Basic Education

KwaZulu-Natal Department of Basic Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS

COMMON TEST

MARCH 2015

NATIONAL
SENIOR CERTIFICATE

GRADE 12

Marks: 100

Time: 2 hours

N.B. This question paper consists of 6 pages and an information sheet.

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INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 6 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

Given: $g(x) = -x^3 + 4x^2 + 11x - 30$
$= -(x-2)(x-5)(x+3)$

1.1 Write down the y – intercept of g. (1)

1.2 Write down the x – intercepts of g. (2)

1.3 Determine $g'(x)$. (3)

1.4 Hence, or otherwise, calculate the coordinates of the turning points of g. (5)

1.5 Sketch a neat graph of g. Clearly indicating the intercepts with the axes and the coordinates of the turning points. (3)

1.6 Determine the value(s) of x for which g is concave up. (3)

[17]

QUESTION 2

2.1 Given: $f(x) = -x^2 + x$, determine $f'(x)$ from first principles. (5)

2.2 Determine:-

2.2.1 $\frac{dy}{dx}$ if $y = \sqrt[3]{8x^{18}} - \frac{3}{4x^2}$ (4)

2.2.2 $f'(x)$ if $f(x) = \frac{3x^4 - 5x^3 + 2x^2}{(3x-2)}$ (4)

[13]

QUESTION 3

Given the equation of the curve $p(x) = x^3 + 3x^2 + 4x - 8$

3.1 Determine the gradient of the tangent to p at $x = -1$. (2)

3.2 Determine the equation of the tangent to p at $x = -1$. (4)

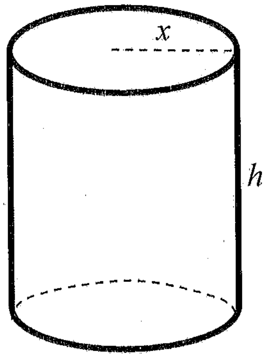
3.3 Show that the function p is increasing for all real value(s) of x. (5)

[11]

QUESTION 4

A container shaped in the form of a cylinder with **no top** has a volume of 340 ml.
It has a radius of x cm and a height of h cm.

Note: $1 \text{ ml} = 1 \text{ cm}^3$.



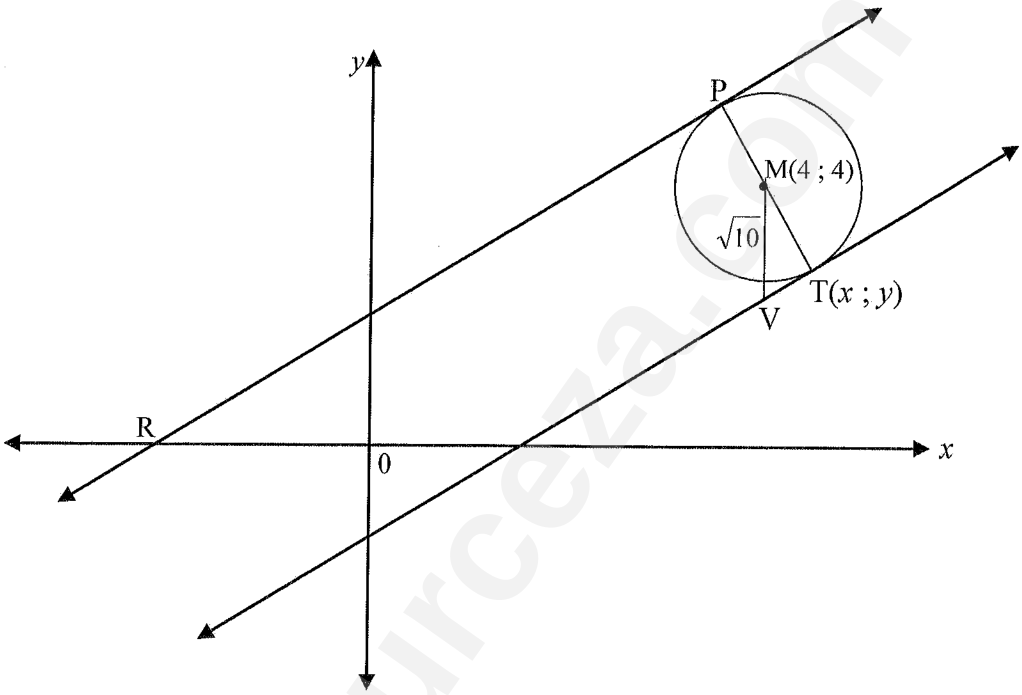
4.1 Write down the height (h) in terms of x. (2)

4.2 Show that the surface area (S) of the cylinder with no top is given by $S = \pi x^2 + \dfrac{680}{x}$. (2)

4.3 Calculate the value of x for which the surface area of the cylinder will be a minimum. (5)
[9]

QUESTION 5

In the diagram below, tangent RP, having equation $y - x - 2 = 0$, touches a circle with centre M(4 ; 4) at P and cuts the x–axis at R. PT is a diameter of the circle.
V is a point on another tangent to the circle that passes through T such that MV = $\sqrt{10}$.



Determine:

5.1 The equation of PT, the diameter of the circle. (4)

5.2 The coordinates of P, the point of contact of tangent PR and circle M. (3)

5.3 The equation of the circle. (3)

5.4 The coordinates of T . (3)

5.5 Calculate the length of TV. (3)

[16]

QUESTION 6

6.1 Make use of $\sin(x+y) = \sin x \cos y + \cos x \sin y$ to derive an expansion of $\cos(x-y)$ in terms of sines and cosines of x and y. (4)

6.2 If $\cos 62^\circ = m$, express $\cos 2^\circ$ in terms of m. (5)

6.3 Prove that:

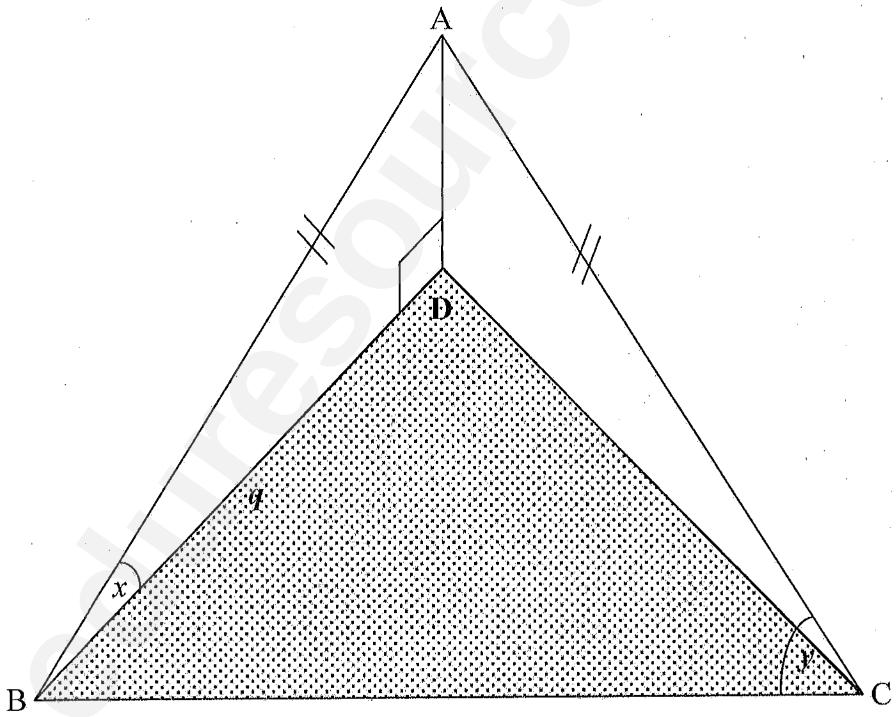
$$\frac{\sin^2(360^\circ - A) + \cos(90^\circ + 2A) + \cos^2(180^\circ - A)}{\sin(90^\circ - 2A)} = \frac{\cos A - \sin A}{\cos A + \sin A}$$ (9)

6.4 Given: $4\sin x + 6\sin x\cos x - 3\cos x = 2$.

6.4.1 Show that $4\sin x + 6\sin x\cos x - 3\cos x = 2$ can be written as $(2\sin x - 1)(2 + 3\cos x) = 0$ (2)

6.4.2 Hence or otherwise find the general solution of $4\sin x + 6\sin x\cos x - 3\cos x = 2$. (5)

6.5 In the figure below, D, B and C are three points in the same horizontal plane. AD is a vertical tower. The angle of elevation from B to A is x. In addition, AB = AC, $A\hat{C}B = y$ and $BD = q$.



6.5.1 Express AB in terms of q and a trigonometric ratio of x. (2)

6.5.2 Determine, with reasons, the size of $B\hat{A}C$ in terms of y. (3)

6.5.3 Hence, show that $BC = \dfrac{2q\cos y}{\cos x}$. (4)

[34]

TOTAL MARKS: [100]

INFORMATION SHEET: MATHEMATICS
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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$(x ; y) \rightarrow (x\cos\theta + y\sin\theta ; y\cos\theta - x\sin\theta)$ $\quad (x ; y) \rightarrow (x\cos\theta - y\sin\theta ; y\cos\theta + x\sin\theta)$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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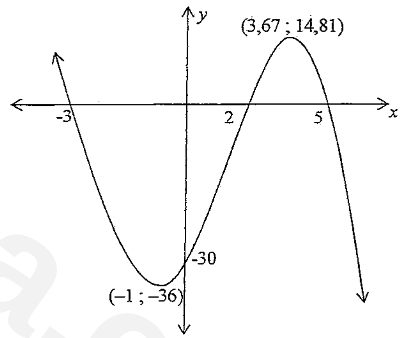
QUESTION 1

1.1	y : int : $(0;-30)$	✓y – intercept	(1)
1.2	x int : $(2;0)$ or $(-3;0)$ or $(5;0)$	✓1 mark for any 1 correct ✓✓ $x=2$ or $x=-3$ or $x=5$	(2)
1.3	$g'(x)=-3x^2+8x+11$	✓ $-3x^2$ ✓ $8x$ ✓ 11	(3)
1.4	$-3x^2+8x+11=0$ $(3x-11)(x+1)=0$ $x=\frac{11}{3}$ or 3,67 or $x=-1$ $y=14{,}81$ or $\frac{400}{27}$ or $y=-36$ $\left(3\frac{2}{3};14\frac{22}{27}\right)$ or $(-1;-36)$	✓derivative =0 ✓factors ✓x values ✓y values ✓coordinates	(5)
1.5		✓turning points ✓intercepts with the axes ✓shape	(3)
1.6	For concave up: $g''(x)>0$ $-6x+8>0$ $x<\frac{4}{3}$	✓ $g''(x)>0$ ✓ $-6x+8>0$ ✓answer	(3)
			[17]

QUESTION 2

2.1	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{-(x+h)^2 + (x+h) - (-x^2 + x)}{h}$ $= \lim_{h \to 0} \frac{-x^2 - 2xh - h^2 + x + h + x^2 - x}{h}$ $= \lim_{h \to 0} \frac{-2xh - h^2 + h}{h}$ $= \lim_{h \to 0} \frac{h(-2x - h + 1)}{h}$ $= -2x + 1$	✓Formula ✓Substitution ✓Simplifying ✓Factorisation ✓Answer (5)
2.2.1	$y = 2x^6 - \frac{3}{4}x^{-2}$ $\frac{dy}{dx} = 12x^5 + \frac{3}{2}x^{-3}$	✓✓each term ✓✓Answers (4)
2.2.2	$f(x) = \frac{3x^4 - 5x^3 + 2x^2}{(3x-2)}$ $f(x) = \frac{x^2(3x^2 - 5x + 2)}{(3x-2)}$ $= \frac{x^2(3x-2)(x-1)}{(3x-2)}$ $= x^3 - x^2$ $f'(x) = 3x^2 - 2x$	✓Factorising numerator ✓ simplification ✓✓Answers (4)
		[13]

QUESTION 3

3.1	$p'(x) = 3x^2 + 6x + 4$ $m = p'(-1) = 3(-1)^2 + 6(-1) + 4$ $= 1$	✓derivative ✓Answer (2)
3.2	$p(-1) = (-1)^3 + 3(-1)^2 + 4(-1) - 8$ $y = -10$ $y = mx + c$ $-10 = 1(-1) + c$ $c = -9$ $y = x - 9$	✓y-value ✓substitution of $m = 1$ & $(-1\,;-10)$ ✓c value ✓answer (4)
3.3	$p'(x) = 3x^2 + 6x + 4$ $= 3\left[x^2 + 2x + \frac{4}{3}\right]$ $= 3\left[(x^2 + 2x + 1) - 1 + \frac{4}{3}\right]$ $= 3\left[(x+1)^2 + \frac{1}{3}\right]$ $= 3(x+1)^2 + 1$ $3(x+1)^2 \geq 0$ for all $x \in R$ $\Rightarrow 3(x+1)^2 + 1 \geq 1$ $\therefore p'(x) \geq 1$ $\therefore p'(x) > 0$ OR $\boxed{p'(x) \text{ is always positive}}$ $\Rightarrow p$ is increasing for all values of x	✓completing the square ✓writing as perfect square ✓ $p'(x) = 3(x+1)^2 + 1$ ✓$3(x+1)^2 \geq 0$ ✓proving $p'(x) > 0$ (5)
		[11]

QUESTION 4

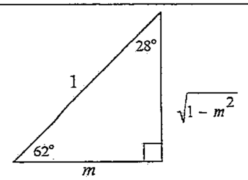
4.1	$V=\pi r^2h$ $\pi r^2h=340$ $h=\dfrac{340}{\pi x^2}$	✓formula ✓answer (2)
4.2	$S = \pi r^2+2\pi rh$ $=\pi x^2+2\pi x\left(\dfrac{340}{\pi x^2}\right)$ $=\pi x^2+\dfrac{680}{x}$	✓Formula ✓Subst. into formula (2)
4.3	$S'(x)=2\pi x-680x^{-2}$ $S'(x)=0$ $2\pi x-\dfrac{680}{x^2}=0$ $\pi x^3=340$ $x^3=\dfrac{340}{\pi}$ $\therefore\ x=\sqrt[3]{\dfrac{340}{\pi}}$ $=4{,}77\ cm$	✓For derivative ✓ equating to 0 ✓for simplifying ✓for x being the subject ✓answer (5)
		[9]

QUESTION 5

5.1	$M_{RP}=1$ $\therefore M_{PT}=-1$ radius $\perp$ tangent $\therefore y=-x+c$ OR $y-y_1=m(x-x_1)$ $\therefore 4=-4+c$ $\quad y-4=-1\ (x-4)$ $\therefore c=8$ $\quad y-4=-x+4$ $\therefore y=-x+8$ $\quad \therefore y=-x+8$	✓ $M_{RP}=1$ ✓ $M_{PT}=-1$ ✓ subst. (4 ; 4) ✓answer (4)
5.2	$x+2=-x+8$ $\therefore 2x=6$ $x=3$ $\therefore y=5$ $\therefore P(3\ ;\ 5)$	✓equating ✓ $x=3$ ✓ $y=5$ (3)
5.3	$(x-a)^2+(y-b)^2=r^2$ $r^2=(4-3)^2+(4-5)^2$ $=2$ $\therefore(x-4)^2+(y-4)^2=2$	✓ $r^2=(4-3)^2+(4-5)^2$ ✓✓ $(x-4)^2+(y-4)^2=2$ (3)
5.4	M is the midpoint of PT $\dfrac{x+3}{2}=4$ $\quad \dfrac{y+5}{2}=4$ $x=8-3$ $\quad y=8-5$ $x=5$ $\quad y=3$ T (5 ; 3)	✓ subst. in midpoint formula ✓ $x=5$ ✓ $y=3$

	OR $(x-4)^2+(y-4)^2=2$ $\therefore (x-4)^2=(-x+8-4)^2=2$ $\therefore (x-4)^2+(-x+4)^2=2$ $\therefore x^2-8x+16+x^2-8x+16=2$ $\therefore 2x^2-16x+30=0$ $\therefore x^2-8x+15=0$ $(x-5)(x-3)=0$ $x=5$ or $x=3$ $y=3$ or $y=5$ T $(5;3)$	✓ Subst. into eqn. of circle ✓ $x=5$ ✓ $y=3$ (3)
5.5	$M\hat{T}V=90°$ (radius ⊥ tangent) $TV^2=MV^2-MT^2$ (Theorem of Pythagoras) $=(\sqrt{10})^2-2$ $TV=\sqrt{8}$ or $2\sqrt{2}$ or 2,83 units	✓ $M\hat{T}V=90°$ ✓ subst. into Pythagoras ✓ answer (3)
		[16]

QUESTION 6

6.1	$\cos(x-y)=\sin[(90°-(x-y)]$ $=\sin[90°-x)-(-y)]$ $=\sin(90°-x).\cos(-y)-\cos(90°-x).\sin(-y)$ $=\cos x.\cos y-\sin x.\sin(-y)$ $=\cos x.\cos y-\sin x(-\sin y)$ $=\cos x\cos y+\sin x\sin y$	✓ expressing cosine in terms of sine ✓ regrouping ✓ expansion ✓ simplifying (4)
6.2	 $\cos 2° = \cos(62°-60°)$ $=\cos 62°\cos 60°+\sin 62°\sin 60°$ $=m.\frac{1}{2}+\frac{\sqrt{1-m^2}}{2}.\frac{\sqrt{3}}{2}$ $=\frac{1}{2}m+\frac{\sqrt{3}}{2}\sqrt{1-m^2}$	✓ sketch ✓ $\sqrt{1-m^2}$ ✓ $2°=62°-60°$ ✓ expansion ✓ substitution (5)

6.3	LHS $=\frac{\sin^2(360°-A)+\cos(90°+2A)+\cos^2(180°-A)}{\sin(90°-2A)}$ $=\frac{(-\sin A)^2+(-\sin 2A)+(-\cos A)^2}{\cos 2A}$ $=\frac{\sin^2 A-2\sin A.\cos A+\cos^2 A}{\cos^2 A-\sin^2 A}$ $=\frac{\cos^2 A-2\sin A\cos A+\sin^2 A}{(\cos A+\sin A)(\cos A-\sin A)}$ $=\frac{(\cos A-\sin A)(\cos A-\sin A)}{(\cos A+\sin A)(\cos A-\sin A)}$ $=\frac{\cos A-\sin A}{\cos A+\sin A}$ = RHS	✓ $(-\sin A)^2$ ✓ $-\sin 2A$ ✓ $(-\cos A)^2$ ✓ $\cos 2A$ ✓ $-\sin 2A=-2\sin A\cos A$ ✓ $\cos 2A=\cos^2 A-\sin^2 A$ ✓ simplifying numerator ✓ factorising denominator ✓ factorising numerator (9)
6.4.1	$4\sin x+6\sin x\cos x-3\cos x=2$ $4\sin x+6\sin x\cos x-3\cos x-2=0$ $(4\sin x-2)+(6\sin x\cos x-3\cos x)=0$ $2(2\sin x-1)+3\cos x(2\sin x-1)=0$ $(2\sin x-1)(2+3\cos x)=0$	✓ grouping ✓ factorising (2)
6.4.2	$2\sin x=1$ or $3\cos x=-2$ $\sin x=\frac{1}{2}$ or $\cos x=-\frac{2}{3}$ $x=30°+k.360°$ or $x=150°+k.360°$; $k\in Z$ $x=131,81°+k.360°$ or $x=228,19°+k.360°$; $k\in Z$	✓ $\sin x=\frac{1}{2}$ or $\cos x=-\frac{2}{3}$ ✓ 30° and 150° ✓ $k.360°$; $k\in Z$ ✓✓ 131,81° and 228,19° (5)
6.5.1	In Δ ABD: $\cos x=\frac{q}{AB}$ $\therefore AB=\frac{q}{\cos x}$	✓ cosine ratio ✓ answer (2)
6.5.2	In Δ ABC: AB = AC ... given $A\hat{B}C=A\hat{C}B=y$ ...... base angles of isosceles Δ $\therefore B\hat{A}C=180°-2y$...sum of ∠s in Δ	✓ $A\hat{B}C=A\hat{C}B=y$ ✓ answer ✓ reasons (3)
6.5.3	In ABC : $\frac{BC}{\sin(180°-2y)}=\frac{AB}{\sin y}$ $\frac{BC}{\sin(180°-2y)}=\frac{q}{\cos x\sin y}$ $\therefore BC=\frac{q\sin 2y}{\cos x\sin y}$ $=\frac{2q\sin y.\cos y}{\cos x.\sin y}$ $=\frac{2\cos y}{\cos x}$	✓ sine rule ✓ substitution ✓ $\sin(180°-2y)=\sin 2y$ ✓ $\sin 2y=2\sin y\cos y$ (4)
		[34]