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Province of the
EASTERN CAPE
EDUCATION

**NATIONAL
SENIOR CERTIFICATE
*NASIONALE
SENIOR SERTIFIKAAT***

GRADE/*GRAAD* 12

SEPTEMBER 2017

**MATHEMATICS P2/*WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN***

MARKS/*PUNTE*: 150

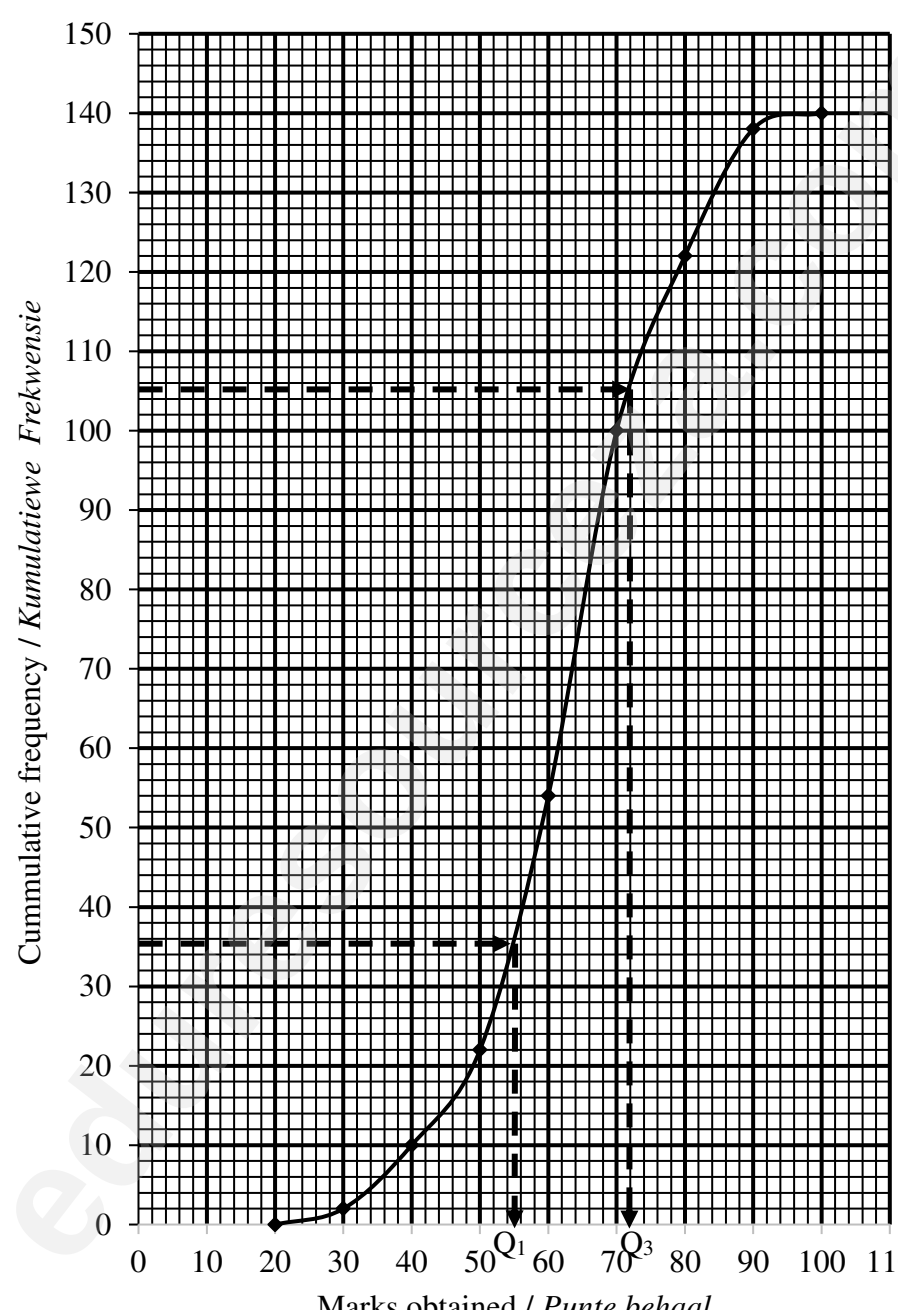
This marking guideline consists of 15 pages.
Hierdie nasienriglyn bestaan uit 15 bladsye.

QUESTION/VRAAG 1

Employee / Werknemer	1	2	3	4	5	6	7	8	9	10															
Hours in training / Ure in opleiding	16	36	20	38	40	30	35	22	40	24															
Productivity (units produced per day) Produktiwiteit (eenhede per dag vervaardig)	45	70	44	56	60	48	75	60	63	38															
1.1	Scatter plot / Spreidiagram <table border="1"><caption>Data points for Scatter plot</caption><thead><tr><th>Hours in training (x)</th><th>Productivity units (y)</th></tr></thead><tbody><tr><td>16</td><td>45</td></tr><tr><td>20</td><td>44</td></tr><tr><td>24</td><td>38</td></tr><tr><td>30</td><td>48</td></tr><tr><td>36</td><td>75</td></tr><tr><td>38</td><td>56</td></tr><tr><td>40</td><td>60</td></tr></tbody></table>						Hours in training (x)	Productivity units (y)	16	45	20	44	24	38	30	48	36	75	38	56	40	60	✓ 2-4 correct points / korrekte punte ✓ 5-7 correct points / korrekte punte ✓ plotting all points / afsteek van alle punte		(3)
Hours in training (x)	Productivity units (y)																								
16	45																								
20	44																								
24	38																								
30	48																								
36	75																								
38	56																								
40	60																								
1.2	$a = 29,22$ $b = 0,89$ $y = 29,22 + 0,89x$			✓ A ✓ B ✓ equation / vergelyking				(3)																	
1.3	$(30,9;55,50)$ y-int / y-afsnit 29,22		✓ mean point / gemiddelde punt $(30,90;55,50)$ and/en y-int/y-afsnit 29,22 ✓ regression line / regressielyn				(2)																		
1.4	$y = 29,22 + 0,89(25)$ $= 51,47$ OR/OF $y = 51,38$ [calculator use]/[sakrekenaar gebruik]			ANSWER ONLY FULL MARKS SLEGS ANTWOORD VOLPUNTE		✓ subst./ verv. ✓ answer / antwoord		(2)																	
1.5	$r = 0,66$						✓ answer / antwoord	(1)																	
1.6	Moderately strong positive correlation / Matige sterk positiewe korrelasie						✓ answer / antwoord	(1)																	

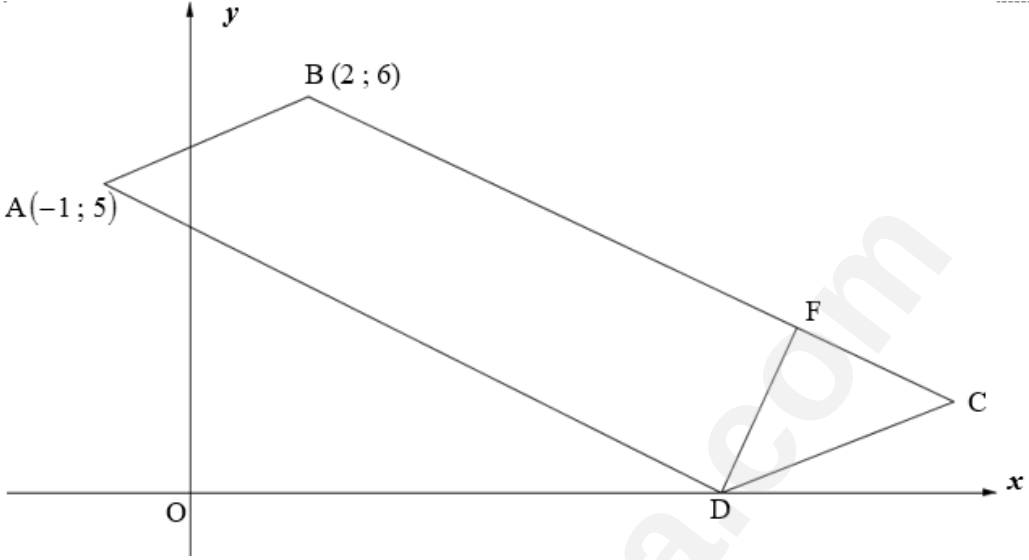
[12]

QUESTION/VRAAG 2

2.1	140 learners / leerders	✓ answer / antwoord	(1)
2.2	$60 < x \leq 70$	✓ answer / antwoord	(1)
2.3	Marks for Mathematics Test / Punte vir Wiskunde-toets  ✓ grounding / anker ✓ cumulative frequency / kumulatiewe frekwensie ✓ plotting against the upper limit / afsteek teen die boonste limiet ✓ shape / vorm		(4)
2.4	$Q_3 - Q_1 = 72 - 55$ $= 17$	✓ Q_3 & Q_1 ✓ IQR	(2)

[8]

QUESTION/VRAAG 3

			
3.1	$BC : x + 2y = 14$ $y = -\frac{1}{2}x + 7$ $m_{BC} = m_{AD} = -\frac{1}{2}$ $y - 5 = -\frac{1}{2}(x + 1)$ $y = -\frac{1}{2}x + \frac{9}{2}$	$\checkmark m_{AD} = -\frac{1}{2}$ $\checkmark$ subst. m and A into correct formula / verv. van m en A in korrekte formule $\checkmark y = -\frac{1}{2}x + \frac{9}{2}$	(3)
3.2	$-\frac{1}{2}x + \frac{9}{2} = 0$ $x = 9$ OR/OF $D(9; 0)$ $m_{AD} = m_{BC}$ $\frac{0 - 5}{x + 1} = -\frac{1}{2}$ $-10 = -x - 1$ $x = 9$ $D(9; 0)$	$\checkmark y = 0$ $\checkmark x = 9$ $\checkmark y = 0$ $\checkmark x = 9$	(2)

3.3	$m_{FD} = \frac{2-0}{10-9}$ $= 2$ $m_{BC} \times m_{FD} = 2 \times -\frac{1}{2}$ $= -1$ $FD \perp BC \quad [m_{BC} \times m_{FD} = -1]$	✓ correct subst. / <i>korrekte verv.</i> ✓ $m_{FD} = 2$ ✓ $m_{BC} \times m_{FD} = -1$	(3)
3.4	$AD = \sqrt{(-1-9)^2 + (5-0)^2}$ $= \sqrt{125}$ $= 5\sqrt{5}$	✓ subst. into correct formula / <i>verv. in korrekte formule</i> ✓ $AD = 5\sqrt{5}$	(2)
3.5	$FD = \sqrt{(9-10)^2 + (0-2)^2}$ $= \sqrt{5}$ $A \text{ of } ABCD = b \times h$ $= 5\sqrt{5} \times \sqrt{5}$ $= 25$	✓ subst. into correct formula / <i>verv. in korrekte formule</i> ✓ $FD = \sqrt{5}$ ✓ subst into correct formula / <i>verv. in korrekte formule</i> ✓ answer / <i>antwoord</i>	(4)
3.6	$m_{AB} = m_{DC}$ $= \frac{6-5}{2-(-1)}$ $= \frac{1}{3}$ $\therefore \text{Inclination of / Helling van } DC = 18,43^\circ$ $\text{Inclination of / Helling van } AD = 180^\circ - \tan^{-1}\left(\frac{1}{2}\right)$ $= 153,43^\circ$ $\hat{ADC} = 153,43^\circ - 18,43^\circ$ $= 135^\circ$ $\therefore \hat{ABC} = 135^\circ \quad [\text{opp } \angle \text{ s of a parm. / teenoorst. } \angle \text{ e van } \parallel^m]$	✓ subst. into correct formula / <i>verv. in korrekte formule</i> ✓ $m_{AB} = \frac{1}{3}$ ✓ $\theta_{DC} = 18,43^\circ$ ✓ $\theta_{AD} = 153,43^\circ$ ✓ $\hat{ADC} = 135^\circ$ ✓ $\hat{ABC} = 135^\circ$	(6)

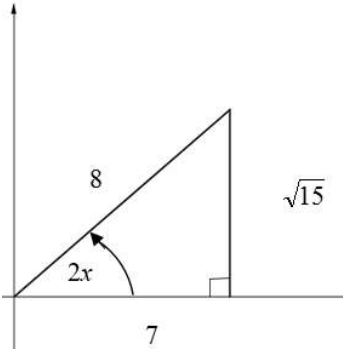
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QUESTION/VRAAG 4

4.1	$x_s = \frac{0+4}{2} = 2$ $y_s = \frac{0-6}{2} = -3$ $S(2; -3)$ Answer only full marks Slegs antwoord volpunte	✓ subst. into correct formula verv. in korrekte formule ✓ both coordinates / beide koördinate	(2)
4.2	$SP = \sqrt{(2-4)^2 + (-3+6)^2}$ $= \sqrt{13}$	✓ subst. into correct formula verv. in korrekte formule ✓ answer / antwoord	(2)
4.3	$(x-2)^2 + (y+3)^2 = 13$	✓ correct subst. centre / korrekte verv. middelpunt ✓ $r^2 = 13$.	(2)
4.4	Tangent $\perp$ radius / Raaklyn $\perp$ radius	✓ answer / antwoord	(1)
4.5	$m_{\text{rad}} = \frac{-3+6}{2-4} = -\frac{3}{2}$ $m_{\text{tan}} = \frac{2}{3} \quad [\text{radius} \perp \text{tan}] \quad [\text{radius} \perp \text{raaklyn}]$ $y+6 = \frac{2}{3}(x-4)$ $= \frac{2}{3}x - \frac{26}{3}$	✓ subst. into correct form. verv. in korrekte formule ✓ $m_{\text{rad}} = -\frac{3}{2}$ ✓ $m_{\text{tan}} = \frac{2}{3}$ ✓ subst. into correct form / verv. in korrekte formule	(4)

4.6	$(0-2)^2 + (y+3)^2 = 13$ $(y+3)^2 = 9$ $y+3 = \pm 3$ $y_T = -6$ $T(0; -6)$ OR / OF $(0-2)^2 + (y+3)^2 = 13$ $y^2 + 6y = 0$ $y(y+6) = 0$ $y_T = -6$ $T(0; -6)$ OR / OF Draw horizontal line $y = -3$ with M on OT <i>Trek horisontale lyn $y = -3$ met M op OT</i> OM = MT [$\perp$ from centre bisect the chord] <i>[$\perp$ vanaf middelpunt halveer koord]</i> $OM = 3$ $\therefore MT = 3$ $\therefore OT = 6$ $\therefore T(0; -6)$	$\checkmark x = 0$ $\checkmark (y+3)^2 = 9$ $\checkmark y+3 = \pm 3$ $\checkmark y = -6$ $\checkmark$ subst. $x = 0$ in eqn of circle <i>verv. $x = 0$ in verg. van sirkel</i> $\checkmark$ standard form / <i>standaardvorm</i> $\checkmark$ factors / <i>faktore</i> $\checkmark y_T = -6$ $\checkmark$ S/R $\checkmark$ length of / <i>lengte van</i> MT $\checkmark$ length of / <i>lengte van</i> OT $\checkmark$ answer / <i>antwoord</i>	(4)
4.7	<i>At U, $y = -\frac{26}{3}$</i> $\therefore TU = \frac{26}{3} - 6$ $= \frac{8}{3}$ $\frac{\text{Area } \triangle OTP}{\text{Area } \triangle PTU} = \frac{\frac{1}{2} \times 6 \times 4}{\frac{1}{2} \times \frac{8}{3} \times 4}$ $= \frac{9}{4}$	$\checkmark U\left(0; -\frac{26}{3}\right)$ $\checkmark$ length of / <i>lengte van</i> TU $\checkmark$ length of / <i>lengte van</i> OT $\checkmark$ correct subst / <i>korrekte verv.</i> $\checkmark$ answer / <i>antwoord</i>	(5)

QUESTION/VRAAG 5

5.1.1	$\sin 2x = \frac{\sqrt{15}}{8}$  $\cos 2x = 2\cos^2 x - 1$ $2\cos^2 x = \cos 2x + 1$ $\cos x = \sqrt{\frac{\cos 2x + 1}{2}}$ $= \sqrt{\frac{\frac{7}{8} + 1}{2}}$ $= \frac{\sqrt{15}}{4}$ OR / OF $\cos 2x = 2\cos^2 x - 1$ $\frac{7}{8} = 2\cos^2 x - 1$ $\frac{15}{8} = \cos^2 x$ $\therefore \cos x = \frac{\sqrt{15}}{4}$	✓ diagram / <i>diagram</i> ✓ identity of $\cos 2x$ <i>identiteit van $\cos 2x$</i> ✓ $\cos x$ subject of formula / <i>$\cos x$ onderwerp v/d formule</i> ✓ $\cos 2x = \frac{7}{8}$ ✓ answer / <i>antwoord</i> ✓ identity / <i>identiteit</i> ✓ $\cos 2x = \frac{7}{8}$ ✓ $\cos^2 x = \frac{15}{16}$ ✓ answer / <i>antwoord</i>	(5)
5.2	$\frac{\sin(180^\circ - \theta) \cdot \sin(540^\circ - \theta) \cdot \cos(\theta - 90^\circ)}{\tan(-\theta) \cdot \sin^2(360^\circ - \theta)}$ $= \frac{(\sin \theta)(\sin \theta)(\sin \theta)}{(-\tan \theta)(-\sin \theta)^2}$ $= \frac{\sin \theta}{\frac{\sin \theta}{\cos \theta}}$ $= -\cos \theta$	✓ $\sin \theta$ ✓ $\sin \theta$ ✓ $\sin \theta$ ✓ $(-\tan \theta)$ ✓ $(-\sin \theta)^2$ ✓ $\frac{\sin \theta}{\cos \theta} = \tan \theta$ ✓ $(-\cos \theta)$	(7)

5.3.1	$\text{LHS} = \frac{\sin 5x \cdot \cos 3x - \cos 5x \cdot \sin 3x}{\tan 2x} - 1$ $= \frac{\sin(5x - 3x)}{\tan 2x} - 1$ $= \frac{\sin 2x}{\sin 2x} - 1$ $= \frac{\cos 2x}{\cos 2x} - 1$ $= \cos 2x - 1$ $= 1 - 2 \sin^2 x - 1$ $= -2 \sin^2 x$ $\therefore \text{LHS} = \text{RHS}$	✓ $\sin(5x - 3x)$ ✓ $\tan 2x = \frac{\sin 2x}{\cos 2x}$ ✓ simplification / vereenvoudiging ✓ identity/identiteit $1 - 2 \sin^2 x$	(4)
5.3.2	$\tan 2x = 0$ $2x = 0^\circ \text{ or / of } 180^\circ$ $x = 0^\circ \text{ or / of } 90^\circ$ OR/OF $\tan 2x$ is undefined / is ongedefinieerd $2x = 90^\circ \text{ or/of } 270^\circ$ $x = 45^\circ \text{ or/of } 135^\circ$	✓ $\tan 2x = 0$ /undefined ongedefinieerd ✓ $0^\circ \text{ or/of } 180^\circ$ ✓ $90^\circ \text{ or/of } 270^\circ$ ✓ answers / antwoorde	(4)

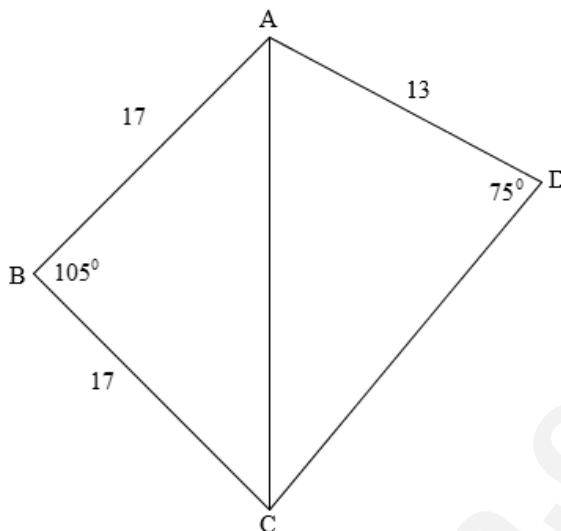
[20]

QUESTION/VRAAG 6

6.1		f ✓ both intercepts / beide afsnitte ✓ asymptote / asimptote ✓ shape / vorm g ✓ intercepts / afsnitte ✓ min & max values / waardes ✓ shape / vorm	(6)
6.2	Period(e) of/van $f\left(\frac{1}{2}x\right) = \frac{180^\circ}{\frac{1}{2}}$ $= 360^\circ$ Answer only full marks Slegs antwoord volpunte	✓ $\frac{180^\circ}{\frac{1}{2}}$ ✓ 360°	(2)
6.3	$0^\circ < x < 30^\circ$	✓ critical values / kritiese waardes ✓ notation / notasie	(2)
6.4	$x = 170^\circ$	✓ answer / antwoord	(1)

[11]

QUESTION/VRAAG 7

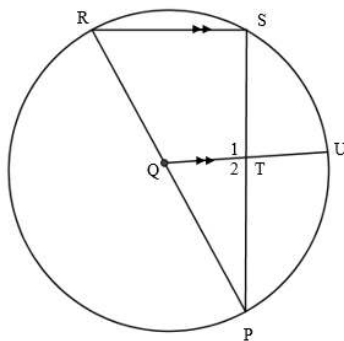


7.1	$\text{Area } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC \cdot \sin \hat{B}$ $= \frac{1}{2} \times 17 \times 17 \times \sin 105^\circ$ $= 139,58$	✓ Area rule formula / Oppervlakte reël formule ✓ correct subst. / korrekte verv. ✓ answer / antwoord	(3)
7.2	$AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos \hat{B}$ $= 17^2 + 17^2 - 2 \cdot 17 \cdot 17 \cdot \cos 105^\circ$ $= 727,5974081$ $\therefore AC = 26,97$	✓ cosine rule formula cosinus-reël formule ✓ correct subst into cosine rule korrekte verv. in cosinus-reël ✓ $AC = 26,97$	(3)
7.3	$\frac{\sin \hat{ACD}}{AD} = \frac{\sin \hat{D}}{AC}$ $\frac{\sin \hat{ACD}}{13} = \frac{\sin 75^\circ}{26,97}$ $\sin \hat{ACD} = 13 \times \frac{\sin 75^\circ}{26,97}$ $= 0,465592721$ $\hat{ACD} = 27,75^\circ$	✓ sine rule formula sinus-reël formule ✓ correct subst. into sine rule korrekte verv. in sinus-reël ✓ answer / antwoord	(3)
7.4	Converse opp $\angle$ s of a cyclic quad Omgekeerde: $\angle$ e van koordevierhoek OR / OF int. opp. $\angle$ s of a quad supp oorst. binne $\angle$ e van koordevierhoek suppl	✓ reason / rede OR/OF ✓ reason / rede	(1)

[10]

QUESTION/VRAAG 8

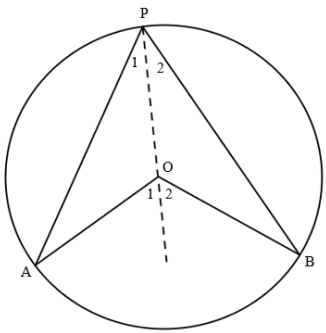
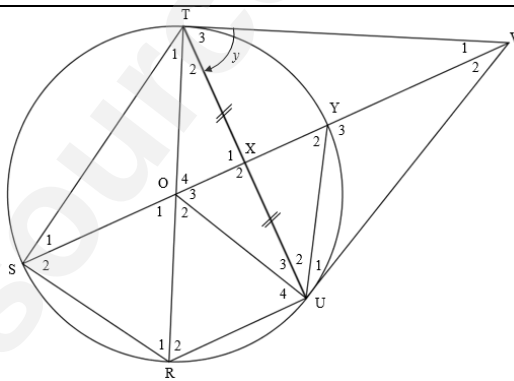
8.1



8.1.1	$\hat{S} = 90^\circ$ [$\angle$ in semi – circle / $\angle$ in semi - sirkel]	✓ S ✓ R	(2)
8.1.2	$\hat{T}_2 = \hat{S} = 90^\circ$ [corresp. $\angle$ s / ooreenk. $\angle$ e, $RS \parallel QU$] $\therefore$ T is the midpt of SP [line from centre $\perp$ to chord] <i>T is die midpt van SP [lyn vanaf middelpunt $\perp$ op koord]</i> $QP = 10$ and/en $TP = 8$ $QT^2 = (10)^2 - (8)^2$ [Pyth. Theorem/Stelling] $\therefore QT = 6$ $QU = QP = 10$ [radii] $\therefore TU = 4$	✓ S/R ✓ S/R ✓ subst. into Pyth. verv. in Pyth. ✓ QT ✓ S/R ✓ TU	(6)
8.2			
8.2.1	$\hat{B}_1 = 30^\circ$ [tan chord theorem] / [raaklyn koord stelling]	✓ S ✓ R	(2)
8.2.2	$\hat{P}_2 = 30^\circ$ [alt $\angle$ s, $BS \parallel PQ$] / [verw. $\angle$ e, $BS \square PQ$]	✓ S ✓ R	(2)
8.2.3	$\hat{R} = 150^\circ$ [opp. $\angle$ s of a cyclic quad] / [oorst. $\angle$ e van 'n koordevierhoek]	✓ S ✓ R	(2)
8.2.4	$\hat{Q}_2 = \hat{S}_3$ [$\angle$ s opp = sides] / [$\angle$ e teenoor = sye] $\hat{Q}_2 = \frac{180^\circ - 150^\circ}{2}$ [sum of $\angle$ s in a Δ] / [som van $\angle$ e in 'n Δ] $= 15^\circ$	✓ S/R ✓ S ✓ answer/antwoord	(3)

[17]

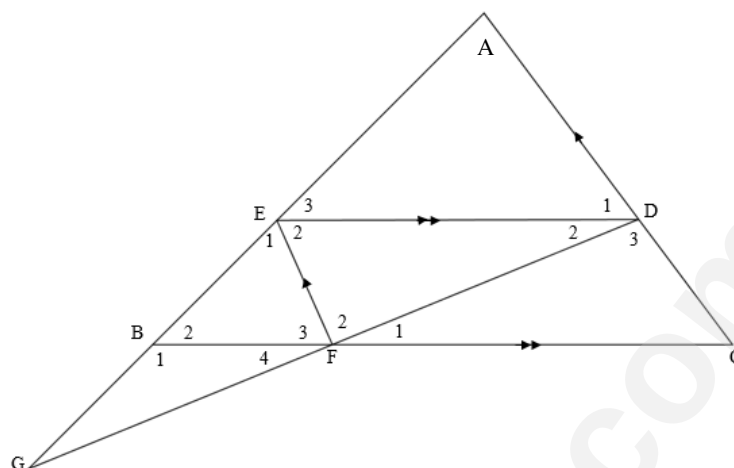
QUESTION/VRAAG 9

				
9.1	$\hat{O}_1 = \hat{P}_1 + A$ and $/ en \hat{O}_2 = \hat{P}_2 + \hat{B}$ [ext $\angle$ of a Δ] / [buite $\angle$ van 'n Δ] $AO = OP = OB$ [radii] / [radiusse] $\hat{P}_1 = \hat{A}$ and $/ en \hat{P}_2 = \hat{B}$ [$\angle$ s opp = sides] / [$\angle$ e teenoor = sye] $\therefore \hat{O}_1 = 2\hat{P}_1$ and $/ en \hat{O}_2 = 2\hat{P}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{P}_1 + 2\hat{P}_2$ $\hat{AOB} = 2(\hat{P}_1 + \hat{P}_2)$ $= 2\hat{APB}$	$\checkmark$ S/R $\checkmark$ S/R $\checkmark$ S/R $\checkmark$ S		(4)
9.2				
9.2.1	$\hat{R}\hat{U}\hat{T} = 90^\circ$ [$\angle$ in a semi-circle] / [$\angle$ in 'n semi-sirkel] $\hat{X}_1 = 90^\circ$ [line from centre to midpoint] / [lyn vanaf middelpunt na middelpunt] $\therefore RU \parallel SY$ [corresp. $\angle$ s =] / [ooreenk. $\angle$ e =] OR/OF $\hat{R}\hat{U}\hat{T} = 90^\circ$ [$\angle$ in a semi-circle] / [$\angle$ in 'n halwe sirkel] $\hat{X}_2 = 90^\circ$ [line from centre to the midpoint] / [lyn vanaf die middelpunt na die middelpunt] $\therefore \hat{R}\hat{U}\hat{T} + \hat{X}_2 = 90^\circ + 90^\circ = 180^\circ$ $\therefore RU \parallel SY$ [co-int $\angle$ s suppl.] / [ko-binne $\angle$ e suppl.]	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ R		(5)

9.2.2	$\hat{R}_2 = y$ [tan chord theorem] / [raaklyn koord stelling] $= \hat{O}_1$ [alt $\angle$ s, $RU \parallel SY$] / [verw. $\angle$ e, $RU \square SY$] $\therefore \hat{T}_1 = \frac{1}{2} \hat{O}_1$ [$\angle$ at centre = twice $\angle$ at circumf.] [middelpunts $\angle$ = tweemaal omtrekshoek] $= \frac{1}{2} y$	$\checkmark S$ $\checkmark R$ $\checkmark S$ $\checkmark R$ $\checkmark R$	(5)
9.2.3	$\hat{O}_4 = y = \hat{O}_1$ [vert opp. $\angle$ s] / [reghoort. $\angle$ e] $\hat{T}\hat{U}\hat{V} = y$ [tan from same point = in length] [raaklyne vanaf dieselfde punt is = in lengte] $\therefore$ TOUV is a cyclic quad [converse same segment] TOUV is 'n koordevierhoek [omgekeerde dieselfde segment] OR/OF $\hat{V}\hat{T}\hat{O} = 90^\circ$ [tan $\perp$ radius] / [raaklyn $\perp$ radius] $\hat{V}\hat{U}\hat{O} = 90^\circ$ [tan $\perp$ radius] / [raaklyn $\perp$ radius] $\therefore \hat{V}\hat{T}\hat{O} + \hat{V}\hat{U}\hat{O} = 180^\circ$ $\therefore$ TOUV is a cyclic quad [converse opp $\angle$ s of cyclic quad. supp] TOUV is 'n koordevierhoek [omgekeerde teenoorst. $\angle$ e van koordevierhoek]	$\checkmark S$ $\checkmark R$ $\checkmark S$ $\checkmark R$ $\checkmark R$ $\checkmark S$ $\checkmark R$ $\checkmark R$	(5)

[19]

QUESTION/VRAAG 10



10.1	$\frac{BA}{EA} = \frac{BC}{FC} \quad [\text{prop theorem; } EF \parallel AC] / [Ewerdigh. stelling; EF \parallel AC]$ $= \frac{CA}{DA} \quad [\text{prop theorem; } ED \parallel BC] / [Ewerdigh. stelling; ED \parallel BC]$ $\therefore \frac{BC}{FC} = \frac{CA}{DA}$	✓S ✓R ✓S ✓R	(4)
10.2	$\hat{B}_2 = \hat{E}_3 \quad [\text{corresp } \angle s, \text{ooreenk. } \angle e: ED \parallel BC]$ $\hat{E}_1 = \hat{A} \quad [\text{corresp } \angle s, \text{ooreenk. } \angle e: EF \parallel AC]$ $\therefore \hat{F}_3 = \hat{D}_1 \quad [\text{sum of } \angle s \text{ of } \Delta / \text{som van } \angle e \text{ van } \Delta]$ $\therefore \triangle BFE \parallel \triangle EDA \quad [\angle \angle \angle]$	✓S/R ✓S/R ✓R	(3)
10.3.1	$\frac{AD}{FE} = \frac{ED}{BF} \quad [\triangle BFE \parallel \triangle EDA]$ $\frac{AD}{2} = \frac{10}{3,5}$ $AD = \frac{40}{7} = 5,71$	✓S ✓R ✓subst verv. ✓AD	(4)
10.3.2	$DC = EF = 2 \quad [\text{opp. sides of a parm}] / [\text{oorst. sye van 'n } ^m]$	✓S ✓R	(2)

[13]

TOTAL/TOTAAL: 150