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GAUTENG DEPARTMENT OF EDUCATION



JOHANNESBURG NORTH DISTRICT

2021 GRADE 12 CONTROL TEST

MATHEMATICS TERM1

MARKS : 100
TIME : 2 hours

This question paper consist of 17 pages

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of **9** questions.
2. Answer **ALL** the questions.
3. Clearly show **ALL** calculations, diagrams, graphs, etc. which was used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. Use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. Where necessary, answers should be rounded off to **TWO** decimal places, unless stated otherwise.
7. Diagrams are **NOT** necessarily drawn to scale.
8. **ANSWER Question 7 on Annexure 7.1 - 7.2.2**
9. **ANSWER Question 8 on Annexure 8.1 - 8.1.3**
10. **ANSWER Question 9 on Annexure 9.1 - 9.2**
11. **Tear off page 12 till page 17 . AND SUBMIT theses pages with your answer scripts .**
12. An **information sheet** is on page 11 of the question paper.
13. Number the questions correctly according to the numbering used in the question paper.
14. Write neatly and legibly.

QUESTION 11.1 Solve for x :

1.1.1 $(x - 5)(x + 1) = 0$ (2)

1.1.2 $2x^2 - 11x + 7 = 0$ (correct to two decimal places) (3)

1.1.3 $x - 5x^{\frac{1}{2}} = -6$ (4)

1.2 Calculate a and b if $\sqrt{\frac{5^{2014} - 5^{2012}}{6}} = a(5^b)$ and a is not a multiple of 5. (4)

1.3 Solve for x and y :

$1 = 3y - x$ and $y^2 + 2xy = 3x^2 - 7$ (7)

[20]

QUESTION 2Given the arithmetic series: $3 + 10 + 17 + \dots + 150$.

2.1 Write down the fourth term in the series. (1)

2.2 Determine the general term of the series. (2)

2.3 Express the series in sigma notation. (1)

[4]

QUESTION 33.1 Consider the progression : $3; \frac{1}{2}; 3; \frac{4}{10}; 3; \frac{16}{50}; \dots$

3.1.1 Write down the next TWO terms of the progression. (1)

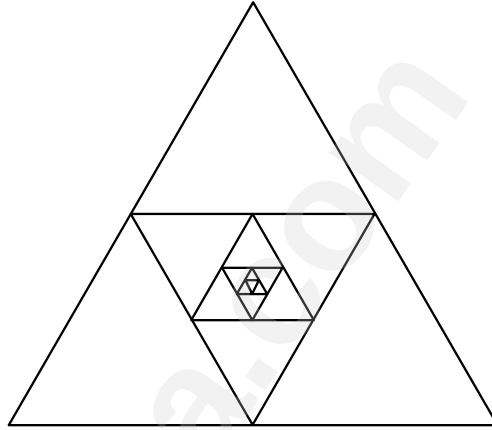
3.1.2 Calculate the sum of the first thirty-five terms of the progression. (5)

3.2 Calculate : $\sum_{n=3}^{\infty} 5(3)^{1-n}$ (4)

[10]

QUESTION 4

In the diagram below, the 1st (outer) triangle is an equilateral triangle with sides of 8cm. A 2nd triangle is drawn within this triangle by joining the midpoints of the sides of the 1st triangle. This process is continued without end.



4.1 What is the perimeter of the 4th triangle? (2)

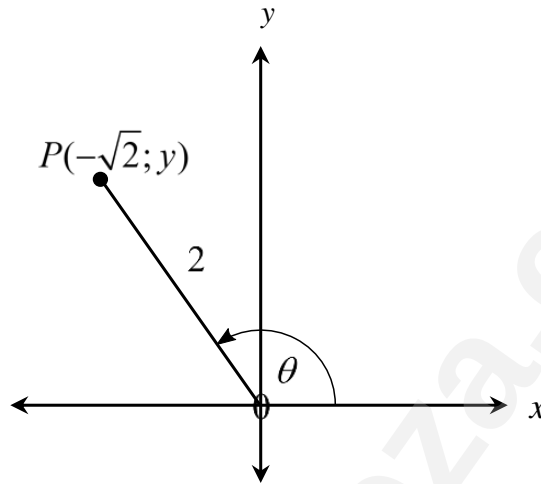
4.2 What is the perimeter of the n^{th} triangle? (3)

[5]

QUESTION 5

5.1 In the sketch below, P is a point on the Cartesian plane, with $P\hat{O}X = \theta$.

Use the sketch to determine the following:



5.1.1 The value of y . (2)

5.1.2 The value of $\frac{2\sin\theta\cos\theta}{\cos^2\theta - 1}$ (5)

5.2 Simplify the following, WITHOUT USING A CALCULATOR:

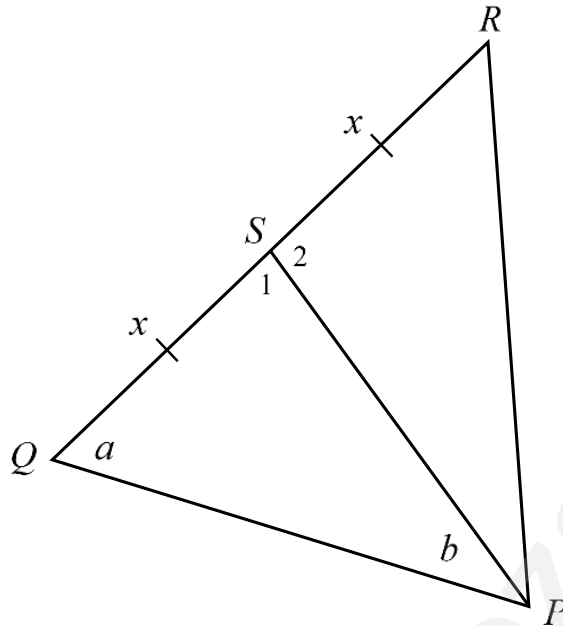
$$\frac{\cos(180^\circ + \theta) \cdot \tan(720^\circ - \theta) \cdot \sin^2(90^\circ - \theta)}{\sin(180^\circ - \theta)} + \sin^2\theta \quad (7)$$

5.3 If $6\sin^2\theta - 4\cos^2\theta = -5\sin\theta \cdot \cos\theta$, determine the general solution for θ . (8)

[22]

QUESTION 6

In the sketch below, PS is the median of $\triangle PQR$, and thus $QS = SR = x$. $\hat{Q} = a$ and $\hat{QPS} = b$



6.1 Show that $PS = \frac{x \sin a}{\sin b}$ (2)

6.2 Express the size of S_2 , in terms of a and b , without reasons. (1)

6.3 Hence, show that: Area of $\triangle PSR = \frac{x^2 \sin a \times \sin(a + b)}{2 \sin b}$ (3)

6.4 Determine the area of $\triangle PSR$, rounded to two decimal places, if $x = 14,2 \text{ cm}$, $a = 34^\circ$ and $b = 41^\circ$. (3)

[9]

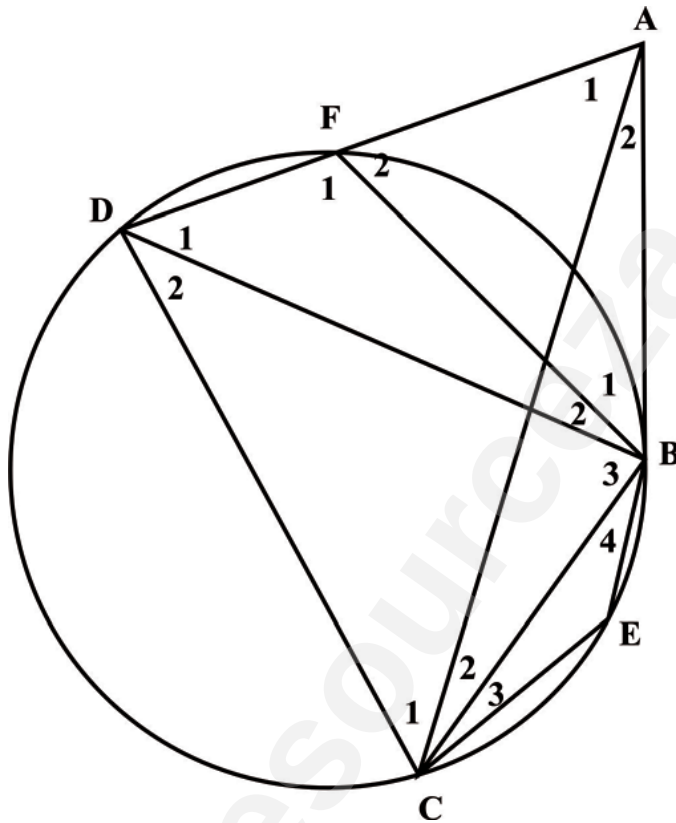
Give reasons for your statements and calculations in QUESTIONS 7, 8 and 9

Use the Annexure's provided to answer QUESTIONS 7, 8 and 9

QUESTION 7

7.1 In the diagram below, AB is a tangent to the circle passing through B, E, C and D

AD cuts the circle at F. AC is drawn.



Give reasons for the following statements:

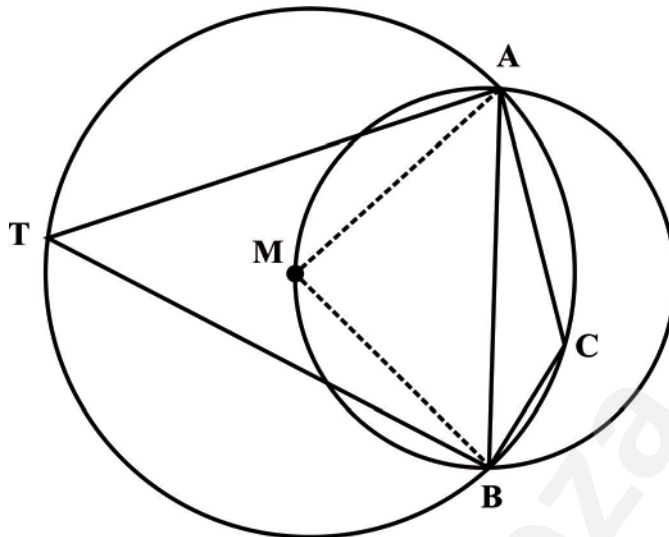
(5)

STATEMENT	REASONS
$\hat{C}_1 + \hat{C}_2 = \hat{F}_2$	
$\hat{D}_2 + \hat{E} = 180^\circ$	
$\hat{B}_1 = \hat{D}_1$	
$\hat{B}_2 + \hat{B}_3 + \hat{D}_1 + \hat{D}_2 = 180^\circ$	
$\hat{B}_2 + \hat{B}_1 = \hat{C}_1 + \hat{C}_2$	

7.2 In the diagram below, circle centre M intersects a second smaller circle at A and B.

A, C, B and T are points on circle M.

AB is the diameter of the smaller circle.



7.2.1 Determine the size of $\hat{C}$. (6)

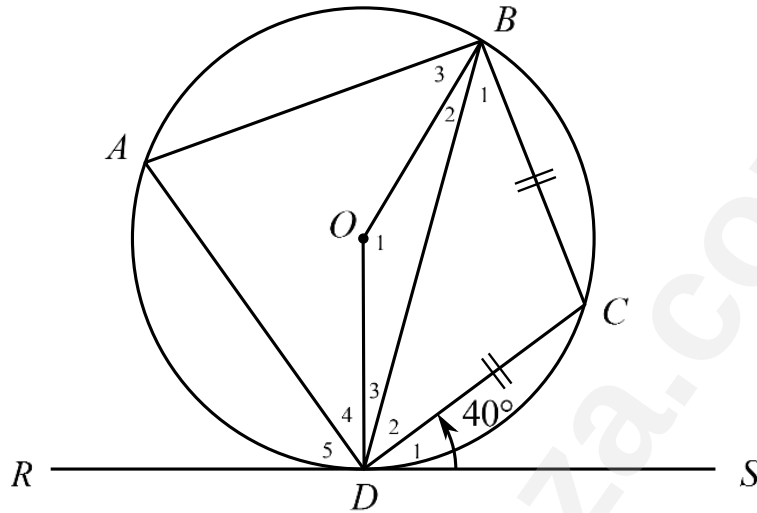
7.2.2 Explain why AMBC is not a cyclic quadrilateral. (1)

[12]

QUESTION 8

In the figure below, RDS is a tangent to circle O at D. $BC = DC$, and $\angle CDS = 40^\circ$.

Thus, calculate the size of the following angles, with reasons.



8.1 $\angle BDC$ (2)

8.2 $\angle C$ (2)

8.3 $\angle A$ (2)

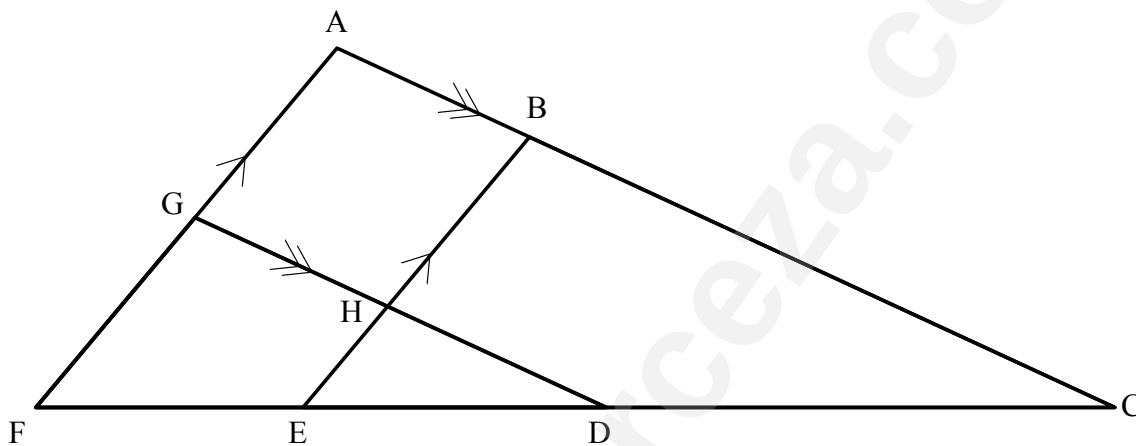
8.4 $\angle O_1$ (1)

[7]

QUESTION 9

The diagram below is the top view design of a new railway system. There are eight stations being built and these are labelled with letters from A- H. You have been asked to do some calculations for the railway company. As the engineer you know that:

- $AF \parallel BE$ and $AC \parallel GD$.
- $\frac{AB}{BC} = \frac{4}{7}$ and $\frac{AG}{AF} = \frac{9}{17}$.



9.1 Calculate

9.1.1 $\frac{FE}{FC}$. (3)

9.1.2 $\frac{CD}{DF}$. (2)

9.2 If the straight line distance of the track from F to C is 374 kilometres and it takes 50 hours to build one kilometre of the track, determine the number of hours it will take to build the section from E to D. (6)

[11]

TOTAL 100 MARKS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC :

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area} \Delta ABC = \frac{1}{2}ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta ; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

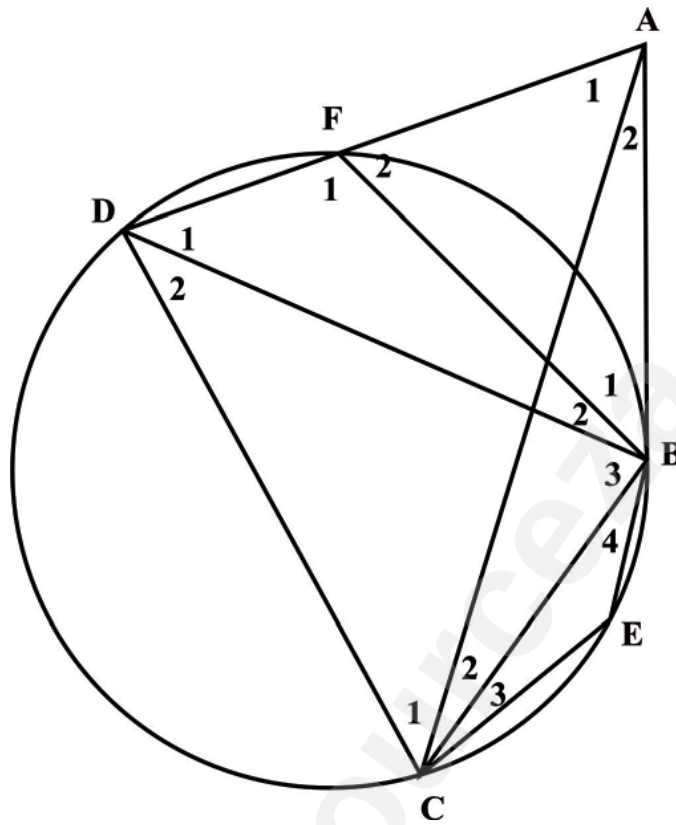
Name: _____

Class: _____

ANNEXURE 7.1 - 7.2.2

QUESTION 7

7.1

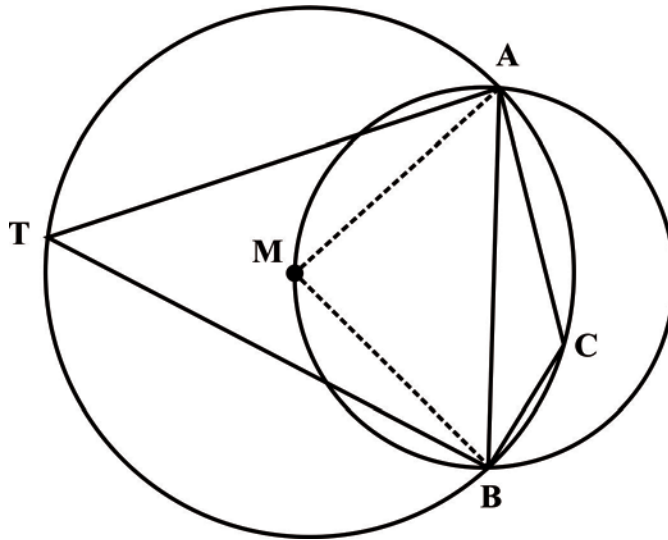


Give reasons for the following statements:

(5)

STATEMENT	REASONS
$\hat{C}_1 + \hat{C}_2 = \hat{F}_2$	
$\hat{D}_2 + \hat{E} = 180^\circ$	
$\hat{B}_1 = \hat{D}_1$	
$\hat{B}_2 + \hat{B}_3 + \hat{D}_1 + \hat{D}_2 = 180^\circ$	
$\hat{B}_2 + \hat{B}_1 = \hat{C}_1 + \hat{C}_2$	

7.2

7.2.1 Determine the size of $\hat{C}$.

(6)

7.2.2 Explain why AMBC is not a cyclic quadrilateral.

(1)

[12]

8.3 $\hat{A}$

(2)

8.4 $\hat{O}_1$

(1)

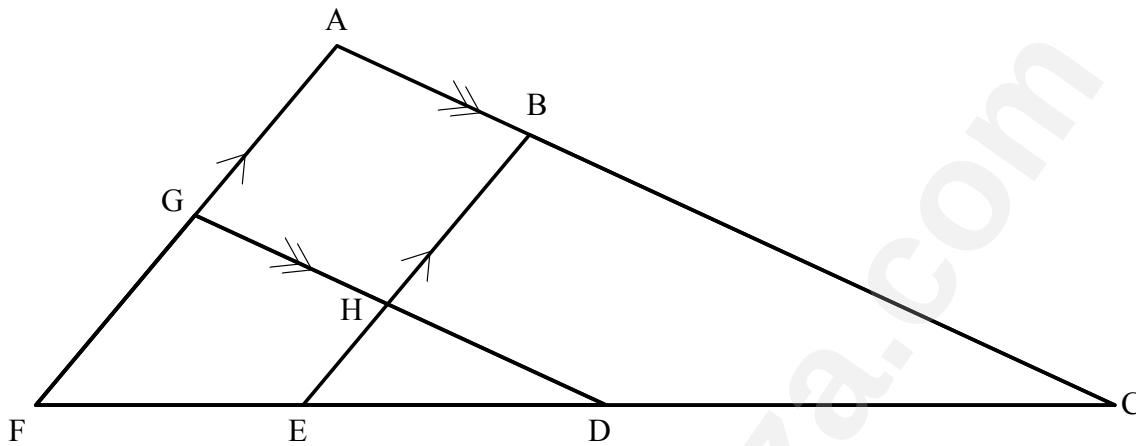
[7]

Name: _____

Class: _____

ANNEXURE 9.1 - 9.2

QUESTION 9



9.1 Calculate

9.1.1 $\frac{FE}{FC}$. (3)

9.1.2 $\frac{CD}{DF}$. (2)

[11]

GAUTENG DEPARTMENT OF EDUCATION



JOHANNESBURG NORTH DISTRICT

2021
GRADE 12
CONTROL TEST

**MATHEMATICS
TERM 1**

MARKING GUIDELINES

MARKS : 100
TIME : 2 hours

QUESTION 1		
1.1.1	$(x - 5)(x + 1) = 0$ $x = 5$ or $x = -1$	$\checkmark x = 5$ $\checkmark x = -1$ (2)
1.1.2	$2x^2 - 11x + 7 = 0$ $x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(2)(7)}}{2(2)}$ $x = 4,77$ or $x = 0,73$	$\checkmark$ Sub $\checkmark x = 4,77$ $\checkmark x = 0,73$ (3)
1.1.3	$x - 5x^{\frac{1}{2}} = -6$ $x - 5x^{\frac{1}{2}} + 6 = 0$ Let $k^2 = x$ and $k = x^{\frac{1}{2}}$ $\therefore k^2 - 5k + 6 = 0$ $(k - 3)(k - 2) = 0$ $k = 3$ or $k = 2$ But: $x^{\frac{1}{2}} = 3$ or $x^{\frac{1}{2}} = 2$ $(x^{\frac{1}{2}})^2 = (3)^2$ or $(x^{\frac{1}{2}})^2 = (2)^2$ $x = 9$ or $x = 4$	$\checkmark$ Standard form $\checkmark$ factors $\checkmark$ squaring $\checkmark$ x values (4)
1.2	$\sqrt{\frac{5^{2014} - 5^{2012}}{6}}$ $= \sqrt{\frac{5^{2012} \cdot 5^2 - 5^{2012}}{6}}$ $= \sqrt{\frac{5^{2012}(25 - 1)}{6}}$ $= \sqrt{4 \cdot 5^{2012}}$ $= 2(5^{1006})$ $\therefore a = 2.$ and $b = 1006$	$\checkmark \sqrt{\frac{5^{2012} \cdot 5^2 - 5^{2012}}{6}}$ $\checkmark \sqrt{4 \cdot 5^{2012}}$ $\checkmark a = 2.$ $\checkmark b = 1006$ (4)

1.3	$1 = 3y - x \dots\dots\dots(1)$ $y^2 + 2xy = 3x^2 - 7 \dots\dots\dots(2)$ $x = 3y - 1 \dots\dots\dots(3)$ Sub (3) into (2) $y^2 + 2y(3y - 1) = 3(3y - 1)^2 - 7$ $y^2 + 6y^2 - 2y = 27y^2 - 18y + 3 - 7$ $0 = 20y^2 - 16y - 4$ $0 = 5y^2 - 4y - 1$ $0 = (y - 1)(5y + 1)$ $y = 1 \quad \text{or} \quad y = \frac{-1}{5}$ Sub y-values into (3) $x = 3(1) - 1 \quad \text{or} \quad x = 3(\frac{-1}{5}) - 1$ $x = 2 \quad \text{or} \quad x = \frac{-8}{5}$	✓ Equation 3 ✓ Sub ✓ Simplification ✓ Standard form ✓ Factors ✓ y-values ✓ x-values (7)
20 MARKS		

QUESTION 2		
2.1	$T_n = 24$	✓ Ans (1)
2.2	$T_n = 3 + (n - 1)(7)$ $T_n = 7n - 4$	✓ Sub ✓ Ans (2)
2.3	$\sum_{k=1}^{22} (7k - 4)$	✓ Ans (1)
4 MARKS		

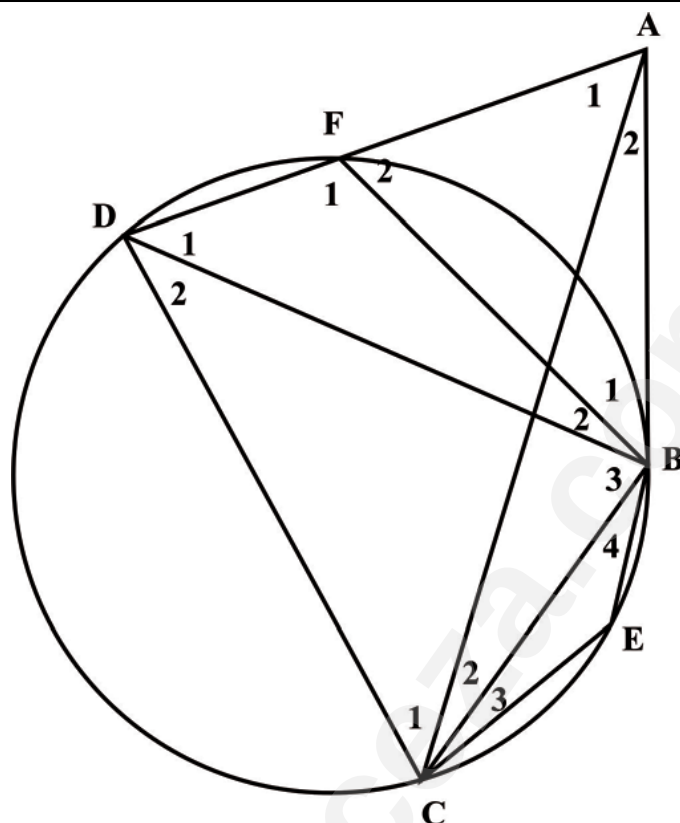
QUESTION 3		
3.1.1	$3; \frac{64}{250}$	✓ Ans (1)
3.1.2	$3 \times 18 = 54$ $S_{17} = \frac{\frac{1}{2}[(\frac{4}{5})^{17} - 1]}{(\frac{4}{5}) - 1}$ $= 2,44$ $\therefore S_{35} = 56,44$	✓ Odd terms ✓ Sub ✓ 2,44 ✓ ✓ $S_{35} = 56,44$ (5)
3.2	$T_1 = \frac{5}{9}$ $T_2 = \frac{5}{27}$ $\therefore r = \frac{1}{3}$ $S_{\infty} = \frac{\frac{5}{9}}{1 - \frac{1}{3}}$ $= \frac{5}{6}$ or 0,83	✓ a ✓ r ✓ Sub ✓ Ans (4)
10 MARKS		

QUESTION 4		
4.1	3cm	✓ ✓ Ans (2)
4.2	$24; 12; 6; 3$ $r = \frac{1}{2}$ $T_n = 24(\frac{1}{2})^{n-1}$	✓ Sequence ✓ ratio ✓ Ans (3)
5 MARKS		

QUESTION 5		
5.1.1	$y^2 = (2)^2 - (-\sqrt{2})^2$ $y = \sqrt{2}$	✓ method ✓ Answer (2)
5.1.2	$\frac{2\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)^2 - 1} = \frac{2\left(-\frac{1}{2}\right)}{\frac{1}{2} - 1} = 2$	✓✓✓ substitution ✓ sign ✓ answer (5)
5.2	$\frac{\cos(180^\circ + \theta) \cdot \tan(720^\circ - \theta) \cdot \sin^2(90^\circ - \theta)}{\sin(180^\circ - \theta)} + \sin^2 \theta$ $= \frac{-\cos \theta \times -\tan \theta \times \cos^2 \theta}{\sin \theta} + \sin^2 \theta$ $= \frac{\cos \theta \times \frac{\sin \theta}{\cos \theta} \times \cos^2 \theta}{\sin \theta} + \sin^2 \theta$ $= \cos^2 \theta + \sin^2 \theta = 1$	✓ $-\cos \theta$ ✓ $-\tan \theta$ ✓ $\cos^2 \theta$ ✓ $\sin \theta$ ✓ Identity: $\tan \theta$ ✓ $\cos^2 \theta + \sin^2 \theta$ ✓ $= 1$ (7)
5.3	$6\sin^2 \theta - 5\sin \theta \cos \theta - 4\cos^2 \theta = 0$ $(2\sin \theta - \cos \theta)(3\sin \theta + 4\cos \theta) = 0$ $\therefore 2\sin \theta = \cos \theta$ $\therefore \tan \theta = \frac{1}{2}$ Ref $\angle = 26,57^\circ$ $\therefore \theta = 26,57^\circ + n180^\circ$ OR: $\theta = 180^\circ + 26,57^\circ + n180^\circ$ $\therefore \theta = 206,57^\circ + n180^\circ$ AND: $3\sin \theta = -4\cos \theta$ $\therefore \tan \theta = -\frac{4}{3}$ Ref $\angle = 53,13^\circ$ $\therefore \theta = 180^\circ - 53,13^\circ + n180^\circ$ $\theta = 126,87^\circ + n180$ OR: $\theta = 360^\circ - 53,13^\circ + n180^\circ$ $\therefore \theta = 306,87^\circ + n180^\circ$	✓ $= 0$ ✓ factors ✓ $\tan \theta$ ✓ solution ✓ solution ✓ $\tan \theta$ ✓ solution ✓ solution (8)
22 MARKS		

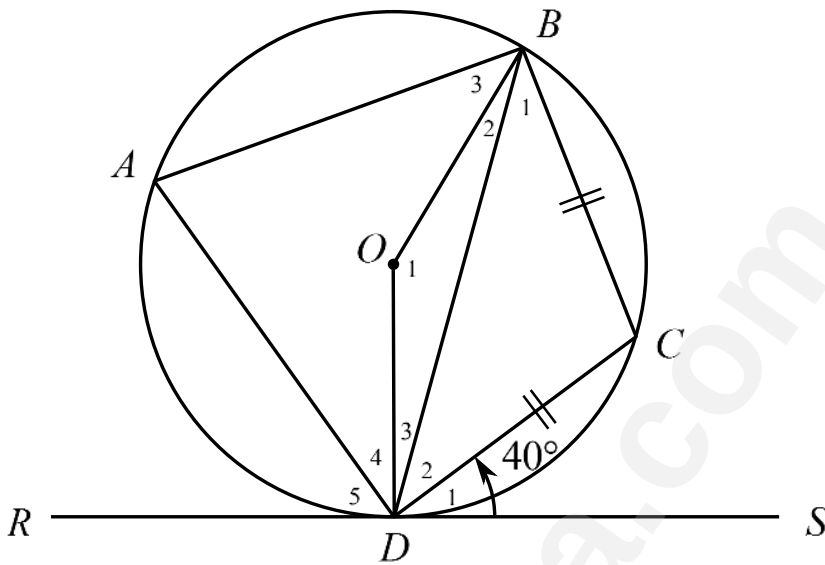
QUESTION 6		
6.1	$\frac{PS}{\sin a} = \frac{x}{\sin b}$ $\therefore PS = \frac{x \sin a}{\sin b}$	✓✓ method sine rule (2)
6.2	$S_2 = a + b$	✓ accuracy (1)
6.3	$Area = \frac{1}{2} PS \times SR \sin S_2$ $= \frac{1}{2} \left(\frac{x \sin a}{\sin b} \right) (x) (\sin(a + b))$ $= \frac{x^2 \sin a \cdot \sin(a + b)}{2 \sin b}$	✓ method: Area rule ✓✓ sub in (3)
6.4	$Area = \frac{(14,2)^2 \times \sin(34^\circ) \times \sin(34^\circ + 41^\circ)}{2 \sin(41^\circ)}$ $= 83,01 cm^2$	✓✓ sub into formula ✓ answer (3)
9 MARKS		

QUESTION 7



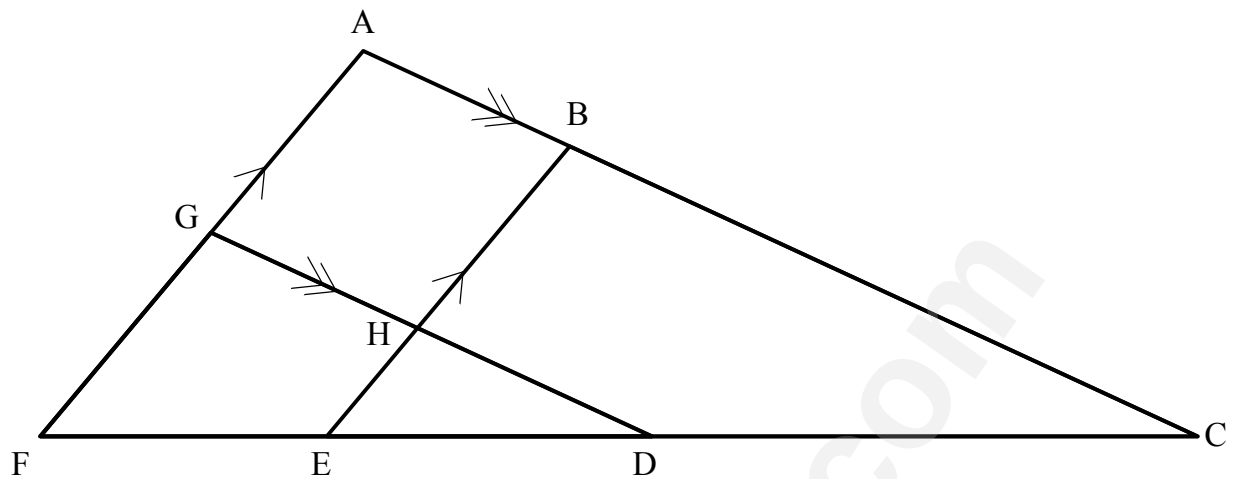
7.1	STATEMENT	REASONS
	$\hat{C}_1 + \hat{C}_2 = \hat{F}_2$	✓ [ext $\angle'$ of cyclic quad]
	$\hat{D}_2 + \hat{E} = 180^0$	✓ [opp $\angle'$ of cyclic quad]
	$\hat{B}_1 = \hat{D}_1$	✓ [tan chord]
	$\hat{B}_2 + \hat{B}_3 + \hat{D}_1 + \hat{D}_2 = 180^0$	✓ [opp $\angle'$ of cyclic quad]
	$\hat{B}_2 + \hat{B}_1 = \hat{C}_1 + \hat{C}_2$	✓ [tan chord]
		(5)
7.2.1	$\hat{A}\hat{M}\hat{B} = 90^0$ ($\angle'$ s in semi circle) $\hat{T} = 45^0$ ($\angle'$ at center = $2 \times \angle$ at circum) $\hat{C} = 135^0$ (opp $\angle'$ s of cyclic quad = 180)	✓ S ✓ R ✓ S ✓ R ✓ S ✓ R
7.2.2	$\hat{M} + \hat{C} \neq 180^0$; opp $\angle'$ s do not add up to 180°	✓ S & R
		(1)
	12 MARKS	

7 MARKS



8.1	$B_1 = 40^\circ$ (tan-chord theorem) $\therefore BDC = 40^\circ$ ($\angle$'s opp = sides)	$\checkmark$ S & R $\checkmark$ S & R (2)
8.2	$C = 100^\circ$ ($\angle$'s in $\triangle$)	$\checkmark$ S $\checkmark$ R (2)
8.3	$\therefore A = 80^\circ$ (opp $\angle$'s in a cyclic quad)	$\checkmark$ S $\checkmark$ R (2)
8.4	$O_1 = 160^\circ$ ($\angle$ at centre = $2\angle$ at circum)	$\checkmark$ S & R (1)

QUESTION 9

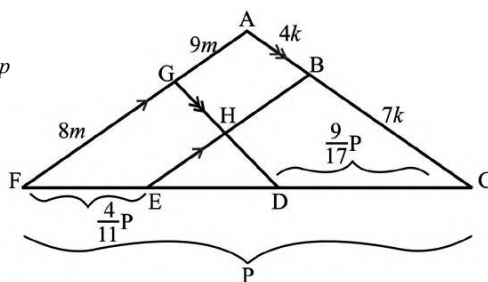


9.1.1	Let $AB = 4k$ and $BC = 7k$ $\therefore \frac{FE}{FC} = \frac{AB}{AC} = \frac{4}{11}$; (Proportionality theorem OR using theorem on diagram)	✓ S ✓ S ✓ R (3)
9.1.2	Let $AG = 9m$ and $AF = 17m$ $\frac{CD}{DF} = \frac{AG}{GF} = \frac{9}{8}$	✓ Reason ✓ $\frac{CD}{DF} = \frac{AG}{GF} = \frac{9}{8}$ (2)

9.2

$$\text{If } FC = p \text{ then } ED = p - \frac{9}{17}p - \frac{4}{11}p$$

$$ED = \frac{20}{187}p$$



The length of ED in kilometres is $\frac{20}{187} \times 374 \text{ km} = 40 \text{ kilometres}$.

It will take 2 000 hours to build the track from E to D.

OR

Alternate:

Let $FE = 4p$ and $EC = 7p$

$FD = 8m$ and $DC = 9m$

$$\therefore 11p = 374 \therefore p = 34$$

$$17m = 374 \therefore m = 22$$

$$\therefore DC = 374 - 4p - 9m$$

$$= 40 \text{ km}$$

$$\therefore 2\,000 \text{ hours}$$

OR

Alternate:

$$FE = \frac{4}{11}(374) = 136$$

$$CD = \frac{9}{17}(374) = 198$$

$$\therefore ED = 374 - 136 - 198$$

$$= 40 \text{ km}$$

$$\therefore 4 \text{ hours} \rightarrow 40 \times 50$$

$$= 2\,000 \text{ hours}$$

✓ ✓

$$\text{If } FC = p \text{ then } ED = p - \frac{9}{17}p - \frac{4}{11}p$$

✓ ✓

$$ED = \frac{20}{187}p$$

✓

The length of ED in kilometres is $\frac{20}{187} \times 374 \text{ km} = 40 \text{ kilometres}$.

✓

It will take 2 000 hours to build the track from E to D.

or

✓

Let $FE = 4p$ and $EC = 7p$

✓

$FD = 8m$ and $DC = 9m$

✓

$$\therefore 11p = 374 \therefore p = 34$$

✓

$$17m = 374 \therefore m = 22$$

✓

$$\therefore DC = 374 - 4p - 9m = 40 \text{ km}$$

$$\therefore 2\,000 \text{ hours}$$

or

$$FE = \frac{4}{11}(374) = 136$$

✓

✓

$$CD = \frac{9}{17}(374) = 198$$

✓ ✓

1 /

$$\therefore ED = 374 - 136 - 198 = 40 \text{ km}$$

✓ ✓

$$\therefore 4 \text{ hours} \rightarrow 40 \times 50 = 2\,000 \text{ hours}$$

(6)

11 MARKS

TOTAL MARKS 100