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LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

**NATIONAL
SENIOR CERTIFICATE**

GRADE/GRAAD 12

**MATHEMATICS PAPER 2/
WISKUNDE VRAESTEL 2**

MEMORANDUM

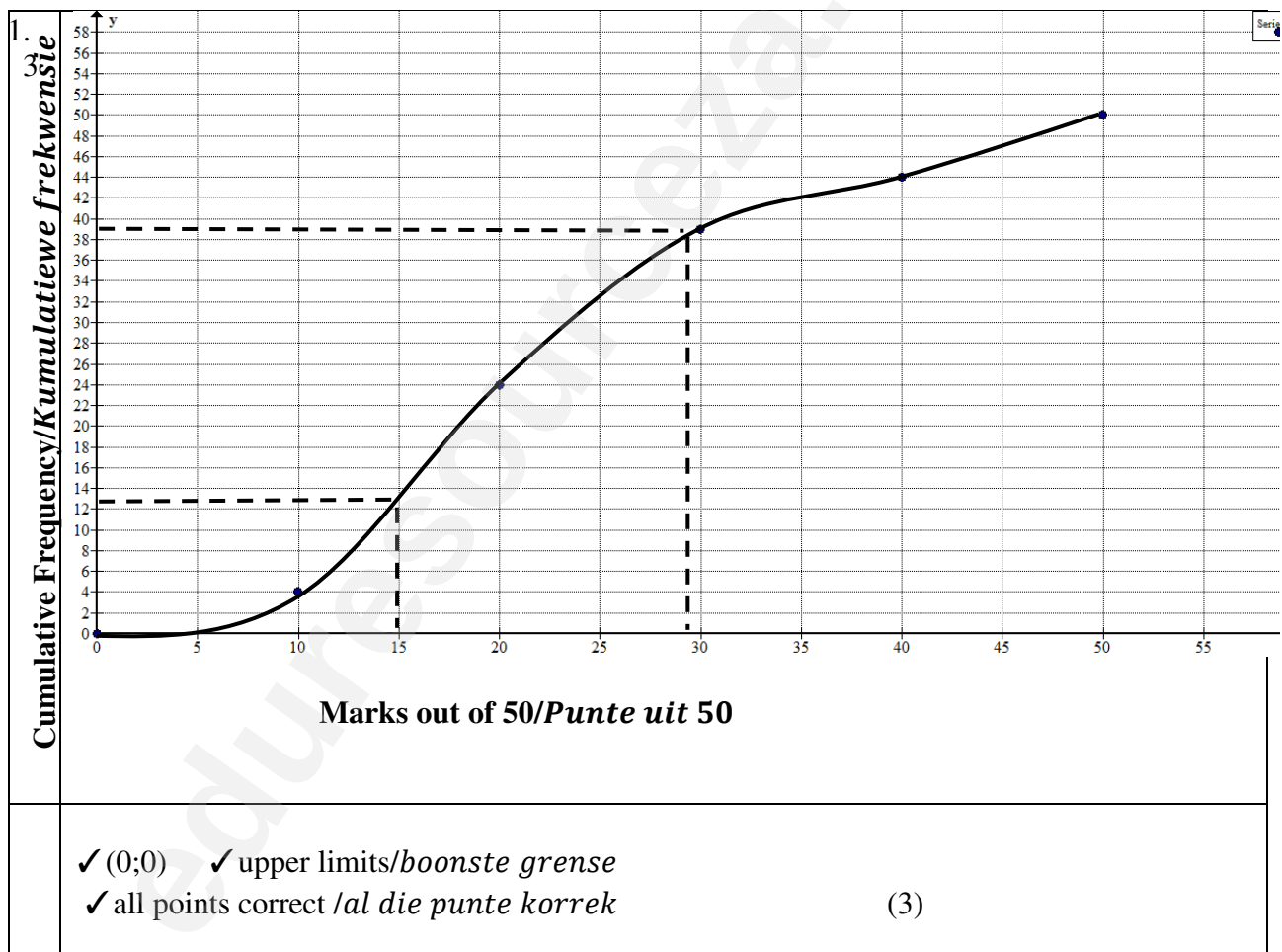
MARKS/PUNTE: 150

TIME/TYD: 3 HOURS/URE

This memorandum consists of 14 pages.
Hierdie memorandum bestaan uit 14 bladsye

QUESTION/VRAAG 1

1.1	$\bar{x} = \frac{1140}{50} = 22,8$			$\checkmark \frac{1140}{50}$ $\checkmark$ answer/ <i>antwoord</i> (2)
1.2	Marks	Frequency	Cumulative Frequency	$\checkmark 4;24;39$ $\checkmark 44;50$ (2)
	$0 \leq x < 10$	4	4	
	$10 \leq x < 20$	20	24	
	$20 \leq x < 30$	15	39	
	$30 \leq x < 40$	5	44	
	$40 \leq x < 50$	6	50	

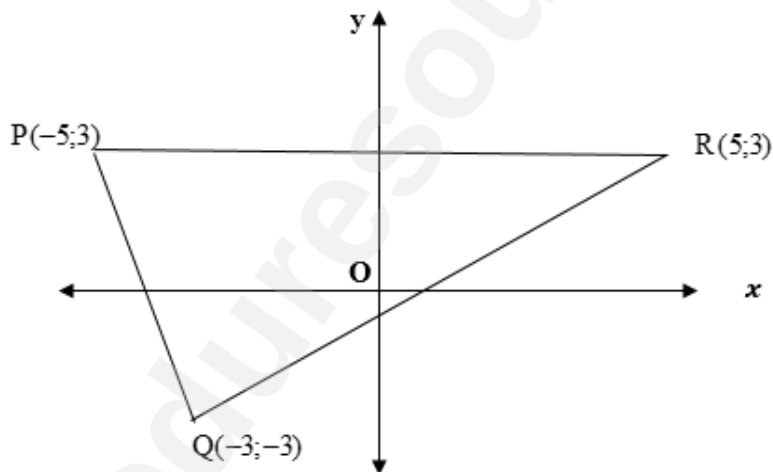


1.4	$Q_1 = 15$ (accept 14 to 16)/ (aanvaar van 14 tot 16) $Q_3 = 29$ (accept 28 to 30)/ (aanvaar van 28 tot 30) $IQR/IKO = 29 - 15 = 14$	$\checkmark Q_1 = 15$ $\checkmark Q_3 = 29$ $\checkmark$ answer <i>/antwoord</i> (3)
[10]		

QUESTION/VRAAG 2

2.1	$\bar{x} = \frac{143}{10} = 14,3$ seconds/ <i>sekondes</i> . $\delta x = 2,87$ seconds/ <i>sekondes</i> .	$\checkmark \bar{x} = \frac{143}{10}$ $\checkmark 14,3$ $\checkmark \delta x = 2,87$ $\checkmark$ accuracy/ <i>akkuraat</i> (4)
2.2	$a = 17,93$ $b = -0,07$ $\hat{y} = -0,07x + 17,93$	$\checkmark a$ $\checkmark b$ $\checkmark$ equation/ <i>vergelyking</i> (3)
2.3	$r = -0,69$	$\checkmark r = -0,69$ (1)
2.4	$y = -0,07(80) + 17,93$ $y = 2,33$ seconds.	$\checkmark$ substitute/ <i>vervang</i> $\checkmark$ answer/ <i>antwoord</i> (2)
		[10]

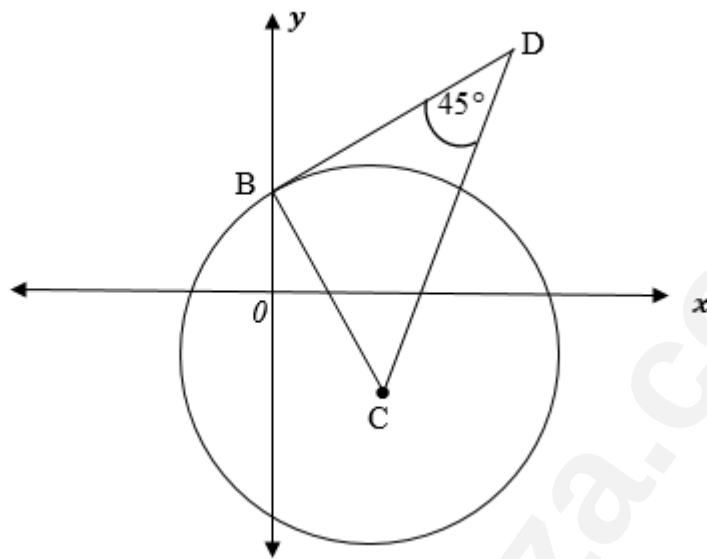
QUESTION/VRAAG 3



3.1	$QR = \sqrt{(5+3)^2 + (3+3)^2}$ $= \sqrt{64 + 36}$ $= \sqrt{100}$ $= 10$	$\checkmark$ substitution/ <i>vervang</i> $\checkmark 10$ (2)
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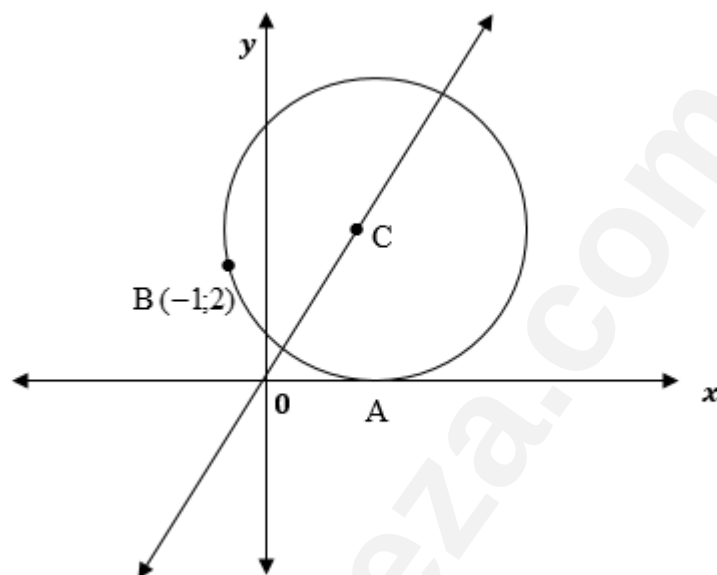
3.2	$M\left(\frac{5-3}{2}; \frac{3-3}{2}\right)$ $= M(1;0)$	✓ x-value/waarde ✓ y-value /waarde (2)
3.3	$P(-5;3) \text{ and } M(1;0) : m = \frac{0-3}{1+5}$ $= \frac{-1}{2}$ $y - 0 = -\frac{1}{2}(x - 1)$ $y = -\frac{1}{2}x + \frac{1}{2}$	✓ m ✓ subst of m and point/ vervang m en punt ✓ equation/vergelyking (3)
3.4	$r = 5; \text{centre } (1;0) :$ $(x-1)^2 + y^2 = 25$	✓ r = 5 and/en (1;0) ✓ LHS/LK ✓ RHS/RK (3)
3.5	$PM = \sqrt{(1+5)^2 + (-3)^2}$ $= \sqrt{45}$ $> \sqrt{25}$ $\therefore P \text{ lies OUTSIDE the circle.}$	✓ $PM = \sqrt{45}$ ✓ $> \sqrt{25}$ ✓ conclusion/gevolgtrekking (3)
3.6	$S(3;9)$	✓ x-value/waarde ✓ y-value/waarde (2)
3.7	$m_{PQ} = \frac{3+3}{-5+3} = -3$ $\tan \theta = -3$ $\theta = 180^\circ - 71,57^\circ$ $= 108,43^\circ$ $\beta = 71,57^\circ$ co-interior angles, $PR \parallel x\text{-axis.}$	✓ $m_{PQ} = -3$ ✓ $\tan \theta = -3$ ✓ $\theta = 108,43^\circ$ ✓ $\beta = 71,57^\circ$ (4)
[19]		

QUESTION/VRAAG 4



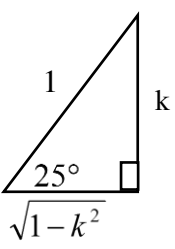
4.1.1	$C(3; -2)$	✓ x -value/waarde ✓ y -value/waarde (2)
4.1.2	$-4y + 8 = 0$ $y = 2$ $B(0; 2)$	✓ x -value/waarde ✓ y -value/waarde(2)
4.1.3	$r^2 = (3 - 0)^2 + (-2 - 2)^2 = 25$ $r = 5$ $x = 3 - 5 = -2$ $x = 3 + 5 = 8$	✓ $r^2 = 25$ ✓ $r = 5$ ✓ $x = -2$ ✓ $x = 8$ (4)
4.1.4	Let $D(x; y) : (x - 0)^2 + (y - 2)^2 = 25$ and $y = \frac{3}{4}x + 2$ $x^2 + (\frac{3}{4}x + 2 - 2)^2 = 25$ $x^2 + \frac{9}{16}x^2 = 25$ $\frac{25x^2}{16} = 25$ $x^2 = 16$ $x = -4$ or $x = 4$ $n/a \quad y = \frac{3}{4}(4) + 2 = 5$ $D(4; 5)$	✓ subst in distance formula/ vervang in afstandformule ✓ $y = \frac{3}{4}x + 2$ ✓ replace y /Vervang y ✓ $x^2 = 16$ ✓ x -value/ waarde ✓ y -value/waarde (6)

4.2



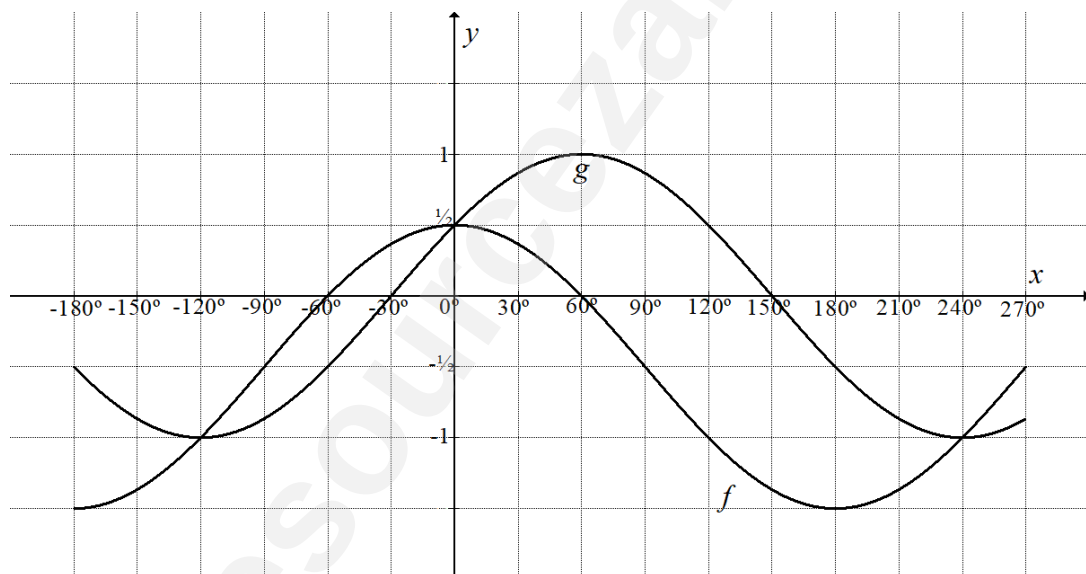
4.2.1	Let $C(x; 2x)$ and $A(x; 0)$ $(x+1)^2 + (2x-2)^2 = (x-x)^2 + (2x-0)^2$ $x^2 + 2x + 1 + 4x^2 - 8x + 4 = 4x^2$ $x^2 - 6x + 5 = 0$ $(x-1)(x-5) = 0$ $x = 1 \text{ or } x = 5$ $C(1; 2) \text{ or } C(5; 10)$	✓ coordinates of C and A/ <i>koördinate A en C</i> ✓ equating two r^2 / <i>Stel twee r^2 gelyk</i> ✓ standard form/ <i>standaardvorm</i> ✓ factors/<i>faktore</i> ✓ $C(1; 2)$ ✓ $C(5; 10)$ (6)
4.2.2	$r = 2 \text{ or } r = 10$	✓ $r = 2$ ✓ $r = 10$ (2)
[22]		

QUESTION/VRAAG 5

5.1.1	$y^2 = 1^2 - (\sqrt{1 - k^2})^2$ $= 1 - 1 + k^2$ $= k^2$ $y = k$ $\sin 25^\circ = k$	 ✓ diagram and/en Pythagoras ✓ $\sin 25^\circ = k$ (2)
5.1.2	$\sin 50^\circ = 2 \sin 25^\circ \cos 25^\circ$ $= 2k\sqrt{1 - k^2}$	✓ double angle expansion/ <i>Brei dubbelhoek uit</i> ✓ substitution/<i>vervang</i> (2)
5.2	$\sqrt{\frac{\tan(-207^\circ)}{\tan 333^\circ} - \frac{\sin^2(x - 360^\circ)}{\sin(x - 90^\circ)\cos x}}$ $= \sqrt{\frac{-\tan 27^\circ}{-\tan 27^\circ} - \frac{\sin^2 x}{-\cos x \cdot \cos x}}$ $= \sqrt{1 + \frac{\sin^2 x}{\cos^2 x}}$ $= \sqrt{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}}$ $= \sqrt{\frac{1}{\cos^2 x}}$ $= \frac{1}{\cos x}$	✓ $-\tan 27^\circ$ ✓ $-\tan 27^\circ$ ✓ $\sin^2 x$ ✓ $-\cos x$ ✓ $= \sqrt{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}}$ ✓ $\sqrt{\frac{1}{\cos^2 x}}$ ✓ answer /<i>antwoord</i> (7)
5.3	RHS: $\frac{\sin 2A}{\cos 2A} \times \frac{\cos A}{\sin A}$ $= \frac{2\sin A \cos A \cdot \cos A}{\sin A \cdot \cos 2A}$ $= \frac{2\cos^2 A}{\cos 2A}$ LHS: $\frac{1 + 2\cos^2 A - 1}{\cos 2A}$ $= \frac{2\cos^2 A}{\cos 2A}$ ∴ LHS = RHS	✓ replace/<i>vervang</i> tan 2A ✓ expansion of $\sin 2A$ <i>/uitbrei van $\sin 2A$</i> ✓ $\frac{2\cos^2 A}{\cos 2A}$ ✓ replacing $\cos 2A$ in numerator/ <i>Vervang $\cos 2A$ in teller</i> ✓ $\frac{2\cos^2 A}{\cos 2A}$ (5)

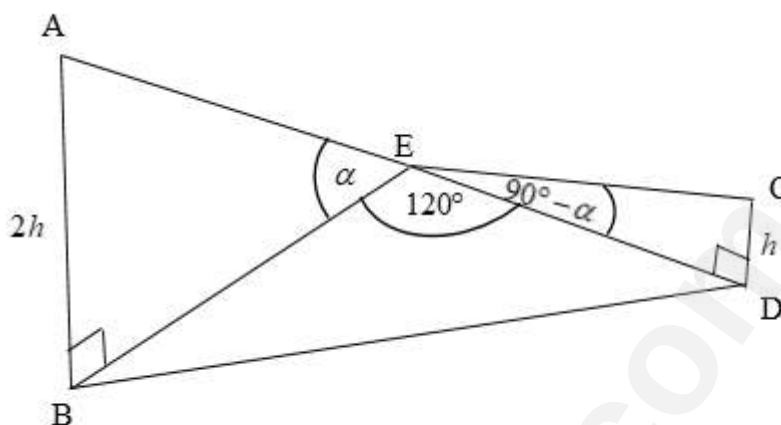
5.4	$2 \sin 2x = -\cos 2x$ $\frac{2 \sin 2x}{\cos 2x} = \frac{-\cos 2x}{\cos 2x}$ $2 \tan 2x = -1$ $\tan 2x = -\frac{1}{2}$ $2x = 153,43^\circ + k180^\circ$ $x = 76,72^\circ + k.90^\circ, k \in \mathbb{Z}$	✓ dividing by $\cos 2x$ / <i>Deel deur $\cos 2x$</i> ✓ $\tan 2x = -\frac{1}{2}$ ✓ $2x = 153,43^\circ + k180^\circ$ ✓ $x = 76,72^\circ + k.90^\circ$ ✓ $k \in \mathbb{Z}$ (5)
		[22]

QUESTION/VRAAG 6



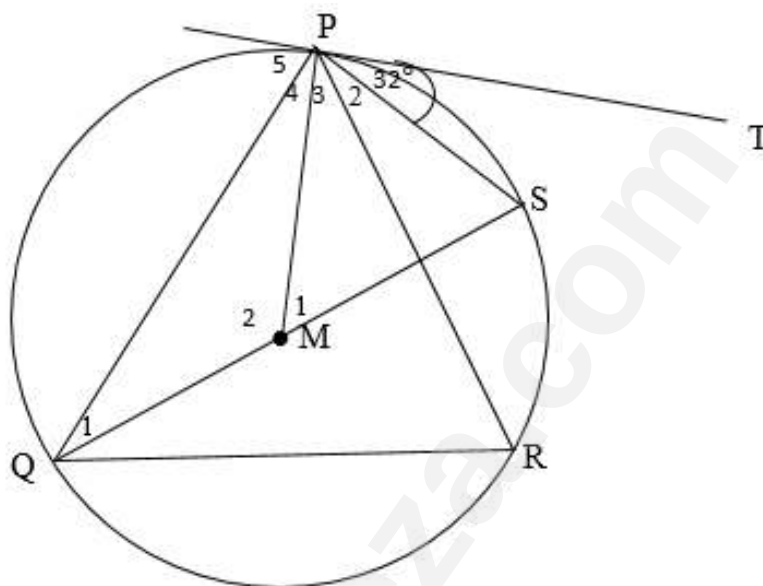
6.1	Graph done/Grafiek geskets	g: ✓ intercepts/as – afsnitte ✓ turning points/draaipunte ✓ shape/vorm (3)
6.2.1	$x \in (-120^\circ; 0^\circ) \cup (240^\circ; 270^\circ]$	✓ $(-120^\circ; 0^\circ)$ ✓ $(240^\circ; 270^\circ]$ ✓ notation /notasie (3)
6.2.2	$x \in [-180^\circ; -60^\circ] \cup [-30^\circ; 60^\circ] \cup [150^\circ; 270^\circ]$	✓ $x \in [-180^\circ; -60^\circ]$ ✓ $[-30^\circ; 60^\circ]$ ✓ $[150^\circ; 270^\circ]$ (3)
6.3	$y = \sin(x + 30^\circ - 120^\circ)$ $= \sin(x - 90^\circ)$ $= -\cos x$	✓ $y = \sin(x + 30^\circ - 120^\circ)$ ✓ answer/antwoord (2)
		[11]

QUESTION/VRAAG 7



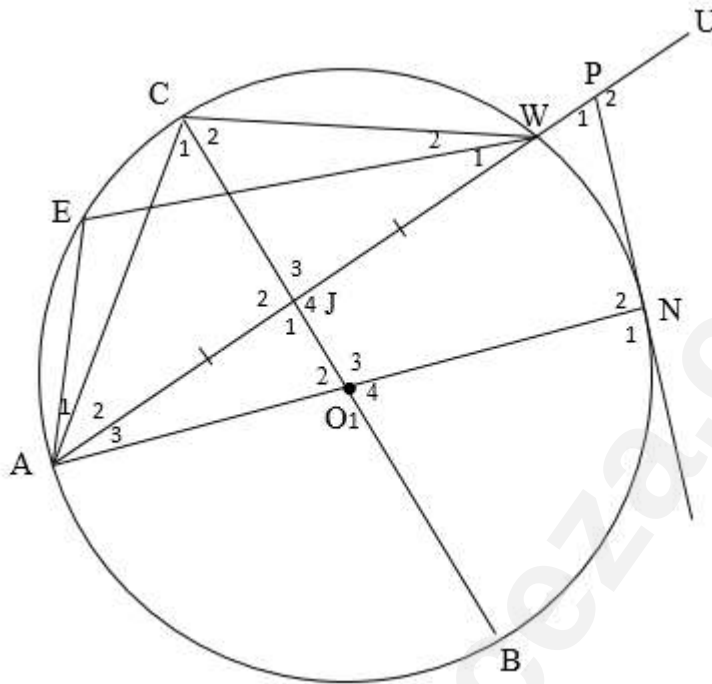
7.1	$\tan \alpha = \frac{2h}{BE}$ $BE = \frac{2h}{\tan \alpha}$	✓ ratio/trig verhouding ✓ $BE = \frac{2h}{\tan \alpha}$ (2)
7.2	$\tan \alpha = \frac{ED}{h}$ $\therefore ED = h \tan \alpha$	✓ $\tan \alpha$ ✓ correct ratio /korrekte verhouding (2)
7.3	$BD^2 = BE^2 + ED^2 - 2BE \cdot DE \cdot \cos \hat{BED}$ $=$ $\left(\frac{2h}{\tan \alpha}\right)^2 + (h \tan \alpha)^2 - 2\left(\frac{2h}{\tan \alpha}\right)(h \tan \alpha)(\cos 120^\circ)$ $= \frac{4h^2}{\tan^2 \alpha} + h^2 \tan^2 \alpha - 4h^2(-\cos 60^\circ)$ $= \frac{4h^2}{\tan^2 \alpha} + h^2 \tan^2 \alpha - 4h^2\left(-\frac{1}{2}\right)$ $= \frac{4h^2}{\tan^2 \alpha} + h^2 \tan^2 \alpha + 2h^2$ $= \frac{4h^2 + h^2 \tan^4 \alpha + 2h^2 \tan^2 \alpha}{\tan^2 \alpha}$ $= \frac{h^2(4 + \tan^4 \alpha + 2 \tan^2 \alpha)}{\tan^2 \alpha}$ $= \frac{h\sqrt{\tan^4 \alpha + 2 \tan^2 \alpha + 4}}{\tan^2 \alpha}$	✓ Correct use of cos rule/korrekte vervanging in kos – reël ✓ substitution/vervanging ✓ simplification/vereenvoudig ✓ simplify /vereenvoudig ✓ h^2 taken out as common factor/haal h^2 gemeenskaplik uit (5)
		[9]

QUESTION/VRAAG 8



8.1	$\hat{Q}_1 = 32^\circ$ (tan-chord theorem/ $\angle$ tuss rkl en krd)	✓S✓R (2)
8.2	$\hat{P}_4 = \hat{Q}_1 = 32^\circ$ ($\angle$'s opp equal sides/ $\angle$ 'e teenoor gelyke sye)	✓S✓R (2)
8.3	$\hat{M}_1 = 64^\circ$ ($\angle$ at centre is $2 \times \angle$ at circumference/ middelpunts $\angle = 2 \times \text{omtreks} \angle$)	✓S✓R (2)
8.4	$\hat{M}_2 = 116^\circ$ ($\angle$'s on a str line / $\angle$ e op 'n rt lyn) $\hat{R} = 58^\circ$ ($\angle$ at centre is $2 \times \angle$ at circumference/ middelpunts $\angle = 2 \times \text{omtreks} \angle$)	✓S✓R ✓S✓R (4)
		[10]

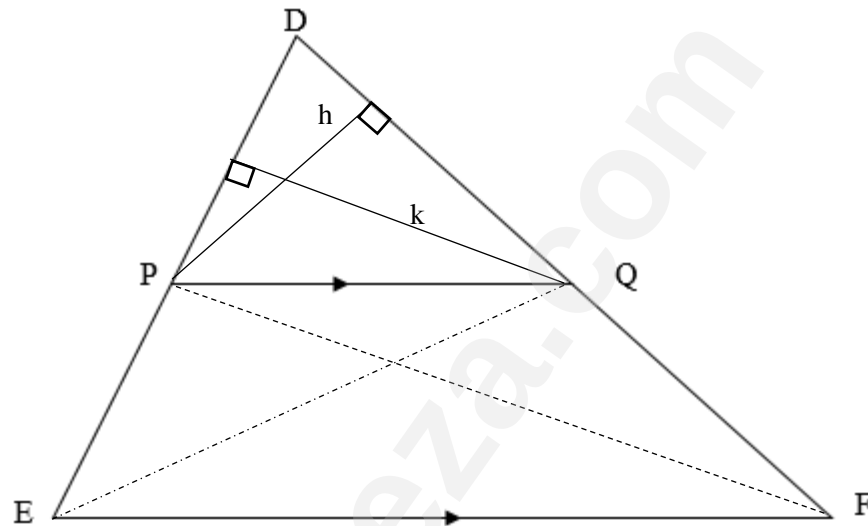
QUESTION/VRAAG 9



9.1	$\hat{J}_3 = 90^\circ$ (line from centre to midpoint/ (lyn van mdpt van sirkel na mdpt van koord) $N_2 = 90^\circ$ (tangent $\perp$ to radius/ $rkl \perp r$) $\hat{J}_3 = \hat{N}_2$ ONPJ is a cyclic quad /koordevierhoek (CONVERSE of ext. $\angle$ of cyclic quad/ OMG van buite $\angle$ van koordevierhoek)	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R (6)
9.2	$\hat{O}_1 = 2\hat{C} = 2x$ ($\angle$ at centre $= 2 \times \angle$ at circumference middelpunts $\angle = 2 \times$ omtreks $\angle$) $\hat{O}_4 = 180^\circ - 2x$ ($\angle$'s on a str line/ $\angle$ e op 'n rt lyn) $\hat{P}_1 = 180^\circ - 2x$ (ext. $\angle$ of a cyclic quad/ buite $\angle$ van koordevierhoek)	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R (6)
		[12]

QUESTION/VRAAG 10

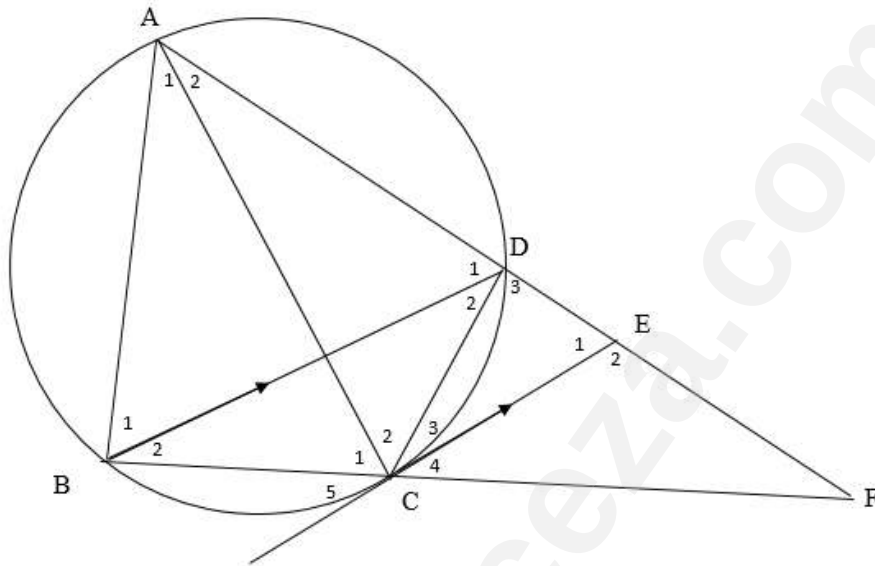
10.1



10.1	Construction: Join PQ and PF and draw altitudes h and k/ <i>Konstruksie: Verbind PQ en PF en trek hoogtelyne h en k</i> $\frac{\text{area}\triangle DPQ}{\text{area}\triangle PEQ} = \frac{\frac{1}{2} DP \times h}{\frac{1}{2} PE \times h} = \frac{DP}{PE}$ $\frac{\text{area}\triangle DPQ}{\text{area}\triangle PQF} = \frac{\frac{1}{2} DQ \times k}{\frac{1}{2} QF \times k} = \frac{DQ}{QF}$ But the $\text{area}\triangle PQE = \text{area}\triangle PQF$ (same base and same height/ <i>dieselfde basis en hoogte</i>) $\therefore \frac{\text{area}\triangle DPQ}{\text{area}\triangle PEQ} = \frac{\text{area}\triangle DPQ}{\text{area}\triangle PQF}$ $\therefore \frac{DP}{PE} = \frac{DQ}{QF}$	✓ construction/ <i>konstruksie</i> ✓ S ✓ S ✓ S ✓ R ✓ S
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(6)

10.2



10.2.1	$\hat{C}_3 = \hat{B}_2$ (tan-chord theorem/ <i>∠ tuss rkl en krd</i>) $\hat{C}_3 = \hat{D}_2$ (alternate/ <i>verwisselende ∠'s</i> ; $BD \parallel CE$) $\hat{B}_2 = \hat{D}_2$ $BC = DC$ (sides opposite equal $\angle$'s/ <i>sye teenoor = ∠e</i>)	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R	(4)
10.2.2	In $\triangle BAF$ and/ en $\triangle DCF$: $\hat{B}_1 + \hat{B}_2 = \hat{D}_3$ (ext. $\angle$ of a cyclic quad/ <i>buite ∠ van kvh</i>) $\hat{A}_1 + \hat{A}_2 = \hat{C}_3 + \hat{C}_4$ (ext. $\angle$ of a cyclic quad/ <i>buite ∠ van kvh</i>) $\hat{F}$ is common/ <i>gemeenskaplik</i> $\therefore \triangle BAF \parallel \triangle DCF$ ($\angle\angle\angle$)	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R	(4)
10.2.3	$\frac{BA}{DC} = \frac{AF}{CF}$ (similar/ <i>gelykvormige ∆'s</i>) $\frac{BA}{AF} = \frac{DC}{CF}$ $\frac{BA}{AF} = \frac{BC}{CF}$ ($BC = DC$) But $\frac{BC}{CF} = \frac{DE}{EF}$ ($BD \parallel CE$; prop theorem/ <i>eweredigh st</i>) $\therefore \frac{BA}{AF} = \frac{DE}{EF}$	$\checkmark$ S $\checkmark$ S $\checkmark$ Replacing/ <i>vervang</i> DC $\checkmark$ S $\checkmark$ R	(5)

10.2.4	In $\triangle ECD$ and $\triangle EAC$: $\hat{E}_1$ s common/<i>gemeenskaplik</i> $\hat{C}_3 = \hat{A}_2$ (tan-chord theorem/<i>∠ tuss rkl en krd</i>) $\hat{D}_3 = \hat{C}_2 + \hat{C}_3$ (∠'s of a Δ/ <i>∠'e van 'n Δ</i>) $\therefore \triangle ECD \parallel \triangle EAC$ ($\angle\angle\angle$)	✓ S ✓ R ✓ S ✓ R (4)
10.2.5	$\frac{EC}{EA} = \frac{ED}{EC}$ ($\parallel \Delta$'s) $EC^2 = EA \cdot ED$ But $ED = \frac{BA \cdot EF}{AF}$ (from/ <i>uit</i> 10.2.3) $EC^2 = \frac{EA \cdot BA \cdot EF}{AF}$	✓ S ✓ S ✓ S (3)
[26]		

TOTAL/TOTAAL: 150