eduresourceza.com

KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

COMMON TEST

JUNE 2023

MARKS: 150

TIME: 3 hours

N.B. This paper consists of 9 pages, 1 information sheet, and 1 diagram sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of **9** questions.
2. Answer **ALL** the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers.
5. Answers only will **NOT** necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to **TWO** decimal places, unless stated otherwise.
8. Diagrams are **NOT** necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

Seven scores for a test out of 50 marks were recorded. However, when recording the data the statistician had some difficulty with his computer. He listed the following conditions that he had remembered,

Scores	a	b	c	d	e	f	g

- Maximum Value is 42
- Range is 35
- Median is 23
- The difference between the median and the upper quartile is 14
- The Inter – Quartile Range is 22
- $e = 2c$
- The mean is 25.

Determine the values of the scores. (Show all working) (10)

[10]

QUESTION 2

The class teacher of a grade 10 class collected the following data for her learners. She recorded the time taken by the learners to complete different tasks assigned.

Time in minutes	**No. of learners**			
$0 < t \leq 10$	5			
$10 < t \leq 20$	8			
$20 < t \leq 30$	18			
$30 < t \leq 40$	7			
$40 < t \leq 50$	2			

2.1 Calculate the estimated mean time. (3)

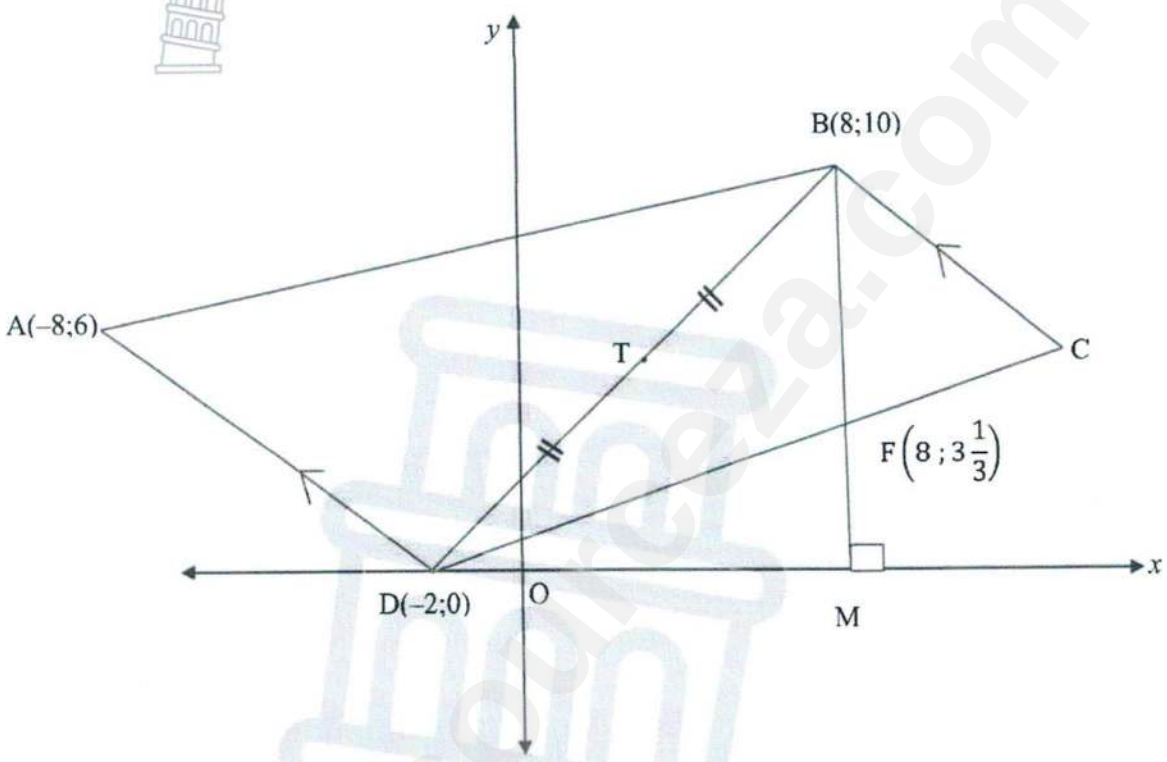
2.2 Draw a frequency polygon (Ogive) curve on the system of axes provided. (5)

2.3 Calculate how many learners took more than 60 % of the time to complete the tasks assigned to them. (3)

[11]

QUESTION 3

In the sketch below, A(-8;6), B(8 ; 10), C and D(–2;0) are the vertices of a trapezium having AD ∥ BC. T is the midpoint of DB. From the vertex B, BM is drawn perpendicular to the x – axis and intersects DC in F $\left(8 ; 3\frac{1}{3}\right)$.



3.1 Calculate the gradient of AD. (2)

3.2 Determine the equation of BC in the form $y = mx + c$. (3)

3.3 Prove that BD ⊥ AD. (3)

3.4 Calculate the size of $B\hat{D}M$. (2)

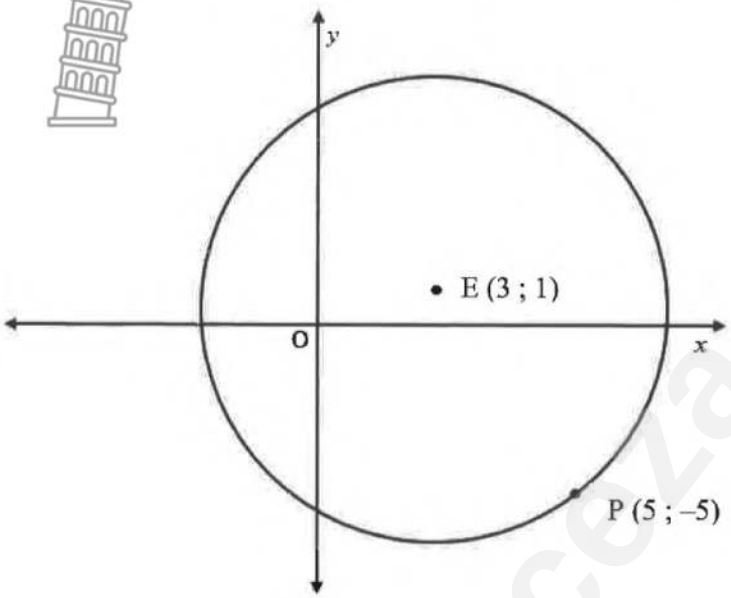
3.5 If TC ∥ DM and points T and C are symmetrical about the line BM. Calculate the coordinates of C. (3)

3.6 Calculate the area of ΔBDF. (5)

[18]

QUESTION 4

In the diagram below, the circle centred at E (3 ; 1) passes through the point, P (5 ; –5).



4.1 Determine the equation of :

4.1.1 The circle in the form $x^2 + y^2 + Ax + By + C = 0$. (4)

4.1.2 The tangent to the circle at P (5 ; –5) in the form $y = mx + c$. (5)

4.2 A smaller circle is drawn inside the circle. Line EP is the diameter of the small circle.

Determine the:

4.2.1 Coordinates of the centre of the smaller circle. (4)

4.2.2 Length of the radius. (2)

4.3 Hence, or otherwise, determine whether the point C(9 ; 3) lies inside, outside or on the circle centre, E. (4)

[19]

QUESTION 5

5.1 If $\tan 12^\circ = q$, then determine the value of the following, without using a calculator:

5.1.1 $\cos 192^\circ$ (3)

5.1.2 $\cos 24^\circ$ (3)

5.1.3 $1 - 2\sin^2 6^\circ$ (2)

5.2 Evaluate the following trigonometric expression without using a calculator:

$$\frac{2\sin^2(x-180^\circ)\cos(180^\circ - x)}{\cos(90^\circ + x)\sin x - \cos(x-90)\sin(720^\circ - x)}$$ (7)

5.3 Given: $(1 - \tan A)\left(\frac{\cos A}{\cos 2A}\right) = \frac{1}{\cos A + \text{Sin } A}$

5.3.1 Prove the identity. (3)

5.3.2 Write down the maximum value of $\frac{1}{\cos A + \text{Sin } A}$ (2)

5.4 Calculate the value of

$$\sum_{38^0}^{52^0} \cos^2 A$$ (4)

[24]

QUESTION 6

6.1 Sketch the graphs of $f(x) = \sin\frac{1}{2}x$ and $g(x) = \cos(x + 60^\circ)$ for $x \in [-180^\circ; 180^\circ]$ on the grid given at the back. (6)

6.2 Use your graphs to answer the following questions:

6.2.1 Write down the range of the graph of f. (2)

6.2.2 State the period of g. (1)

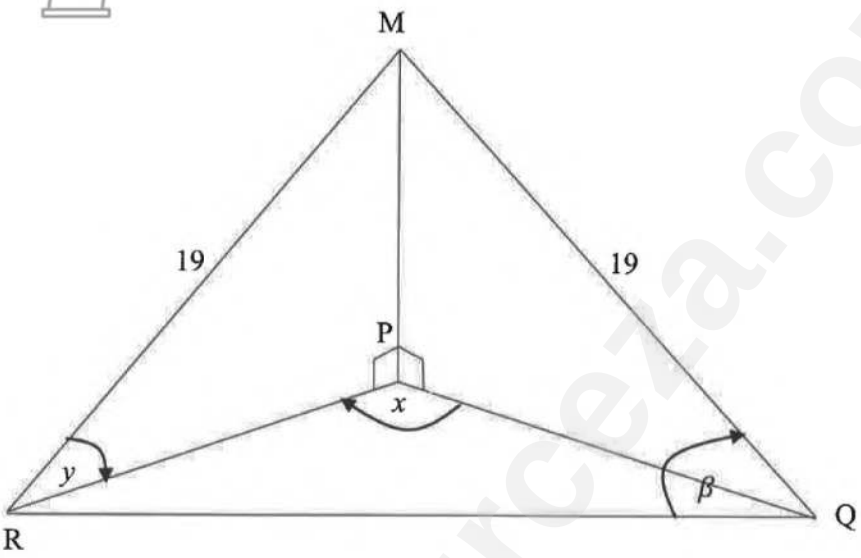
6.2.3 Determine the values of x for which $f(x) = g(x)$. (4)

6.2.4 If $h(x) = g(x + 30^\circ)$, write down the equation of h. (3)

[16]

QUESTION 7

In the diagram below, MP is a vertical tower. MQ and MR are wire ropes used to stabilize MP and are each 19 metres in length. R, P and Q are in the same horizontal plane.
The angle of elevation of M from R is y.
$\hat{RPQ} = x$ *and* $\hat{MQR} = \beta$



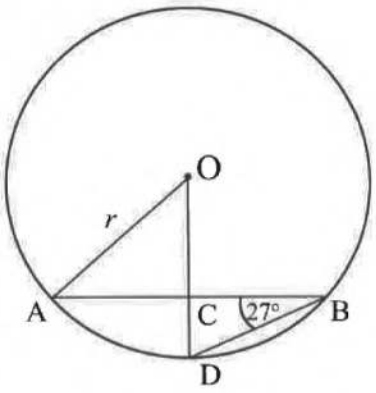
7.1 Prove that the Area of Δ PQR $= \frac{361 \sin x \cos^2 y}{2}$ (5)

7.2 Show that, RQ $= 38 \cos \beta$ (6)

[11]

QUESTION 8

8.1 O is the centre of the circle, radius r, and chord AB = $\sqrt{128}$ cm. OCD ⊥ AB and OC : CD = 3 : 2. $A\hat{B}D = 27°$



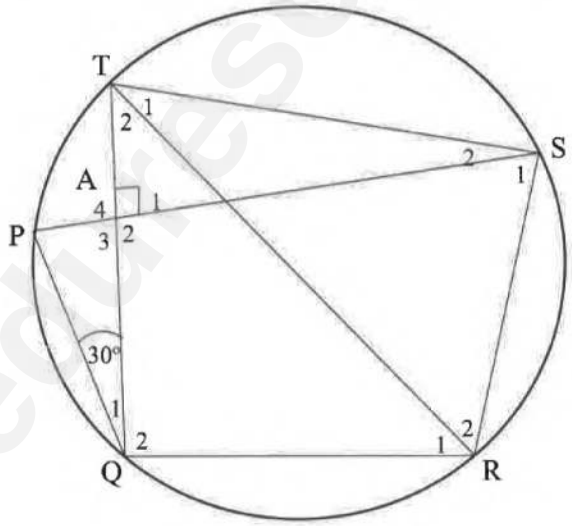
8.1.1 Calculate the length of the radius of the circle, without using a calculator. (5)

8.1.2 Calculate, with reason and $A\hat{O}D$. (2)

8.2 Complete the following statement: (2)

8.2.1 The exterior angle of a cyclic quadrilateral is .....

8.2.2 TQ is a chord of the circle PQRST. QAT ⊥ PAS. $\hat{Q}_1 = 30°$ and $\hat{P} = \hat{S}_1$.



a) Name 3 angles each equal to 60°. (3)

b) Calculate the size of $Q\hat{R}S$. (2)

c) Prove that PS || QR. (2)

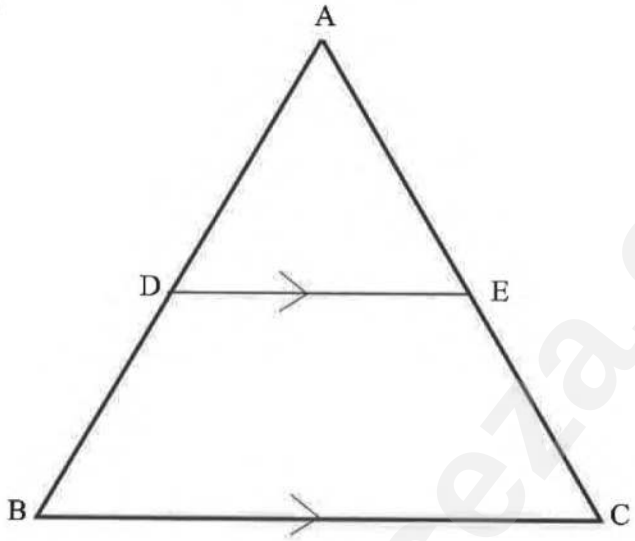
d) Prove that TR is a diameter of the circle . (2)

[18]

QUESTION 9

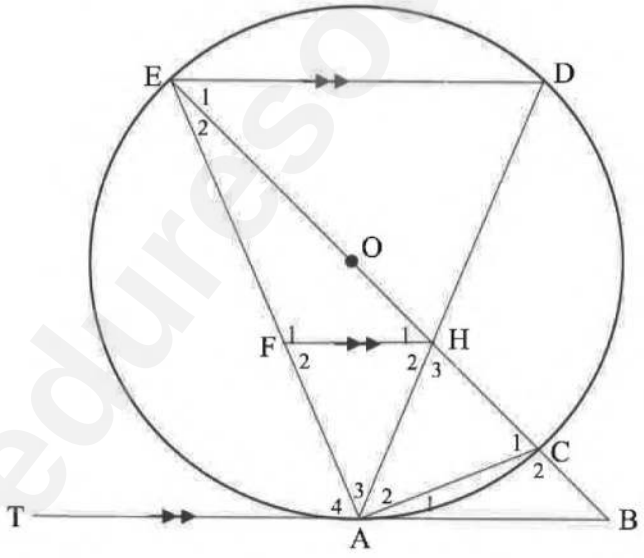
9.1 In Δ ABC, DE || BC. Prove :

$$\frac{AD}{DB} = \frac{AE}{EC}$$



(6)

9.2 In the diagram, O is the centre of the circle ACDE. EC is produced to B. TAB is a tangent to the circle at A. EC and AD intersect at H. F is a point on EA. ED || FH || TAB. EOHCB is a straight line. AH : HD = 5: 7.



9.2.1 Prove ΔABC ||| ΔEBA (4)

9.2.2 Calculate r, the radius of the circle, if AB = 5 metres and BC = $\frac{2r}{3}$ metres. (4)

9.2.3 Write down, with reason the value of AF : FE. (2)

9.2.4 Determine $\frac{\text{Area of } \Delta AFH}{\text{Area of } \Delta AED}$. (4)

9.2.5 Calculate the length of OH. (3)

[23]

TOTAL MARKS: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \; ; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$In\ \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum f.x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$\mathrm{P}(A) = \frac{n(\mathrm{A})}{n(\mathrm{S})} \qquad \mathrm{P(A\ or\ B)} = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A\ and\ B)}$$

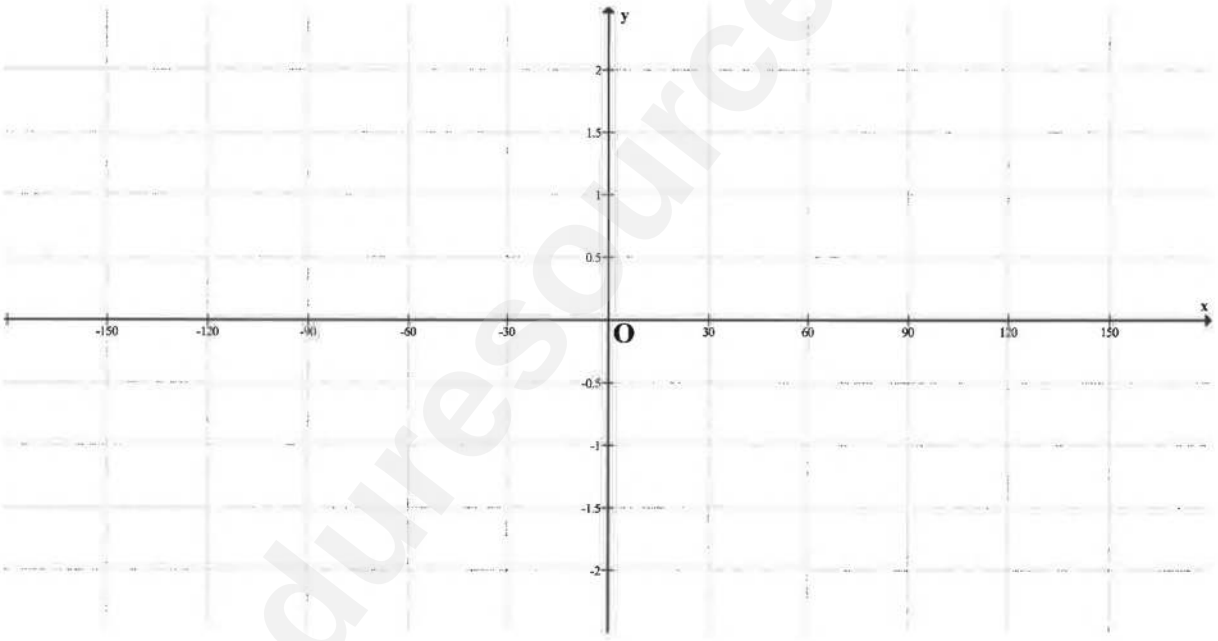
$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

DIAGRAM SHEET

QUESTION 2

Time in minutes	No. of learners	
$0 < t \leq 10$	5	
$10 < t \leq 20$	8	
$20 < t \leq 30$	18	
$30 < t \leq 40$	7	
$40 < t \leq 50$	2	

QUESTION 6.1



FINAL

KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

MARKING GUIDELINE

COMMON TEST

JUNE 2023

MARKS: 150

N.B. This marking guideline consists of 16 pages.

QUESTION 1

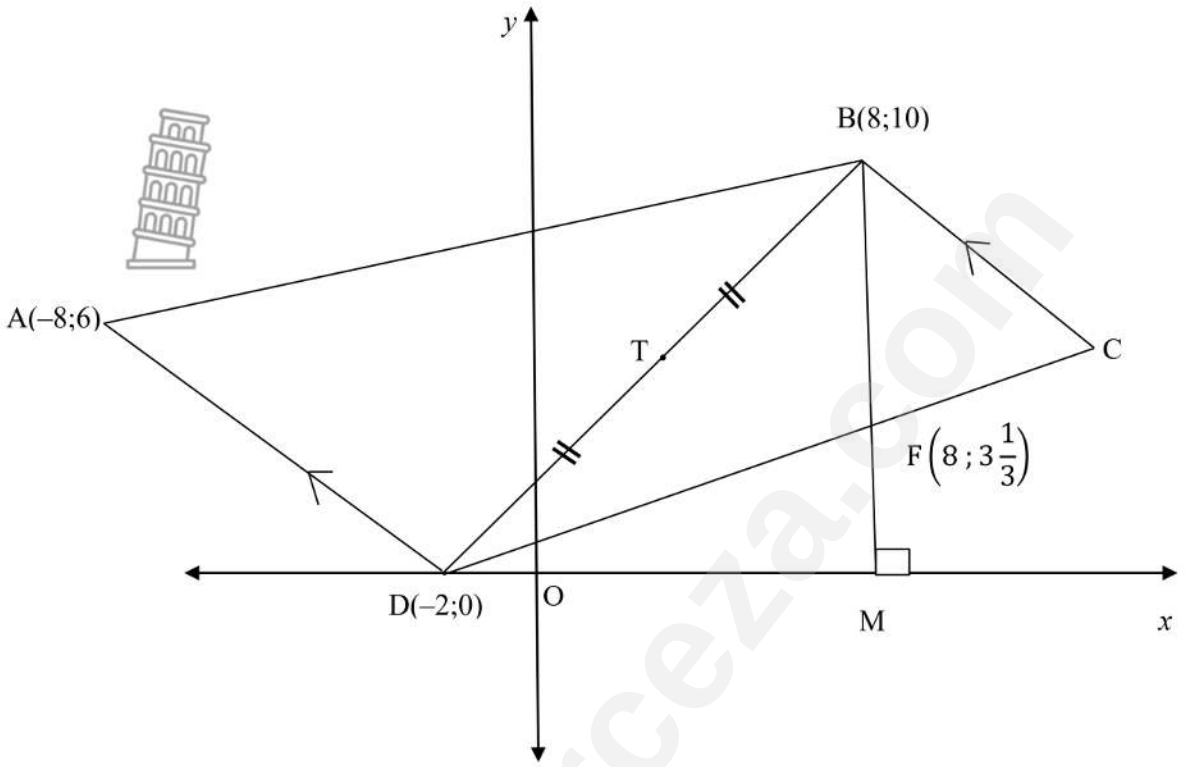
1.1	$g = 42$ Minimum : $a = 42 - 35 = 7$ $d = 23$ $f - 23 = 14$ ∴ Therefore $f = 37$ $f - b = 22$ ∴ Therefore $b = 15$ $\frac{7 + 15 + c + 23 + 2c + 37 + 42}{7} = 25$ $\frac{3c + 124}{7} = 25$ $3c + 124 = 175$ $3c = 51$ $c = 17$ Therefore $e = 34$	A✓ Value of g A✓ Value of a A✓ Value of f A✓ Value of d CA✓ Value of b CA✓ Setting up equation CA✓ Simplifying numerator CA✓ equation in c CA✓ Value of c CA✓ Value of e	(10)
			[10]

QUESTION 2

Time (in minutes)	Number of Learners	Midpoint of Interval(x)	f.x	Cf
$0 < t \leq 10$	5	**5**	**25**	**5**
$10 < t \leq 20$	8	**15**	**120**	**13**
$20 < t \leq 30$	18	**25**	**450**	**31**
$30 < t \leq 40$	7	**35**	**245**	**38**
$40 < t \leq 50$	2	**45**	**90**	**40**
Total	**40**		**930**	

2.1	Estimated Mean = $\frac{930}{40} = 23{,}25$	A✓930 A✓40 CA✓Answer (only if denominator is 40) **(Answer only full marks)**	(3)
2.2	DO NOT MARK THIS QUESTION		
2.3	60 % of 50 minutes = 30 minutes 9 learners	A✓Calculation A✓30 minutes A✓Answer **(AO full marks)**	(3)

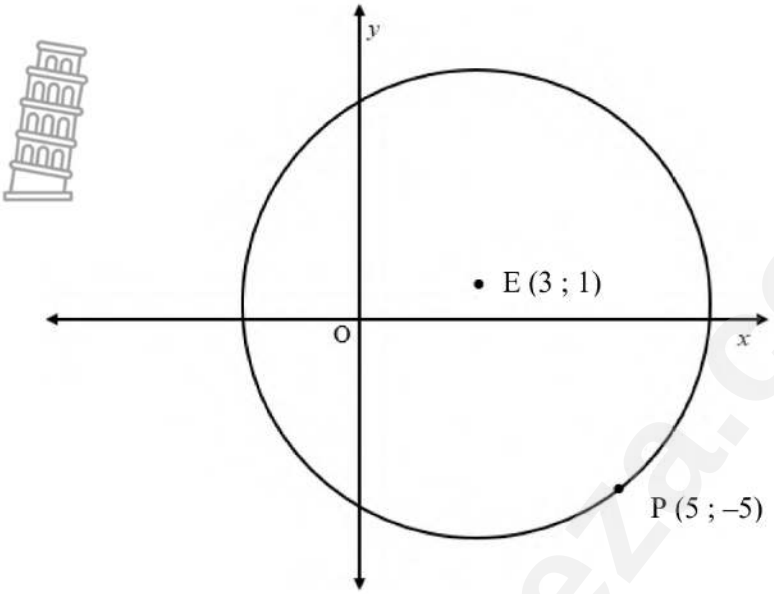
QUESTION 3



3.1	$m\text{AD} = \frac{6-0}{-8+2} = \frac{6}{-6} = -1$	A✓subst. of points A and D CA✓Answer	(2)
3.2	$m\text{AD} = m\text{BC} = -1$ …..(AD ∥ BC) $y = mx + c$ $10 = -1(8) + c$ $18 = c$ $y = -x + 18$	CA✓gradient of BC CA✓subst. of point C and gradient CA✓Answer	(3)
3.3	$m\text{BD} = \frac{10-0}{8+2} = \frac{10}{10} = 1$ Since the product of the gradients of lines AD and BD = –1 Therefore BD ⊥ AD	A✓subst. of points A and D A✓Gradient of BD A✓Justification	(3)
3.4	$\tan\theta = 1$ $\text{B}\hat{\text{D}}\text{M} = 45^\circ$	CA✓$\tan\theta = 1$ CA✓Answer	(2)
3.5	T(3 ; 5) … midpoint C(13 ; 5)	A ✓coordinates CA CA✓✓coordinates of C	(3)

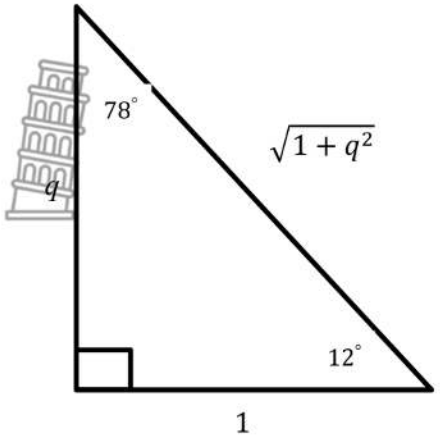
3.6	$BF = \sqrt{(8-8)^2 + \left(10 - 3\frac{1}{3}\right)^2} = 6\frac{2}{3} = \frac{20}{3}$ Area of $\Delta BDF = \frac{1}{2}(10)\left(\frac{20}{3}\right) = \frac{100}{3} = 33\frac{1}{3}$ square units. **OR** Area of ΔBDF = Area of Δ DBM – area of Δ DFM $= \frac{1}{2}(10)(10) - \frac{1}{2}(10)\left(\frac{10}{3}\right)$ $= \frac{100}{33} = 33{,}3 \text{ units}^2$ **OR** $m_{DF} = \frac{1}{3}$ $\tan F\hat{D}M = \frac{1}{3}$ $F\hat{D}M = 18.43^\circ$ $BD = 10\sqrt{2}$ $DF = \frac{10\sqrt{10}}{3}$ $\therefore$ Area of $\Delta BDF = \frac{1}{2}\, 10\sqrt{2} \times \frac{10\sqrt{10}}{3} \sin 26.57^\circ$ $= 33{,}3 \text{ units}^2$	A✓Substitution A✓ BF value CA✓Subst. into formula CA✓Simplification CA✓Answer	(5)
			[18]

QUESTION 4

4.1			
4.1.1	$EP = \sqrt{(3-5)^2 + (1+5)^2}$ $EP = \sqrt{(-2)^2 + (6)^2}$ $EP = \sqrt{40}$ $(x-3)^2 + (y-1)^2 = 40$ $x^2 - 6x + 9 + y^2 - 2y + 1 = 40$ $x^2 + y^2 - 6x - 2y - 30 = 0$	A✓Substitution CA✓EP value CA✓Substitution CA✓Answer	(4)
4.1.2	Radius: $m = \frac{1+5}{3-5} = -3$ Tangent: $m = \frac{1}{3}$ $y = mx + c$ $-5 = \frac{1}{3}(5) + c$ $-15 = 5 + 3c$ $-20 = 3c$ $c = -6\frac{2}{3}$ $y = \frac{1}{3}x - 6\frac{2}{3}$	A✓Gradient of radius CA✓Gradient of tangent (provided it is positive) CA✓Subst. Of point and gradient. CA✓c – value CA✓Answer	(5)

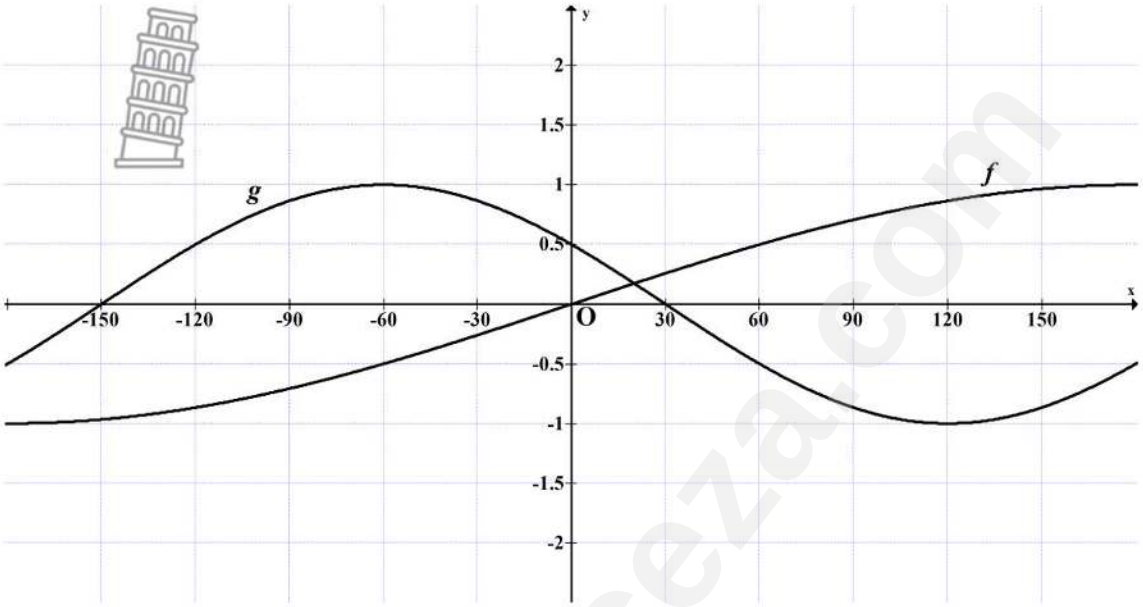
4.2.1	Coordinates of Centre of smaller circle: $\left(\frac{3+5}{2};\frac{1-5}{2}\right)=(4;-2)$	A✓ midpoint formula A✓Substitution CA CA✓✓Answer **(AO full marks)**	(4)
4.2.2	Radius of larger circle: $2\sqrt{10}$ Radius of smaller circle: $r=\frac{1}{2}(2\sqrt{10})=\sqrt{10}$ units **OR** Radius $=\sqrt{(4-3)^2+(-2-1)^2}$ $=\sqrt{10}$ **OR** Raduis $=\sqrt{(4-5)^2+(-2-5)^2}$ $=\sqrt{10}$	A ✓ $\frac{1}{2}(2\sqrt{10})$ CA ✓ Answer	(2)
4.3	EC $=\sqrt{(9-3)^2+(3-1)^2}$ $=\sqrt{6^2+2^2}$ $=\sqrt{36+4}$ $=\sqrt{40}$ $\therefore$ EC = radius $\therefore$ C lies on the circle, centre E **OR** $x^2-6x+9+y^2-2y+1=40$ $x^2+y^2-6x-2y-30=0$ **OR** $(x-3)^2+(y-1)^2=40$ LHS $=(9-3)^2+(3-1)^2=40$ RHS $=40$ $\therefore$ LHS = RHS $\therefore$ It lies on the circle	A✓ Substitution CA ✓ EC $=\sqrt{40}$ CA ✓ equating to radius CA ✓ Conclusion	(4)
			[19]

QUESTION 5

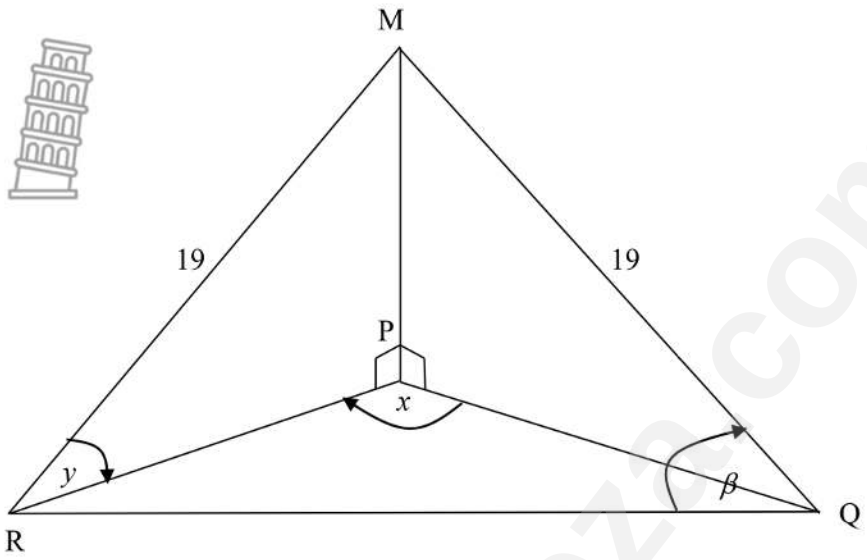
5.1			
5.1.1	$\cos 192^\circ$ $= -\cos 12^\circ$ $= -\frac{1}{\sqrt{1+q^2}}$	A✓ $\sqrt{1+q^2}$ A✓$-\cos 12^\circ$ CA✓Answer	(3)
5.1.2	$\cos 24^\circ$ $= 2\cos^2 12^\circ - 1$ $= 2\left(\frac{1}{\sqrt{1+q^2}}\right)^2 - 1$ $= \frac{2}{1+q^2} - 1$ **OR** $\text{Cos } 24^\circ = 1 - 2\sin^2 12^\circ$ $= 1 - 2\left(\frac{q}{\sqrt{1+q^2}}\right)^2$ $= 1 - \frac{2q}{1+q^2}$ **OR** $\text{Cos } 24^\circ = \left(\frac{1}{\sqrt{1+q^2}}\right)^2 - \left(\frac{q}{\sqrt{1+q^2}}\right)^2$ $= \frac{1}{1+q^2} - \frac{q^2}{1+q^2}$ $= \frac{1-q^2}{1+q^2}$	A✓Double angle Expansion CA✓Substitution CA✓Answer	(3)

5.1.3	$1-2\sin^2 6^\circ$ $=\cos 12^\circ$ $=\frac{1}{\sqrt{1+q^2}}$	A✓Double angle CA✓Answer	(2)
5.2	$\frac{2\sin^2(x-180^\circ)\cos(180^\circ-x)}{\cos(90^\circ+x)\sin x-\cos(x-90)\sin(720^\circ-x)}$ $=\frac{2(-sinx)^2.(-\cos x)}{-\sin x.\sin x-(\sin x.-\sin x)}$ $=\frac{-2\sin^2 x\cos x}{0}$ $=undefined$	A✓$(-sinx)^2$ A✓$-\cos x$ A✓$-\sin x$ A✓$\sin x$ A✓$-\sin x$ CA✓$\frac{-2\sin^2 x\cos x}{0}$ CA✓Answer	(7)
5.3.1	LHS: $(1-\tan A)\left(\frac{\cos A}{\cos 2A}\right)$ $=\left(1-\frac{\sin A}{\cos A}\right)\left(\frac{\cos A}{\cos^2 A-\sin^2 A}\right)$ $=\left(\frac{\cos A-\sin A}{\cos A}\right)\frac{\cos A}{(\cos A-\sin A)(\cos A+\sin A)}$ $=\frac{1}{\cos A+\sin A}$ = RHS	A ✓ $\frac{\sin A}{\cos A}$ A ✓ $\cos^2 A-\sin^2 A$ A ✓ simplification	(3)
5.3.2	DO NOT MARK THIS QUESTION		
5.4	DO NOT MARK THIS QUESTION		
			[18]

QUESTION 6

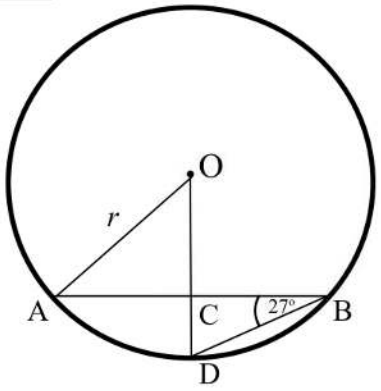
6.1			
	Graph of f: A 1 mark for x – intercepts A 1 marks for minimum and maximum points A 1 mark for shape Graph of g: A 1 mark for end points A 1 mark for x – intercepts A 1 mark for y – intercept		(6)
6.2.1	$y \in [-1\,;1]$	CA✓ $[-1\,;1]$ A ✓notation	(2)
6.2.2	360°	A✓Answer	(1)
6.2.3	$\sin\frac{1}{2}x = \cos(x+60^\circ)$ $\sin\frac{1}{2}x = \sin[90^\circ - (x+60^\circ)]$ $\sin\frac{1}{2}x = \sin[30^\circ - x]$ $\frac{1}{2}x = 30^\circ - x$ $\frac{3}{2}x = 30^\circ$ $x = 20^\circ$	A✓Co - Ratio A✓Equation in sine only A✓$\frac{1}{2}x = 30^\circ - x$ A✓Answer CA(**AO full marks**)	(4)
6.2.4	$h(x) = g(x+30)$ $h(x) = cos(x+30+60)$ $h(x) = cos(x+90)$ $h(x) = -\sin x$	A ✓substitution A✓ $cos(x+90)$ A✓Answer (**AO full marks**)	(3)
			[14]

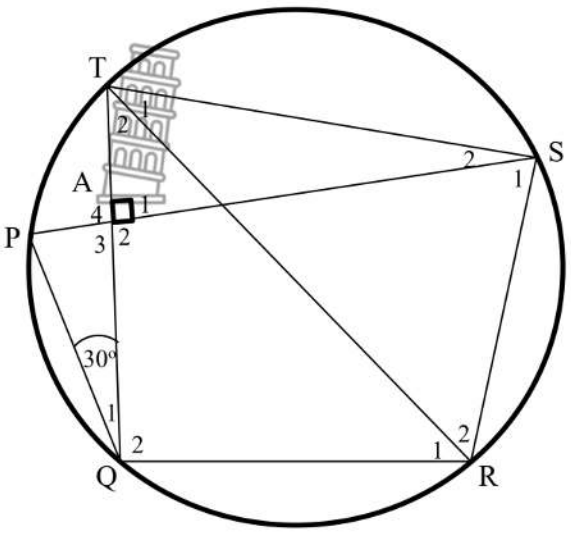
QUESTION 7



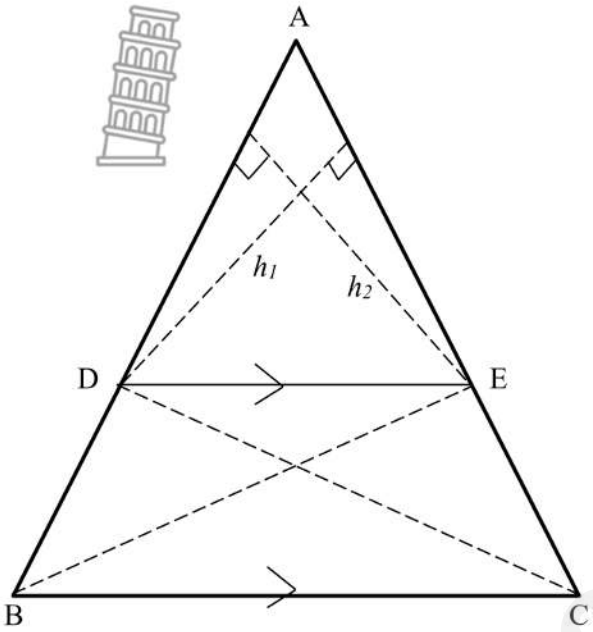
7.1	$\frac{PR}{19} = \cos y$ $PR = 19\cos y$ Now PR = PQ …..(ΔMPR ≡ ΔMPQ …90deg HS) Area of Δ PQR $= \frac{1}{2}(19\cos y)(19\cos y)\sin x$ $= \frac{361\sin x\cos^2 y}{2}$	A✓Trig ratio A✓Length of PR A✓Congruent triangles A✓Reason SAS A✓Substitution into Area Formula	(5)
7.2	$RQ^2 = 19^2 + 19^2 - 2(19)(19)\cos(180^\circ - 2\beta)$ $RQ^2 = 361 + 361 - 722(-\cos 2\beta)$ $RQ^2 = 722 + 722\cos 2\beta$ $RQ^2 = 722 + 722(2\cos^2\beta - 1)$ $RQ^2 = 1444\cos^2\beta$ $RQ = 38\cos\beta$ **OR** $MR^2 = RQ^2 + MQ^2 - 2RQ\,.\,MQ\cos\beta$ $19^2 = RQ^2 + 19^2 - 2(RQ)(19)\cos\beta$ $0 = RQ^2 - 38\,RQ\,.\cos\beta$ $\frac{38\,RQ\cos\beta}{RQ} = \frac{RQ^2}{RQ}$ $RQ = 38\cos\beta$	A✓Size of angle RMQ A✓Subst. Cosine rule A✓Reduction A✓Simplifying A✓Double angle expansion A✓$1444\cos^2\beta$	(6)
			[11]

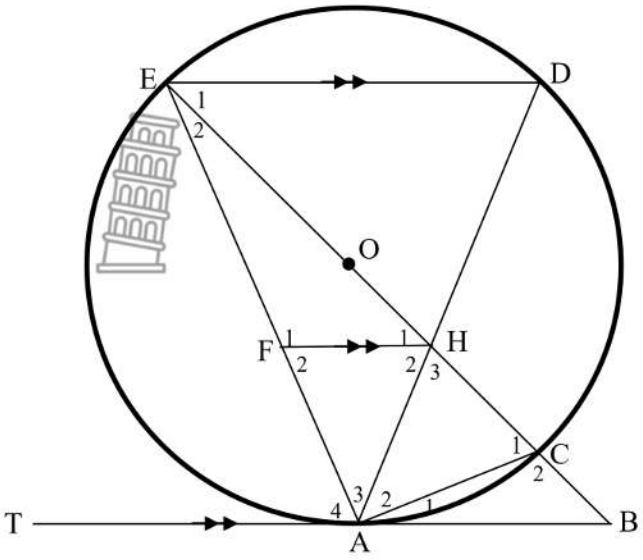
QUESTION 8

8.1	O is the centre of the circle, radius r, and chord AB = $\sqrt{128}$ cm. OCD ⊥ AB and OC : CD = 3 : 2. $A\hat{B}D = 27°$  		
8.1.1	$\frac{OC}{CD} = \frac{3}{2} = \frac{3r}{2r}$ AC = CB = $4\sqrt{2}$ cm … (line from centre ⊥ chord) In ΔAOC: $r^2 = \left(\frac{3}{5}r\right)^2 + \left(4\sqrt{2}\right)^2$ … Pythagoras $\frac{16}{25}r^2 = 32$ $r^2 = \frac{32}{1} \times \frac{25}{16} = 50\ cm^2$ $r = 5\sqrt{2}$ cm	A✓ OC and CD in terms of r. A✓ S/R A✓ Pythagoras (S/R) CA✓ Simplifying CA✓ Answer	(5)
8.1.2	$A\hat{O}D = 54°$ (angle at the centre 2 x angle at the circum)	A ✓ S A✓ R	(2)

8.2.1	(Exterior angle of cyclic quad = int. opp. A)	S ✓✓	(2)
8.2.2			
a)	$\hat{P}$ $Q\hat{T}S$ $\hat{S}_1$	S✓ S✓ S✓	(3)
b)	$Q\hat{R}S = 120^\circ$…[Opposite angles of cyclic quad.]	S✓ R✓	(2)
c)	$Q\hat{R}S + \hat{S}_1 = 120° + 60° = 180^\circ$ PS ∥ QR (Co-Interior angles are supplementary)	S✓ R✓	(2)
d)	$\hat{Q}_2 = 90^\circ$ …..(Corresponding Angles ; PS ∥ QR) TR is a diameter of the circle. (Conv. Angle in the semi -circle) **OR** $\hat{S}_1 = 60°$ ... given $\hat{Q}_1 = 30°$ ... given $\hat{Q}_1 + \hat{Q}_2 + \hat{S}_1 = 180°$ opp ∠'s of a cyclic quad $\therefore \hat{Q}_2 = 90°$ ∴ TR is a diameter (∠'s subt by 90°) Conv ∠'s on semi circle)	S/R✓ R✓	(2)
			[18]

QUESTION 9

9.1	 Proof: $\frac{\text{Area } \Delta \text{ ADE}}{\text{Area } \Delta \text{ DBE}} = \frac{\frac{1}{2} AD \text{ x } h_2}{\frac{1}{2} \text{DB x } h_2} = \frac{\text{AD}}{\text{DB}}$ (Common vertex E, same height h_2) $\frac{\text{Area } \Delta \text{ AED}}{\text{Area } \Delta \text{ DBE}} = \frac{\frac{1}{2} AE \text{ x } h_1}{\frac{1}{2} EC \text{ x } h_1} = \frac{\text{AE}}{\text{EC}}$ (Common vertex D, same height h_1) Area Δ DBE = Area Δ ECD (Common base DE, same height, DE // BC) $\frac{\text{Area } \Delta \text{ ADE}}{\text{Area } \Delta \text{ DBE}} = \frac{\text{Area } \Delta \text{ AED}}{\text{Area } \Delta \text{ ECD}}$ $\therefore \frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$ Area of Δ ADE is common and Area Δ DBE = Area Δ ECD	S✓ Construction ✓S✓R S/R✓ ✓S✓R **(No Marks if no construction indicated in drawing or words)**	(6)

9.2			
9.2.1	In Δ's ABC and EBA 1) $\hat{B} = \hat{B}$ ….. [Common] 2) $\hat{A}_1 = \hat{E}_2$ … [Tangent – Chord Theorem] 3) $\hat{C}_2 = E\hat{A}B$ …. [Remaining Angles of Δ's] ΔABC ||| ΔEBA…[AAA]	S/R✓ S✓R✓ R✓	(4)
9.2.2	ΔABC ||| ΔEBA $\frac{AB}{EB} = \frac{BC}{AB} = \frac{AC}{EA}$ similar triangles $\frac{5}{2r + \frac{2r}{3}} = \frac{\frac{2r}{3}}{5}$ $\frac{2r}{3}\left(2r + \frac{2r}{3}\right) = 25$ $\frac{4r^2}{3} + \frac{4r^2}{9} = 25$ $\frac{16r^2}{9} = 25$ $r^2 = 25 \times \frac{9}{16}$ $r = \sqrt{25 \times \frac{9}{16}} = \frac{15}{4}$ metres	S/R✓ S✓Substitution S✓Multiplication S✓Answer	(4)
9.2.3	Let AH = 5*a* and HD = 7*a* $\frac{AF}{FE} = \frac{AH}{HD} = \frac{5a}{7a} = \frac{5}{7}$ ….. [Prop. Theorem; FH || ED]	✓S ✓R	(2)

9.2.4	$\Delta AFH = \frac{5}{12}\Delta AHE$ …. [Common Vertex; Equal Heights] $\Delta AFH = \frac{5}{12}\left(\frac{5}{12}\Delta AED\right)$ …. [Common Vertex; Equal Heights] $\frac{\Delta AFH}{\Delta AED} = \frac{25}{144}$ **OR** $\frac{\text{Area } \Delta \text{ AFH}}{\text{Area } \Delta \text{ AED}} = \frac{\frac{1}{2}\text{ AF . AH Sin } \hat{A}_3}{\frac{1}{2}\text{ AE . AD Sin } A_3}$ $\therefore \left(\frac{AF}{AE}\right)^2$ since $\frac{AF}{FE} = \frac{AH}{AD}$ $= \left(\frac{5a}{12a}\right)^2$ $= \frac{25}{144}$	✓S ✓R ✓S/R ✓S	(4)
9.2.5	Let OH = x Then HC = $r - x = \frac{15}{4} - x$ metres $\frac{AF}{FE} = \frac{BH}{HE}$ …. [Prop. Theorem; FH || AB $\frac{5}{7} = \frac{\frac{15}{4} - x}{\frac{15}{4} + x}$ $5\left(\frac{15}{4} + x\right) = 7\left(\frac{15}{4} - x\right)$ $\left(\frac{75}{4} + 5x\right) = \left(\frac{105}{4} - 7x\right)$ $75 + 20x = 105 - 28x$ $48x = 30$ $x = \frac{30}{48} = \frac{5}{8}$ metres	✓S/R ✓S ✓S	(3)
			[22]

TOTAL MARKS: 150

THIS PAPER SHOULD BE MARKED OUT OF 139