NATIONAL SENIOR CERTIFICATE EXAMINATION
SUPPLEMENTARY EXAMINATION – MARCH 2016

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours 150 marks

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Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13
18	10	9	18	12	9	13	11	10	10	10	14	6

TOTAL	/150

SECTION A

QUESTION 1

(a) $$\text{AB} = \sqrt{(6-1)^2 + (-1+3)^2}$$
$$= \sqrt{29} \approx 5{,}4$$ (2)

(b) $$\text{M} = \left(\frac{-3+5}{2}; \frac{1+1}{2}\right)$$
$$\text{M} = (1; 1)$$ (2)

(c) $$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m_{AB} = \frac{5}{2}$$
$$m_{AD} = \frac{6-1}{-1-5} = -\frac{5}{6}$$ (3)

(d) $$\tan\theta = m_{AB} = \frac{5}{2}$$
$$\theta \approx 68°$$
$$\tan(180° - \alpha) = m_{AD} = -\frac{5}{6}$$
$$180° - \alpha = 140°$$
$$\alpha = 140°$$
$$\hat{\text{A}} \approx 180° - (68° + 40°)$$
$$\approx 72°$$ (5)

(e) The diagonals of a parallelogram bisect each other
$\therefore$ M = Midpoint of AE
$$(1; 1) = \left(\frac{-1+x_E}{2}; \frac{6+y_E}{2}\right)$$
$$-1 + x_E = 2,\ x_E = 3$$
$$6 + y_E = 2,\ y_E = -4$$
E = (3; –4) (4)

(f) $$m_{AC} = \frac{6+4}{1+5} = \frac{5}{2} = m_{AB}$$
$\therefore$ A, B and C lie on a straight line. (2)

[18]

QUESTION 2

(a) (1) $(\max)-(\min)$
$\text{Range}=98-10$
$\text{Range}=88$ (2)

(2) Median $=52$ (1)

(3) $\text{IQR}=Q_3-Q_1$
$\text{IQR}=70-44$
$\text{IQR}=26$ (2)

(b) Science, higher median, higher minimum. (2)

(c) Mathematics, Range is 88% and the IQR is 30 compared to Science with a Range of 58 and IQR of 26. (2)

(d) 25% or 0,25 (1)
[10]

QUESTION 3

(a) $y=a+bx$
$y=51{,}6478+0{,}7776x$ (3)

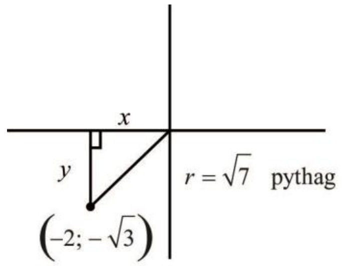
(b) $y=a+bx$
$y=51{,}6478+0{,}7776\ (100)$
$y=129$ (2)

(c) This prediction is not reliable as the maximum that could be obtained is 100. (1)

(d) r = 0,8 which indicates a very strong positive correlation. (3)
[9]

QUESTION 4

(a) (1)



$\sin\theta = \dfrac{-\sqrt{3}}{\sqrt{7}}$

$\cos\theta = \dfrac{-2}{\sqrt{7}}$

(2)

(2) $\sin 2\theta$

$= 2\sin\theta\cos\theta$

$= 2\left(\dfrac{-\sqrt{3}}{\sqrt{7}}\right)\left(\dfrac{-2}{\sqrt{7}}\right)$

$= \dfrac{4\sqrt{3}}{7}$ (2)

(3) $\cos^2(90° + \theta)$

$= (-\sin\theta)^2$

$= \sin^2\theta$

$= \left(\dfrac{-\sqrt{3}}{\sqrt{7}}\right)^2$

$= \dfrac{3}{7}$ (2)

(b) $\dfrac{\tan(180° - \theta).\cos(-\theta).\sin 390°}{\cos 300°.\sin\theta - \cos 450°}$

$= \dfrac{(-\tan\theta)(\cos\theta).\sin 390°}{\cos 300°.\sin\theta - \cos 450°}$

$= \dfrac{(-\tan\theta)(\cos\theta).\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)\sin\theta - 0}$

$= \dfrac{\left(\dfrac{-\sin\theta}{\cos\theta}\right)(\cos\theta)\left(\cancel{\frac{1}{2}}\right)}{\left(\cancel{\frac{1}{2}}\right)\sin\theta} = -1$ (5)

(c) (1) $\cos(A - 45°)$

$= \cos A\cdot\cos 45° + \sin A\cdot\sin 45°$

$= \dfrac{\sqrt{2}}{2}\cos A + \dfrac{\sqrt{2}}{2}\sin A$

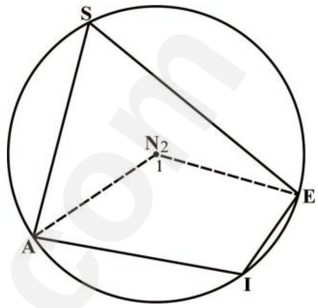
$= \dfrac{\sqrt{2}}{2}(\cos A + \sin A)$

$\therefore \cos(A - 45°) = \dfrac{\sqrt{2}}{2}k$ (4)

(2) $1 + \sin 2A$
$= \sin^2 A + 2\sin A\cos A + \cos^2 A$
$= (\cos A + \sin A)^2$
$= k^2$ (3)

[18]

QUESTION 5



(a) $\hat{N}_1 = 2x$ *∠ at centre*

$\therefore\ \hat{N}_2 = 360° - 2x$ *angles around a point*

$\therefore\ \hat{I} = 180° - x$ *angle at centre*

$\therefore\ \hat{S} + \hat{I} = x + \hat{I} = 180°$ (6)

(b) $\hat{I}_1 = x\ ;\ \hat{I}_2 = y$

$\hat{K}_1 = x \quad ;\ (GI = GK)$

$\hat{K}_2 = y \quad (JI = JK)$

$180° = \hat{I}_1 + \hat{I}_2 + \hat{K}_1 + \hat{K}_2$ *opposite angles of a cyclic quadrilateral*

$180° = x + y + x + y$
$180° = 2(x + y)$
$\therefore\ x + y = 90°$
$\therefore$ GJ is a diameter
GJ subtends $\angle$ of 90° at circumference

Alternate:

Since GI = GK, G lies on the perpendicular bisector of IK.
Since IJ = JK, J lies on the perpendicular bisector of IK.
$\therefore$ GJ is the perpendicular bisector of IK.
Hence the centre lies on GJ, proving GJ is a diameter of the circle. (6)

[12]

QUESTION 6

$O\hat{L}M = 90°$ Tan $\perp$ Radius

$\hat{O}_2 = 64°$ Int. $\angle$s of ΔLOM

$\hat{P} = \hat{L}_2$ $OP = OL$

$\hat{P} + \hat{L}_2 = 64°$ ext. $\angle$ of Δ

$\therefore \hat{L}_2 = 32°$

$\hat{L}_5 = \hat{P}$; tan / chord theorem

$\hat{P} = \hat{L}_2 = 32°$

$\therefore \hat{L}_5 = 32°$

$R\hat{L}O = 90°$ tan $\perp$ radius

$\therefore \hat{L}_1 = 90° - 32° = 58°$

$\hat{Q}_1 = \hat{L}_1 = 58°$ tan-chord theorem

$\hat{N} = 32°$ tan-chord theorem
or $\angle$ in same segment
or $\angle$ at centre

NB:

Mark allocation

$\hat{O}_2$ (2)

$\hat{L}_2$ (3)

$\hat{L}_5$ (2)

$\hat{Q}_1$ (1)

$\hat{N}$ (1)

(9)

[9]

76 marks

SECTION B

QUESTION 7

(a) (1) Sub $x = \frac{7}{5}$ and $y = -\frac{9}{5}$

$$\left(\frac{7}{5}\right)^2 + \left(-\frac{9}{5}\right)^2 - 4\left(\frac{7}{5}\right) + 2\left(-\frac{9}{5}\right) + 4$$

$= 0$

$\therefore \left(\frac{7}{5}\,;-\frac{9}{5}\right)$ is a point on circle (2)

(2) Eq. of tangent: $y = mx + c$ at point $\left(\frac{7}{5};\frac{-9}{5}\right)$

$x^2 - 4x + y^2 + 2y = -4$

$(x-2)^2 + (y+1)^2 = -4 + 4 + 1$

$(x-2)^2 + (y+1)^2 = 1$

$\therefore$ Centre $(2;-1)$

$$m_{normal} = \frac{\frac{-9}{5}+1}{\frac{7}{5}-2}$$

$m_{normal} = \frac{4}{3}$ $\therefore$ $m_{\tan} = \frac{-3}{4}$

$y = \frac{-3}{4}x + c$ sub point $\left(\frac{7}{5};\frac{-9}{5}\right)$

$c = \frac{-3}{4}$

Eq. of tangent: $y = \frac{-3}{4}x - \frac{3}{4}$ or $4y = -3x - 3$ (8)

(b) Centre of circle = (4; –3)

Distance from (2; –1) to the centre

$= \sqrt{(4-2)^2 + (-3+1)^2}$

$= \sqrt{8}$

$\sqrt{8} > \sqrt{7}$

$\therefore$ Point lies outside (3)

[13]

QUESTION 8

(a) In ΔOCA: Let OC $= x$

$\therefore$ EC $= (x+2) =$ CA radii

OA = 8 units

$(x+2)^2 = x^2 + 8^2$ pythag

$\therefore \quad x = 15$

$\therefore$ C(0 ; -15) and radius = CP = 17

Sub. into: $(x-a)^2 + (y-b)^2 = r^2$

$(x-0)^2 + (y+15)^2 = 17^2$

$x^2 + y^2 + 30y - 64 = 0$ (8)

(b) $\mathrm{P}\left(\dfrac{x_1 + x_2}{2}; \dfrac{y_1 + y_2}{2}\right)$

$\mathrm{P}\left(\dfrac{-8+0}{2}; \dfrac{0+0}{2}\right)$

$\mathrm{P}(-4;0)$

$\therefore \mathrm{Q}(-4; \mathrm{y})$ sub $x = -4$ in circle equation

$(-4)^2 + (y+15)^2 = 17^2$

$(y+15)^2 = 289 - 16$

$y + 15 \approx 16{,}5$

$y \approx 1{,}5$

Height of the pillar is 1,5 metres (3)

[11]

QUESTION 9

(a) $\text{PT} = \frac{5}{8}(144)$

$\text{PT} = 90 \text{ m}$

$\text{T}\hat{\text{Q}}\text{P} = 51°$ Ext. $\angle$ of ΔQPT

$$\frac{\text{QT}}{\sin 63°} = \frac{90}{\sin 51°}$$

$\text{QT} \approx 103 \text{ m}$ (4)

(b) In ΔQTS: $\text{TS} = 54 \text{ m}$ and $\text{QT} = 103 \text{ m}$

$\text{QS}^2 = \text{TS}^2 + \text{QT}^2 - 2.\text{TS}.\text{QT}.\cos 114°$

$\text{QS}^2 = (54)^2 + (103)^2 - [2(54)(103)(\cos 114°)]$

$\text{QS} = \sqrt{18049{,}53842}$

$\text{QS} = 134{,}3485706$

$$\tan \text{Q}\hat{\text{S}}\text{R} = \frac{70}{134{,}3485706}$$

$\text{Q}\hat{\text{S}}\text{R} \approx 28°$ which is the angle of elevation of R from S (6)

[10]

QUESTION 10

(a)

$$\frac{\sin 3\alpha}{\sin\alpha}$$
$$= \frac{\sin(2\alpha+\alpha)}{\sin\alpha}$$
$$= \frac{(\sin 2\alpha)(\cos\alpha)+(\cos 2\alpha)(\sin\alpha)}{\sin\alpha}$$
$$= \frac{(2\sin\alpha.\cos\alpha)(\cos\alpha)+(\cos^2\alpha-\sin^2\alpha)(\sin\alpha)}{\sin\alpha}$$
$$= \frac{\sin\alpha(2\cos\alpha)(\cos\alpha)+(\cos^2\alpha-\sin^2\alpha)(\sin\alpha)}{\sin\alpha}$$
$$= (2\cos\alpha)(\cos\alpha)+(\cos^2\alpha-\sin^2\alpha)$$
$$= 2\cos^2\alpha+\cos^2\alpha-\sin^2\alpha$$
$$= 3\cos^2\alpha-\sin^2\alpha$$
$$= 3(1\text{-}\sin^2\alpha)-\sin^2\alpha$$
$$= 3\text{-}3\sin^2\alpha-\sin^2\alpha$$
$$= 3\text{-}4\sin^2\alpha$$
$$= \text{RHS}$$

OR

$$\frac{\sin 3\alpha}{\sin\alpha}$$
$$= \frac{\sin 2\alpha\ \cos\alpha + \cos 2\alpha.\sin\alpha}{\sin\alpha}$$
$$= \frac{2\sin\alpha\ \cos^2\alpha + \left(1-2\sin^2\alpha\right).\sin\alpha}{\sin\alpha}$$
$$= 2\cos^2\alpha + 1 - 2\sin^2\alpha$$
$$= 2 - 2\sin^2\alpha + 1 - 2\sin^2\alpha$$
$$= 3 - 4\sin^2\alpha$$
$$= \text{RHS}$$

(6)

(b)

$$3-4\sin^2\alpha = 2$$
$$4\sin^2\alpha = 1$$
$$\sin\alpha = \frac{1}{2} \text{ OR } -\frac{1}{2}$$
$$\alpha = 30° + 360k;\quad k\in\mathbb{Z}$$

OR

$$\alpha = 150° + 360k;\quad k\in\mathbb{Z}$$

(4)

[10]

QUESTION 11

(a) $S\hat{W}T = 90°$ $\angle$ in semi-circle

$V\hat{U}S = 90°$ given

$\therefore$ WVUT is a cyclic quad. Converse of Ext angle of cyclic quadrilateral (4)

(b) $U\hat{W}T = \hat{S}$ tan-chord theorem

$U\hat{W}T = \hat{V}_1$ $\angle$s in same segment

$\therefore \hat{V}_1 = S$

$\therefore$ VU is a tangent ; converse of tan/chord theorem (6)

[10]

QUESTION 12

(a) In ΔSWU

$$\frac{ST}{18} = \frac{8}{12}$$ *line parallel to one side of triangle*

$$\therefore ST = \frac{8 \times 18}{12} = 12$$

In ΔXUS

$$\frac{XW}{20} = \frac{12}{18}$$ *line parallel to one side of triangle*

$$\therefore XW = \frac{20 \times 12}{18} = \frac{40}{3}$$ (6)

(b) Determine Area ΔSXU:

$$\frac{VT}{20} = \frac{12}{20}$$ *($\Delta VUT /// \Delta WUS$)*

$$\therefore VT = 12$$

$$(12)^2 = (12)^2 + (18)^2 - 2(12)(18)\cos \hat{U}$$

$$\cos \hat{U} = \frac{3}{4}$$

$$\sin \hat{U} = \sqrt{1 - \cos 2\hat{U}} = \frac{\sqrt{7}}{4}$$

$$\text{Area} = \frac{1}{2}\left(33\frac{1}{3}\right)(30)\left(\frac{\sqrt{7}}{4}\right)$$

$$= 125\sqrt{7}$$ (8)

[14]

QUESTION 13

(a) $OR = ON = p$ radii

$\therefore O\hat{R}N = \hat{N}_1 = \alpha$ $\angle$ opp = sides

$\therefore R\hat{O}N = 180° - 2\alpha$ int. $\angle$s of Δ

Using the Cosine Rule:

$(RN)^2 = p^2 + p^2 - [2.p.p.\cos(180° - 2\alpha)]$

$(RN)^2 = 2p^2 - 2p^2(-\cos 2\alpha)$

$(RN)^2 = 2p^2(1 + \cos 2\alpha)$

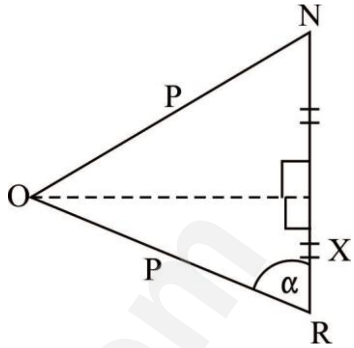
$(RN)^2 = 2p^2[1 + (2\cos^2\alpha - 1)]$

$(RN)^2 = 2p^2(2\cos^2\alpha)$

$(RN)^2 = 4p^2\cos^2\alpha$

$RN = 2p\cos\alpha$

OR



$\cos\alpha = \dfrac{x}{p}$

$x = p\cos\alpha$ (3)

$\therefore RN = 2p\cos\alpha$

(b) RN = 20cos45°

$= 10\sqrt{2}$

Perpendicular height = circumference of sphere = 25 m

Volume of pyramid = $\frac{1}{3}\left(10\sqrt{2} \times 10\sqrt{2}\right)(25)$

Volume of pyramid = 1 667 m^3

Volume of hemisphere = $\frac{2}{3} \times \pi \times 25^3$

= 3 2 724,92

$\therefore$ volume of water = 31 058 m^3 (3)

[6]

74 marks

Total: 150 marks