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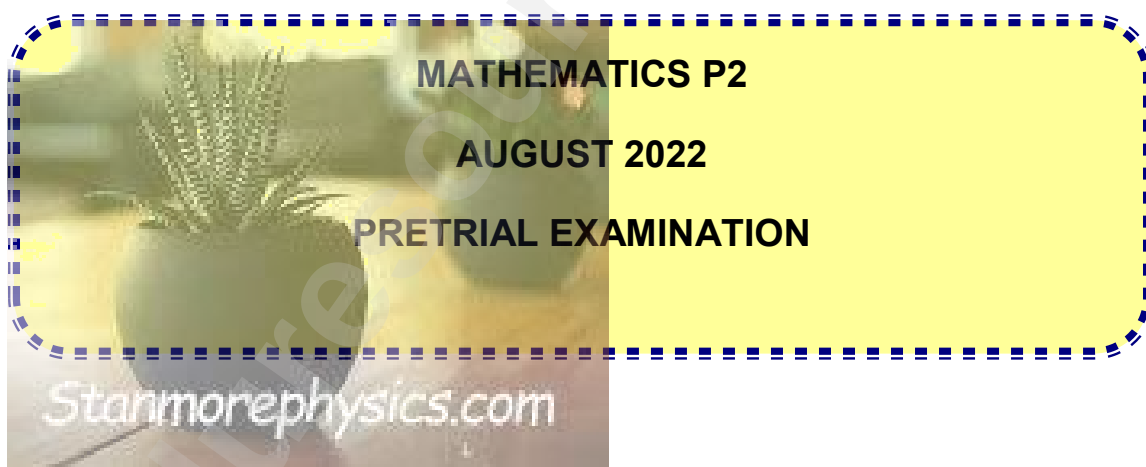


LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12



MARKS: 150

TIME: 3 HOUR

This question paper consists of 15 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.

Diagrams are NOT necessarily drawn to scale.

7. An information sheet with formulae is included at the end of the question paper.
8. Write neatly and legibly.
- 9.

QUESTION 1

Each child in a group of four-year-old children was given the same puzzle to complete. The time taken (in minutes) by each child to complete the puzzle is shown in the table below.

TIME TAKEN (t) (IN MINUTES)	NUMBER OF CHILDREN
$2 < t \leq 6$	2
$6 < t \leq 10$	10
$10 < t \leq 14$	9
$14 < t \leq 18$	7
$18 < t \leq 22$	8
$22 < t \leq 26$	7
$26 < t \leq 30$	2

- 1.1 How many children completed the puzzle? (1)
- 1.2 Calculate the estimated mean time taken to complete the puzzle. (2)
- 1.3 Complete the cumulative frequency column in the table given in the ANSWER BOOK. (2)
- 1.4 Draw a cumulative frequency graph (ogive) to represent the data on the grid provided in the ANSWER BOOK. (3)
- 1.5 Use the graph to determine the median time taken to complete the puzzle. (2)

[10]

QUESTION 2

Learners who scored a mark below 50% in a Mathematics test were selected to use a computer-based programme as part of an intervention strategy. On completing the programme, these learners wrote a second test to determine the effectiveness of the intervention strategy. The mark (as a percentage) scored by 15 of these learners in both tests is given in the table below.

LEARNER	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15
TEST 1 (%)	10	18	23	24	27	34	34	36	37	39	40	44	45	48	49
TEST 2 (%)	33	21	32	20	58	43	49	48	41	55	50	45	62	68	60

2.1 Determine the equation of the least squares regression line. (3)

2.2 A learner's mark in the first test was 15 out of a maximum of 50 marks.

2.2.1 Write down the learner's mark for this test as a percentage. (1)

2.2.2 Predict the learner's mark for the second test. Give your answer to the nearest integer. (2)

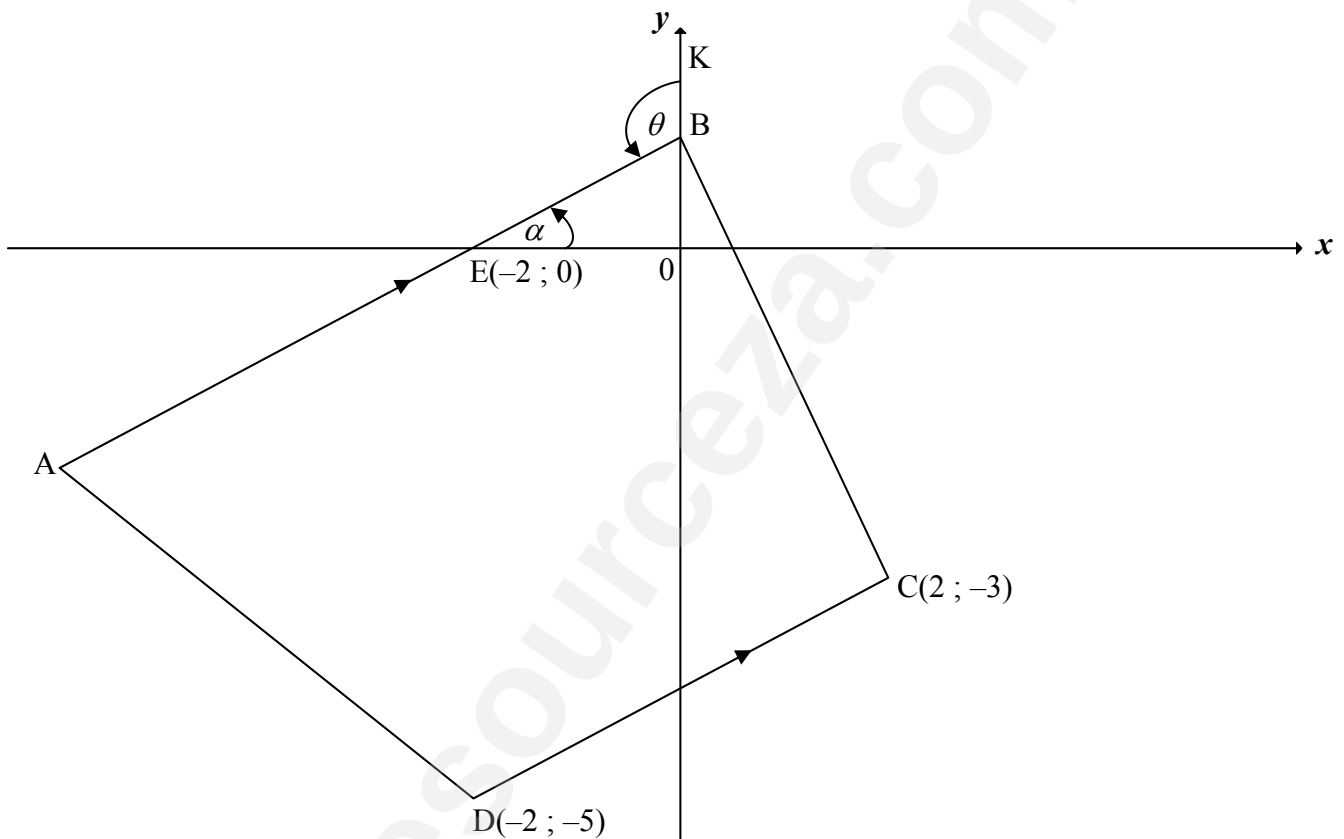
2.3 For the 15 learners above, the mean mark of the second test is 45,67% and the standard deviation is 13,88%. The teacher discovered that he forgot to add the marks of the last question to the total mark of each of these learners. All the learners scored full marks in the last question. When the marks of the last question are added, the new mean mark is 50,67%.

2.3.1 What is the standard deviation after the marks for the last question are added to each learner's total? (2)

2.3.2 What is the total mark of the last question? (2)

QUESTION 3

In the diagram, A, B, C(2 ; -3) and D(-2 ; -5) are vertices of a trapezium with $AB \parallel DC$. E(-2 ; 0) is the x-intercept of AB. The inclination of AB is α . K lies on the y-axis and $\widehat{KBE} = \theta$.



3.1 Determine:

3.1.1 The midpoint of EC (2)

3.1.2 The gradient of DC (2)

3.1.3 The equation of AB in the form $y = mx + c$ (3)

3.1.4 The size of θ (3)

3.2 Prove that $AB \perp BC$. (3)

3.3 The points E, B and C lie on the circumference of a circle. Determine:

3.3.1 The centre of the circle (1)

3.3.2 The equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (4)

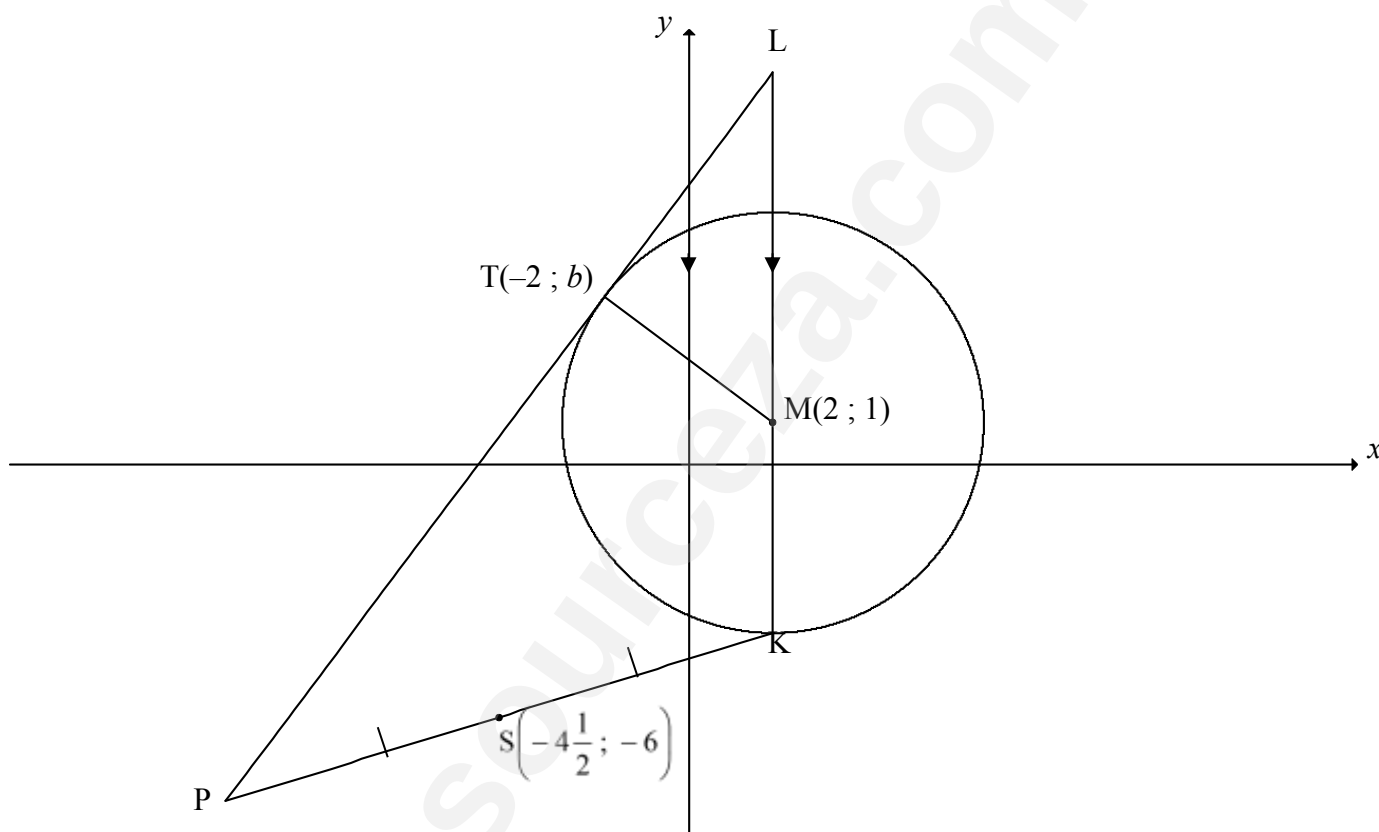
[18]

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QUESTION 4

In the diagram, the circle is centred at $M(2; 1)$. Radius KM is produced to L , a point outside the circle, such that $KML \parallel y$ -axis. LTP is a tangent to the circle at $T(-2; b)$.

$S\left(-4\frac{1}{2}; -6\right)$ is the midpoint of PK .



4.1 Given that the radius of the circle is 5 units, show that $b = 4$. (4)

4.2 Determine:

4.2.1 The coordinates of K (2)

4.2.2 The equation of the tangent LTP in the form $y = mx + c$ (4)

4.2.3 The area of $\triangle LPK$ (7)

4.3 Another circle with equation $(x - 2)^2 + (y - n)^2 = 25$ is drawn. Determine, with an explanation, the value(s) of n for which the two circles will touch each other externally. (4)

[21]

QUESTION 5

5.1 **Without using a calculator**, write the following expressions in terms of $\sin 11^\circ$:

5.1.1 $\sin 191^\circ$ (1)

5.1.2 $\cos 22^\circ$ (1)

5.2 Simplify $\cos(x - 180^\circ) + \sqrt{2} \sin(x + 45^\circ)$ to a single trigonometric ratio. (5)

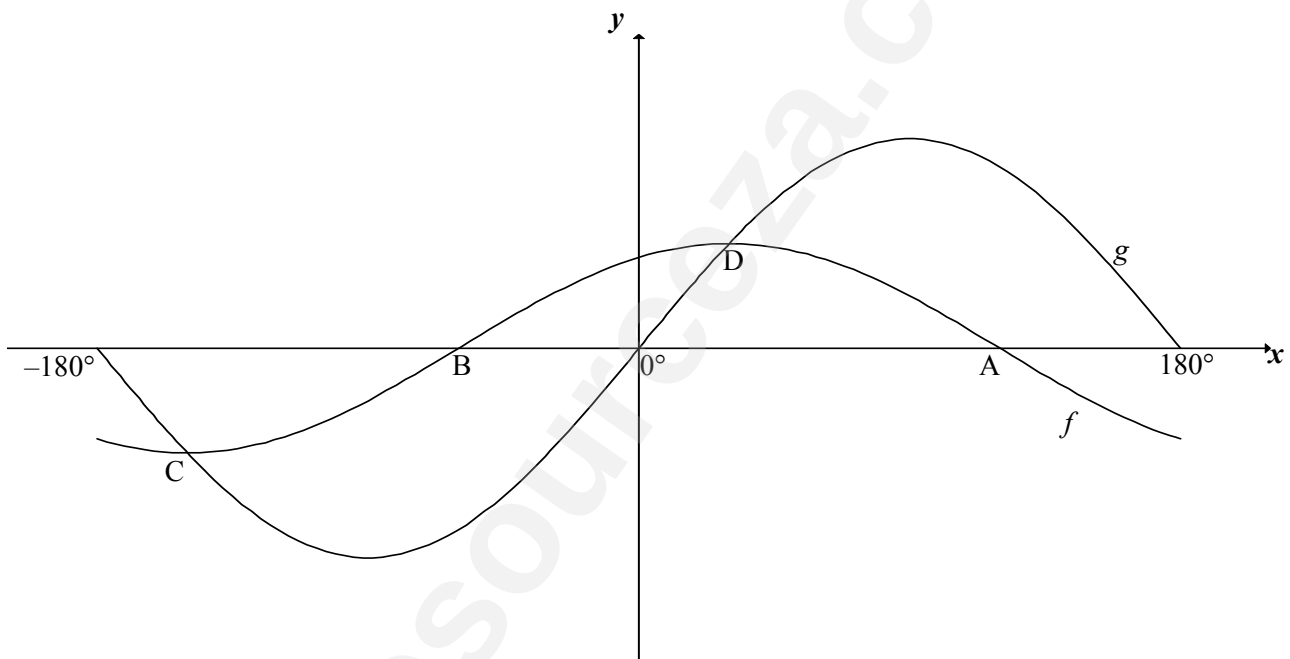
5.3 Given: $\sin P + \sin Q = \frac{7}{5}$ and $\hat{P} + \hat{Q} = 90^\circ$
Without using a calculator, determine the value of $\sin 2P$. (5)

[12]

QUESTION 6

6.1 Determine the general solution of $\cos(x - 30^\circ) = 2 \sin x$. (6)

6.2 In the diagram, the graphs of $f(x) = \cos(x - 30^\circ)$ and $g(x) = 2 \sin x$ are drawn for the interval $x \in [-180^\circ; 180^\circ]$. A and B are the x -intercepts of f . The two graphs intersect at C and D, the minimum and maximum turning points respectively of f .



6.2.1 Write down the coordinates of:

(a) A (1)

(b) C (2)

6.2.2 Determine the values of x in the interval $x \in [-180^\circ; 180^\circ]$, for which:

(a) Both graphs are increasing (2)

(b) $f(x + 10^\circ) > g(x + 10^\circ)$ (2)

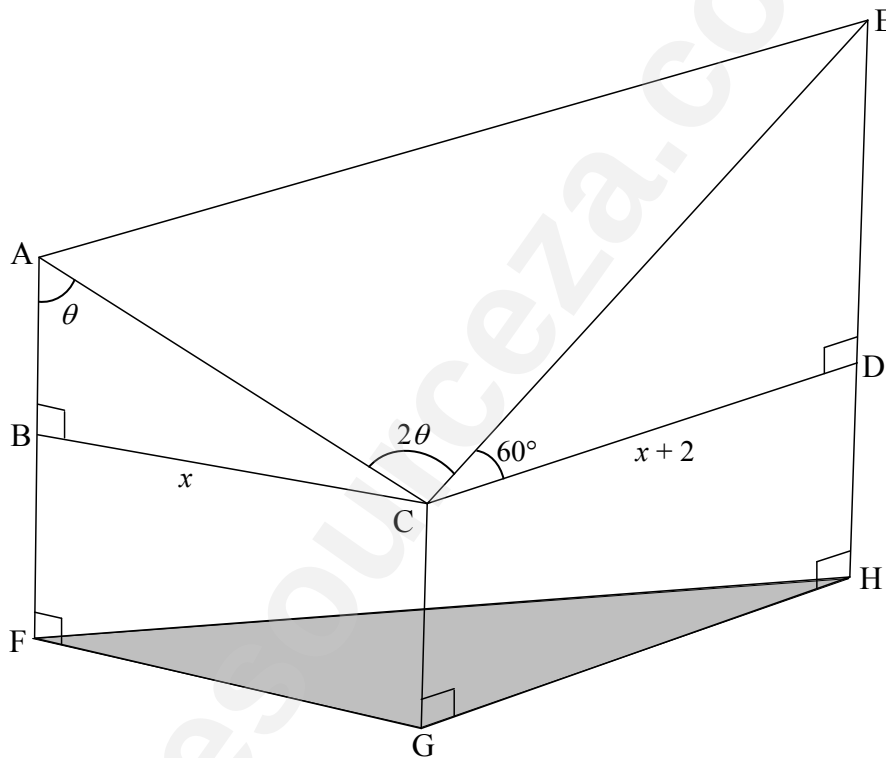
6.2.3 Determine the range of $y = 2^{2 \sin x + 3}$ (5)

[18]

QUESTION 7

In the diagram below, CGFB and CGHD are fixed walls that are rectangular in shape and vertical to the horizontal plane FGH. Steel poles erected along FB and HD extend to A and E respectively. $\triangle ACE$ forms the roof of an entertainment centre.

$BC = x$, $CD = x + 2$, $\hat{BAC} = \theta$, $\hat{ACE} = 2\theta$ and $\hat{ECD} = 60^\circ$



7.1 Calculate the length of:

7.1.1 AC in terms of x and θ (2)

7.1.2 CE in terms of x (2)

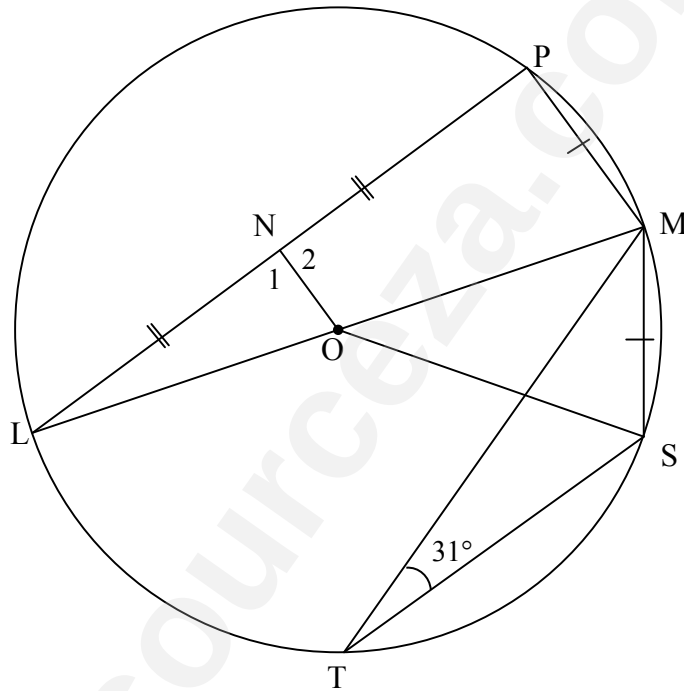
7.2 Show that the area of the roof $\triangle ACE$ is given by $2x(x+2)\cos\theta$. (3)

7.3 If $\theta = 55^\circ$ and $BC = 12$ metres, calculate the length of AE. (4)

[11]

QUESTION 8

- 8.1 In the diagram, O is the centre of the circle and LOM is a diameter of the circle. ON bisects chord LP at N. T and S are points on the circle on the other side of LM with respect to P. Chords PM, MS, MT and ST are drawn. $PM = MS$ and $\hat{M\hat{T}S} = 31^\circ$



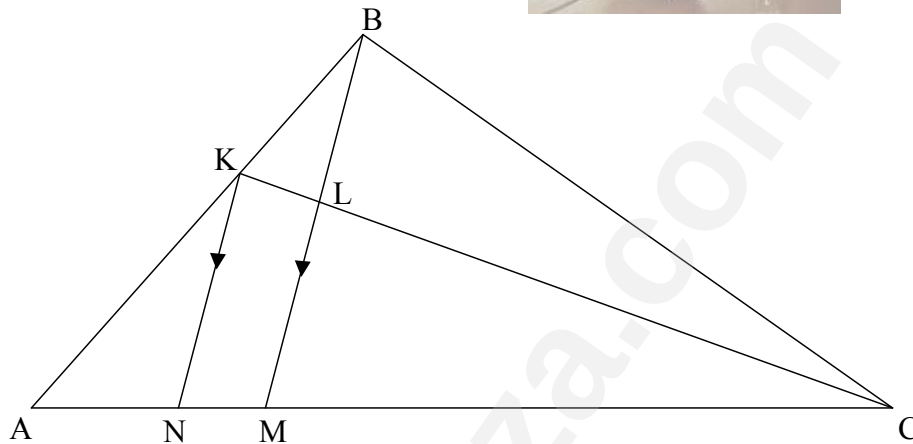
- 8.1.1 Determine, with reasons, the size of each of the following angles:

(a) $\hat{M\hat{O}S}$ (2)

(b) $\hat{L}$ (2)

- 8.1.2 Prove that $ON = \frac{1}{2} MS$. (4)

- 8.2 In $\triangle ABC$ in the diagram, K is a point on AB such that $AK : KB = 3 : 2$. N
 and M are points on AC such that $KN \parallel BM$. BM intersects KC at L . AM
 $AM : MC = 10 : 23$.



Determine, with reasons, the ratio of:

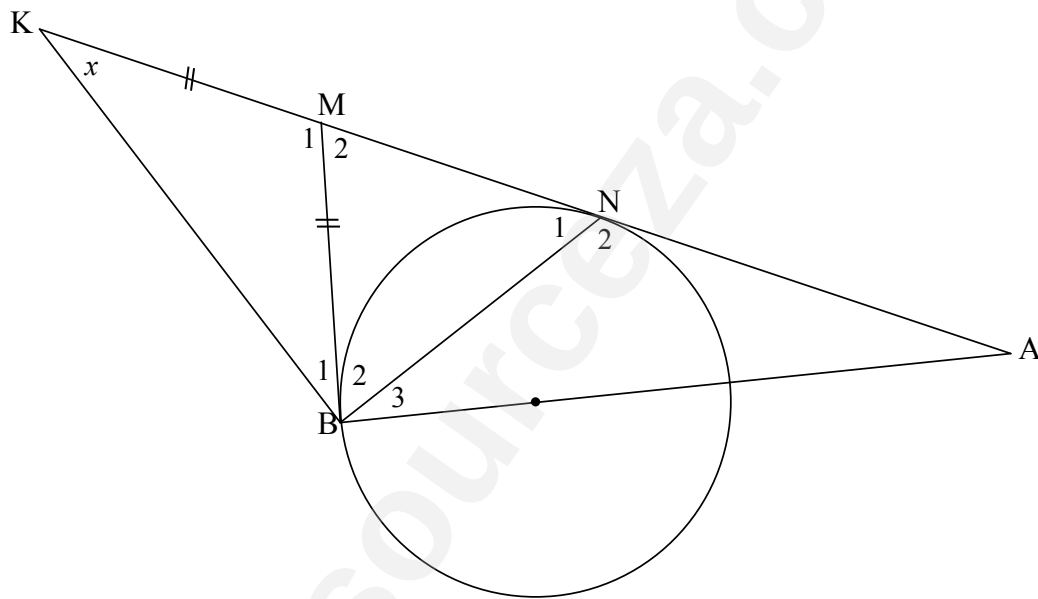
8.2.1 $\frac{AN}{AM}$ (2)

8.2.2 $\frac{CL}{LK}$ (3)

[13]

QUESTION 9

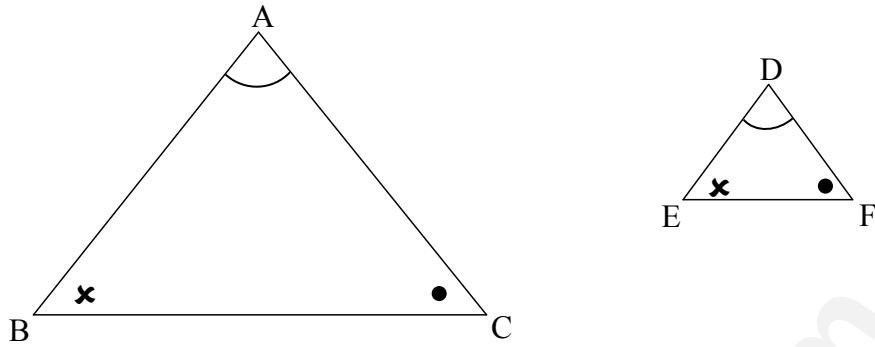
In the diagram, tangents are drawn from point M outside the circle, to touch the circle at B and N . The straight line from B passing through the centre of the circle meets MN produced in A . NM is produced to K such that $BM = MK$. BK and BN are drawn. Let $\hat{K} = x$.



- 9.1 Determine, with reasons, the size of $\hat{N}_1$ in terms of x . (6)
- 9.2 Prove that BA is a tangent to the circle passing through K , B and N . (5)
- [11]

QUESTION 10

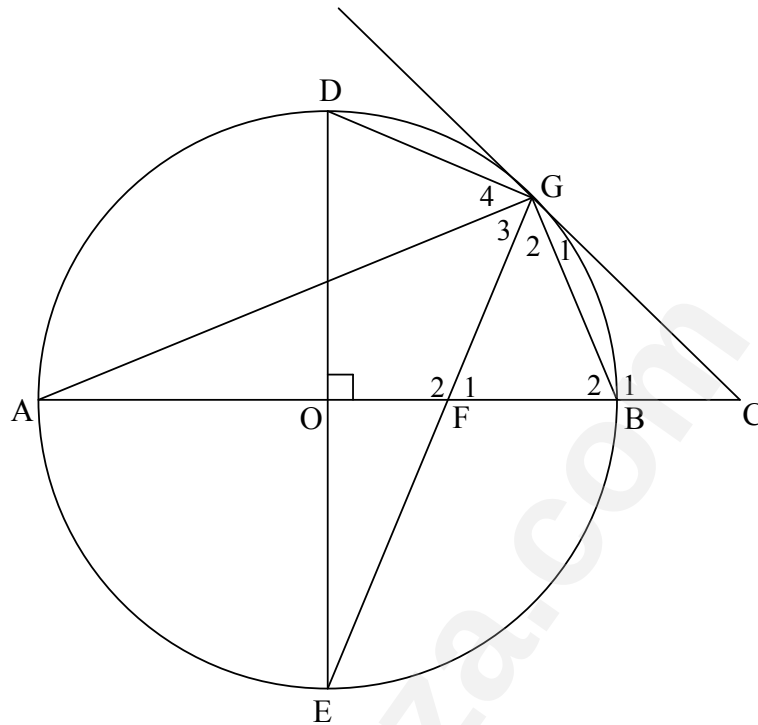
- 10.1 In the diagram, $\triangle ABC$ and $\triangle DEF$ are drawn such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{AB}{DE} = \frac{AC}{DF}$.

(6)

- 10.2 In the diagram, O is the centre of the circle and CG is a tangent to the circle at G . The straight line from C passing through O cuts the circle at A and B . Diameter DOE is perpendicular to CA . GE and CA intersect at F . Chords DG , BG and AG are drawn.



10.2.1 Prove that:

(a) $DGFO$ is a cyclic quadrilateral (3)

(b) $GC = CF$ (5)

10.2.2 If it is further given that $CO = 11$ units and $DE = 14$ units, calculate:

(a) The length of BC (3)

(b) The length of CG (5)

(c) The size of $\hat{E}$. (4)

[26]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In $\triangle ABC$:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



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GRADE 12

**MATHEMATICS P2 MARKING GUIDE
WISKUNDE V2 NASIENRIGLYNE**

AUGUST 2022

MARKS/PUNTE: 150

These marking guidelines consist of 20 pages/Hierdie nasieriglyne bestaan uit 20 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • MEETKUNDE	
S	A mark for a correct statement (A statement mark is independent of a reason)
	<i>'n Punt vir 'n korrekte bewering</i> (<i>'n Punt vir 'n bewering is onafhanklik van die rede</i>)
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
	<i>'n Punt vir 'n korrekte rede</i> (<i>'n Punt word slegs vir die rede toegeken as die bewering korrek is</i>)
S/R	Award a mark if statement AND reason are both correct
	<i>Ken 'n punt toe as die bewering EN rede beide korrek is</i>

QUESTION/VRAAG 1

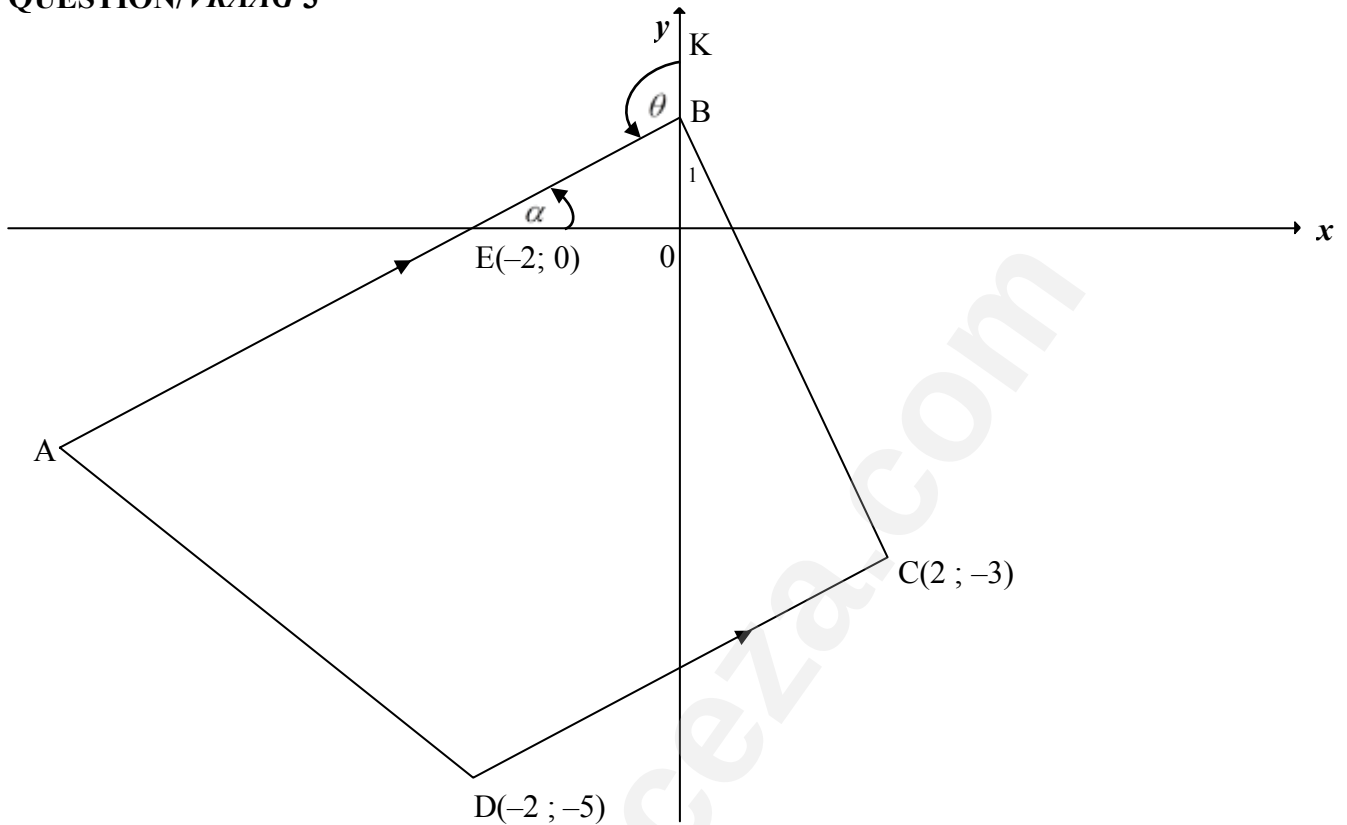
1.1	45 children	✓ answer (1)																								
1.2	$= \frac{\sum fx}{n} = \frac{(4 \times 2) + (8 \times 10) + (12 \times 9) + (16 \times 7) + (20 \times 8) + (24 \times 7) + (28 \times 2)}{45}$ $= \frac{692}{45}$ OR $x = 15,38$ minutes Answer only: full marks	✓ 692 ✓ answer (2)																								
1.3	<table border="1"> <thead> <tr> <th>Time taken (t) (in minutes)</th><th>Number of children</th><th>Cumulative frequency</th></tr> </thead> <tbody> <tr> <td>$2 < t \leq 6$</td><td>2</td><td>2</td></tr> <tr> <td>$6 < t \leq 10$</td><td>10</td><td>12</td></tr> <tr> <td>$10 < t \leq 14$</td><td>9</td><td>21</td></tr> <tr> <td>$14 < t \leq 18$</td><td>7</td><td>28</td></tr> <tr> <td>$18 < t \leq 22$</td><td>8</td><td>36</td></tr> <tr> <td>$22 < t \leq 26$</td><td>7</td><td>43</td></tr> <tr> <td>$26 < t \leq 30$</td><td>2</td><td>45</td></tr> </tbody> </table>	Time taken (t) (in minutes)	Number of children	Cumulative frequency	$2 < t \leq 6$	2	2	$6 < t \leq 10$	10	12	$10 < t \leq 14$	9	21	$14 < t \leq 18$	7	28	$18 < t \leq 22$	8	36	$22 < t \leq 26$	7	43	$26 < t \leq 30$	2	45	✓ first 4 cum freq correct ✓ last 3 cum freq correct (2)
Time taken (t) (in minutes)	Number of children	Cumulative frequency																								
$2 < t \leq 6$	2	2																								
$6 < t \leq 10$	10	12																								
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$18 < t \leq 22$	8	36																								
$22 < t \leq 26$	7	43																								
$26 < t \leq 30$	2	45																								
1.4	CUMULATIVE FREQUENCY GRAPH (OGIVE)	✓ plotting cum freq at upper limits correctly (all points) ✓ shape (smooth) ✓ grounding (2;0)																								

			(3)
1.5	On graph at the y -value of 22,5 or 23 Median = ± 15 minutes.	Answer only: full marks	✓ graph ✓ answer (2)
			[10]

QUESTION/VRAAG 2

2.1	$a = 12,44$ $b = 0,98$ Answer only: full marks $y = 12,44 + 0,98x$	✓ value of a ✓ value of b ✓ equation (3)
2.2.1	$\frac{15}{50} \times 100$ Percentage = = 30%	✓ answer (1)
2.2.2	$\hat{y} = 12,44 + 0,98x$ $= 12,44 + 0,98(30)$ $\hat{y} = 41,84$ $= 42$ Answer only: full marks OR $\hat{y} = 41,87$ (if using calculator) $\hat{y}$ $= 42$ OR $\frac{21}{50}$ $\hat{y} =$	✓ substitution of 30 ✓ answer as integer (2) ✓ value of y ✓ answer as integer (2) ✓ ✓ answer (2)
2.3.1	standard deviation = 13,88	✓ ✓ answer (2)
2.3.2	$x = 50,67 - 45,67$ Answer only: full marks = 5%	✓ 50,67 - 45,67 ✓ answer (2)
		[10]

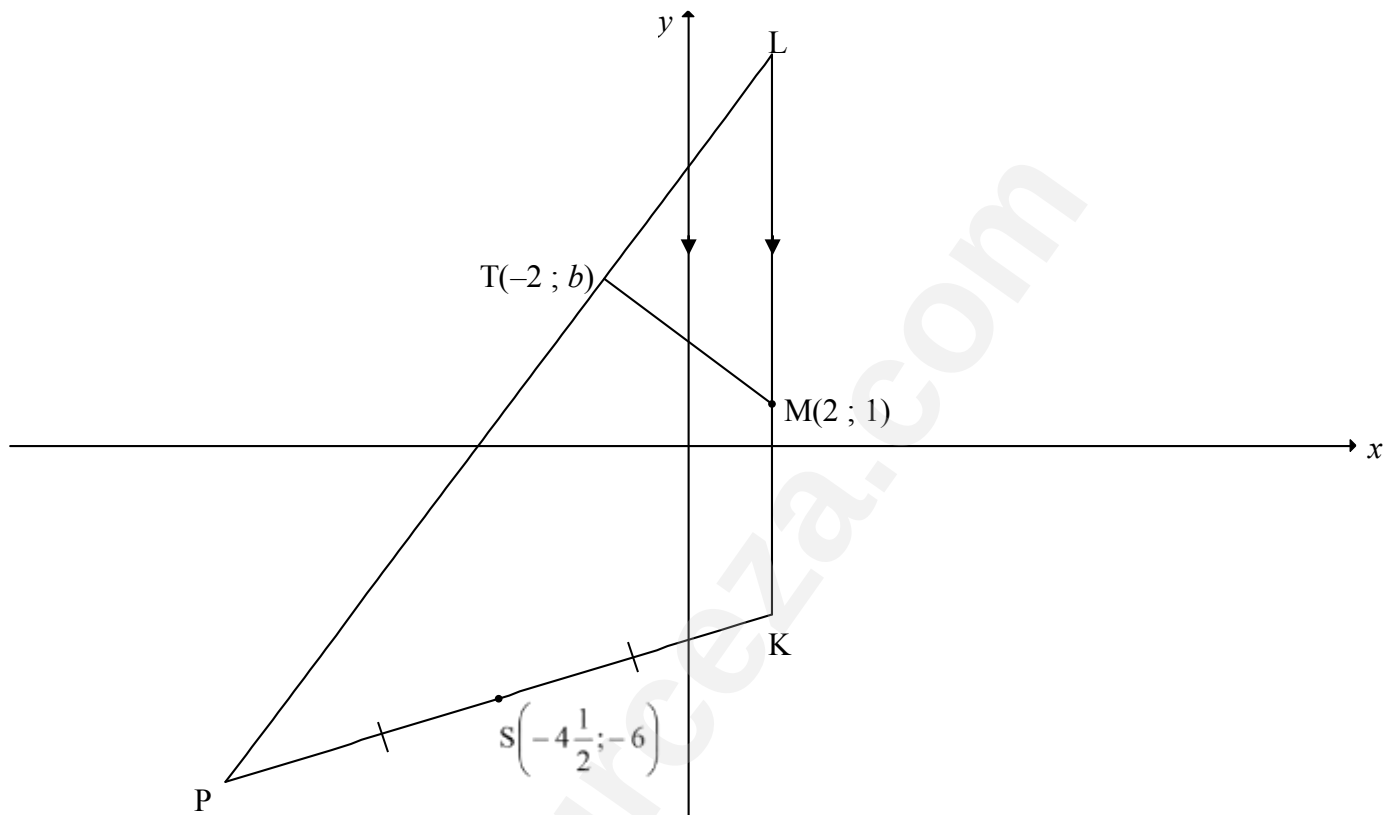
QUESTION/VRAAG 3



3.1.1	Midpoint of EC: $= \left(\frac{12+2}{2}, \frac{0+(-3)}{2} \right) = \left(0, -\frac{3}{2} \right)$	✓ x value ✓ y value (2)
3.1.2	$m_{DC} = \frac{-3 - (-5)}{2 - (-2)} \text{ OR } \frac{-5 - (-3)}{-2 - 2}$ $= \frac{2}{4} = \frac{1}{2}$	✓ substitution ✓ answer (2)
3.1.3	$m_{AB} = \frac{1}{2} = [AB \parallel DC]$ $y = \frac{1}{2}x + c$ $0 = \frac{1}{2}(-2) + c$ $c = 1$ $\therefore y = \frac{1}{2}x + 1$ $y - y_1 = \frac{1}{2}(x - x_1)$ $y - 0 = \frac{1}{2}(x - (-2))$	$m_{AB} = \frac{1}{2}$ ✓ substitution of (-2;0) ✓ equation (3)

3.1.4	$\tan \alpha = \frac{1}{2} m_{AB}$ $\alpha = 26,57^\circ$ $\theta = 90^\circ + 26,57^\circ$ $= 116,57^\circ$ [ext $\angle$ of Δ]	✓ $\tan \alpha = \frac{1}{2}$ ✓ value of α ✓ value of θ (3)
3.2	B(0 ; 1) $m_{BC} = \frac{1 - (-3)}{0 - 2} \quad \text{OR} \quad m_{BC} = \frac{(-3) - 1}{2 - 0}$ $= -2 \quad \quad \quad = -2$ $m_{AB} \times m_{BC} = \frac{1}{2} \times -2$ $= -1$ $\therefore AB \perp BC$	✓ coordinates of B ✓ $m_{BC} = -2$ ✓ product of gradients = -1 (3)
3.3.1	$\hat{A}B C = 90^\circ$ $\therefore EC$ is diameter [converse: $\angle$ in semi circle] $\therefore$ centre of circle = $\left(0; -\frac{3}{2}\right)$	✓ answer (1)
3.3.2	$(x-0)^2 + \left(y + \frac{3}{2}\right)^2 = r^2$ $(0-0)^2 + \left(0 + \frac{3}{2}\right)^2 = r^2 \quad \text{or} \quad (2-0)^2 + \left(-3 - \left(-\frac{3}{2}\right)\right)^2 = r^2$ $\text{or } R = \frac{EC}{2} = \frac{\sqrt{(2-2)^2 + (1-(-3))^2}}{2}$ $\text{or } r = 1 - \left(-\frac{3}{2}\right)$ $r^2 = \frac{25}{4} \quad \text{or} \quad r = \frac{5}{2}$ $x^2 + \left(y + \frac{3}{2}\right)^2 = \frac{25}{4}$	✓ substitution of centre ✓ correct substitution of E(-1 ; 0), B(0 ; 1) or C(2 ; -3) to calculate r^2 or r ✓ value of r^2 or r ✓ equation (4)
		[18]

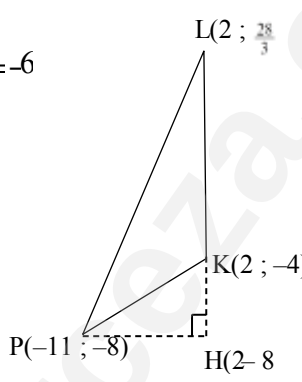
QUESTION/VRAG 4



4.1	$(x-2)^2 + (y-1)^2 = 25$ $(-2-2)^2 + (b-1)^2 = 25$ $(b-1)^2 = 9$ OF $b-1 = \pm 3$ $\therefore b = 4 \quad \text{or} \quad b = -2$	$(x-2)^2 + (y-1)^2 = 25$ $(-2-2)^2 + (b-1)^2 = 25$ $16 + b^2 - 2b + 1 = 25$ $b^2 - 2b - 8 = 0$ $\therefore b = 4 \quad \text{or} \quad b = -2$	✓ equation of the circle ✓ substitution of point T ✓ simplification ✓ answer (4)
4.2.1	$K(2; 1-5)$ $\therefore K(2; -4)$	Answer only: full marks	✓ x value ✓ y value (2)

4.2.2	$\frac{4-1}{2-2} = \frac{3}{4}$ $m_{MT} m_{PL}$ [radius $\perp$ tangent] $= \frac{4}{3}y + c$ $= \frac{4}{3}x + c$ $4 = \frac{4}{3}(-2) + c$ $= \frac{20}{3}$ $c = \frac{20}{3}$ $= \frac{4}{3}x + \frac{20}{3}$	✓ m_{MT} ✓ $m_{PL} = \frac{4}{3}$ ✓ substitution of m_{PL} and the point T ✓ equation (4)
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OR	$= \frac{4-1}{-2-2} = -\frac{3}{4}$ m_{MT} $m_{PL} = \frac{4}{3}$ [radius $\perp$ tangent] $-y_1 = \frac{4}{3}(x - x_1)$ $-4 = \frac{4}{3}(x + 2)$ $= \frac{4}{3}x + \frac{20}{3}$ OR $P(-11; -8)$ $m_{PL} = \frac{4 - (-8)}{-2 - (-11)}$ $= \frac{4}{3}$ $y = \frac{4}{3}x + c$ $-8 = \frac{4}{3}x + c$ $c = \frac{20}{3}$ $y = \frac{4}{3}x + \frac{20}{3}$	✓ m_{MT} ✓ $m_{PL} = \frac{4}{3}$ ✓ substitution of m_{PL} and the point T ✓ equation (4) ✓ coordinates of P ✓ $m_{PL} = \frac{4}{3}$ ✓ substitution of m_{PL} and the point P or T ✓ equation (4)
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4.2.3	$y_L = \frac{4}{3}(2) + \frac{20}{3} = \frac{28}{3}$ $L\left(2; \frac{28}{3}\right) \text{ and } K(2; 4)$ $LK = \frac{28}{3} - (-4)$ $= \frac{40}{3}$ <u>Coordinates of P:</u> $\frac{x+2}{2} = -4 \quad \text{and} \quad \frac{y-4}{2} = -6$ $\therefore x = -11 \quad y = -8$ $\therefore P(-11; -8)$ $\perp \text{ height (PH)} = 2 - (-11) = 13$ $\text{Area } \triangle PKL = \frac{1}{2} LK \times PH$ $= \frac{1}{2} \left(\frac{40}{3} \right) \times (13)$ $= \frac{260}{3} \text{ OR } 86,67$ 	$\checkmark \quad y = \frac{28}{3}$ $\checkmark \quad \text{length of LK}$ $\checkmark \quad x_P \quad \checkmark \quad y_P$ $\checkmark \quad \text{length of } \perp \text{ height}$ $\checkmark \quad \text{substitution into the area formula}$ $\checkmark \quad \text{answer}$ (7)
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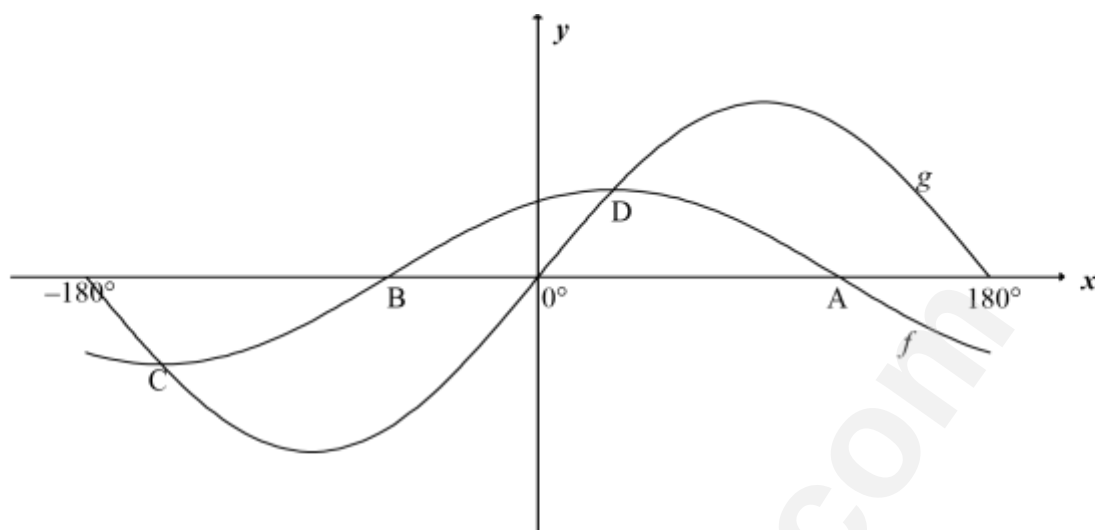
4.2.3	OR $y_L = \frac{4}{3}(2) + \frac{20}{3} = \frac{28}{3}$ $L\left(2; \frac{28}{3}\right) \text{ and } K(2; 4)$ $LK = \frac{28}{3} - (-4)$ $= \frac{40}{3}$ <u>Coordinates of P:</u> $\frac{x+2}{2} = -4 \quad \text{and} \quad \frac{y-4}{2}$ $\therefore x = -11 \quad y = -8$ $\therefore P(-11; -8)$ $PK^2 = (-11 - 2)^2 + (-8 - (-4))^2$ $PK = \sqrt{185} \text{ units}$ $m_{PK} = \frac{-8 - (-4)}{-11 - 2} = \frac{4}{13}$ $\tan \theta = \frac{4}{13} \therefore \theta = 17,1027\dots^\circ$ $\therefore \hat{PKL} = 90^\circ + 17,1027\dots^\circ = 107,1^\circ$ $\text{Area } \triangle PKL = \frac{1}{2} LK \times PK \sin \hat{PKL}$ $= \frac{1}{2} \left(\frac{40}{3} \right) \times (\sqrt{185}) \sin 107,1^\circ$ $= \frac{269}{3} \text{ OR } 86,67$	$\checkmark y_L = \frac{28}{3}$ $\checkmark$ length of LK $\checkmark x_P \checkmark y_P$ $\checkmark \hat{PKL}$ $\checkmark$ substitution into the area rule $\checkmark$ answer (7)
4.3	The centres of the two circles lie on the same vertical line $x = 2$. and the sum of the radii = 10 $n-1 = 10 \quad 1-n = 10$ or $n=11 \quad n = -9$ Answer only: full marks	$\checkmark$ correct method $\checkmark$ sum of radii = 10 $\checkmark n=11 \checkmark n = -9$ (4)
	[21]	

QUESTION/VRAAG 5

5.1.1	$\sin 191^\circ$ $= -\sin 11^\circ$	$\checkmark -\sin 11^\circ$ (1)
5.1.2	$\cos 22^\circ$ $= \cos(2 \times 11^\circ)$ $= 1 - 2\sin^2 11^\circ$	$\checkmark$ answer (1)
5.2	$\cos(x - 180^\circ) + \sqrt{2} \sin(x + 45^\circ)$ $= -\cos x + \sqrt{2}(\sin x \cos 45^\circ + \cos x \sin 45^\circ)$ $= -\cos x + \sqrt{2}\left(\sin x \left(\frac{1}{\sqrt{2}}\right) + \cos x \left(\frac{1}{\sqrt{2}}\right)\right)$ $= -\cos x + \sin x + \cos x$ $= \sin x$ OR $\cos(x - 180^\circ) + \sqrt{2} \sin(x + 45^\circ)$ $= -\cos x + \sqrt{2}(\sin x \cos 45^\circ + \cos x \sin 45^\circ)$ $= -\cos x + \sqrt{2}\left(\sin x \left(\frac{\sqrt{2}}{2}\right) + \cos x \left(\frac{\sqrt{2}}{2}\right)\right)$ $= -\cos x + \sin x + \cos x$ $= \sin x$	$\checkmark -\cos x$ $\checkmark$ expansion $\checkmark$ special angle ratios $\checkmark$ simplification of last 2 terms $\checkmark$ answer (5) $\checkmark -\cos x$ $\checkmark$ expansion $\checkmark$ special angle ratios $\checkmark$ simplification of last 2 terms $\checkmark$ answer (5)
5.3	$\sin P + \sin Q = \sin P + \cos P$ $(\sin P + \cos P)^2 = \left(\frac{7}{5}\right)^2$ $\sin^2 P + 2 \sin P \cos P + \cos^2 P = \frac{49}{25}$ $2 \sin P \cos P = \frac{49}{25} - 1$ $\sin 2P = \left(\frac{49}{25} - \frac{25}{25}\right)$ $= \frac{24}{25}$	$\checkmark \sin Q = \cos P$ $\checkmark$ squaring $\checkmark$ expansion $\checkmark \sin^2 P + \cos^2 P = 1$ $\checkmark$ answer (5)
		[12]

QUESTION/VRAG 6

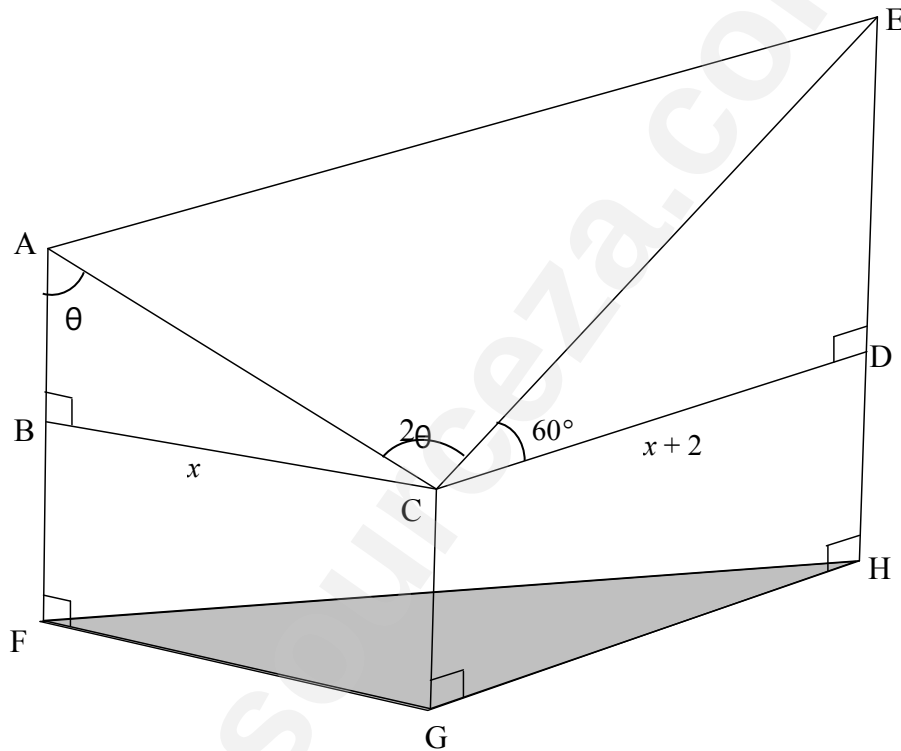
6.1	$\cos(x-30^\circ) = 2\sin x \cos x \cos 30^\circ + \sin$ $\frac{\sqrt{3}}{2} \sin 30^\circ = 2\sin x \cos x + \frac{1}{2}\sin x =$ $\frac{\sqrt{3}}{2} \cos x = \frac{3}{2}\sin x$ $\tan x = \frac{\sqrt{3}}{3}$ $x = 30^\circ + k \cdot 180^\circ; k \in \mathbb{Z}$ OR $x = 30^\circ + k \cdot 360^\circ \text{ or } x = 210^\circ + k \cdot 360^\circ;$ $k \in \mathbb{Z}$	✓ expansion ✓ special $\angle$s ✓ simplification ✓ equation in tan ✓ 30° ✓ $k \cdot 180^\circ; k \in \mathbb{Z}$ OR ✓ 30° and 210° ✓ $k \cdot 360^\circ; k \in \mathbb{Z}$ (6)
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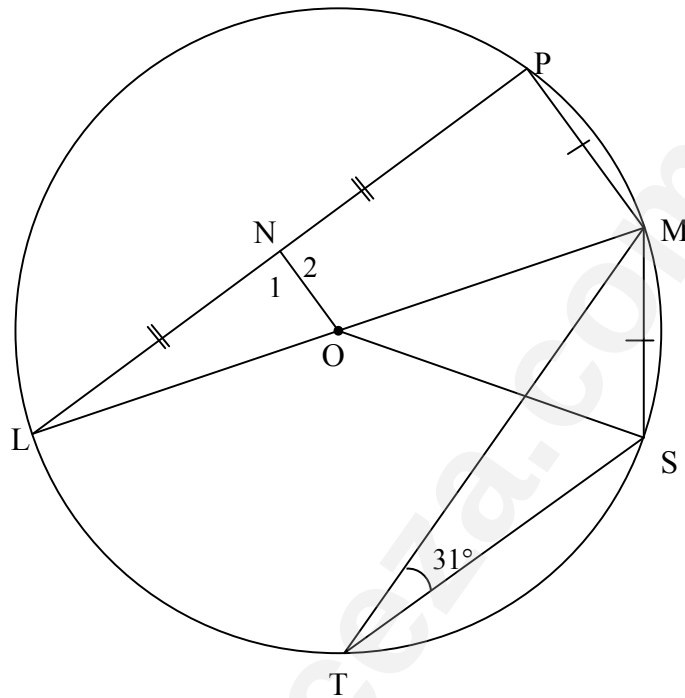
6.2.1(a)	A(120° ; 0)	✓ answer (1)
6.2.1(b)	C(-150° ; -1)	✓ x value ✓ y value (2)
6.2.2(a)	$x \in (-90^\circ ; 30^\circ)$ OR $-90^\circ < x < 30^\circ$	✓ endpoints ✓ correct interval (2)
6.2.2(b)	$x \in (-160^\circ ; 20^\circ)$ OR $-160^\circ < x < 20^\circ$	✓ endpoints ✓ correct interval (2)
6.2.3	$y = 2^{2\sin x + 3}$ Range of $y = 2\sin x$: $y \in [-2 ; 2]$ OR $-2 \leq y \leq 2$ Range of $y = 2\sin x + 3$: $y \in [1 ; 5]$ OR $1 \leq y \leq 5$ Range: $y = 2^{2\sin x + 3}$: $y \in [2 ; 32]$ OR $2 \leq y \leq 32$ Answer only: full marks	✓ 1 ✓ 5 ✓ 2 ✓ 32 ✓ correct interval (5)
		[18]

7.1.1	$\sin \theta = \frac{x}{AC}$ $AC = \frac{x}{\sin \theta}$ OR $\frac{\sin \theta}{x} = \frac{\sin 90^\circ}{AC}$ $AC = \frac{x}{\sin \theta}$	✓ trig ratio ✓ simplification (2)
7.1.2	$\cos 60^\circ = \frac{x+2}{CE}$ $CE = \frac{x+2}{\cos 60^\circ}$ $CE = \frac{x+2}{\frac{1}{2}} = 2(x+2)$ OR $\frac{\sin 30^\circ}{x+2} = \frac{\sin 90^\circ}{CE}$ $CE = \frac{x+2}{\sin 30^\circ} = 2(x+2)$	✓ trig ratio ✓ making CE the subject (2)
7.2	$\text{Area } \triangle ACE = \frac{1}{2} AC \cdot EC \cdot \sin \hat{A} C E$ $= \frac{1}{2} \cdot x \cdot (2(x+2)) \sin 2\theta$ $= \frac{1}{2} \cdot x \cdot (2(x+2)) \cdot \sin \theta \cdot \cos \theta$ $= \frac{x(x+2) \sin \theta \cos \theta}{\sin \theta}$	✓ use area rule correctly ✓ substitution of x $\frac{x}{\sin \theta} (2(x+2))$ ✓ substitution of $\sin 2\theta$ (3)

7.3	$EC = 2(12 + 2) = 28$ $AE^2 = AC^2 + EC^2 - 2AC \cdot EC \cos \hat{ACE}$ $= \left(\frac{12}{\sin 55^\circ} \right)^2 + 28^2 - 2 \left(\left(\frac{12}{\sin 55^\circ} \right) \cdot 28 \right) \cos 110^\circ$ so $AE = 35,77m$	✓ EC ✓ use cosine rule correctly ✓ substitution ✓ answer (4)
		[11]



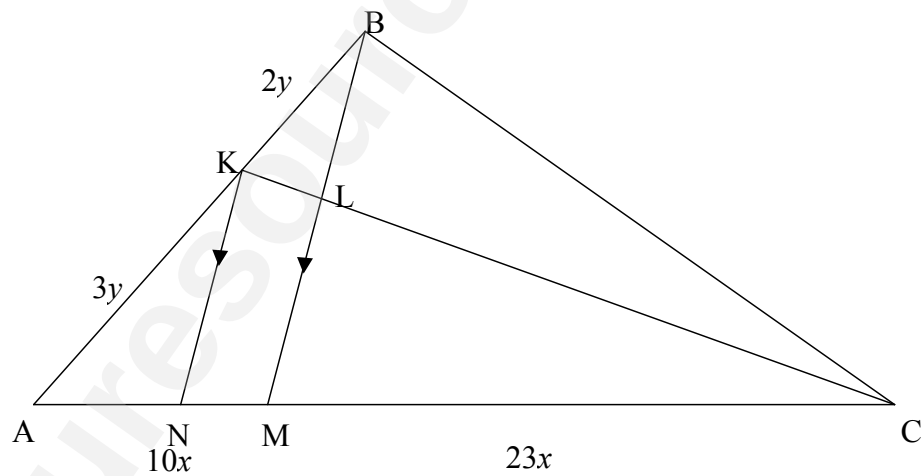
8.1



8.1.1(a)	$\angle MOS = 62^\circ$ [$\angle$ at centre = $2 \times \angle$ at circumf/middelpunt $\angle = 2 \times$ omtreks $\angle$]	✓ S ✓ R (2)
8.1.1(b)	$\angle L = 31^\circ$ [equal chords; equal $\angle$ s / = koorde; = $\angle$ e]	✓ S ✓ R (2)
8.1.2	LN = NP and LO = OM $\therefore ON = \frac{1}{2} PM$ [midpoint theorem/middelpuntstelling] $\therefore ON = \frac{1}{2} MS$ [PM = MS] OR $\angle LNP = 90^\circ$ [line from centre to midpt chord/lyn v midpt na midpt kd] $\angle PMS = 90^\circ$ [$\angle$ in semi-circle/$\angle$ in halfsirkel] $\angle L$ is common/gemeen $\therefore \triangle NLO \sim \triangle PLM$ ($\angle \angle \angle$) $\frac{NL}{PL} = \frac{NO}{PM} = \frac{1}{2}$ $\therefore ON = \frac{1}{2} PM$	✓ LO = OM ✓ S ✓ R ✓ S (4) ✓ S R ✓ S/R ✓ S (4)

	$\therefore ON = \frac{1}{2} MS$ [PM = MS] b	✓ S
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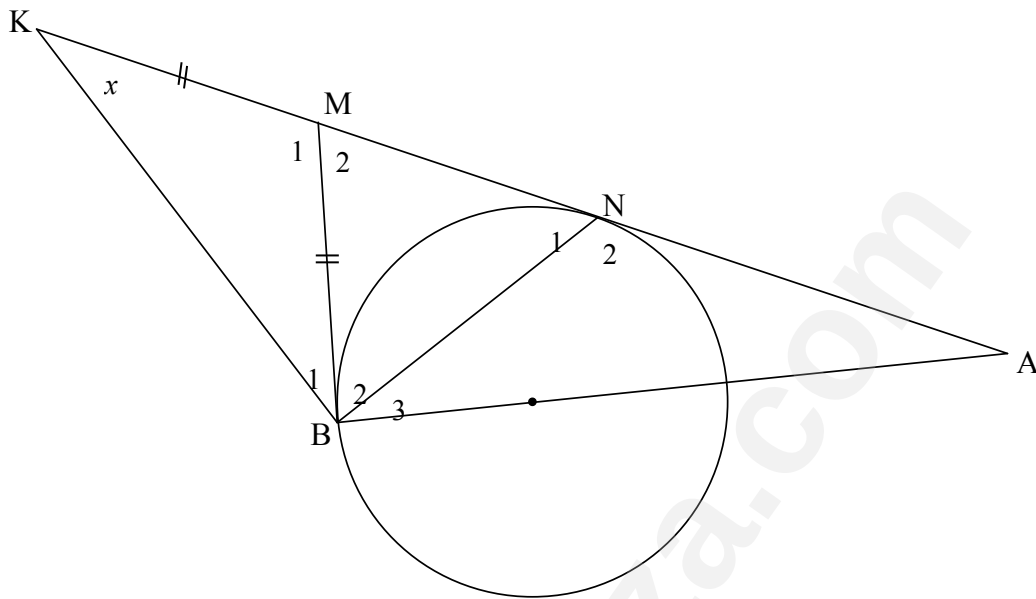
8.2



8.2.1	$\frac{AN}{AM} = \frac{AK}{AB}$ [line one side of Δ OR prop theorem; $KN \parallel BM$ / <i>lyn sy van Δ OR eweredigheidst; $KN \parallel BM$]</i>	✓ R
	$\frac{AN}{AM} = \frac{3y}{5y} = \frac{3}{5}$	✓ S

(2)

8.2.2	AM $10x$ $\frac{\text{AM}}{\text{MC}} = \frac{10x}{23x}$ [given] AM $= 5y = 10x \therefore y=2x$ LC MC $\frac{\text{LC}}{\text{KL}} = \frac{\text{MC}}{\text{NM}}$ [line $\parallel$ one side of Δ OR prop theorem; $\text{KN} \parallel \text{LM}/$ $\text{lyn} \parallel \text{sy van } \Delta$ OR eweredigheidst; $\text{KN} \parallel \text{BM}$] $= \frac{23x}{2y} = \frac{23x}{4x} = \frac{23}{4}$ OR AM $10x$ $\frac{\text{AM}}{\text{MC}} = \frac{10x}{23x}$ [given] MC $23x$ AN $3y$ $6x$ $\frac{\text{AN}}{\text{MN}} = \frac{3y}{2y} = \frac{6x}{4x}$ LC MC $\frac{\text{LC}}{\text{KL}} = \frac{\text{MC}}{\text{NM}}$ [line $\parallel$ one side of Δ OR prop theorem; $\text{KN} \parallel \text{LM}/$ $\text{lyn} \parallel \text{sy van } \Delta$ OR eweredigheidst; $\text{KN} \parallel \text{BM}$] $= \frac{23x}{2y} = \frac{23x}{4x} = \frac{23}{4}$	✓ S ✓ R ✓ S ✓ R ✓ S (3) (3) [13]
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✓S ✓R

✓S

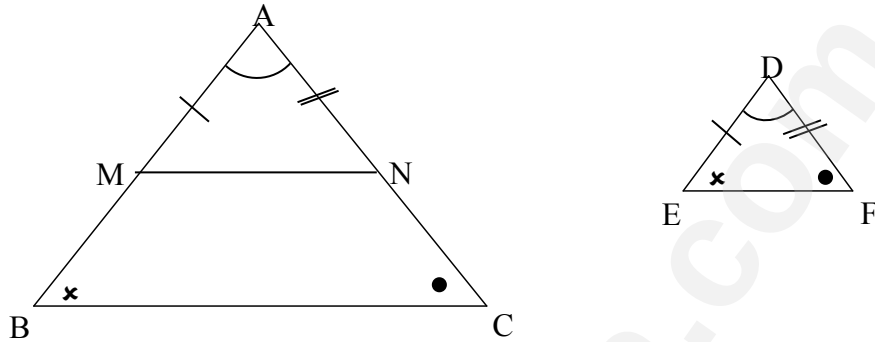
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9.2	$\angle MBA = \angle 2 + \angle 3 = 90^\circ \text{ [tangent} \perp \text{diameter/raaklyn} \perp \text{middellyn]}$ $\angle 3 = 90^\circ - \angle 2$ $= 90^\circ - (90^\circ - x) = x$ $\angle 3 = \angle K = x$ $\therefore$ AB is a tangent/raaklyn converse tan-chord theorem/ omgekeerde raakl koordst]]	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ S $\checkmark$ R (5)
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	OR $\angle 2 = \angle N_1$ $\angle 1 + \angle 2 = x + (90^\circ - x) = 90^\circ$ $\therefore$ KN is diameter/middellyn [converse $\angle$ in semi-circle/ omgekeerde $\angle$ in halfsirkel] $\angle MBA = \angle 2 + \angle 3 = 90^\circ \text{ [tangent} \perp \text{diameter]}$ $\therefore$ AB is a tangent/raaklyn converse tan-chord theorem/ omgekeerde raakl koordst]]	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ R (5)
		[11]

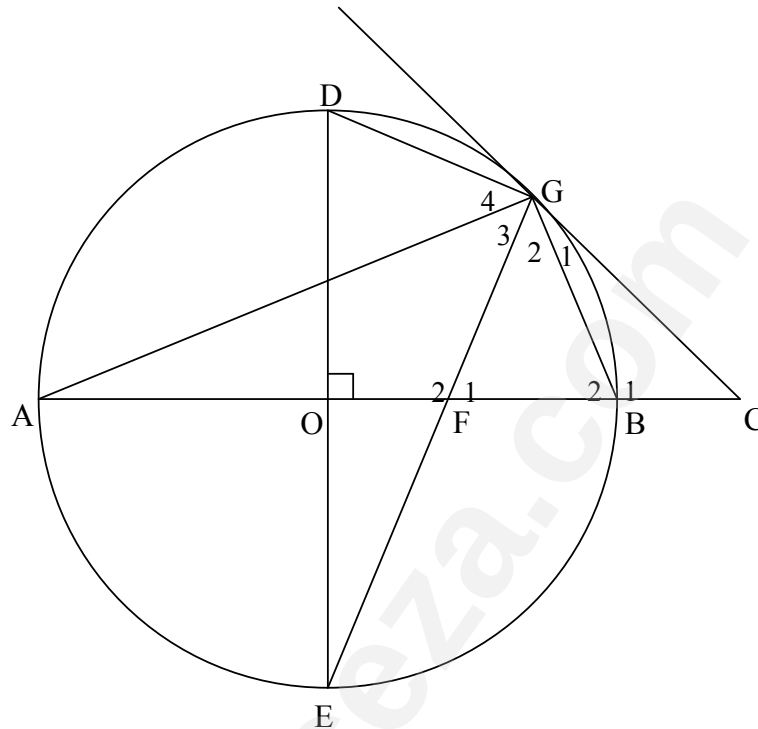
QUESTION 10

10.1



10.1	Constr: Let M and N lie on AB and AC respectively such that $AM = DE$ and $AN = DF$. Draw MN. Konst: Merk M en N op AB en AC onderskeidelik af sodanig dat $AM = DE$ en $AN = DF$. Verbind MN. Proof: In $\triangle AMN$ and $\triangle DEF$ $AM = DE$ [Constr] $AN = DF$ [Constr] $\hat{A} = \hat{D}$ [Given] $\therefore \triangle AMN \equiv \triangle DEF$ (SAS) $\therefore \hat{AMN} = \hat{EDF}$ $MN \parallel BC$ [corresp $\angle$'s are equal/ooreenkomstige $\angle$'e =] $\frac{AM}{AB} = \frac{AN}{AC}$ = [line $\parallel$ one side of $\triangle$ OR prop theorem; $MN \parallel BC$] $\frac{AM}{AB} = \frac{AN}{AC}$ $\therefore \frac{AM}{DE} = \frac{AN}{DF}$ [AM=DE and AN=DF]	✓ Constr / Konst ✓ $\triangle AMN \equiv \triangle DEF$ ✓ SAS ✓ $MN \parallel BC$ and R ✓ $\frac{AM}{AB} = \frac{AN}{AC}$ ✓ R (6)
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10.2



10.2.1(a)	$\angle DOB = 90^\circ$ $\angle DGF = \angle G_3 + \angle G_4 = 90^\circ$ [$\angle$ in semi-circle / $\angle$ in halfsirkel] $\angle DOB + \angle DGF = 180^\circ$ $\therefore$ DGFO is a cyclic quad. [converse: opp $\angle$s of cyclic quad / omgekeerde teenoorst $\angle$e v koordevh] OR $\angle$s of quad = 180° / $\angle$e van koordevh = 180°] OR $\angle EOB = 90^\circ$ $\angle DGF = \angle G_3 + \angle G_4 = 90^\circ$ [$\angle$ in semi-circle / $\angle$ in halfsirkel] $\angle EOB = \angle DGF$ $\therefore$ DGFO is a cyclic quad. [converse: ext $\angle$ = opp int $\angle$ / omgekeerde buite $\angle$ = teenoorst $\angle$] OR ext $\angle$ of quad = opp int $\angle$ / buite $\angle$ v vh = teenoorst $\angle$	$\checkmark$ S $\checkmark$ R $\checkmark$ R $\checkmark$ S $\checkmark$ R (3) $\checkmark$ R (3)
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10.2.1(b)	$\hat{F}_1 = \hat{D}$ [ext $\angle$ of cyclic quad/buite $\angle$ v koordevh] $\hat{G}_1 + \hat{G}_2 = \hat{D}$ [tan-chord theorem/raakl koordst] $\therefore \hat{F}_1 = \hat{G}_1 + \hat{G}_2$ $\therefore GC = CF$ [sides opp equal $\angle$ s/sye teenoor = $\angle$ e]	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ R (5)
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10.2.2(a)	$AB = DE = 14$ [diameters/ <i>middellynne</i>] $\therefore OB = 7$ units $\therefore BC = OC - OB = 11 - 7$ $= 4$ units Answer only: full marks	✓ S ✓ S ✓ S (3)
10.2.2(b)	In $\triangle CGB$ and $\triangle CAG$] $\hat{G} = \hat{A} = x$ [tan-chord theorem/ <i>raak koords</i>] $\hat{C} = \hat{C}$ [common] AG $\triangle CGB \parallel \triangle CAG$ $\angle, \angle, \angle$ $\frac{CG}{CA} = \frac{CB}{CG}$ $\frac{CG}{18} = \frac{4}{CG}$ $CG^2 = 72$ $CG = \sqrt{72}$ or $6\sqrt{2}$ or 8,49 units	✓ S/R ✓ S ✓ S ✓ CA = 18 ✓ answer (5)
10.2.2(c)	$OF = OC - FC$ $= 11 - \sqrt{72}$ $\tan E = \frac{OF}{OE}$ $= \frac{11 - \sqrt{72}}{7}$ $= 0,36$ $\hat{E} = 19,76^\circ$ OR $OF = OC - FC$ $= 11 - \sqrt{72}$ $FE^2 = OE^2 + OF^2$ $= 7^2 + (11 - \sqrt{72})^2$ $FE = 7,437.. = 7,44$ $\cos E = \frac{OE}{FE}$ $\sin E = \frac{OF}{FE}$ $= \frac{11 - \sqrt{72}}{7,44}$ $\hat{E} = 19,76^\circ = 0,338$	✓ OF ✓ trig ratio ✓ substitution ✓ answer (4) ✓ OF ✓ trig ratio

	$= \frac{7}{7,44} = 0,94$ $\hat{E} = 19,76^\circ$	✓ substitution ✓ answer (4)
		[26]
	TOTAL/TOTAAL: 150	