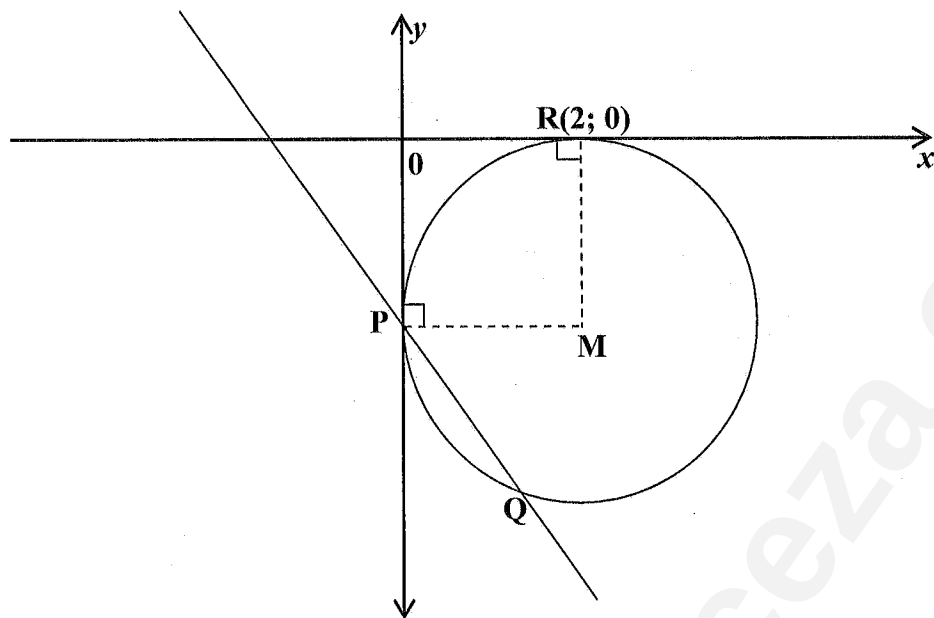
5.2 Calculate the value of p if $KM = LM$ (4)

5.3 Determine α , correct to ONE decimal digit. (2)

[10]

QUESTION 6

In the sketch below, the circle with centre M touches the y -axis at P and the x -axis at R(2;0). The straight line defined by the equation $y = -x - 2$ cuts the circle at point Q and passes through point P.



- 6.1 Write down the co-ordinates of P. (1)
- 6.2 Write down the co-ordinates of M, the centre of the circle. (1)
- 6.3 Show that the equation of the circle with centre M is : $x^2 + y^2 - 4x + 4y + 4 = 0$. (5)
- 6.4 The straight line with equation $y = -x + c$ is a tangent to the circle with centre M. Calculate the numerical values of c . (5)
- [10]**

QUESTION 7

- 7.1 If $4 \tan \alpha - 3 = 0$ and $90^\circ \leq \alpha \leq 360^\circ$, determine without the use of a calculator the value of $\cos^2 \alpha - \sin \alpha$. (5)
- 7.2 Simplify, without using a calculator:
- 7.2.1 $\frac{\sin 61^\circ \cdot \sin (90^\circ - \theta)}{\cos 29^\circ \cdot \sin (180^\circ - \theta)}$ (4)
- 7.2.2 $\sin 15^\circ \cos 15^\circ$ (3)
- 7.3 Prove the following identity:

$$\frac{\sin A - \cos A}{\sin A + \cos A} + \frac{\sin A + \cos A}{\sin A - \cos A} = \frac{-2}{\cos 2A} \quad (6)$$

[18]

QUESTION 8

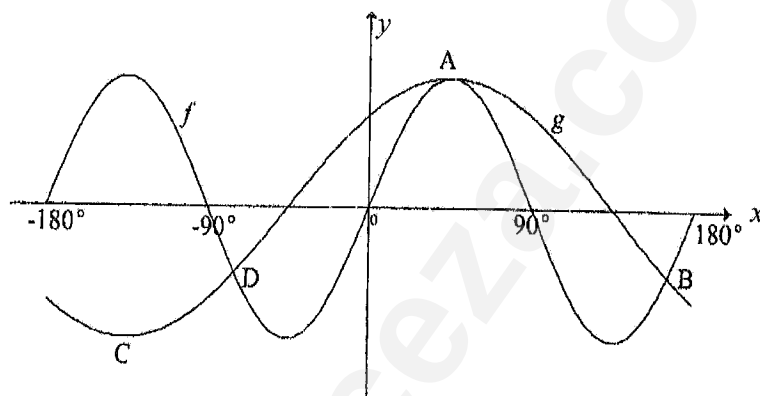
8.1 Determine the general solution of $\cos 2x + \cos x - 2 = 0$ (5)

8.2 The sketch below, shows the graphs of :

$$f = \{(x; y) / y = \sin px\} \text{ and}$$

$$g = \{(x; y) / y = \cos(x + q); x \in [-180^\circ; 180^\circ]\}$$

A $(45^\circ; 1)$ and B $(165^\circ; -\frac{1}{2})$ are two points of intersection of f and g .



8.2.1 Determine the value(s) of p and q . (4)

8.2.2 What is the period of g ? (1)

8.2.3 Write down the co-ordinates of C, the turning point of the curve g . (1)

8.2.4 Write down the co-ordinates of D, a point of intersection of f and g . (1)

[12]

TOTAL: 100

INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1-r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta + y \sin \theta; y \cos \theta - x \sin \theta)$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

Basic EducationKwaZulu-Natal Department of Education
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MATHEMATICS

COMMON TEST

MARCH 2016

MEMORANDUM

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MARKS: 100

This memorandum consists of 11 pages.

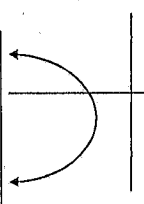
QUESTION 1

1.1	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-5}{x+h} - \frac{-5}{x}$ $= \lim_{h \rightarrow 0} \frac{-5x + 5x + 5h}{x(x+h)}$ $= \lim_{h \rightarrow 0} \frac{5h}{x(x+h)} \times \frac{1}{h}$ $= \frac{5}{x^2}$ Penalise 1 mark for incorrect notation	✓ Formula ✓ Substitution ✓ Simplifying ✓ Simplifying ✓ Answer	(5)
1.2.1	$y = x^2 + 3x^2$ $\frac{dy}{dx} = \frac{5}{2}x^2 + \frac{3}{2}x^{-\frac{1}{2}}$ OR $y = (x^2 + 3)^{\frac{1}{2}}$ $\frac{dy}{dx} = (x^2 + 3)^{-\frac{1}{2}} \times \frac{1}{2} \times 2x$ $= \frac{1}{2}x^{\frac{1}{2}} + \frac{3}{2}x^{-\frac{1}{2}} + 2x^{\frac{3}{2}}$ $= \frac{5}{2}x^{\frac{1}{2}} + \frac{3}{2}x^{-\frac{1}{2}}$ Product Rule	✓✓ each term ✓✓ each answer ✓✓ $(x^2 + 3)^{-\frac{1}{2}} \times \frac{1}{2} \times x^{\frac{1}{2}}$ ✓✓ $x^{\frac{1}{2}} \times 2x$	(4)
1.2.2	$f(x) = \frac{(x-2)(x^2 + 2x + 4)}{-(x-2)}$ $= -x^2 - 2x - 4$ $f'(x) = -2x - 2$ If second bracket is linear - break down	✓ Factorising numerator ✓ $-x^2 - 2x - 4$ ✓ Answers	(4) [13]

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QUESTION 2

2.1	$h'(x) = 3x^2 + 6x + 15$ $h'(-3) = 3(-3)^2 + 6(-3) + 15 = 24$ $h(-3) = (-3)^3 + 3(-3)^2 + 15(-3) = -45$ $y - y_1 = m(x - x_1)$ $y + 45 = 24(x + 3)$ $y = 24x + 27$ OR $h'(x) = 3x^2 + 6x + 15$ $h'(-3) = 3(-3)^2 + 6(-3) + 15 = 24$ $h(-3) = (-3)^3 + 3(-3)^2 + 15(-3) = -45$ $y = mx + c$ $-45 = 24(-3) + c$ $c = 27$ $y = 24x + 27$	✓ derivative ✓ gradient value ✓ y - value ✓ substitution of gradient = 24 and point (-3, -45) ✓ answer (5) OR ✓ derivative ✓ gradient value ✓ y - value ✓ substitution of gradient = 24 and point (-3, -45) ✓ answer (5)
2.2	$h'(x) = 3x^2 + 6x + 15$ $= 3x^2 + 6x + 3 + 12$ $= 3(x^2 + 2x + 1) + 12$ $= 3(x+1)^2 + 12$ $3(x+1)^2 \geq 0$ for all $x \in \mathbb{R}$ $\Rightarrow 3(x+1)^2 + 12 \geq 12$ $\therefore h'(x) \geq 12 \Rightarrow h'(x) > 0$ h is increasing for all values of x OR $h'(x) = 3x^2 + 6x + 15$ $= 3[x^2 + 2x + 5]$ $= 3[x^2 + 2x + 1 - 1 + 5]$ $= 3[(x+1)^2 + 4]$ $= 3(x+1)^2 + 12$ $3(x+1)^2 \geq 0$ for all $x \in \mathbb{R}$ $\Rightarrow 3(x+1)^2 + 12 \geq 12$ $\therefore h'(x) \geq 12 \Rightarrow h'(x) > 0$ h is increasing for all values of x	Graph Method full marks if concluded $h'(x) > 0$  writing 15 as 3+12 factorising writing as perfect square $3(x+1)^2 \geq 0$ proving $h'(x) > 0$ (5)

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QUESTION 3

3.1	$x = 1$ or $x = 3$	✓ answers ✓ $x = 2$	(2)
3.2	Midpoint of x values of TP's $x = \frac{1+3}{2} = 2$ Using symmetry $x = 4$	Answer only full marks	
3.3	$x < 1$ or $x > 3$	✓ Using symmetry ✓ answer ✓ $x < 1$ ✓ $x > 3$ ✓ or	(3)
3.4	$x = 2$	✓ answer	(1)

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QUESTION 6

6.1	P(0; -2)	✓ correct co-ordinates	(1)
6.2	M(2; -2)	✓ correct co-ordinates	(1)
6.3	Centre M(2; -2) and radius 2 units $(x - 2)^2 + (y + 2)^2 = (2)^2$ $x^2 - 4x + 4 + y^2 + 4y + 4 = 4$ $x^2 + y^2 - 4x + 4y + 8 = 4$ $x^2 + y^2 - 4x + 4y + 4 = 0$	✓ substitution of centre ✓ substitution of radius $x^2 + y^2 - 4x + 4y + 8 = 4$	(3)
6.4	$y = -x + c$ substituting $y = -x + c$ into the equation of the circle $x^2 + (-x + c)^2 - 4x + 4(-x + c) + 4 = 0$ $x^2 + x^2 - 2xc + c^2 - 4x - 4x + 4c + 4 = 0$ $2x^2 - 2xc + c^2 - 8x + 4c + 4 = 0$ $2x^2 - (2c + 8)x + (c^2 + 4c + 4) = 0$ Since the line is a tangent to the circle, the above equation must have equal root ($\Delta = 0$) $\Delta = b^2 - 4ac = 0$ $0 = (2c + 8)^2 - 4(2)(c^2 + 4c + 4)$ $0 = 4c^2 + 32c + 64 - 8c^2 - 32c - 32$ $0 = -4c^2 + 32$ $\frac{4c^2}{4} = \frac{-32}{4}$ $c^2 = 8$ $c = \pm\sqrt{8}$ $= \pm 2\sqrt{2}$ or $\pm 2,82$ OR Let points of contact of the tangent with the circle be W and W'	✓ Subst $y = -x + c$ ✓ writing the equation in a standard form ✓ using $\Delta = 0$ ✓ subst ✓ values of c	(5)

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QUESTION 5

5.1.1	$\text{mx}_1 = \frac{-3-3}{-5-(-2)} = \frac{-6}{-3} = 2$ $y = 2x + c$ $(-2; 3): 3 = 2(-2) + c$ $7 = c$ $y = 2x + 7$	✓ gradient ✓ substitution ✓ equation of the line	(3)
5.1.2	N($-\frac{7}{2}; 0$)	✓ Co-ordinates of N	(1)
5.2	$KM^2 = LM^2$ $(-5 - (-2))^2 + (-3 - 3)^2 = (-5 - p)^2 + (-3 - 0)^2$ $(-5 + 2)^2 + (-6)^2 = (-5 - p)^2 + (3)^2$ $9 + 36 = (-5 - p)^2 + 9$ $45 = (-5 - p)^2 + 9$ $(-5 - p)^2 = 45 - 9$ $(-5 - p)^2 = 36$ $25 + 10p + p^2 = 36$ $p^2 + 10p + 25 - 36 = 0$ $p^2 + 10p - 11 = 0$ $(p + 11)(p - 1) = 0$ $p = -11$ or $p = 1$ $\therefore p = 1$	✓ substitution ✓ simplifying the square ✓ simplifying ✓ $p = 1$	(4)
5.3	$\text{th}_{KM} = 2 = \tan \alpha$ $\tan^{-1} \alpha = 2$ $\alpha = 63,4^\circ$	✓ $\tan^{-1} \alpha = 2$ ✓ answer	(2)
			[10]

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m of tangent = -1 m of radius = 1 $MS = WS = a$ $a^2 + a^2 = 2^2$ $2a^2 = 4$ $a^2 = 2$ $a = \pm\sqrt{2}$ $\therefore W(2-\sqrt{2}, -2-\sqrt{2})$ and $W'(2+\sqrt{2}, -2+\sqrt{2})$ For tangent at W: $y = -x + c$ $-2 - \sqrt{2} = -(2 - \sqrt{2}) + c$ $c = -2\sqrt{2}$ For tangent at W': $y = -x + c$ $-2 + \sqrt{2} = -(2 + \sqrt{2}) + c$ $c = 2\sqrt{2}$	✓ gradient of radius ✓ theorem of pythagoras ✓ both values of a ✓ substitution into the equation of the tangent ✓ both answers (5)
---	--

7.1 $4 \tan \alpha - 3 = 0$ $4 \tan \alpha = 3$ $\tan \alpha = \frac{3}{4}$ and $90^\circ \leq \alpha \leq 360^\circ$ implies that α lies in the third quadrant. $\therefore \tan \alpha = \frac{3}{4} = \frac{-3}{-4}$ $\therefore x = -3$ and $y = -4$ $r^2 = (-3)^2 + (-4)^2 = 25$ $\therefore r = 5$ $\cos^2 \alpha - \sin \alpha$ $= \left(\frac{-4}{5}\right)^2 - \left(\frac{-3}{5}\right)$ $= \frac{16}{25} + \frac{3}{5}$ $= \frac{16 + 15}{25}$ $= \frac{31}{25}$ or $1 \frac{6}{25}$	✓ correct quadrant ✓ $r = 5$ ✓ subst ✓ simplifying ✓ answer (5)	
7.2.1 $\frac{\sin 61^\circ \cdot \sin(90^\circ - \theta)}{\cos 29^\circ \cdot \sin(180^\circ - \theta)}$ $= \frac{\sin(90^\circ - 29^\circ) \cdot \sin(90^\circ - \theta)}{\cos 29^\circ \cdot \sin \theta}$ $= \frac{\cos 29^\circ \cdot \cos \theta}{\cos 29^\circ \cdot \sin \theta}$ $= \frac{1}{\tan \theta}$	✓ $\cos 29^\circ$ in the numerator ✓ reduction of $\cos \theta$ ✓ reduction of $\sin \theta$ ✓ $\frac{1}{\tan \theta}$ (4)	
7.2.2 $\sin 15^\circ \cdot \cos 15^\circ$ $= \frac{1}{2} (2 \sin 15^\circ \cdot \cos 15^\circ)$ $= \frac{1}{2} \sin 30^\circ$ $= \frac{1}{2} \cdot \frac{1}{2}$ $= \frac{1}{4}$	✓ writing as double angle ✓ $\sin 30^\circ = \frac{1}{2}$ ✓ answer (3)	

Mathematics	11	NSC-MEMORANDUM	March 2016 Common Test	
7.3	$\frac{\sin A - \cos A + \sin A + \cos A}{\sin A - \cos A + \sin A - \cos A} = \frac{-2}{\cos 2A}$ $\text{LHS} = \frac{\sin A - \cos A}{\sin A - \cos A} + \frac{\sin A + \cos A}{\sin A - \cos A}$ $= \frac{(\sin A - \cos A)(\sin A - \cos A) + (\sin A + \cos A)(\sin A + \cos A)}{(\sin A - \cos A)(\sin A - \cos A)}$ $= \frac{\sin^2 A - 2\sin A \cos A + \cos^2 A + \sin^2 A + 2\sin A \cos A + \cos^2 A}{(\sin A - \cos A)(\sin A - \cos A)}$ $= \frac{2\sin^2 A + 2\cos^2 A}{\sin^2 A - \cos^2 A}$ $= \frac{2(\sin^2 A + \cos^2 A)}{-(\cos^2 A - \sin^2 A)}$ $= \frac{-2(1)}{\cos 2A}$ $= \frac{-2}{\cos 2A}$ $= \text{RHS}$		✓ adding using LCD ✓ simplifying ✓ simplifying denominator ✓ factorising numerator ✓ $\sin^2 A + \cos^2 A = 1$ ✓ writing $1 - 2\cos^2 A$ as $-\cos 2A$	(6)
[18]				

QUESTION 8

8.1	$2\cos^2 x - 1 + \cos x - 2 = 0$ $2\cos^2 x + \cos x - 3 = 0$ $(2\cos x + 3)(\cos x - 1) = 0$ $\cos x = \frac{-3}{2} \quad \text{or} \quad \cos x = 1$ N/A $x = 0^\circ + n \cdot 360^\circ, n \in \mathbb{Z}$	✓ $2\cos^2 x - 1$ ✓ factorising ✓ $\cos x = 1$ ✓ rejecting the solution ✓ $x = n \cdot 360^\circ, n \in \mathbb{Z}$	(5)
8.2.1	Substituting $A(45^\circ; 1)$ in $f: y = \sin px$ $1 = \sin(p \cdot 45^\circ)$ $p \cdot 45^\circ = 90^\circ$ $\therefore p = 2$	✓ subst ✓ $p = 2$	
8.2.2	Substituting $A(45^\circ; 1)$ in $g: y = \cos(x+q)$ $1 = \cos(45^\circ + q)$ $\therefore 45^\circ + q = 0^\circ$ $\therefore q = -45^\circ$	✓ subst ✓ $q = -45^\circ$	(4)
8.2.3	360° $C(-135^\circ; -1)$	✓ 360° ✓ both co-ordinates	(1)
8.2.4	$D(-75^\circ; -\frac{1}{2})$	✓ both co-ordinates	(1)
[12]			

TOTAL MARKS: 100

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TAXONOMY GRID
COMMON TEST – MARCH 2016
MATHEMATICS GRADE 12

QUESTION NUMBER	LEVEL	LEVEL 2	LEVEL 3	LEVEL 4
1.1		5		
1.2.1		4		
1.2.2		4		
2.1		5		
2.2				5
3.1	2			
3.2	3			
3.3			3	
3.4	1			
3.5			7	
4.1	2			
4.2	2			
4.3			2	
5.1.1	3		5	
5.1.2	1			
5.2		4		
5.3	2			
6.1	1			
6.2	1			
6.3		4		
6.4			5	5
7.1		4		
7.2.1				3
7.2.2			6	
7.3			5	
8.1		4		
8.2.1	1			
8.2.2	1			
8.2.3	1			
8.2.4	1			
TOTAL	20	34	33	13