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Department of
Education
FREE STATE PROVINCE

PREPARATORY EXAMINATION

GRADE 12

MATHEMATICS P2

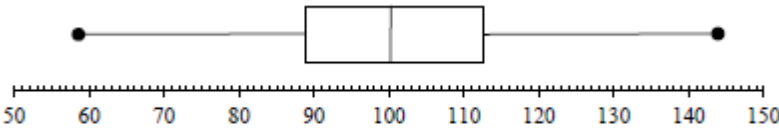
SEPTEMBER 2018

MARKS: 150

MARKING GUIDELINE

This marking guideline consists of 14 pages.

QUESTION 1

1.1	Lower quartile : $Q_1 = 89$ Upper quartile : $Q_3 = 113$	✓ 89 ✓ 113 (2)
1.2		✓ Min = 58 and max = 145 ✓ $Q_1 = 89$ and $Q_3 = 113$ ✓ $Q_2 = 100$ (3)
1.3	Symmetrical Marginally skewed to the right accept both	✓ Answer (1)
1.4	Mean : $\bar{x} = \frac{1522}{15}$ $\bar{x} = 101,47$	✓✓ $\frac{1522}{15}$ (2)
1.5	Standard deviation : $\sigma = 19,07$	✓ $\sigma = 19,07$ (1)
1.6	$(\bar{x} - \sigma; \bar{x} + \sigma)$ $= (82,4; 120,54)$ 2 days	✓ 82,4 ✓ 120,54 ✓ 2 days (3)
		[12]

QUESTION 2

2.1	$r = -0,95$	✓ ✓ value of r (2)
2.2	There is a strong negative relationship.	✓ strong negative (1)
2.3	$a = 11,71323529$ $b = -1,11764$ $\hat{y} = 11,71 - 1,12x$	✓ $a = 11,71$ ✓ $b = -1,12$ ✓ correct equation (3)
2.4	$\hat{y} = 11,71 - 1,12(6)$ $\hat{y} = 4,99$ $\hat{y} \approx 5$ times OR $\hat{y} = 5,007$ $= 5$ times	✓ Substitution ✓ $y \approx 5$ times ✓ calculator ✓ $\hat{y} \approx 5$ times (2)
		[8]

QUESTION 3

3.1.1	$M_{EF} = M_{FG}$ $\frac{-1-3}{0-4} = \frac{1-(-1)}{t}$ $\frac{-4}{-4} = \frac{1+1}{t}$ $1 = \frac{2}{t}$ $t = 2$	✓ $M_{EF} = M_{FG}$ ✓ Substitution into the correct formula ✓ $1 = \frac{2}{t}$ ✓ Answer (4)
3.1.2	$M_{EF} \times M_{FG} = -1$ $1 \times \frac{2}{t} = -1$ $t = -2$	✓ $1 \times \frac{2}{t} = -1$ ✓ Answer (2)

3.2.1	$y = \frac{1}{2}x + 8$ $M_{QR} = M_{PS} = \frac{1}{2}$ QR//PS $y - 5 = \frac{1}{2}(x - 4)$ $y = \frac{1}{2}x - 2 + 5$ $\therefore y = \frac{1}{2}x + 3$	✓ rewrite equation of PR ✓ $M_{PS} = \frac{1}{2}$ ✓ Substitute (4; 5) correctly ✓ Equation (4)
3.2.2	$0 = \frac{1}{2}x + 3$ $x = -6$ $\therefore S(-6; 0)$	✓ $y = 0$ ✓ $S(-6; 0)$ (2)
3.2.3	$M\left(\frac{2 + (-6)}{2}, \frac{6 + 0}{2}\right)$ $= M(-2; 3)$	✓ $\frac{2 + (-6)}{2}$ ✓ $\frac{6 + 0}{2}$ (2)
3.2.4	Midpoint of QS = midpoint of PR (diagonals bisect) $-2 = \frac{x + 4}{2}$ $x = -8$ $3 = \frac{y + 5}{2}$ $y = 1$ $\therefore R(-8; 1)$ OR $P \rightarrow Q : (x - 2; y + 1)$ $S \rightarrow R : (x - 2; y + 1)$ $\therefore R(-6 - 2; 0 + 1)$ $\therefore R(-8; 1)$	✓ $-2 = \frac{x + 4}{2}$ ✓ $3 = \frac{y + 5}{2}$ ✓ $R(-8; 1)$ (3) ✓ $\rightarrow R : (x - 2; y + 1)$ ✓✓ $\therefore R(-8; 1)$ (3)
3.2.5	$m_{PS} = \frac{1}{2}$ PS // QR $\tan \hat{OSP} = m_{PS} = \frac{1}{2}$ $\therefore \hat{OSP} = 26,57^\circ$ $m_{RS} = -\frac{1}{2}$ $R\hat{SO} = 180^\circ - 26,57^\circ$ $= 153,43^\circ$ $\therefore R\hat{SP} = 153,43^\circ - 26,57^\circ$ $= 126,86^\circ$	✓ $\tan \hat{OSP} = m_{PS} = \frac{1}{2}$ ✓ $\hat{OSP} = 26,57^\circ$ ✓ $m_{RS} = -\frac{1}{2}$ ✓ $R\hat{SO} = 153,43^\circ$ ✓ $126,86^\circ$ (5)

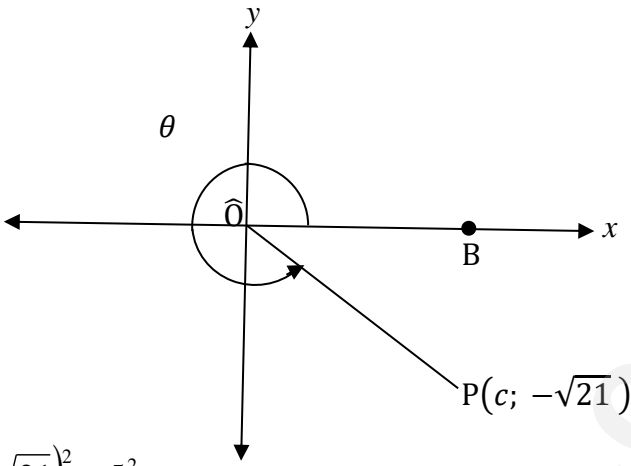
		[22]
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QUESTION 4

4.1	$M\left(\frac{-2+4}{2}; \frac{3+(-7)}{2}\right)$ $= M(1; -2)$ $\therefore$ centre of the circle is $(1; -2)$ $(x-1)^2 + (y-(-2))^2 = r^2$ $(-2-1)^2 + (3+2)^2 = r^2$ $9 + 25 = r^2$ $\therefore 34 = r^2$ $(x-1)^2 + (y+2)^2 = 34$	✓ Subst. into the correct formula ✓ Centre of the circle ✓ Subst. $N(-2; 3)$ ✓ $34 = r^2$ ✓ Equa of the circle (5)
4.2	$M_{rad} = \frac{-2-3}{1-(-2)}$ $= \frac{-5}{3}$ $M_{tan} = \frac{3}{5}$ radius $\perp$ tangent $y-3 = \frac{3}{5}(x-(-2))$ $\therefore y = \frac{3}{5}x + \frac{6}{5} + 3$ $= \frac{3}{5}x + \frac{21}{5}$	✓ Subst. into the correct formula ✓ $M_{rad} = \frac{-5}{3}$ ✓ $M_{tan} = \frac{3}{5}$ ✓ Subst. $N(-2; 3)$ and gradient ✓ Equation (5)

4.3	Let the angle of inclination of line NP be α and the angle adjacent to it be θ $\tan \alpha = \frac{-5}{3}$ $\alpha = 180^\circ - \tan^{-1}\left(\frac{5}{3}\right)$ $\therefore \alpha = 180^\circ - 59,036243\dots^\circ$ $= 120,96^\circ$ $\theta = 59,04^\circ$ OR $\widehat{NPQ} = 180^\circ - (59,04^\circ + 90^\circ)$ $= 30,96^\circ$ OR $\widehat{NQP} = 90^\circ \quad \text{diameter subtends a right angle}$ $NQ = 6 \text{ and } PQ = 10$ $\therefore \tan \widehat{NPQ} = \frac{6}{10}$ $\therefore \widehat{NPQ} = 30,96^\circ$	$\checkmark \tan \alpha = -\frac{5}{3}$ $\checkmark \alpha = 120,96^\circ$ $\checkmark \theta = 59,04^\circ$ $\checkmark \widehat{NPQ} = 30,96^\circ$ (4) $\checkmark \widehat{NQP} = 90^\circ$ $\checkmark NQ = 6 \text{ and } PQ = 10$ $\checkmark \tan \widehat{NPQ} = \frac{6}{10}$ $\checkmark \widehat{NPQ} = 30,96^\circ$ (4)
4.4	QP = 10 units and NQ = 6 units Area $\Delta NPQ = \frac{1}{2} (6)(10) \checkmark$ $= 30 \text{ square units}$ $\therefore \frac{\text{Area } \Delta NPQ}{\text{Area of circle}} = \frac{30 \checkmark}{34\pi \checkmark}$ $= 0,28 \checkmark$	(4)
		[17]

QUESTION 5

5.1.1	Reflex $\widehat{BOP} = \theta$  $c^2 + (\sqrt{21})^2 = 5^2$ $c^2 = 25 - 21$ $c^2 = 4$ $\therefore c = 2$	✓ subst into pyth ✓ Answer (2)
5.1.2	a) $\cos \theta = \frac{2}{5}$	✓ Answer (1)
	b) $= \frac{-\sqrt{21}}{2} + \left(\frac{-\sqrt{21}}{5} \right)^2$ $= \frac{-25\sqrt{21} + 42}{50}$	✓ $\frac{\sqrt{21}}{2} + \left(\frac{\sqrt{21}}{5} \right)^2$ for corr subst ✓ Answer (2)
	c) $2 \sin \theta \cos \theta$ $= 2 \left(\frac{-\sqrt{21}}{5} \right) \left(\frac{2}{5} \right)$ $= \frac{-4\sqrt{21}}{25}$	✓ Identity ✓ Answer (2)
5.2	$\frac{(-\sin x) \cdot \tan x \cdot \cos(360^\circ - 30^\circ)}{(\sin x)^2}$ $= \frac{-\sin x \cdot \tan x (\cos 30^\circ)}{\sin^2 x}$ $= \frac{-\sqrt{3}}{2} \tan x \div \sin x$ $= \frac{-\sqrt{3} \sin x}{2 \cos x} \times \frac{1}{\sin x}$ $= \frac{-\sqrt{3}}{2 \cos x}$	✓ $-\sin x$ ✓ $(\sin x)^2$ ✓ $\cos 30^\circ$ ✓ $\frac{\sin x}{\cos x}$ ✓ answer (5)
		[12]

QUESTION 6

6.1	$LHS = \frac{2 \sin^2 x}{2 \frac{\sin x}{\cos x} - 2 \sin x \cos x}$ $= \frac{2 \sin^2 x}{2 \sin x \left(\frac{1}{\cos x} - \cos x \right)}$ $= \frac{\sin x}{1 - \cos^2 x}$ $= \frac{\sin x}{\cos x}$ $= \sin x \times \frac{\cos x}{\sin^2 x}$ $= \frac{\cos x}{\sin x}$ $\therefore LHS = RHS$	$\checkmark \frac{\sin x}{\cos x}$ $\checkmark 2 \sin x \cos x$ $\checkmark 2 \sin x \left(\frac{1}{\cos x} - \cos x \right)$ $\checkmark \frac{\sin x}{1 - \cos^2 x}$ $\checkmark \sin^2 x$ (5)
6.2	This identity will be undefined when $2 + 2 \cos 2x = 0$ $\cos 2x = -1$ $\therefore 2x = 180^\circ + k \cdot 360^\circ \quad k \in \mathbb{Z}$ $x = 90^\circ + k \cdot 180^\circ$ NB: Penalise 1 mark for leaving out $k \in \mathbb{Z}$ OR $2 + 2 \cos 2x = 0$ $1 + \cos 2x = 0$ $1 + 2 \cos^2 x - 1 = 0$ $\cos x = 0$ $\therefore x = 90^\circ$ $x = \pm 90^\circ + 360^\circ k; k \in \mathbb{Z}$	$\checkmark \cos 2x = -1$ $\checkmark 2x = 180^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ $\checkmark x = 90^\circ + k \cdot 180^\circ; k \in \mathbb{Z}$ (4) $\checkmark$ Identity $\checkmark \cos x = 0$ $\checkmark \pm 90^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ (4)
		[9]

QUESTION 7

7.1	$D\hat{E}F = 180^\circ - \theta$ opp angle of cyclic quad	✓ Answer (1)
7.2	$DF = \sqrt{r^2 + r^2 - 2r^2 \cos(180^\circ - \theta)}$ $= \sqrt{2r^2 + 2r^2 \cos \theta}$ $= r\sqrt{2 + 2 \cos \theta} \quad \text{given}$	✓ correct sub into cosine rule ✓ simplification (2)
7.3	$D\hat{F}G = 90^\circ$ angl in semi circle In $\triangle DFG$: $\sin \theta = \frac{DF}{DG}$ $\sin \theta = \frac{r\sqrt{2 + 2 \cos \theta}}{2r}$ $(2 \sin \theta)^2 = (\sqrt{2 + 2 \cos \theta})^2$ $4 \sin^2 \theta = 2 + 2 \cos \theta$ $2 \sin^2 \theta = 1 + \cos \theta$	$\checkmark \sin \theta = \frac{r\sqrt{2 + 2 \cos \theta}}{2r}$ $\checkmark 4 \sin^2 \theta = 2 + 2 \cos \theta$ $\checkmark 2 \sin^2 \theta = 1 + \cos \theta$ (3)
7.4	$2(1 - \cos^2 \theta) = 1 + \cos \theta$ $2 - 2 \cos^2 \theta = 1 + \cos \theta$ $-2 \cos^2 \theta - \cos \theta + 1 = 0$ $2 \cos^2 \theta + \cos \theta - 1 = 0$ $(2 \cos \theta - 1)(\cos \theta + 1) = 0$ $\cos \theta = \frac{1}{2} \quad \text{or } \cos \theta \neq -1$ $\therefore \theta = 60^\circ \quad \theta \neq 180^\circ$	$\checkmark 2(1 - \cos^2 \theta) = 1 + \cos \theta$ $\checkmark \text{standard form}$ $\checkmark \text{factors}$ $\checkmark \text{both equations}$ $\checkmark \theta = 60^\circ$ (5)
		[11]

QUESTION 8

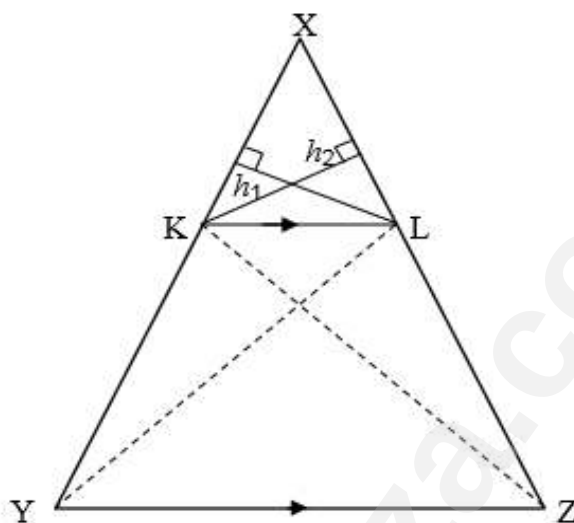
8.1	$a = 1$ $b = 2$ $c = 2$ $d = 1$	$\checkmark a = 1$ $\checkmark b = 2$ $\checkmark c = 2$ $\checkmark d = 1$ (4)
8.2	$P(21,44^\circ; 0,73)$	$\checkmark$ correct substitution (2)
8.3.1	$x = 90^\circ$	$\checkmark 90^\circ$ (1)
8.3.2	$x \in [45^\circ; 135^\circ]$ OR $45^\circ \leq x \leq 135^\circ$	$\checkmark 45^\circ$ and 135° $\checkmark$ Notation (2)
		[9]

QUESTION 9

9.1.1	$\hat{C} = \hat{A}_1 = 78^\circ$ alt $\angle$'s AP // CB	$\checkmark$ R (1)
9.1.2	$\hat{A}\hat{B}\hat{C} = \hat{A}_1 = 78^\circ$ tan chord theorem	$\checkmark$ S $\checkmark$ R (2)
9.1.3	$\hat{A}_3 = \hat{C} = 78^\circ$ tan chord theorem	$\checkmark$ S (1)
9.1.4	$\hat{A}_1 + \hat{A}_2 + \hat{A}_3 = 180^\circ$ straight line $78^\circ + \hat{A}_2 + 18^\circ = 180^\circ$ $\hat{A}_2 = 24^\circ$	$\checkmark$ S (1)

9.2.1	$\hat{Y}_1 = a$ $\hat{Y}_1 = \widehat{YXP} = a$ $\widehat{YXP} = \hat{Q} = a$ $\hat{Y}_2 = \hat{Q} = a$	tan chord theorem alt angle $XY \parallel QT$ $=$ angles opp $=$ sides angles in same circle segment	$\checkmark$ S $\checkmark$ R $\checkmark$ $\checkmark$ S $\checkmark$ S $\checkmark$ S $\checkmark$ R $\checkmark$	(6)
9.2.2	$\hat{T}_2 = \widehat{XYT}$ $= \hat{Y}_1 + \hat{Y}_2$ $= \hat{T}_1 + \hat{T}_1$ $= 2\hat{T}_1$	alt angles; $XY \parallel QT$	$\checkmark$ S/R $\checkmark$	(2)
9.2.3	$\hat{T}_2 + \hat{T}_1 = 90^\circ$ $\hat{T}_2 = 90^\circ - a$	angle in $\frac{1}{2}\odot$	S $\checkmark$ R $\checkmark$	(2)
9.2.4	$\hat{T}_2 = 90^\circ - a$ But $\widehat{YQO} = 2\hat{T}_2$ $= 2(90^\circ - a)$ $= 180^\circ - 2a$ $\widehat{QOR} = 180^\circ - 180^\circ + 2a$ $= 2a$ $\therefore$ SORT is a cyclic quad ext angl = opp int angle OR $\widehat{XYO} = 90^\circ$ $\widehat{XOY} = 90^\circ - a$ $\widehat{XOY} = \hat{T}_2$ $\therefore$ SORT is a cyclic quad/converse: ext. $\angle$ s of cyclic quad OR $\widehat{XOY} = 2\hat{T}_1$ $\widehat{XOY} = \hat{T}_2$ $\therefore$ SORT is a cyclic quad/converse: ext. $\angle$ of cyclic quad	proven angle at centre = 2 angl on circle tan $\perp$ radius sum of $\angle$ s of Δ $\angle$ at centre = 2 $\angle$ at circum.	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R	(2)
9.2.5	$\hat{T}_1 + \hat{T}_2 = 90^\circ$ $a + 2a = 90^\circ$ $3a = 90^\circ$ $\therefore a = 30^\circ$		$\checkmark$ S $\checkmark$ answer	(2)
9.2.6	$\widehat{XYO} = 90^\circ$ $\hat{R}_2 = 90^\circ$ $\therefore TR = RQ$ chord	rad $\perp$ tangent alt.angles; $XY \parallel TQ$ line from centre of circle $\perp$	$\checkmark$ S $\checkmark$ S $\checkmark$ R	(3)
				[22]

QUESTION 10



10.1.1	Const: Join KZ & LY & draw h_1 from K $\perp$ XL & h_2 from L $\perp$ XK Proof $\frac{\text{Area } \Delta XKL}{\text{Area } \Delta LYK} = \frac{\frac{1}{2}XK \times h_1}{\frac{1}{2}KY \times h_1} = \frac{XK}{KY}$ $\frac{\text{Area } \Delta XKL}{\text{Area } \Delta KLZ} = \frac{\frac{1}{2}XL \times h_2}{\frac{1}{2}LZ \times h_2} = \frac{XL}{LZ}$ Area of $\Delta XKL = \text{Area } \Delta XKL$ common But Area $\Delta LKY = \text{Area } \Delta KLZ$ same base & height; LK // YZ $\frac{\text{Area } \Delta XKL}{\text{Area } \Delta LYK} = \frac{\text{Area } \Delta XKL}{\text{Area } \Delta KLZ}$ $\therefore \frac{XK}{KY} = \frac{XL}{LZ}$	✓ const $\checkmark \frac{\text{Area } \Delta XKL}{\text{Area } \Delta LYK} = \frac{XK}{KY}$ $\checkmark \frac{\text{Area } \Delta XKL}{\text{Area } \Delta KLZ} = \frac{XL}{LZ}$ ✓ R ✓ S (5)
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10.2.1	$\frac{ED}{DC} = \frac{AT}{AC} = \frac{2}{3}$ $DE = \frac{2}{3} \times 9$ $= 6$ $\therefore D$ is the midpoint of BE line // to one side of $\triangle$	$\checkmark$ S $\checkmark$ R $\checkmark$ DE = 6 (3)
10.2.2	$BF = FT$ line from midpt drawn // to 2 nd side bisct 3 rd $FD = \frac{1}{2}TE$ midpoint theorem $\therefore TE = 4$	$\checkmark$ S/R $\checkmark$ R $\checkmark$ TE = 4 (3)
10.2.3(a)	$\frac{\text{Area of } \triangle ADC}{\text{Area of } \triangle ABD}$ $= \frac{DC}{BD}$ same height $= \frac{3y}{2y}$ $= \frac{3}{2}$	$\checkmark$ same height $\checkmark \frac{3}{2}$ (2)
10.2.3(b)	$\frac{\frac{1}{2} \times TC \times EC \times \sin C}{\frac{1}{2} \times AC \times BC \times \sin C}$ $= \frac{(x)(y)}{(3x)(5y)}$ $= \frac{1}{15}$	$\checkmark \frac{\frac{1}{2} \times TC \times EC \times \sin C}{\frac{1}{2} \times AC \times BC \times \sin C}$ $\checkmark \frac{(x)(y)}{(3x)(5y)}$ $\checkmark \frac{1}{15}$ (3)
		[16]

QUESTION 11

11.1	In $\triangle TPS$ and $\triangle QSR$ $\frac{PS}{QS} = \frac{1,5}{4} = \frac{3}{8}$ $\frac{TP}{SR} = \frac{4,5}{12} = \frac{3}{8}$ $\frac{TS}{QR} = \frac{3,6}{9,6} = \frac{3}{8}$ $\therefore \triangle TPS \sim \triangle QSR$ sides of triangles in proportion $\therefore \hat{P}TS = \hat{R}$ $\triangle$s are equiangular $\therefore TP$ is a tangent converse of tan chord theorem	$\checkmark \frac{PS}{QS} = \frac{1,5}{4}$ $\checkmark \frac{TP}{SR} = \frac{4,5}{12}$ $\checkmark \frac{TS}{QR} = \frac{3,6}{9,6}$ $\checkmark \checkmark S/R$ $\checkmark \hat{P}TS = \hat{R}$ $\checkmark R$ (7)
11.2	$\hat{P} = \hat{Q}SR$ $\triangle$s are equiangular $QS \parallel TP$ corres. $\angle$s are = $\frac{TQ}{9,6} = \frac{1,5}{12}$ proportion theorem; $PT \parallel SQ$ $\therefore TQ = 1,2$	$\checkmark S/R$ $\checkmark S/R$ $\checkmark \checkmark S/R$ $\checkmark TQ = 1,2$ (5)
		[12]

TOTAL: 150