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Department of
Education
FREE STATE PROVINCE

PREPARATORY EXAMINATION

GRADE 12

MATHEMATICS P2

TIME: 3 HOURS

SEPTEMBER 2023

MARKS: 150

This question paper consists of 13 pages, 1 information sheet
and an answer book of 19 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. Unless stated otherwise, you may use an approved scientific calculator (non-programmable and non-graphical).
6. If necessary, round off answers to TWO decimal places unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.



QUESTION 1

To celebrate Pi Day at school, learners participate in a competition to write down the value of Pi (π), up to the most correct decimal places. Eleven learners make it to the final round of the competition, where their number of correct decimal places is counted.

The judges stop counting after the first mistake. The results of the eleven learners are shown in the table below.

63	79	50	74	75	66	150	86	72	74	60
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1.1 Calculate the:

1.1.1 Mean of the data (2)

1.1.2 The standard deviation for the given data (1)

1.1.3 Number of results that lie outside ONE standard deviation of the mean (3)

1.2 Identify the outlier in the given results. (1)

1.3 The result with the number of the most correct decimal places is increased by $k\%$, while the result with the number of the lowest correct decimal places is decreased by $t\%$. The other nine results remain unchanged.

Only one of the options below correctly reflects the new range of the data in terms of k and t . Choose the answer and write only the letter next to the question number in the ANSWER BOOK

A. $100 + k - t$

B. $150k - 50t$

C. $150k + 50t$

D. $100 + \frac{3}{2}k + \frac{1}{2}t$ (2)

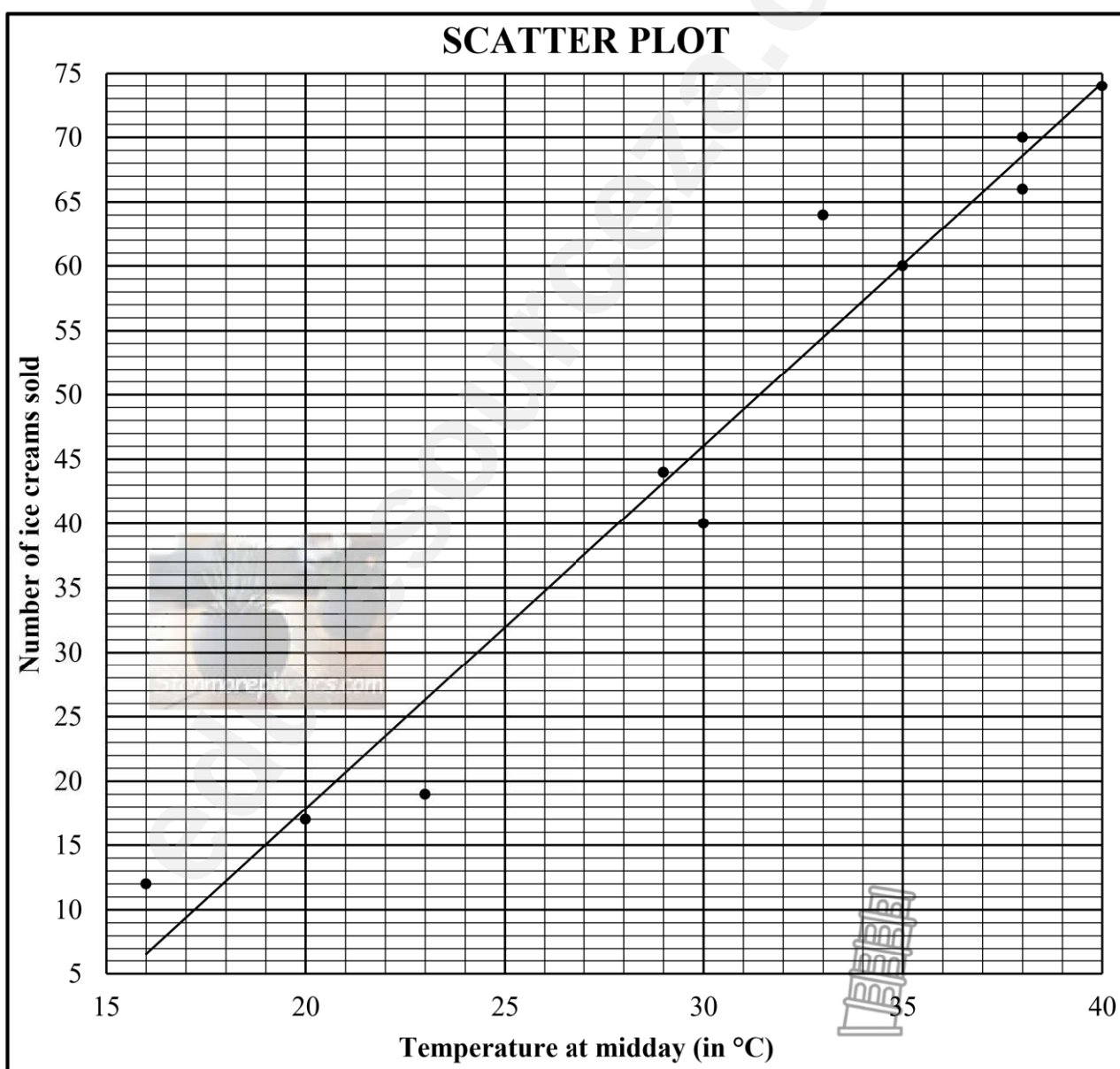
1.4 It was established that a judge made a mistake with one of the six lowest results. The result was corrected and changed to double its original value. How will this change impact the median of the data? Motivate your answer. (2)

[11]

QUESTION 2

On the first Saturday of a month, for a period of ten months, information was recorded about the temperature at midday (in $^{\circ}\text{C}$); and the number of ice creams sold at an ice cream stand at a particular beach. The data is shown in the table below and represented on the scatter plot. This data's least squares regression line is drawn on the scatter plot.

Temperature at midday (in $^{\circ}\text{C}$)	16	20	23	29	33	38	40	38	35	30
Number of ice creams sold	12	17	19	44	64	70	74	66	60	40

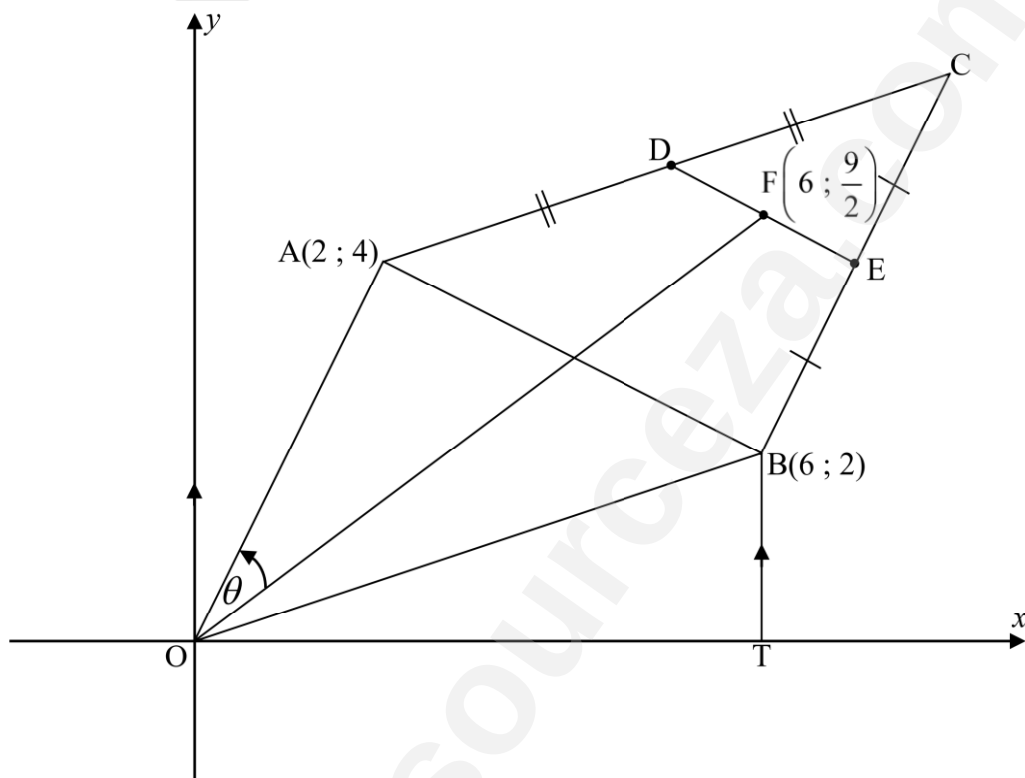


- 2.1 Refer to the scatter plot. Would you say that the relationship between the temperature at midday and the number of ice creams sold is weak or strong? Motivate your answer. (2)
- 2.2 Determine the equation of the least squares regression line. (3)
- 2.3 Predict the number of ice creams that will be sold on a Saturday if the temperature is 26°C at midday. (2)
- 2.4 On another first Saturday of the month, the temperature at midday was 24°C and 40 ice creams were sold. If this data is added to the data set, how will the prediction of the number of ice creams sold within the given domain be affected? Motivate your answer. (2)
- [9]



QUESTION 3

In the diagram below, $A(2 ; 4)$, O , $B(6 ; 2)$ and C are the vertices of a quadrilateral. D and E are the midpoints of AC and BC , respectively. $F\left(6 ; \frac{9}{2}\right)$ is a point on DE . From B the straight line drawn parallel to the y -axis cuts the x -axis in T . $\angle AOF = \theta$

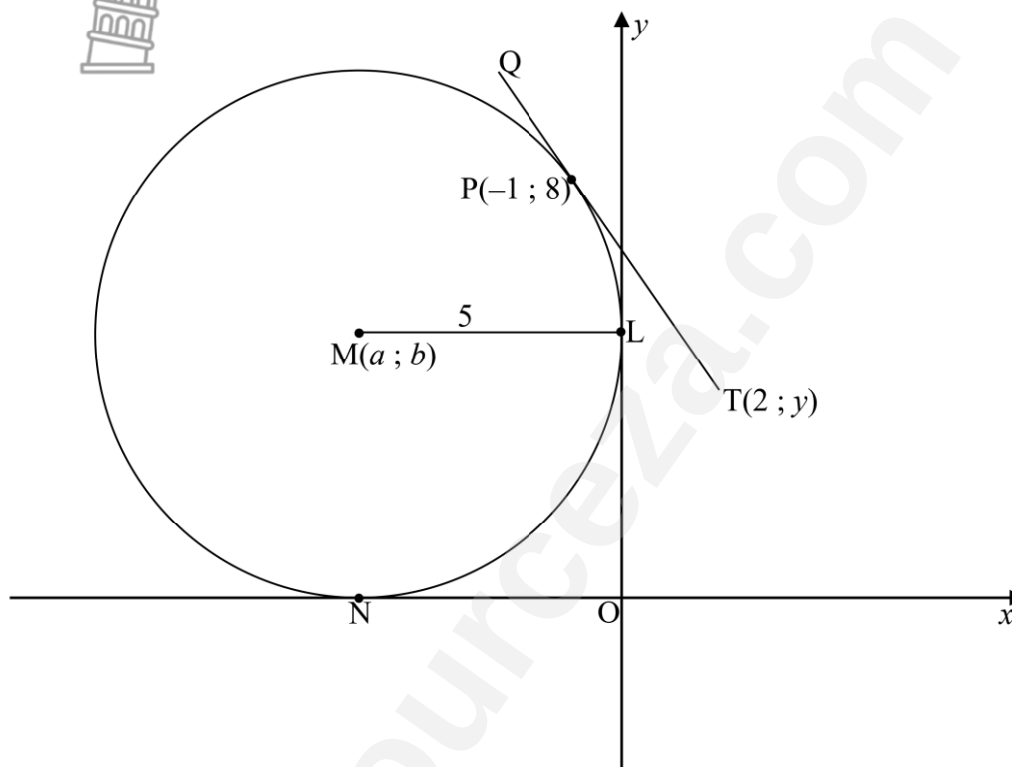


- 3.1 Calculate:
- 3.1.1 The length of AB. Leave your answer in surd form (2)
- 3.1.2 The gradient of AB (2)
- 3.2 Prove that $OA \perp AB$ (2)
- 3.3 Determine the equation of DE. (4)
- 3.4 Determine the coordinates of C such that AOCB, in this order, is a parallelogram. (3)
- 3.5 Calculate the:
- 3.5.1 Size of θ (5)
- 3.5.2 Area of $\triangle ABT$, if A and T are joined to form $\triangle ABT$ (4)

[22]

QUESTION 4

In the diagram, a circle centred at $M(a; b)$ with a radius of 5 units touches the x -axis and the y -axis at points N and L , respectively. QPT is a tangent to this circle at $P(-1; 8)$. The coordinates of T are $(2; y)$.



4.1 Give a reason why $ML \perp y$ -axis. (1)

4.2 Determine the:

4.2.1 Coordinates of M (2)

4.2.2 Equation of the circle having centre M (2)

4.2.3 Equation of the tangent QPT in the form $y = mx + c$ (5)

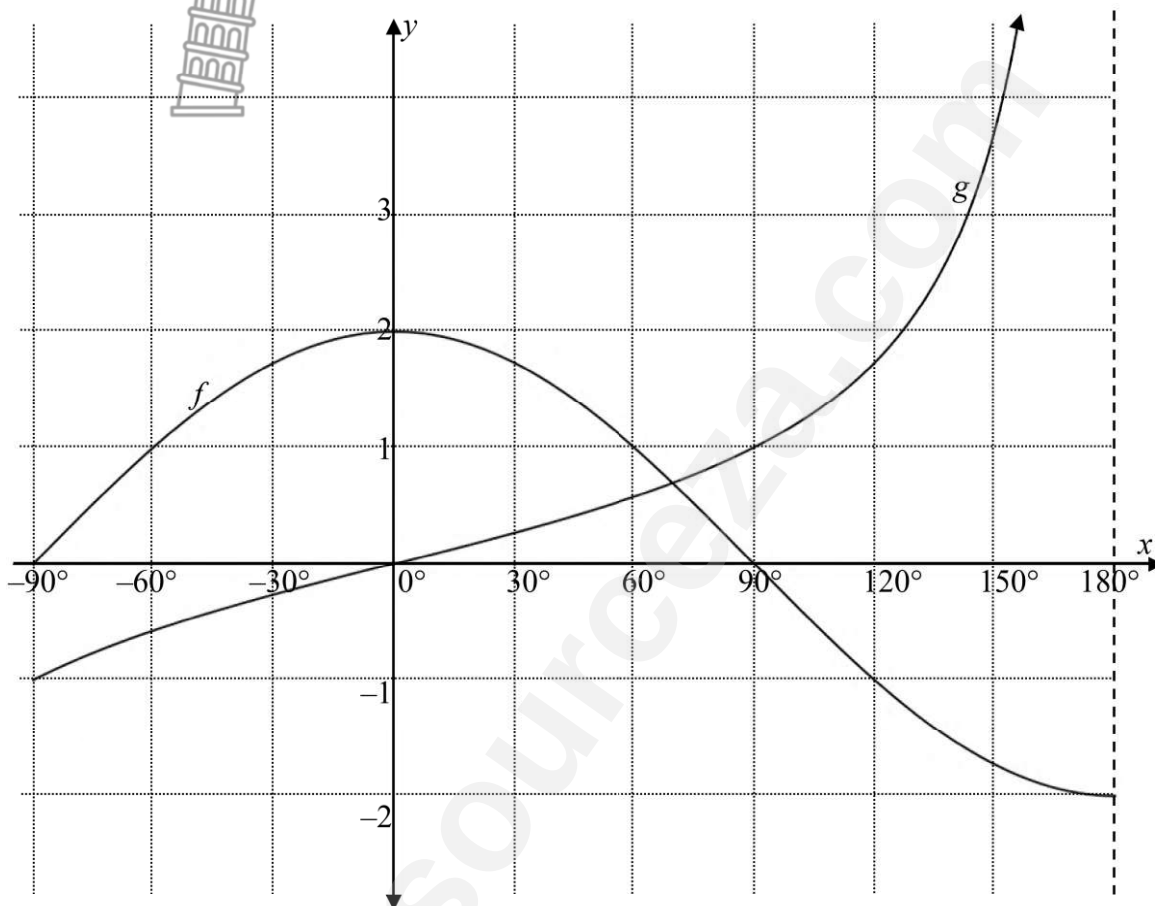
4.3 Another circle, having point T as the centre, touches the circle, having M as the centre externally. Determine the equation of the circle centred at T in the form $(x - h)^2 + (y - k)^2 = r^2$. (6)

4.4 The circle with centre M is translated across the Cartesian plane in such a way that both horizontal and vertical axes remain tangents to the circle simultaneously. Write down all the possible coordinates of the centres of the newly translated circles, given that $\sqrt{xy}$ must be real for ALL values of x and y . (4)

[20]

QUESTION 5

In the diagram below, the graphs of $f(x) = 2\cos x$ and $g(x) = \tan bx$ are drawn for the interval $x \in [-90^\circ; 180^\circ]$.



Use the graphs to answer the following questions.

- 5.1 Write down the value of b . (1)
- 5.2 Write down the range of g for the interval $x \in [-90^\circ; 180^\circ]$. (2)
- 5.3 Write down the period of g . (1)
- 5.4 Write down a value of x , in the given interval, where $g(x + 5^\circ) - f(x + 5^\circ) = 1$. (1)
- 5.5 Write down TWO values of x in the given interval, where $\frac{g(x)}{f'(x)}$ is undefined. (2)
- 5.6 Write down the value of p if $\sum_{x=0^\circ}^p 2\cos x = 0$ (2)

[9]

QUESTION 6

6.1 **WITHOUT using a calculator**, determine the following in terms of $\sin 25^\circ$:

6.1.1 $\sin 335^\circ$ (1)

6.1.2 $\cos 50^\circ$ (2)

6.2 Simplify the following expression to ONE trigonometric ratio:

$$\frac{\sin(-2x) \cdot (1 - \sin^2 x)}{\sin(90^\circ + x) \cdot \tan x} \quad (6)$$

6.3 **WITHOUT using a calculator**, simplify $(p \tan 30^\circ + q \sin 60^\circ)^2$ to a single fraction in terms of p and q . (3)

6.4 Given: $\cos(A - B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

6.4.1 Use the formula for $\cos(A - B)$ to derive a formula for $\sin(A - B)$. (4)

6.4.2 Prove that $\sin 9A + \sin A = 2 \sin 5A \cdot \cos 4A$ (3)

6.4.3 Write down the maximum value of $3^{2 \sin 5A \cos 4A}$ (2)

6.5 Determine the general solution of $\cos 2x - 5 \cos x - 2 = 0$ (6)

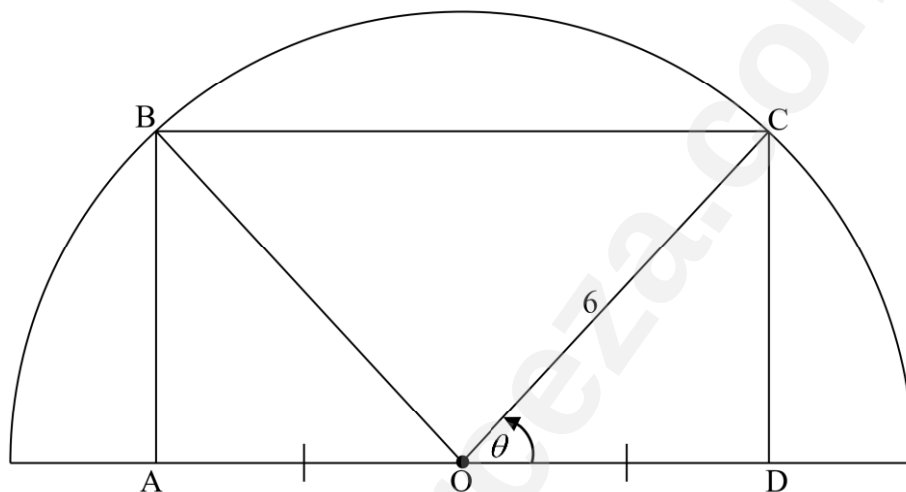
6.6 Given: $\tan x = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots}}}}$, with $x \in [0^\circ; 90^\circ)$

WITHOUT using a calculator, show that: $2 \sin^2 x = \sin 2x (\cos x + 1)$ (5)
[32]



QUESTION 7

In the diagram below, O is the centre of the circle. A, B, C and D are points on the semi-circle such that ABCD is a rectangle. The radius of the semi-circle is 6 units, $\angle COD = \theta$ and $AO = OD$.

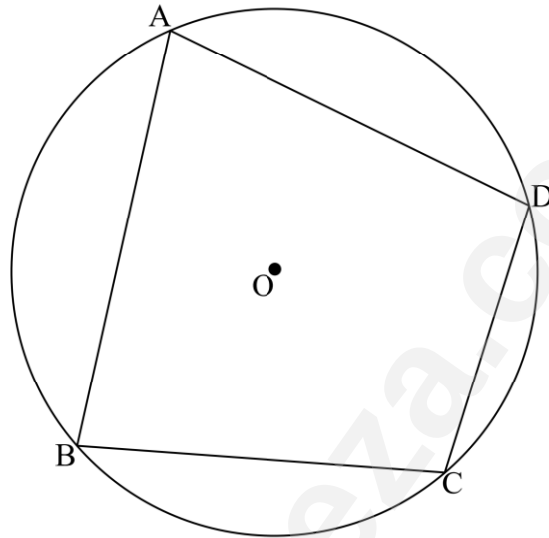


- 7.1 Write $\angle BOC$ in terms of θ (2)
- 7.2 If $\theta = 43^\circ$, calculate the length of BC. (3)
- 7.3 Points A, B, C and D are shifted along the semi-circle. Calculate the value of θ if ABCD now forms a square. (4)
- [9]

Give reasons for your statements in QUESTIONS 8, 9 and 10.

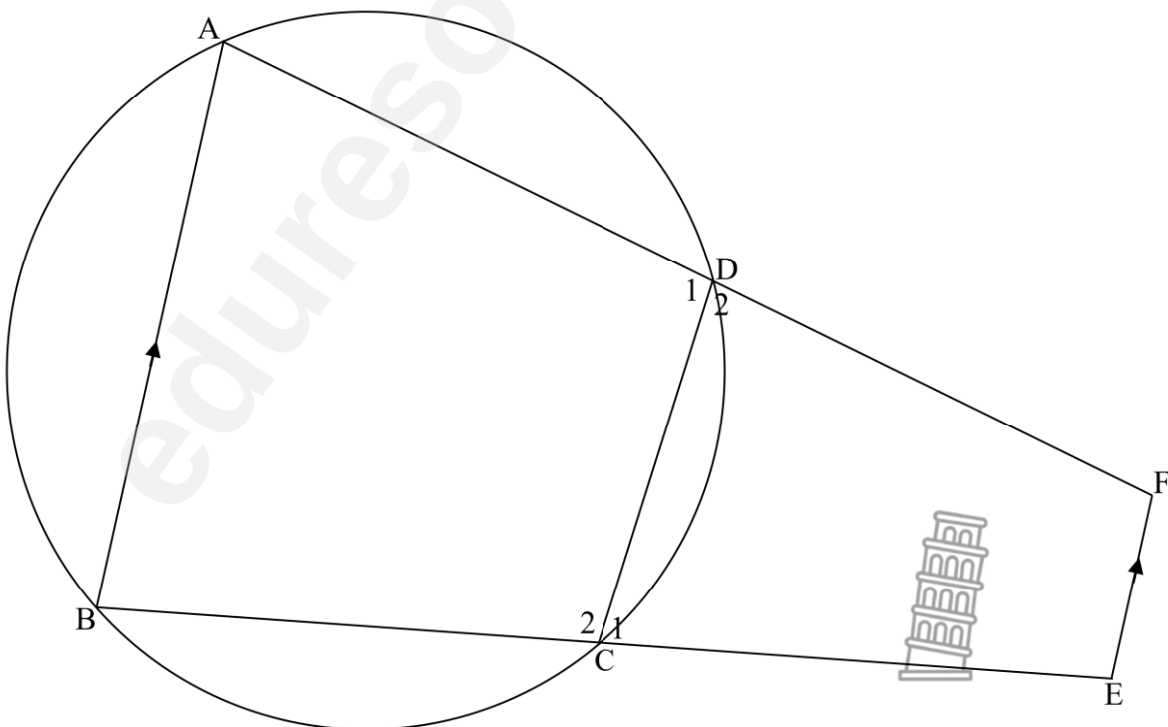
QUESTION 8

8.1 In the diagram, ABCD is a cyclic quadrilateral, and the circle has a centre O.



Prove the theorem which states that $\hat{A} + \hat{C} = 180^\circ$. (5)

8.2 In the diagram below, ABCD is a cyclic quadrilateral. Chords AD and BC are produced to F and E, respectively. F and E are joined such that $EF \parallel AB$.



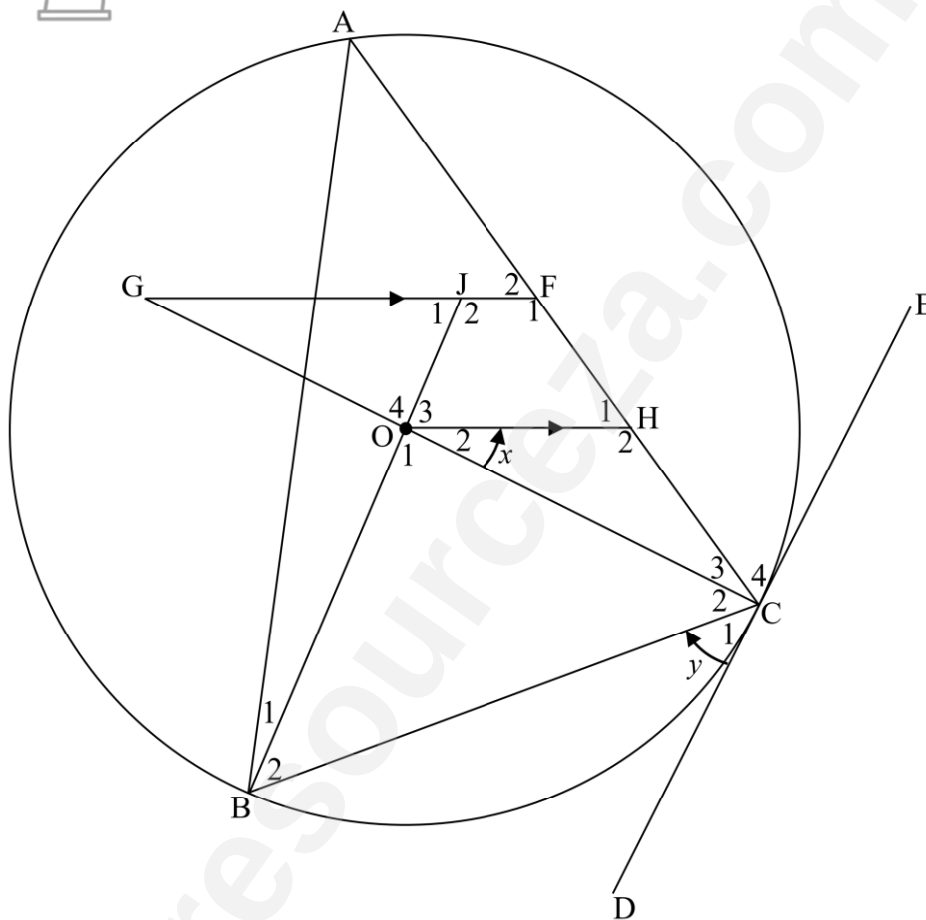
Prove that CEFD is a cyclic quadrilateral.

(5)
[10]

QUESTION 9

In the diagram below, O is the centre of the circle, with points A, B and C on the circle. DCE is a tangent to the circle at C. GOC, BOJ, and GJF are straight lines. F and H are points on AC such that $GF \parallel OH$.

$\hat{C}_1 = y$, $\hat{O}_2 = x$ and $FH : HC = 2 : 3$



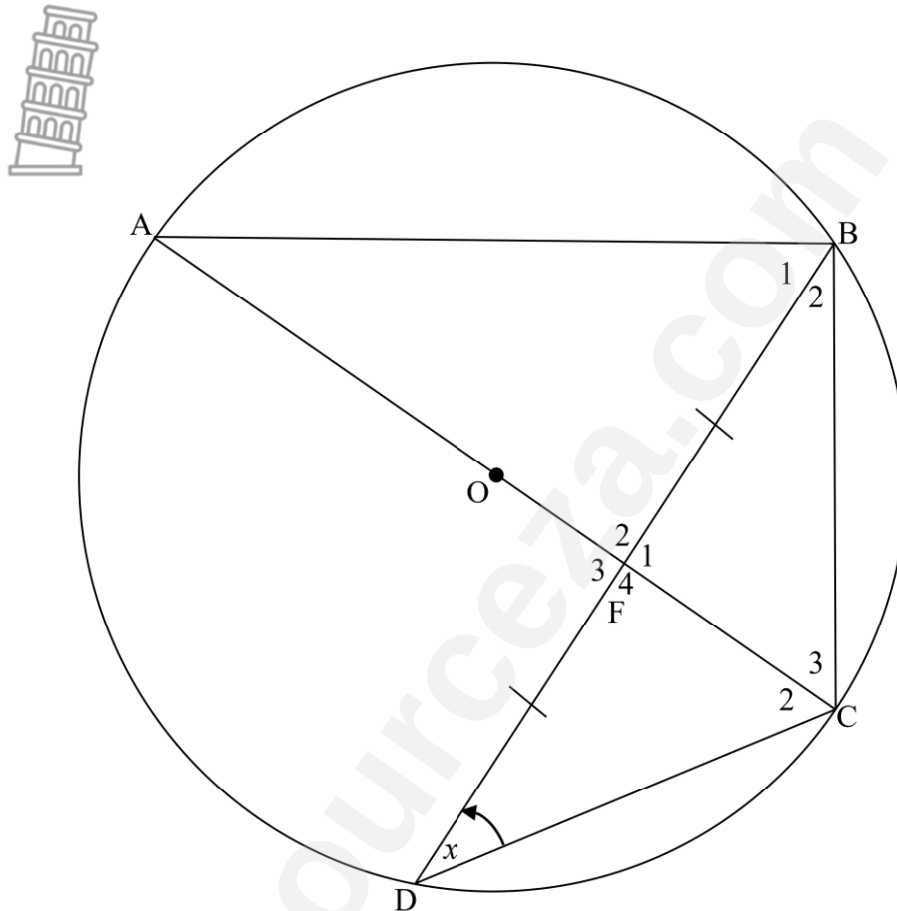
9.1 Calculate, giving reasons, $\hat{J}_1$ in terms of x and y . (6)

9.2 Determine, giving reasons, the value of $\frac{GO}{GC}$. (3)

[9]

QUESTION 10

In the diagram below, A, B, C, and D lie on the circle with centre O.
AOFC and DFB are straight lines, $DF = FB$ and $\hat{D} = x$.



10.1 Determine, with reasons, the size of EACH of the following in terms of x :

10.1.1 $\hat{A}$ (2)

10.1.2 $\hat{C}_3$ (3)

10.2 Prove, giving reasons, that:

10.2.1 $\hat{F}_2 = \hat{F}_3$ (2)

10.2.2 $\triangle CFB \parallel \triangle CBA$ (3)

10.2.3 $DC^2 = FC.AC$ (4)

10.2.4 $\frac{FC}{AC} = \left(1 - \frac{AB}{AO + OC}\right) \left(1 + \frac{AB}{AO + OC}\right)$ (5)

[19]

TOTAL: 150

INFORMATION SHEET:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



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FREE STATE PROVINCE

PREPARATORY EXAMINATION/ *VOORBEREIDENDE EKSAMEN*



GRADE/GRAAD 12

MATHEMATICS P2/ *WISKUNDE V2*

SEPTEMBER 2023

MARKS/PUNTE: 150

MARKING GUIDELINES/ *NASIENRIGLYNE*



These marking guidelines consist of 20 pages and a 3-page cognitive grid/
Hierdie nasienriglyne bestaan uit 20 bladsye en 'n 3 bladsye kognitiewe tabel.

- NOTA:**

- ### QUESTION/VRAAG 1

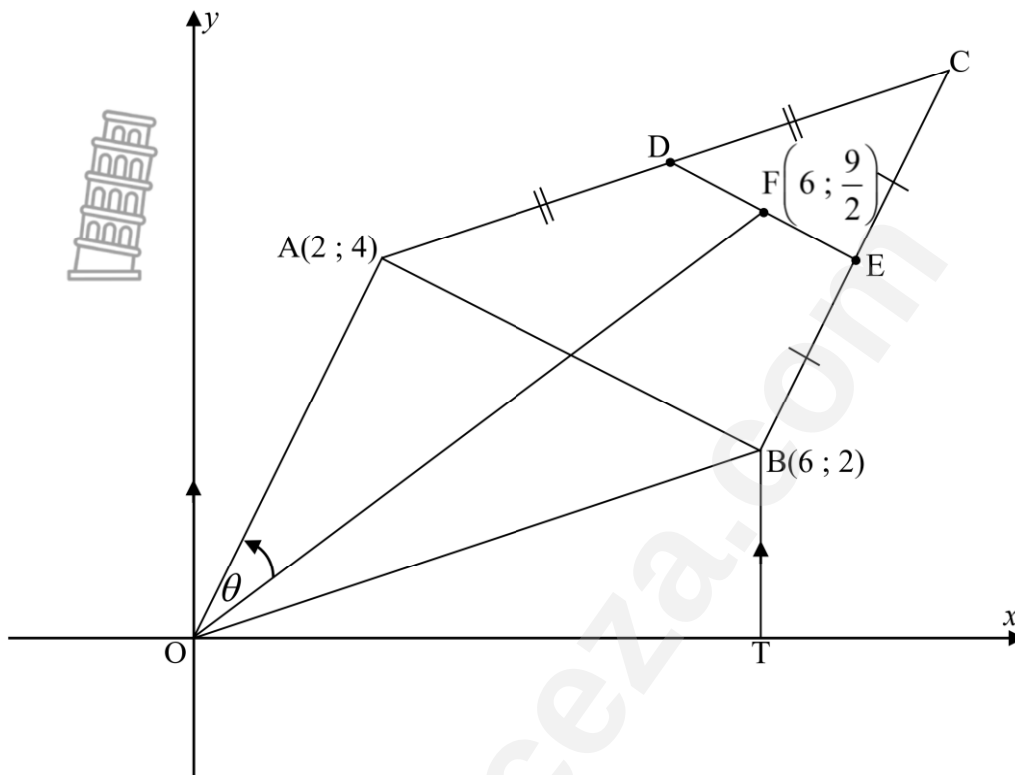
1.1.1	$\bar{x} = \frac{849}{11}$ $= 77,18$	Answer only: full marks	✓ 849 (addition of results/ <i>optel van uitslae</i>) ✓ answer/antwoord (CA if/as ÷ 11) (2)
1.1.2	$\sigma = 24,86$	No penalty for rounding:	✓ answer/antwoord (A) (1)
1.1.3	$(\bar{x} - \sigma ; \bar{x} + \sigma)$ $= (52,32 ; 102,04)$ $\therefore$ 2 results/uitslae	Answer only: full marks provided 1.1.1 & 1.1.2 both correct	✓ 52,32 ✓ 102,04 ✓ answer/antwoord (3)
1.2	150		✓ answer/antwoord (1)
1.3	D		✓✓ answer/antwoord (2)
1.4	By doubling any of the six lowest results, the new result will be more than 74 and lie to the right hand side of 74/<i>Deur om enige uitslag van die ses laagste uitslae te verdubbel sal die nuwe uitslag meer as 74 wees en regs van 74 lê.</i> Because 74 is also the mode, the new median will still be 74/<i>Omdat 74 die modus is sal die nuwe mediaan 74 bly.</i> The median therefore has not changed/<i>Die mediaan het dus nie verander nie.</i>		✓ 74 mode/modus ✓ no change/geen verandering (2)
			[11]

QUESTION/VRAAG 2

Temperature at midday (in °C)/ Middag-temperatuur (in °C)	16	20	23	29	33	38	40	38	35	30
Number of ice creams/Aantal roomyse	12	17	19	44	64	70	74	66	60	40

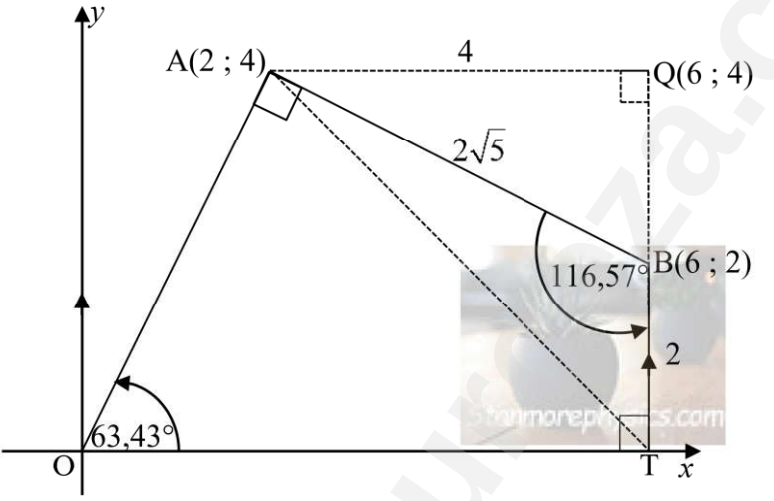
2.1	Strong/Goed The majority of the points lie close to the regression line/ Die meerderheid punte lê naby die regressielyn OR/OF Strong/Sterk $r = 0,98$	✓ strong/sterk ✓ reason/rede (2) ✓ strong/sterk ✓ $r = 0,98$ (2)
2.2	$a = -38,4828...$ $b = 2,8173...$ $\hat{y} = -38,48 + 2,82x$ Answer only: full marks, but if a and b are swopped only 1/3 marks/ maar as a en b omgeruil is, slegs 1/3 punte.	✓ a ✓ b ✓ equation/vergelyking (3)
2.3	$\hat{y} = -38,48 + 2,82(26)$ $= 34,84$ $\therefore 34$ ice creams / roomyse OR/OF $26\hat{y} = 34,77$ (calculator / sakrekenaar) $\therefore 34$ ice cream / roomyse Answer only: full marks Can also accept 35	✓ substitute 26 into eq./ vervang 26 in vgl. ✓ 34 (2) ✓✓ 34 (2)
2.4	Regression line will be pulled slightly upwards/regressielyn sal effe opwaarts getrek word The prediction will be that more ice cream will be sold/ Die voorspelling is dat meer roomyse verkoop sal word.	✓ explanation/verduideliking ✓ more ice cream/meer roomyse (2)
		[9]

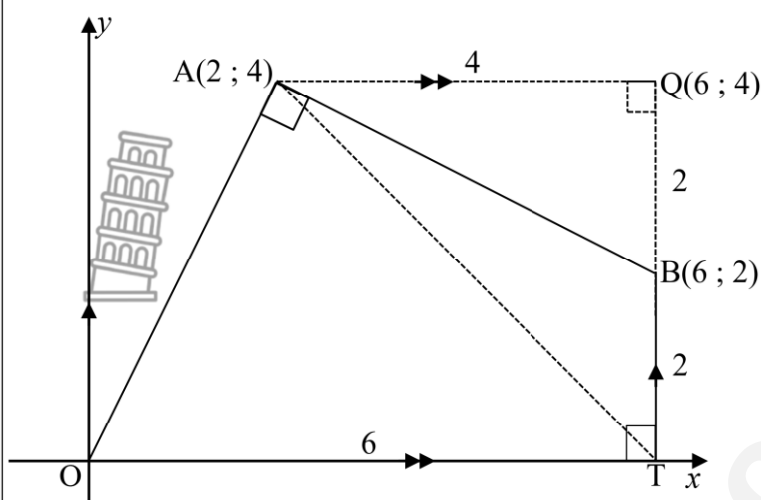
QUESTION/VRAAG 3



3.1.1	$AB = \sqrt{(6 - 2)^2 + (2 - 4)^2}$ $= 2\sqrt{5}$	✓ subst./vervang ✓ answer/antwoord (2)
3.1.2	$m_{AB} = \frac{2 - 4}{6 - 2}$ $= -\frac{1}{2}$ Answer only: full marks	✓ subst./vervang ✓ answer/antwoord (2)
3.2	$m_{OA} = 2$ But / maar : $m_{OA} \times m_{AB}$ $= 2 \left(-\frac{1}{2} \right)$ $= -1$ $\therefore OA \perp AB$	✓ $m_{OA} = 2$ $\checkmark 2 \left(-\frac{1}{2} \right) = -1$ (2)

3.3	DE AB [midpt.theorem / midpt.-stelling] $\therefore m_{DE} = -\frac{1}{2}$ through / deur $\left(6; \frac{9}{2}\right)$ $y - \frac{9}{2} = -\frac{1}{2}(x - 6)$ $\therefore y = -\frac{1}{2}x + \frac{15}{2}$	✓ S OR/OF R ✓ m_{DE} ✓ subst./vervang $\left(6; \frac{9}{2}\right)$ ✓ equation/vergelyking (4)
3.4	If / As AOBC is parm: OA BC and / en OB AC $x_O \rightarrow x_B = x_O + 6$ (translation / translasie) $\therefore x_A \rightarrow x_C = 2 + 6$ $\therefore x_C = 8$ in the same way / op dieselfde wyse: $y_A \rightarrow y_C = 4 + 2$ $\therefore y_C = 6$ $\therefore C(8; 6)$ OR/OF midpt. AB = $(4; 3)$ $\therefore$ midpt. OA = $(4; 3)$ [diag./hoekln. parm.] $\frac{0 + x_C}{2} = 4$ and / en $\frac{0 + y_C}{2} = 3$ $\therefore x_C = 8$ $\therefore y_C = 6$ $\therefore C(8; 6)$ OR/OF eq./vgl. BC: $y = 2x - 10$ eq./vgl. AC: $y = \frac{1}{3}x + \frac{10}{3}$ $2x - 10 = \frac{1}{3}x + \frac{10}{3}$ $5x = 40$ $x = 8$ and / en: $y = 2(8) - 10 = 6$ $\therefore C(8; 6)$	✓ method/metode ✓ $x_C = 8$ ✓ $y_C = 6$ (3) ✓ method/metode ✓ $x_C = 8$ ✓ $y_C = 6$ (3) (3)

3.5.1	$\tan \hat{AOT} = 2$ $\therefore \hat{AOT} = 63,43^\circ$ $\tan \hat{FOT} = \frac{3}{4}$ $\therefore \hat{FOT} = 36,87^\circ$  $\theta = 63,43^\circ - 36,87^\circ$ $\therefore \theta = 26,56^\circ$	$\checkmark \tan \hat{AOT} = m_{OA}$ $\checkmark \text{answer/antwoord}$ $\checkmark \tan \hat{FOT} = \frac{3}{4}$ $\checkmark \text{answer/antwoord}$ $\checkmark \text{answer/antwoord}$ (5)
3.5.2	 $BT = 2 \text{ units/eenhede}$ $\perp_h = 4$ $\text{area } \triangle ABT = \frac{1}{2}(2)(4)$ $= 4 \text{ units}^2 / \text{eenhede}^2$ OR/OF $BT = 2 \text{ units/eenhede}$ $\hat{ABT} = 116,57^\circ \quad [\angle \text{s of quad.} / \angle \text{e van vierhk.}]$ $\text{area } \triangle ABT = \frac{1}{2}(2\sqrt{5})(2)\sin 116,57^\circ$ $= 3,9998\dots$ $\approx 4 \text{ units}^2 / \text{eenhede}^2$ OR/OF	$\checkmark BT = 2$ $\checkmark \perp_h = 4$ $\checkmark \text{subst./vervang}$ $\checkmark \text{answer/antwoord}$ (4) $\checkmark BT = 2$ $\checkmark \hat{ABT}$ $\checkmark \text{subst./vervang}$  $\checkmark \text{answer/antwoord}$ (4)



$$\begin{aligned}\text{Area } \triangle ABT &= \text{Area AOTQ} - \text{Area } \triangle AOT - \text{Area } \triangle ABQ \\ &= \frac{1}{2}(4 + 6) \times 4 - \frac{1}{2}(6)(4) - \frac{1}{2}(2)(4) \\ &= 4 \text{ units}^2 / \text{eenhede}^2\end{aligned}$$

✓ method/metode

$$\checkmark \frac{1}{2}(4 + 6) \times 4$$

$$\checkmark \frac{1}{2}(6)(4) \& \frac{1}{2}(2)(4)$$

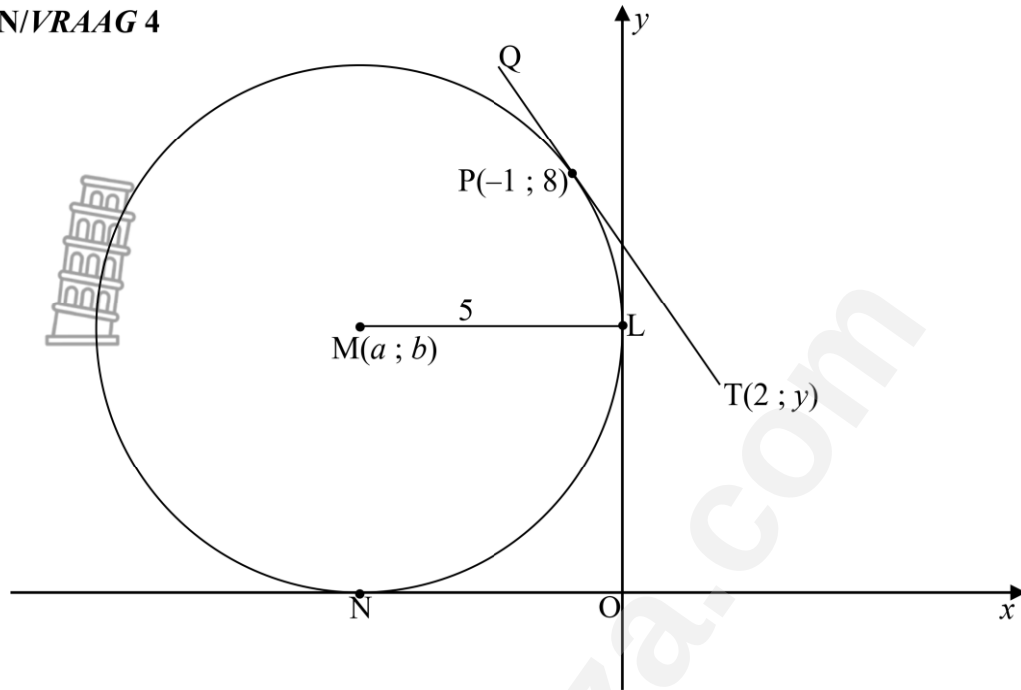
✓ answer/antwoord

(4)

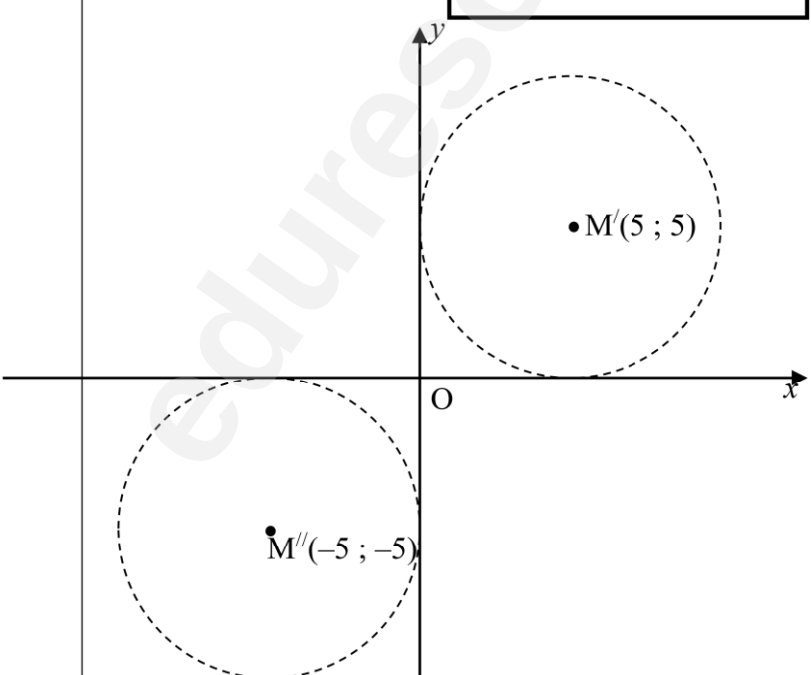
[22]



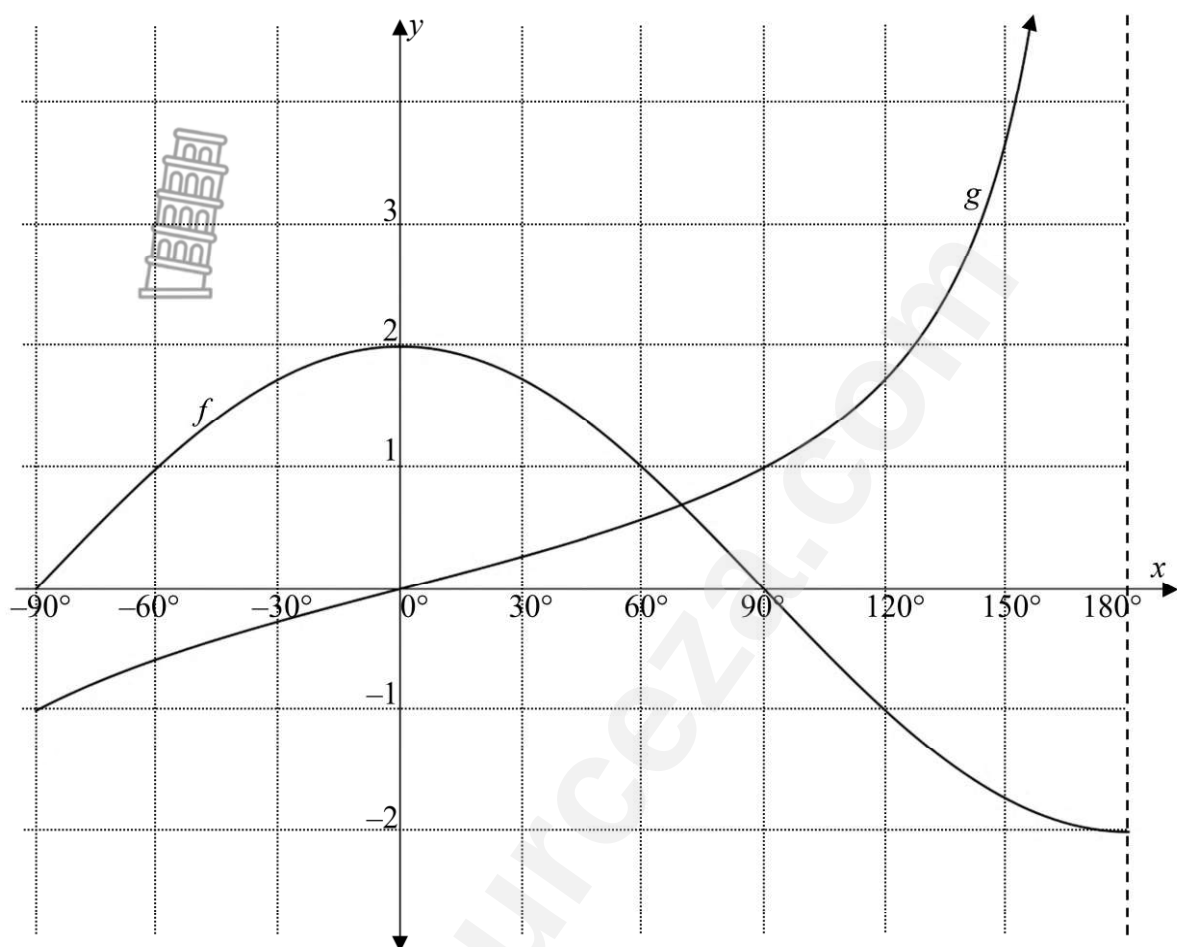
QUESTION/VRAAG 4



4.1	radius $\perp$ tangent / <i>radius $\perp$ raaklyn</i>	✓rede/reason (1)
4.2.1	M(-5 ; 5)	✓ $x = -5$ ✓ $y = 5$ (2)
4.2.2	$(x + 5)^2 + (y - 5)^2 = 25$	✓ $(x + 5)^2 + (y - 5)^2$ ✓ $r^2 = 25$ (2)
4.2.3	$m_{MP} = \frac{8-5}{-1-(-5)}$ $= \frac{3}{4}$ $\therefore m_{QT} = -\frac{4}{3} \text{ [rad } \perp \text{ tangent / rad } \perp \text{ raaklyn]}$ $8 = -\frac{4}{3}(-1) + c \quad \text{OR / OF} \quad y - 8 = -\frac{4}{3}(x - (-1))$ $c = \frac{20}{3}$ $y = -\frac{4}{3}x + \frac{20}{3}$	✓subst./vervangings ✓ $m_{MP} = \frac{3}{4}$ ✓ $m_{QT} = -\frac{4}{3}$ ✓subst./vervang m & (-1 ; 8) ✓equation/vergelyking (5)

4.3	$y = -\frac{4}{3}(2) + \frac{20}{3}$ $y = 4$ $\therefore T(2;4)$ $MT = \sqrt{(2 - (-5))^2 + (4 - 5)^2}$ $= 5\sqrt{2}$ radius circle T / radius sirkel T = $5\sqrt{2} - 5$ $(x - 2)^2 + (y - 4)^2 = 75 - 50\sqrt{2}$ $\approx 4,29$	✓ subst./vervangings ✓ $y_T = 4$ ✓ subst./vervangings ✓ answer/antwoord ✓ MT – 5 ✓ equation/vergelyking (6)
4.4	If the x-axis and y-axis have to remain simultaneously as tangents to the circle M, then the circle is reflected across the axes/As die x-as en y-as gelyktydig as raaklyne aan die sirkel M bly, dan word die sirkel oor die asse gereflekteer. The only quadrants where $xy \geq 0$, are quadrants 1 and 3/ Die enigste kwadrante waar $xy \geq 0$, is kwadrante 1 en 3 $M'(5 ; 5)$ & $M''(-5 ; -5)$ Accuracy / Akkuraatheid But CA from 4.2.1 / Maar CA van 4.2.1 	✓✓ $M'(5 ; 5)$ ✓✓ $M''(-5 ; -5)$ (4) [20]

QUESTION/VRAAG 5



5.1	$b = \frac{1}{2}$	✓ answer/antwoord (1)
5.2	$y \geq -1$ OR/OF $y \in [-1; \infty)$	✓ critical values/kritiese waardes ✓ notation/notasie (2)
5.3	360°	✓ 360° (1)
5.4	$x = 85^\circ$	✓ 85° (1)
5.5	$x = 0^\circ$ or/of $x = 180^\circ$	✓ 0° ✓ 180° (2)
5.6	$p = 180^\circ$	✓✓ 180° (2)
		[9]

QUESTION/VRAAG 6

6.1.1	$\sin 335^\circ$ $= -\sin 25^\circ$	$\checkmark -\sin 25^\circ$ (1)
6.1.2	$\cos 50^\circ$ $= \cos 2(25^\circ)$ $= 1 - 2\sin^2 25$ Answer only: full marks	$\checkmark \cos 2(25^\circ)$ $\checkmark 1 - 2\sin^2 25$ (2)
6.2	$\frac{\sin(-2x) \cdot (1 - \sin^2 x)}{\sin(90^\circ + x) \cdot \tan x}$ $= \frac{(-\sin 2x)(\cos^2 x)}{(\cos x)\left(\frac{\sin x}{\cos x}\right)}$ $= \frac{-2 \sin x \cdot \cos x \cdot \cos^2 x}{\sin x}$ $= -2 \cos^3 x$	$\checkmark -\sin 2x$ $\checkmark \cos^2 x$ $\checkmark \cos x$ $\checkmark \frac{\sin x}{\cos x}$ $\checkmark -2 \sin x \cdot \cos x$ $\checkmark -2 \cos^3 x$ (6)
6.3	$(p \tan 30^\circ + q \sin 60^\circ)^2$ $= \left(p \cdot \frac{1}{\sqrt{3}} + q \cdot \frac{\sqrt{3}}{2}\right)^2$ $= \left(\frac{p}{\sqrt{3}} + \frac{\sqrt{3}q}{2}\right)^2$ $= \frac{p^2}{3} + pq + \frac{3q^2}{4}$ $= \frac{4p^2 + 12pq + 9q^2}{12}$	$\checkmark$ substitution/vervanging $\checkmark$ expansion/uitbreiding $\checkmark$ answer/antwoord (3)



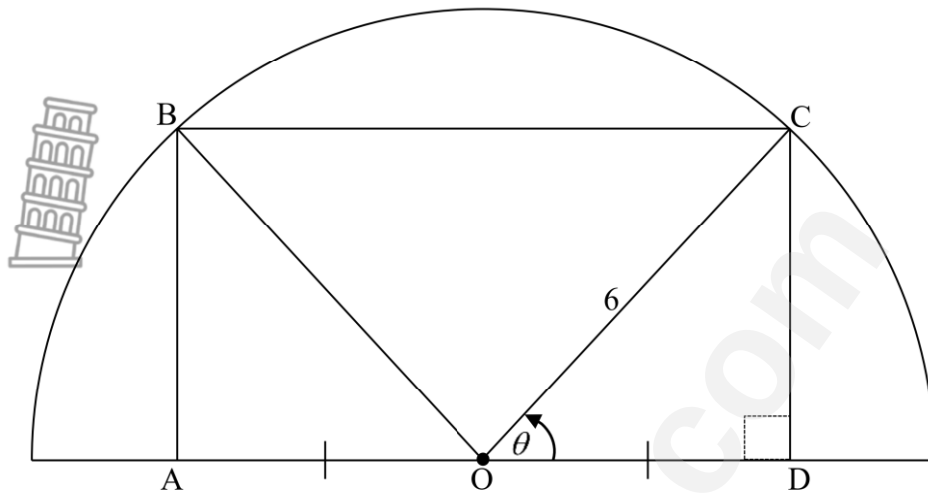
6.4.1	$\sin(A - B)$ $= \cos[90^\circ - (A - B)]$ $= \cos[(90^\circ + B) - A]$ $= \cos(90^\circ + B) \cdot \cos A + \sin(90^\circ + B) \cdot \sin A$ $= (-\sin B) \cdot \cos A + \cos B \cdot \sin A$ $= \sin A \cdot \cos B - \sin B \cdot \cos A$ OR/OF $\sin(A - B)$ $= \cos[90^\circ - (A - B)]$ $= \cos[(90^\circ - A) - (-B)]$ $= \cos(90^\circ - A) \cdot \cos(-B) + \sin(90^\circ - A) \cdot \sin(-B)$ $= \sin A \cdot \cos B + \cos A \cdot (-\sin B)$ $= \sin A \cdot \cos B - \sin B \cdot \cos A$	✓ co-ratio/ko-verhouding ✓ writing as difference of A and B/skryf as die verskil van A en B ✓ expansion/uitbreiding ✓ all reductions/alle reduksies (4)
6.4.2	$LHS/LK = \sin 9A + \sin A$ $= \sin(5A + 4A) + \sin(5A - 4A)$ $= \sin 5A \cdot \cos 4A + \sin 4A \cdot \cos 5A + \sin 5A \cdot \cos 4A - \sin 4A \cdot \cos 5A$ $= 2 \sin 5A \cdot \cos 4A$	✓ $\sin(5A + 4A)$ ✓ $\sin(5A - 4A)$ ✓ expansion/uitbreiding (3)
6.4.3	Max value of/Maks waarde van: $2 \sin 5A \cdot \cos 4A = 2$ $\therefore 3^2 = 9$ $\therefore \text{max value / maks. waarde: } 3^{2 \sin 5A \cdot \cos 4A} = 9$	✓ max value/maks. waarde $2 \sin 5A \cdot \cos 4A = 2$ ✓ 9 (2)



6.5	$\cos 2x - 5 \cos x - 2 = 0$ $2 \cos^2 x - 1 - 5 \cos x - 2 = 0$ $2 \cos^2 x - 5 \cos x - 3 = 0$ $(2 \cos x + 1)(\cos x - 3) = 0$ $\cos x = -\frac{1}{2}$ or / of $\cos x = 3$ $\text{ref. } \angle = 60^\circ$ $\cos x \neq 3$ $x = 120^\circ + k.360^\circ$; or / of $x = 240^\circ + k.360^\circ$; $k \in \mathbb{Z}$	$\checkmark 2 \cos^2 x - 1$ $\checkmark$ factors/faktore $\checkmark$ both equations/beide vergls. $\checkmark \cos x \neq 3$ $\checkmark x = 120^\circ$ & $x = 240^\circ$ $\checkmark + k.360^\circ$, $k \in \mathbb{Z}$ (6)
6.6	$\tan x = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots}}}}$ $\tan^2 x = \sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots}}}}$ $\tan^2 x = \sin x + \tan x$ $\frac{\sin^2 x}{\cos^2 x} = \sin x + \frac{\sin x}{\cos x}$ $\sin^2 x = \sin x \cdot \cos^2 x + \sin x \cdot \cos x$ $2 \sin^2 x = 2 \sin x \cdot \cos^2 x + 2 \sin x \cdot \cos x$ $2 \sin^2 x = \sin 2x \cdot \cos x + \sin 2x$ $2 \sin^2 x = \sin 2x (\cos x + 1)$	$\checkmark$ $\sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots}}}$ $\checkmark \tan^2 x = \sin x + \tan x$ $\checkmark \frac{\sin^2 x}{\cos^2 x} = \sin x + \frac{\sin x}{\cos x}$ $\checkmark$ multiply by/vermenigvuldig met $2 \cos^2 x$ $\checkmark$ double angle identity/ dubbelhoekidentiteit (5)
		[32]



QUESTION/VRAAG 7

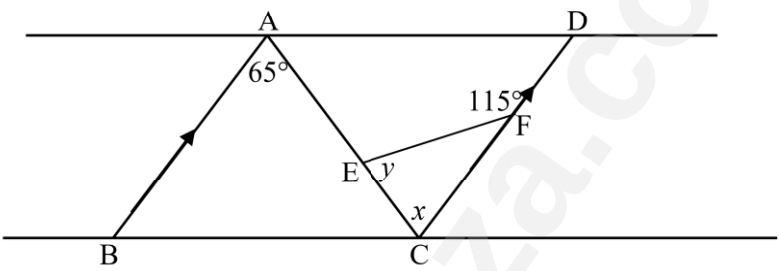


7.1	$\hat{BOA} = \theta$ $\therefore \hat{BOC} = 180^\circ - 2\theta$	$\checkmark \hat{BOA} = \theta$ $\checkmark$ answer/antwoord (2)
7.2	$\hat{BOC} = 94^\circ$ $BC^2 = OB^2 + OC^2 - 2OB \cdot OC \cdot \cos \hat{BOC}$ $BC^2 = 6^2 + 6^2 - 2(6)(6)\cos 94^\circ$ $BC^2 = 77,0224...$ $\therefore BC = 8,78$ units / eenhede	$\checkmark \hat{BOC}$ $\checkmark$ Subst. in cosine rule correctly/vervang korrek in cos-reël $\checkmark$ answer/antwoord (3)
7.3	For ABCD to be a square/Vir ABCD om vierkant te wees: $AD = DC$ $\therefore DC = 2OD$ In $\triangle ODC$ $\tan \theta = \frac{DC}{OD}$ $= \frac{2OD}{OD}$ $= 2$ $\therefore \theta = \tan^{-1}(2)$ $\theta = 63,43^\circ$	$\checkmark DC = 2OD$ $\checkmark \tan \theta = \frac{DC}{OD}$ $\checkmark \tan \theta = 2$ $\checkmark \theta = 63,43^\circ$ (4)
		[9]

GEOMETRY/MEETKUNDE

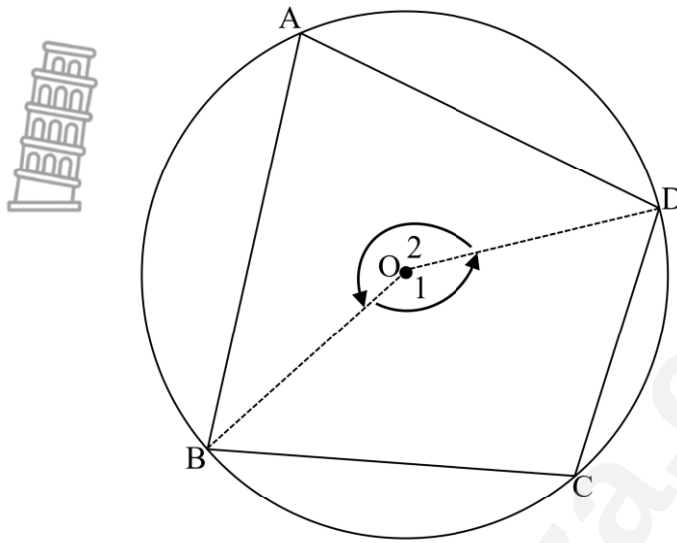
Please read carefully through the following table before marking **QUESTION 8–10/**

Lees asseblief sorgvuldig deur die volgende tabel alvorens **VRAAG 8–10** nagesien word.

	The order in which the candidate answers a geometry question must follow logically/Die volgorde waarin 'n kandidaat 'n meetkundevraag beantwoord moet logies volg. Example/Voorbeeld Given/Gegee $AB \parallel CD$ and/en $\hat{EFD} = 115^\circ$  The candidate first needs to calculate x BEFORE he/she can calculate y/Die kandidaat moet eerste vir x bereken VOORDAT hy/sy vir y kan bereken.
S	A mark for a correct statement (A statement mark is independent of a reason) 'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct) 'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)
S/R	Award a mark if the statement AND reason are both correct (Both MUST be correct to get one mark) Ken 'n punt toe as die bewering EN rede beide korrek is (Beide MOET korrek wees om een punt te kry)



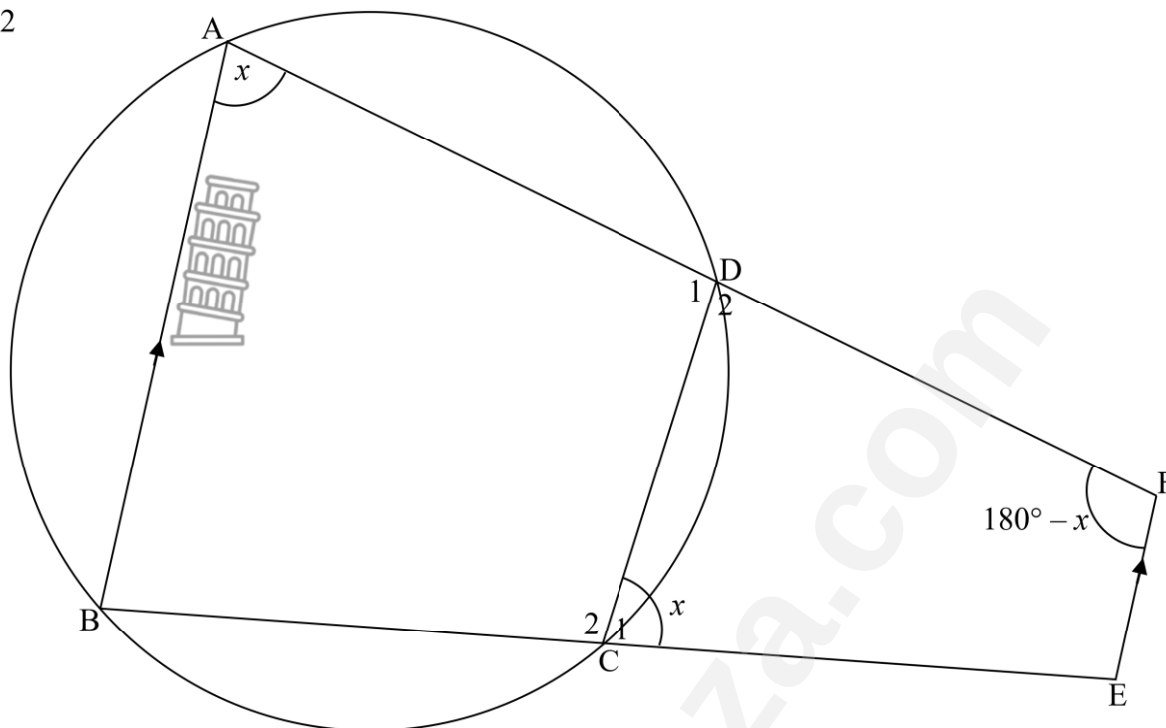
QUESTION/VRAAG 8



8.1	Construction/Konstr.: Draw BO and OD/Trek BO en OD. $\hat{O}_1 = 2\hat{A}$ [$\angle$ at centre = $2 \times \angle$ at circ./midpts $\angle = 2 \times$ omtreks $\angle$] $\hat{O}_2 = 2\hat{C}$ [$\angle$ at centre = $2 \times \angle$ at circ./midpts $\angle = 2 \times$ omtreks $\angle$] $\hat{O}_1 + \hat{O}_2 = 360^\circ$ [$\angle$s around a point/$\angle$e om 'n punt] $\therefore 2\hat{A} + 2\hat{C} = 360^\circ$ $\therefore 2(\hat{A} + \hat{C}) = 360^\circ$ $\therefore \hat{A} + \hat{C} = 180^\circ$ The last statement mark / Die laaste punt vir bewering: $2\hat{A} + 2\hat{C} = 360^\circ$ OR / OF $2(\hat{A} + \hat{C}) = 360^\circ$	✓ constr./konstr. ✓ S/R ✓ S ✓ S/R ✓ S
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(5)

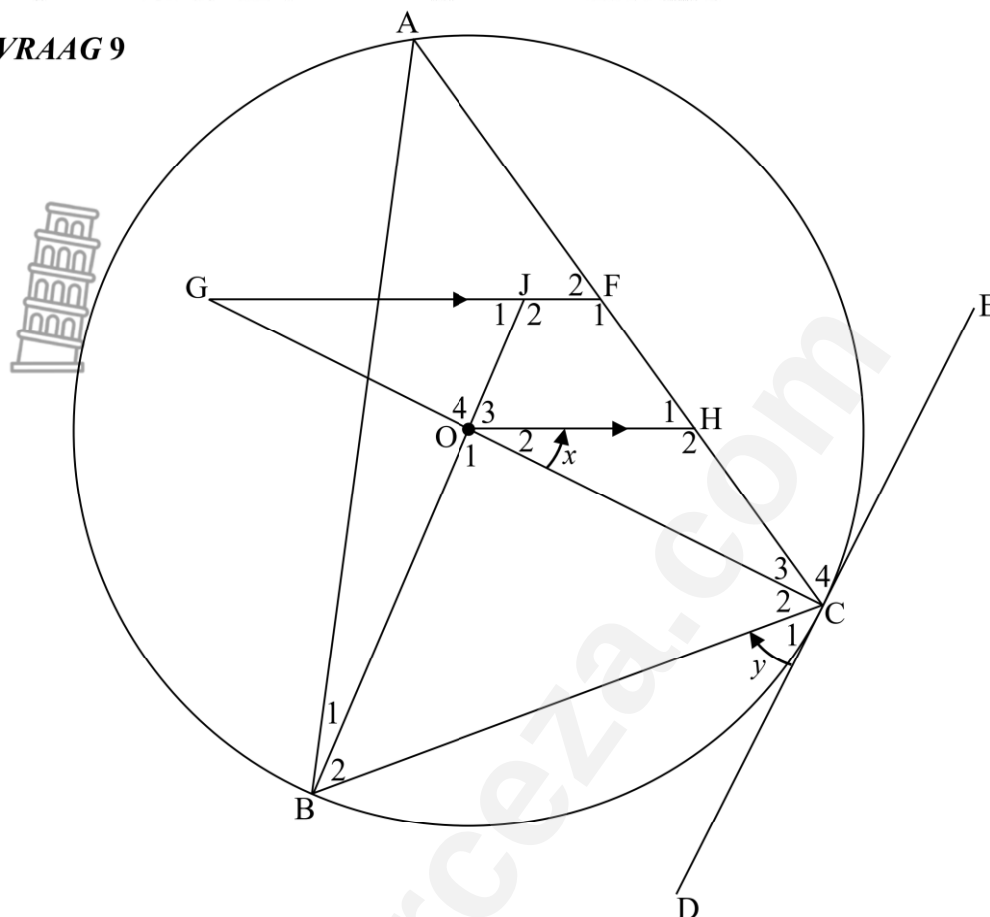
8.2



Let/*Laat*: $\hat{A} = x$

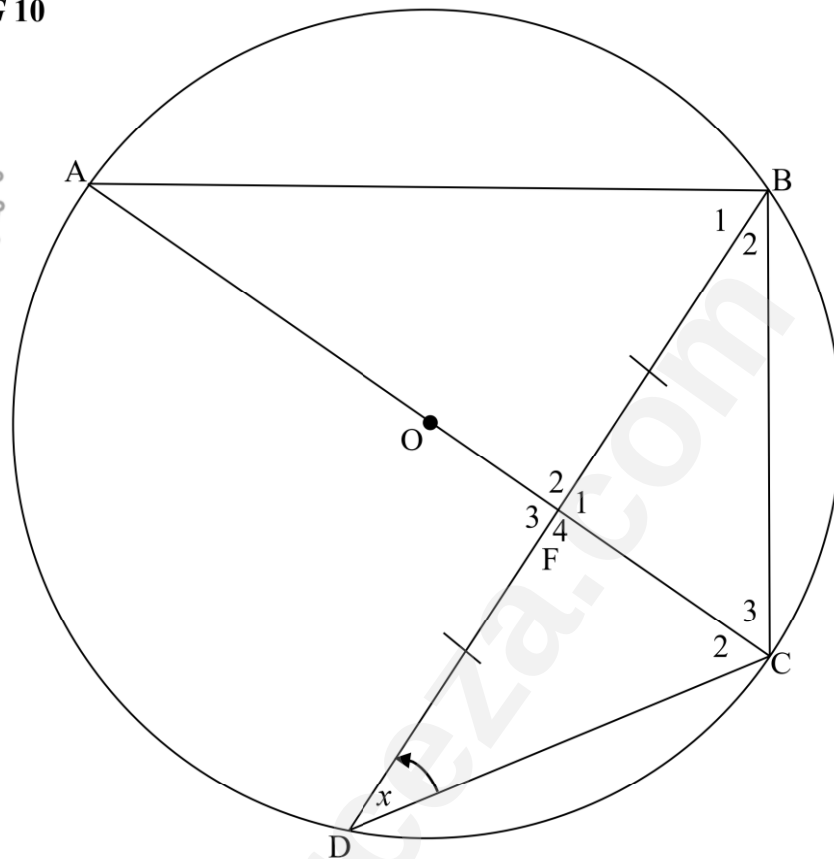
8.2	$\hat{C}_1 = \hat{A} = x$ [ext. $\angle$ cyclic quad./buite $\angle$ kvh.] $\hat{F} = 180^\circ - \hat{A} = 180^\circ - x$ [co-interior $\angle$ s/ko – binne $\angle$ e; AB FE] $\hat{C}_1 + \hat{F} = x + 180^\circ - x = 180^\circ$ $\therefore$ CEFD is a cyclic quad/is 'n kvh. [converse opp. $\angle$ s cyclic quad/omgekeerde teenoorst. $\angle$ e kvh] / [ext. $\angle$ of quad = opp. interior $\angle$ /buite $\angle$ van vierhoek = teenoorst. binne $\angle$]	✓S✓R ✓S/R ✓S ✓R (5)
	OR / OF $\hat{C}_2 = 180^\circ - \hat{A} = 180^\circ - x$ [opp. $\angle$ s cyclic quad/teenoorst. $\angle$ e kvh] $\hat{F} = 180^\circ - \hat{A} = 180^\circ - x$ [co-interior $\angle$ s/ko – binne $\angle$ e; AB FE] $\hat{C}_2 = \hat{F} = 180^\circ - x$ $\therefore$ CEFD is a cyclic quad/is 'n kvh. [converse ext. $\angle$ cyclic quad/omgekeerde buite. $\angle$ kvh]/ [opp. $\angle$ s of quad supplementary/teenoorst. $\angle$ e van vierhoek supplementêr]	✓S✓R ✓S/R ✓S ✓R (5)
		[10]

QUESTION/VRAAG 9



9.1	$\hat{C}_1 = \hat{A} = y$ [tan-chord theorem/raaklyn-koordstelling] $\hat{O}_1 = 2\hat{A} = 2y$ [$\angle$ at centre = $2 \times \angle$ at circ./ midpts $\angle = 2 \times$ omtreks $\angle$] $\hat{O}_3 = 180^\circ - 2y - x$ [$\angle$ s on straight line / $\angle$ e op reguit lyn] $\hat{J}_1 = 180^\circ - 2y - x$ [corresponding $\angle$ s / ooreenkomstige $\angle$ e; GF $\parallel$ OH] OR/OF $\hat{C}_1 = \hat{A} = y$ [tan-chord theorem/raaklyn-koordstelling] $\hat{O}_1 = 2\hat{A} = 2y$ [$\angle$ at centre = $2 \times \angle$ at circ./ midpts $\angle = 2 \times$ omtreks $\angle$] $\hat{G} = x$ [corresponding $\angle$ s / ooreenkomstige $\angle$ e; GF $\parallel$ OH] $\hat{O}_4 = 2y$ [opp. $\angle$ s = / regoorst. $\angle$ e =] $\hat{J}_1 = 180^\circ - 2y - x$ [sum of $\angle$ s in Δ / som van $\angle$ e in Δ]	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ S/R $\checkmark$ S/R $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ S/R $\checkmark$ S/R (6) (6)
9.2	$\frac{FH}{FC} = \frac{GO}{GC}$ [prop.th / eweredigh.st ; GF $\parallel$ OH] $= \frac{2}{5}$	$\checkmark$ S $\checkmark$ R $\checkmark$ answer/ antwoord (3)
		[9]

QUESTION/VRAAG 10



10.2.3	$\frac{CF}{CB} = \frac{CB}{CA} \text{ [from / vanuit } \parallel \Delta]$ $\therefore CB^2 = AC \cdot FC$ but / maar : $\triangle DFC \equiv \triangle FBC \text{ [S } \angle \text{ S]}$ $\therefore DC = BC$ $\therefore DC^2 = AC \cdot FC$	✓S/R ✓S ✓S/R ✓S (4)
10.2.4	$AC^2 = AB^2 + BC^2 \text{ [Pythagoras]}$ $AC^2 = AB^2 + FC \cdot AC \text{ [} BC^2 = DC^2 \text{]}$ $1 = \frac{AB^2}{AC^2} + \frac{FC}{AC}$ $\frac{FC}{AC} = 1 - \frac{AB^2}{AC^2}$ $\frac{FC}{AC} = \left(1 - \frac{AB}{AC}\right) \left(1 + \frac{AB}{AC}\right)$ but / maar $AC = AO + OC$ $\therefore \frac{FC}{AC} = \left(1 - \frac{AB}{AO + OC}\right) \left(1 + \frac{AB}{AO + OC}\right)$	✓S ✓ $BC^2 = FC \cdot AC$ ✓ $\div AC^2$ ✓ $\frac{FC}{AC} = 1 - \frac{AB^2}{AC^2}$ ✓factorise/faktoriseer (5)
		[19]

TOTAL/TOTAAL: 150



GRID-Analysis (According to BLOOMS TAXONOMY)													
Question	Description	KNOWLEDGE			ROUTINE PROCEDURES			COMPLEXED PROCEDURES			PROBLEM SOLVING		
		Low	Mod	High	Low	Mod	High	Low	Mod	High	Low	Mod	High
1.1.1	Mean	2											
1.1.2	Standard deviation (calculator)				1								
1.1.3	Outside one SD					3							
1.2	Outlier	1											
1.3	Complex range							2					
1.4	Critical thinking median										2		
2													
2.1	Regression(Gr.12) [9]												
2.2	Interpret scatter plot + regr. line			2									
2.3	Equation of regression line				3								
2.4	Estimation by using regr. line	2											
2.5	Critical thinking SD										2		
3													
3.1	Analytical (Gr.11) [22]												
3.1.1	Distance		2										
3.1.2	Gradient		2										
3.2	Prove lines perpendicular				2								
3.3	Equation str.line + midpt.th							4					
3.4	4 th coordinate of parm						3						
3.5.1	Inclination							5					
3.5.2	Integrating trig to find area Δ							4					
4													
4.1	Analytical(Gr.12) [20]												
4.2.1	Geometry theorem			1									
4.2.2	Coordinates centre			1	1								
4.2.3	Eq. of circle			2									
4.3	Equation of tangent					5							
4.4	Eq. of new circle with specific conditions							6					
4.5	Critical thinking new equation											4	

Question	Description	KNOWLEDGE			ROUTINE PROCEDURES			COMPLEX PROCEDURES			PROBLEM SOLVING		
		Low	Mod	High	Low	Mod	High	Low	Mod	High	Low	Mod	High
5	Trig graphs [9]												
5.1	Parameters		1										
5.2	Range				2								
5.3	Period	1											
5.4	Translation					1							
5.5	Undefined							2					
5.6	Critical thinking + integrating concepts											2	
6	Trigonometry [32]												
6.1.1	Reduction	1											
6.1.2	Double angle		2										
6.2.	Simplification + combination of identities		2					4					
6.3	Special angles + combining algebra			2			1						
6.4.1	Derive formula $\sin(A - B)$				4								
6.4.2	Identity						3						
6.4.3	Max value										2		
6.5	General solution						6						5
6.6	Critical thinking + evaluation												
7	2D/3D Trig [9]												
7.1	Angle in terms of θ			2									
7.2.	Cosine rule					3							
7.3	Transforming a quadrilateral									4			
8	Geometry [10]												
8.1	Theorem				5								
8.2	Prove quad. to be cyclic quad.							5					



[illegible]