



GCSE Similarity

GCSE Specification

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Pack Ref	Description
171	Solve problems involving finding lengths in similar shapes.
172	Understand the effect of enlargement for perimeter, area and volume of shapes and solids. Know the relationships between linear, area and volume scale factors of mathematically similar shapes and solids
131	Convert between units of area
137	Convert between volume measures, including cubic centimetres and cubic metres

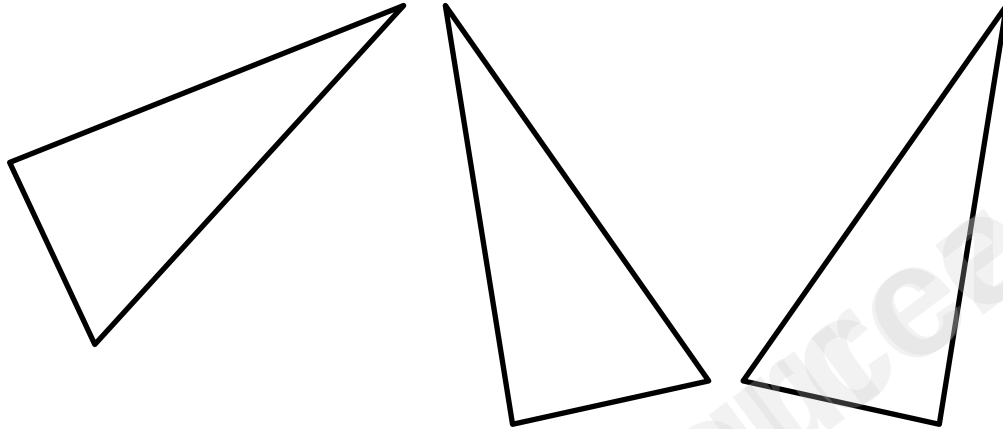
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Similarity vs Congruence

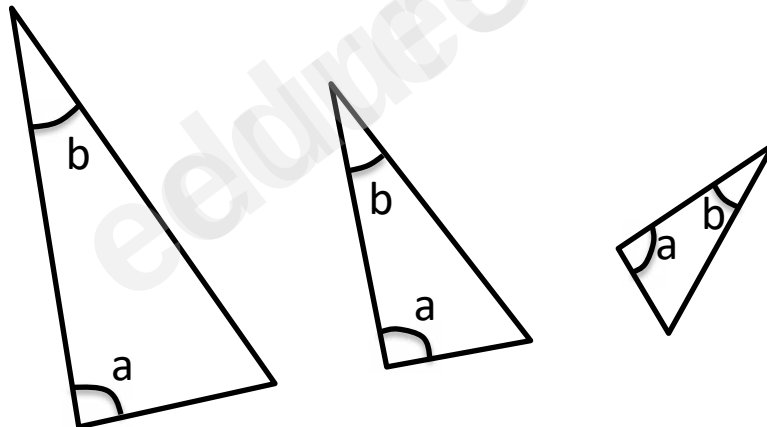
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Two shapes are congruent if:



Two shapes are similar if:

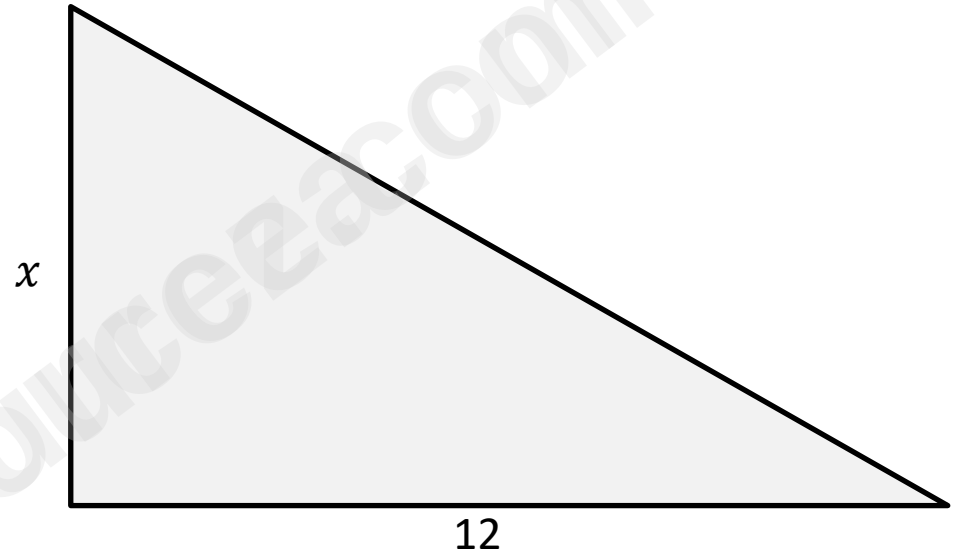
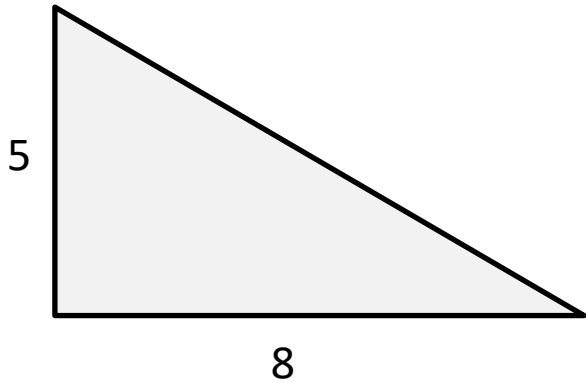


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Similarity

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The two shapes are similar. Determine the length x .



There's two ways we could solve this:

Scale factor
approach?

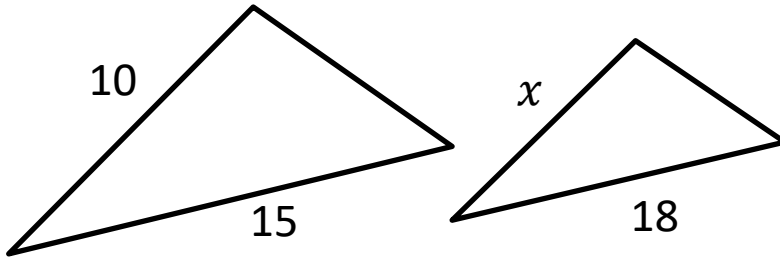
Two fractions approach ?

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Basic Examples

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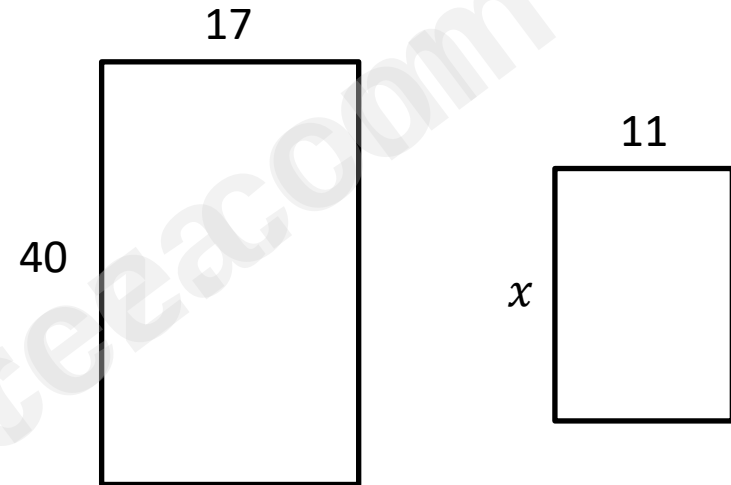
1



Try using 'scale factor' method:



2



Try using 'two fractions' method:



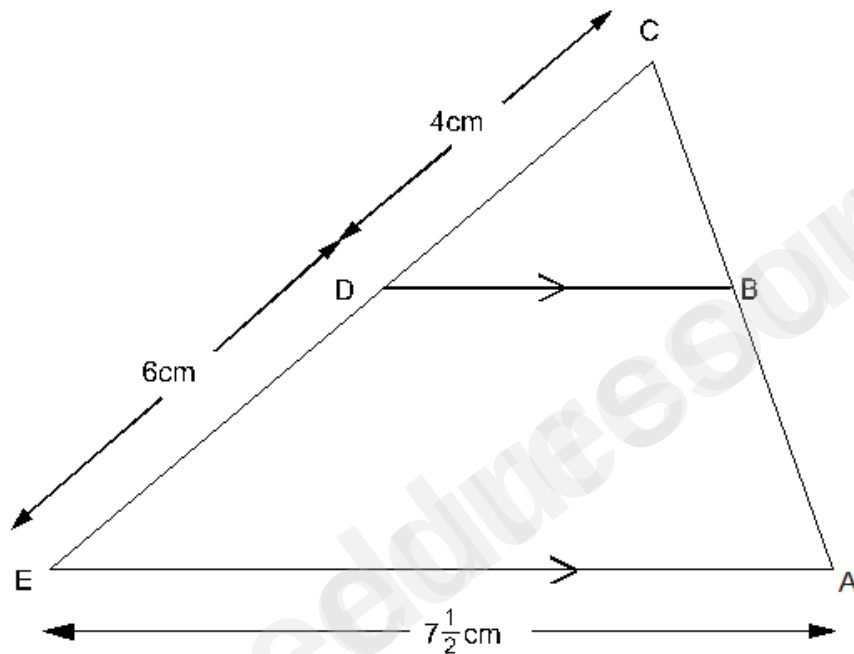
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Harder Problems

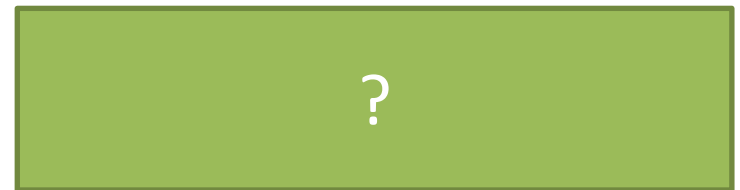
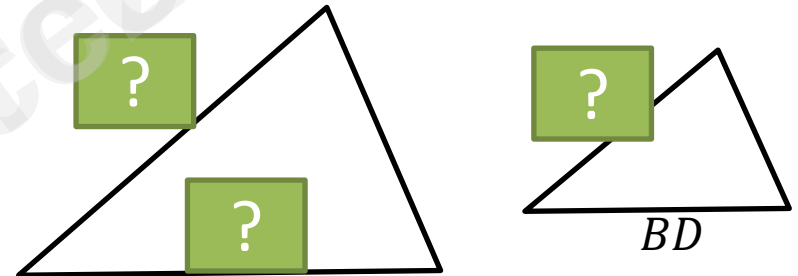
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3

In the diagram BCD is similar to triangle ACE. Work out the length of BD.



Bro Tip: For some students it may help to draw out the two similar triangles separately.



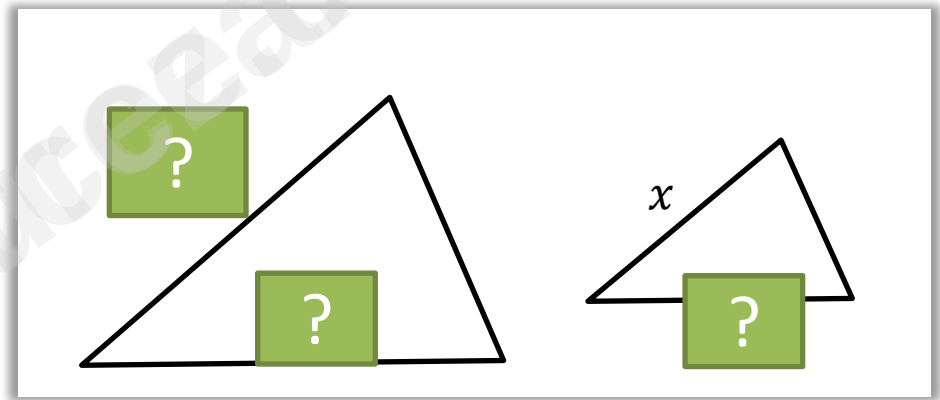
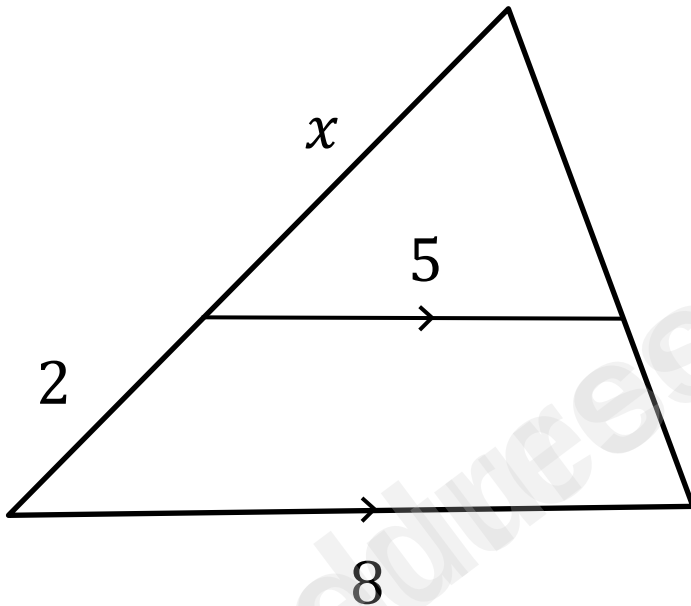
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Harder Problems

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4

In the diagram BCD is similar to triangle ACE. Work out the length of BD.

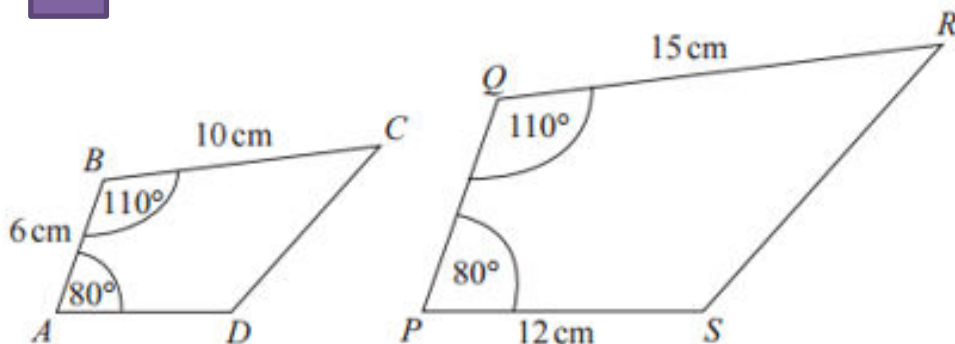


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Test Your Understanding

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1 [Nov 2008 4H Q22]

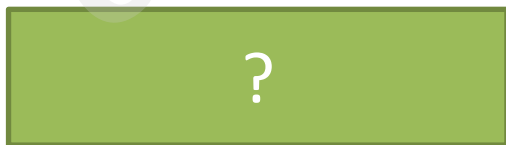


$ABCD$ and $PQRS$ are mathematically similar.

(a) Find the length of PQ .



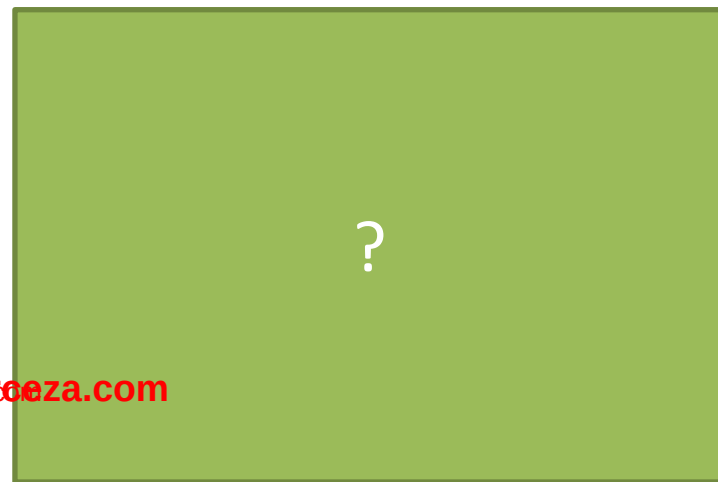
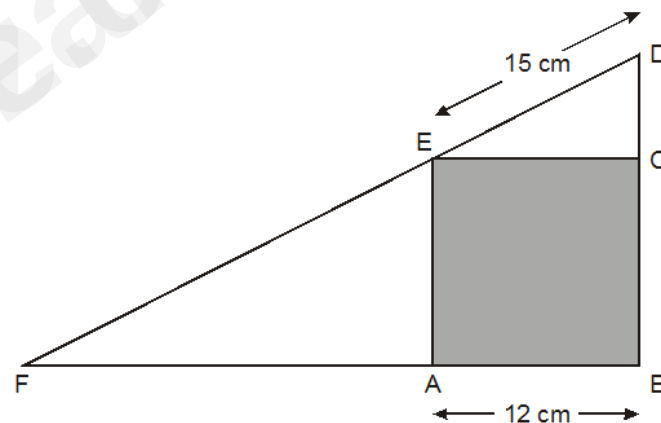
(b) Find the length of AD .



2

The diagram shows a square inside a triangle. DEF is a straight line. What is length EF ?

(Hint: you'll need to use Pythag at some point)



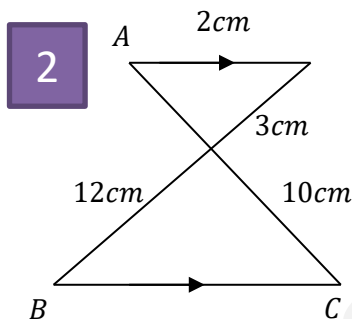
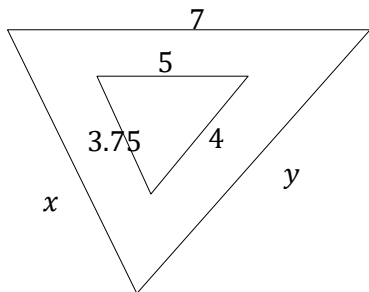
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Exercise 1

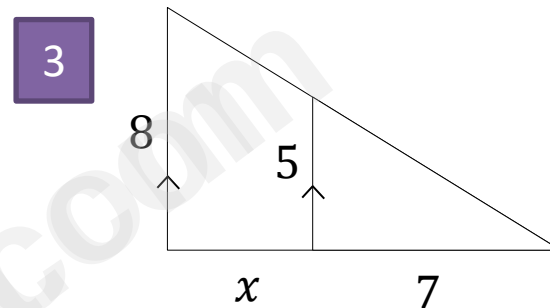
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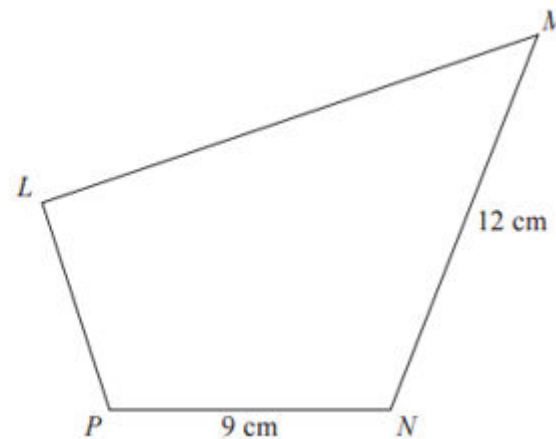
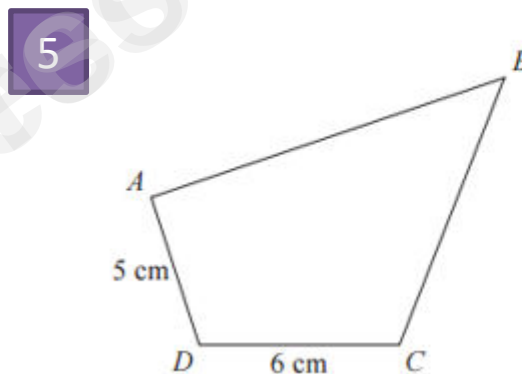
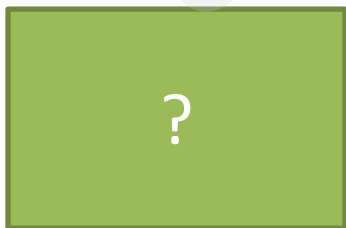
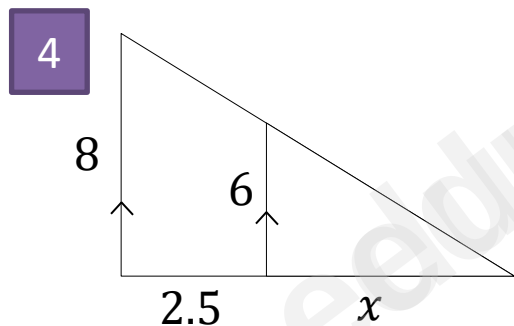
- 1 The two triangles are mathematically similar. Find x and y .



$$\begin{array}{l} BC = \text{?} \\ AC = \text{?} \end{array}$$

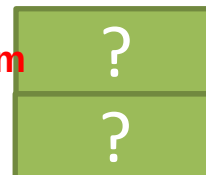


$$x = \text{?}$$



[June 2014 2H Q17] Quadrilaterals $ABCD$ and $LMNP$ are mathematically similar.

- (a) Determine the length of LP .
(b) Determine the length of BC .



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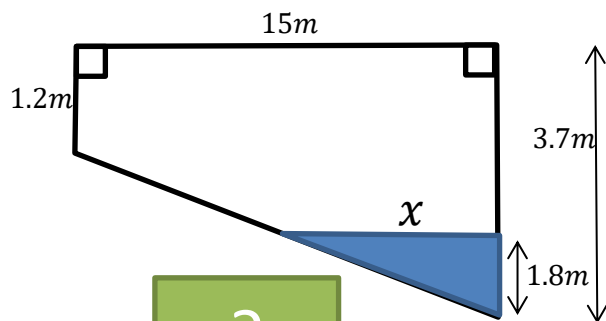
Exercise 1

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6

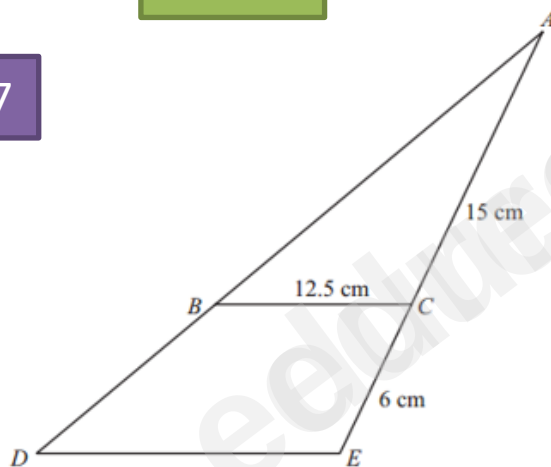
A swimming pool is filled with water. Find x .



$x =$

?

7

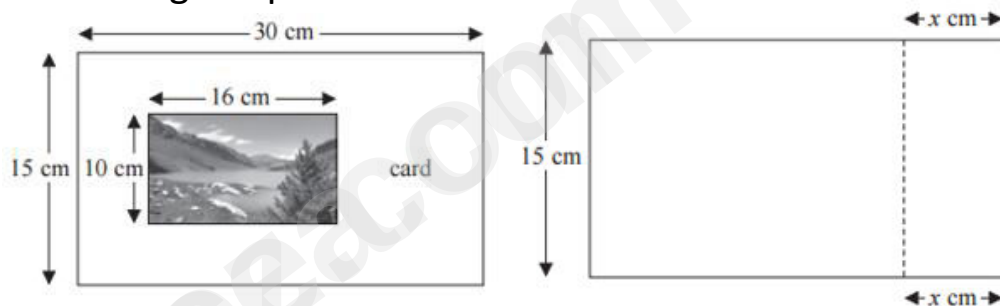


Triangle ABC is similar to triangle ADE .
Work out the length of DE .

?

8

[June 2014 1H Q20] Steve has a photo and a rectangular piece of card.

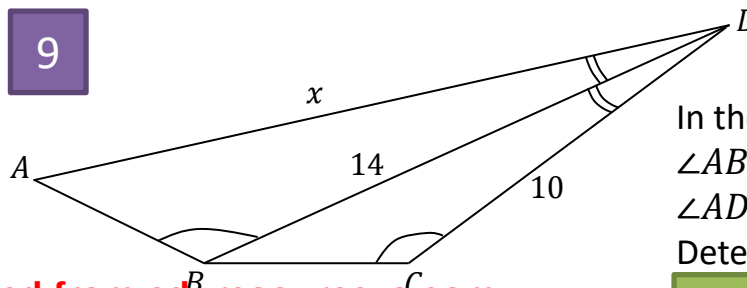


Steve cuts the card along the dotted line shown in the second diagram.

Steve throws away the piece of card that is 15cm by x cm. The piece of card he has left is mathematically similar to the photo. Work out the value of x .

?

9



In the following diagram
 $\angle ABC = \angle BCD$ and
 $\angle ADB = \angle BDC$.
Determine x .

?

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Exercise 1

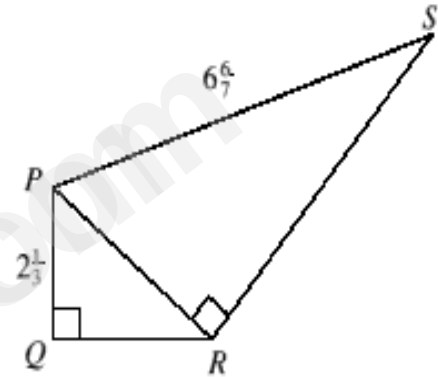
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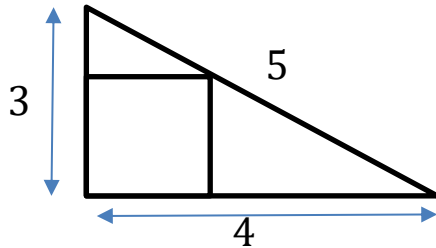
10

[IMC 2006 Q23] In the figure, $PQ = 2\frac{1}{3}$, $PS = 6\frac{6}{7}$ and $\angle QPR = \angle RPS$. How long is PR ?

?



1



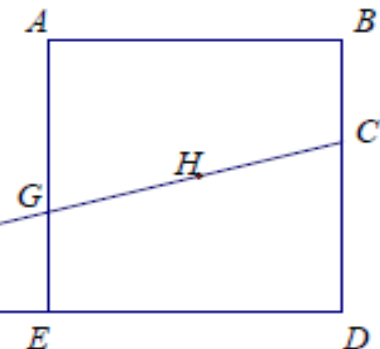
[IMO] A square is inscribed in a 3-4-5 right-angled triangle as shown. What is the side-length of the square?

?

2

[STMC Regional 2009/10 Q10] $ABDE$ is a square with centre H . The base of the square DE is extended so that it meets the straight line CF which passes through H . If $BC = 3$ cm and $CD = 4$ cm find the area of the triangle CDF .

Solution: 56 cm^2



not to scale

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Exercise 1

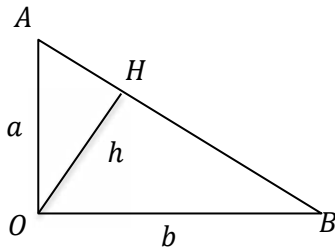
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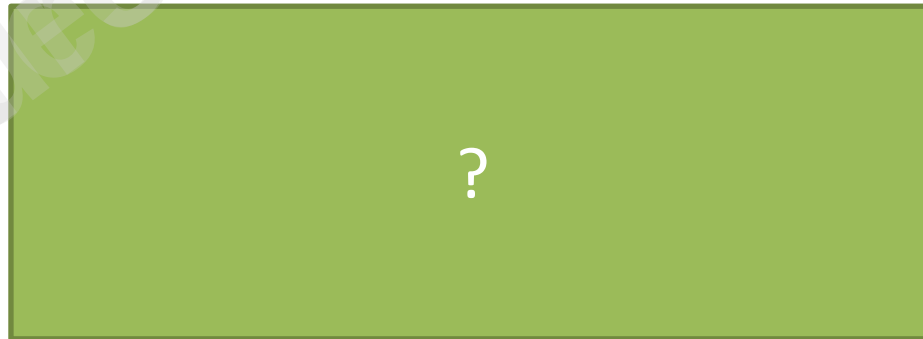
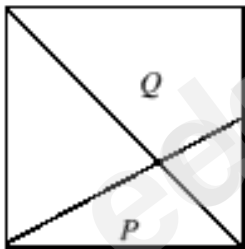
3

Let a and b be the lengths of the two shorter sides of a right-angled triangle, and let h be the distance from the right angle to the hypotenuse. Prove $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{h^2}$



4

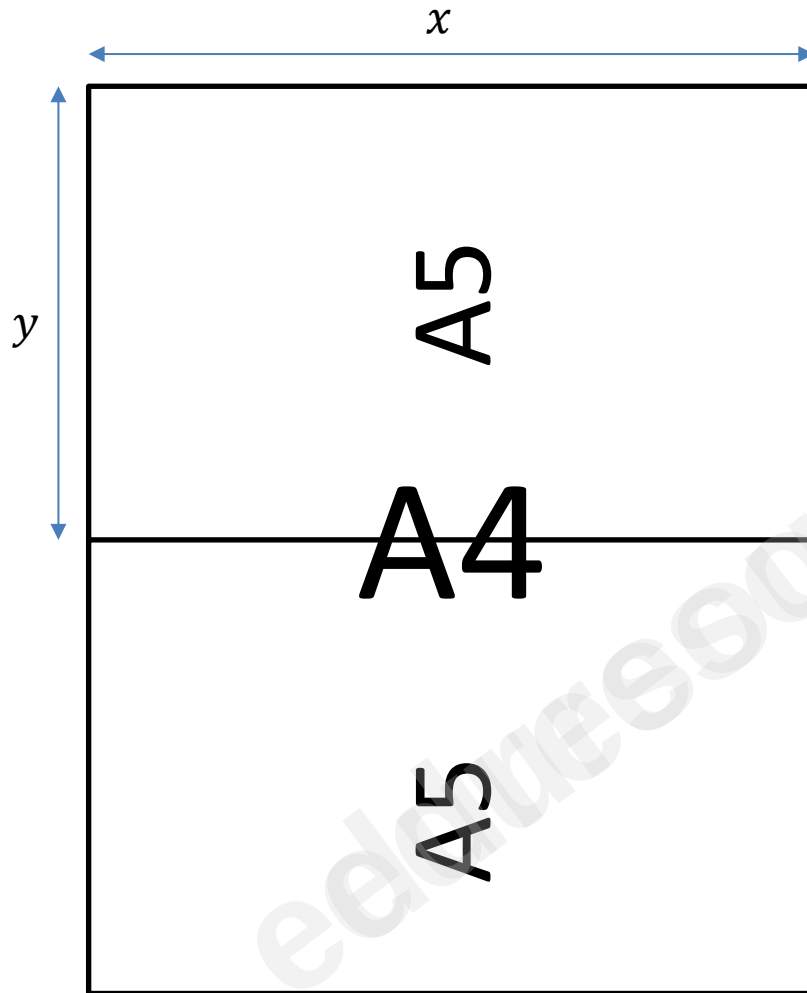
[Source: IMC] The diagram shows a square, a diagonal and a line joining a vertex to the midpoint of a side. What is the ratio of area P to area Q ?



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A4/A3/A2 paper puzzle...

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“A” sizes of paper (A4, A3, etc.) have the special property that what two sheets of one size paper are put together, the combined sheet is mathematically similar to each individual sheet.

What therefore is the ratio of length to width?



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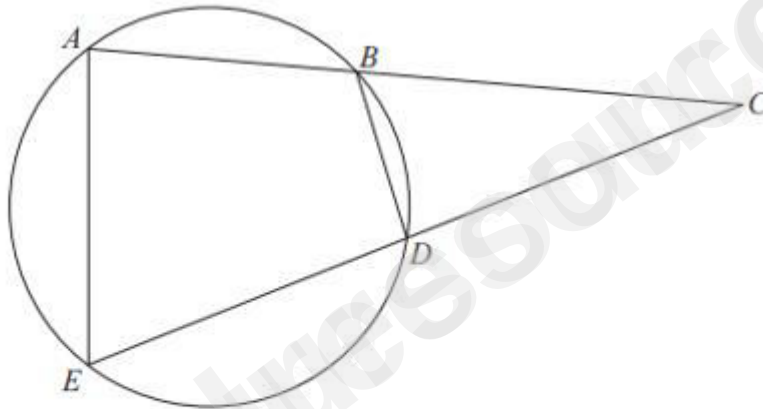
Similar Triangle Proofs

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What do we need to compare to prove that two triangles are similar?

?

November 2015 1H Q22



A , B , D and E are points on a circle.
 ABC and EDC are straight lines.

Prove that triangle BCD is similar to triangle ECA .
You must give reasons for your working.

?

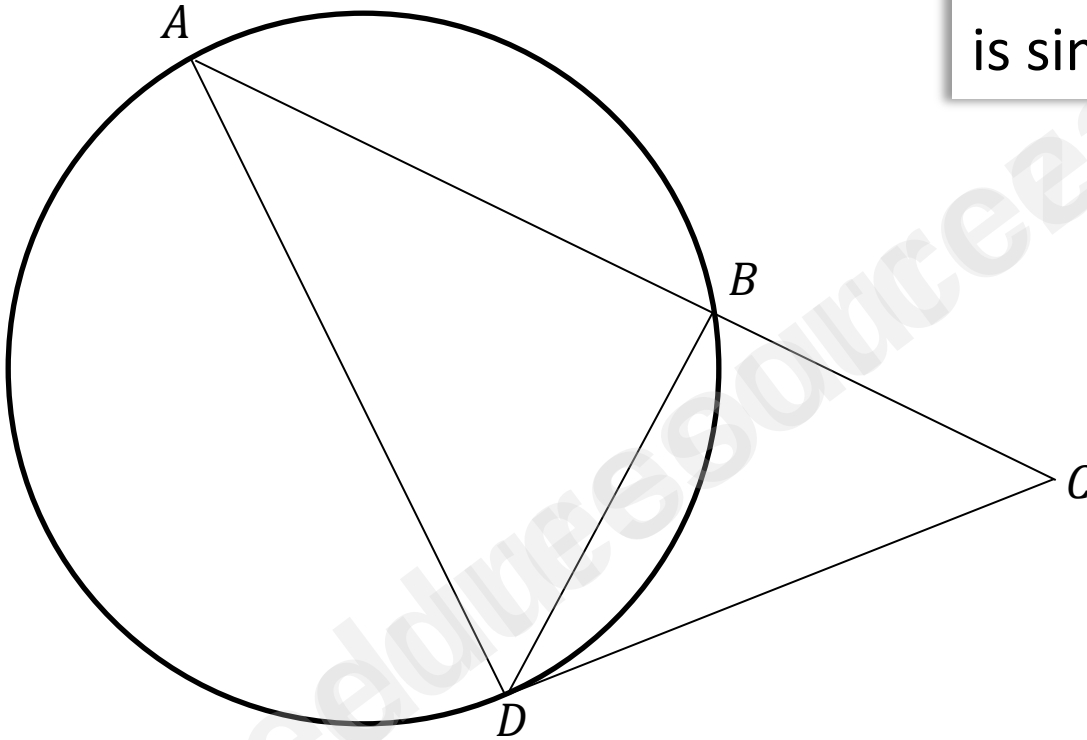
Required Knowledge:
Circle Theorems

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Test Your Understanding

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The line CD is tangent to the circle. Prove that BCD is similar to ABC .

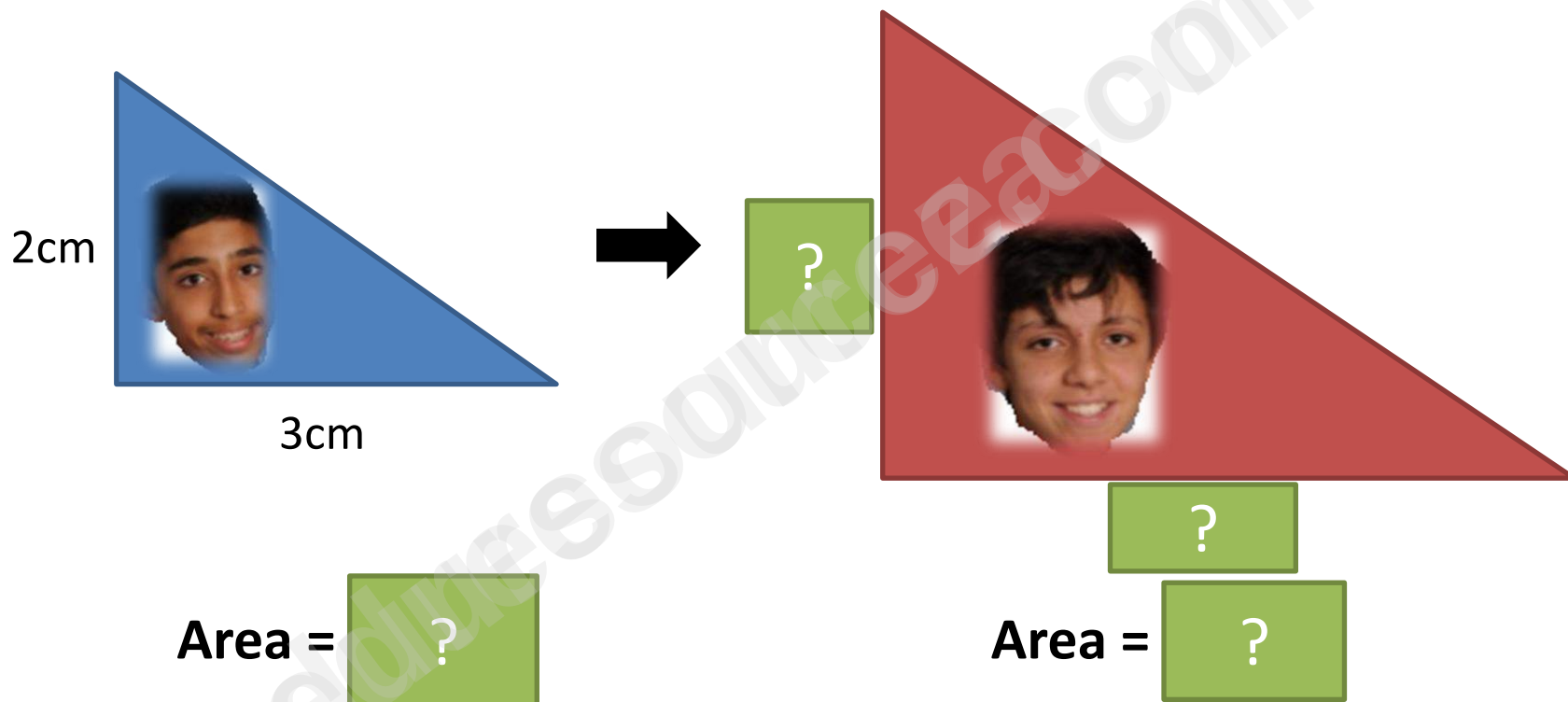


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Scaling areas and volumes

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A Savvy-Triangle is enlarged by a **scale factor of 3** to form a Yusutriangle.



Length increased by a factor of ?

Area increased by a factor of ?

We can see that if the length increases by a scale factor k , the area increases by this squared, i.e. k^2 .

Scaling areas and volumes

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For area, the scale factor is **squared**.

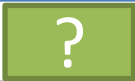
For volume, the scale factor is **cubed**.

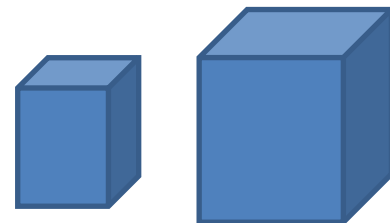
Example: A shape X is enlarged by a scale factor of 5 to produce a shape Y. The area of shape X is 3m^2 . What is the area of shape Y?

	Shape X		Shape Y
Length:		$\times 5$	
Area:	3m^2		

Bro Tip: This is my own way of working out questions like this. You really can't go wrong with this method!

Example: Shape A is enlarged to form shape B. The surface area of shape A is 30cm^2 and the surface area of B is 120cm^2 . If shape A has length 5cm , what length does shape B have?

	Shape A		Shape B
Length:	5cm		
Area:	30cm^2		120cm^2



Scaling areas and volumes

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For area, the scale factor is **squared**.

For volume, the scale factor is **cubed**.

Example 3: Shape A is enlarged to form shape B. The surface area of shape A is 30cm^2 and the surface area of B is 270cm^2 . If the volume of shape A is 80cm^3 , what is the volume of shape B?

	Shape A		Shape B
Length:			
			
Area:	30cm^2		270cm^2
			
Volume:	80cm^3		
			

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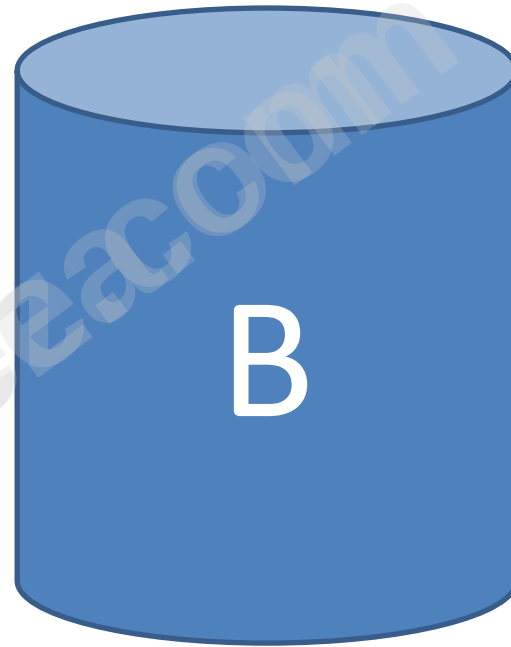
Test Your Understanding

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These 3D shapes are mathematically similar.
If the surface area of solid A is 20cm^2 . What is
the surface area of solid B?



Volume = 10cm^3



Volume = 640cm^3

?

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Exercise 2

(on provided sheet)

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- 1 Copy the table and determine the missing values.

	Shape A	Shape B
Length:	3cm	$\times 2 \rightarrow$ 6cm
Area:	5cm ²	?
Volume:	10cm ³	?

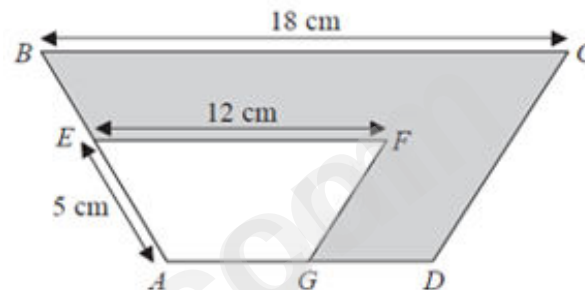
- 2 Determine the missing values.

	Shape A	Shape B
Length:	5m	?
Area:	8m ²	?
Volume:	12m ³	?

- 3 [Edexcel 2003] Cylinder A and cylinder B are mathematically similar. The length of cylinder A is 4 cm and the length of cylinder B is 6 cm. The volume of cylinder A is 80cm³. Calculate the volume of cylinder B.

?

4



[Edexcel Nov 2014] $ABCD$ and $AEGF$ are mathematically similar trapeziums. Trapezium $AEGF$ has an area of 36cm^2 . Work out the area of the shaded region.

?

5

[2007] Two cones, P and Q, are mathematically similar. The total surface area of cone P is 24cm^2 . The total surface area of cone Q is 96cm^2 . The height of cone P is 4 cm.

(a) Work out the height of cone Q.

?

(b) The volume of cone P is 12cm^3 . Work out the volume of cone Q.

?

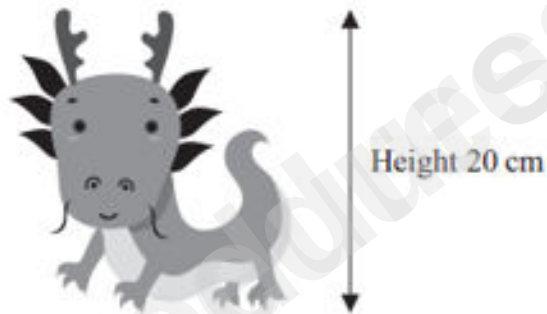
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Exercise 2

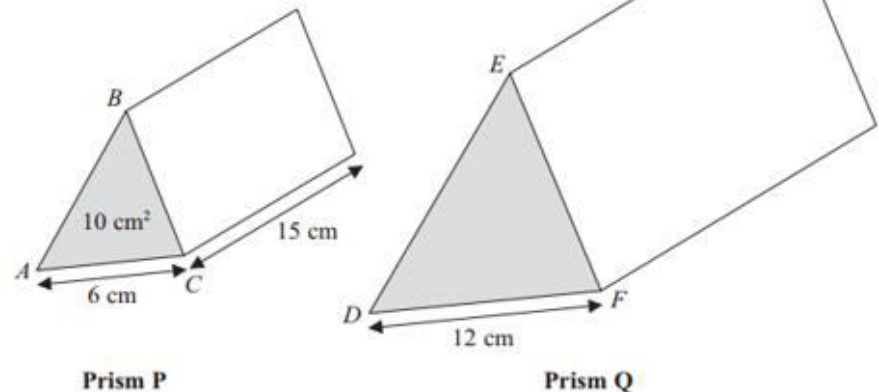
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- 6 [Nov 2013 1H Q16] A company makes monsters. The company makes small monsters with a height of 20cm. A small monster has a surface area of 300cm^2 . The company also makes large monsters with a height of 120cm. A small monster and a large monster are mathematically similar. Work out the surface area of a large monster.



- 7 [June 2013 1H Q22] P and Q are two triangular prisms that are mathematically similar. The area of the cross section of P is 10cm^2 . The length of P is 15cm. Work out the volume of prism Q .



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Exercise 2

(on provided sheet)

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- 8 [Nov 2012 Q25] The diagram shows two similar solids, A and B .

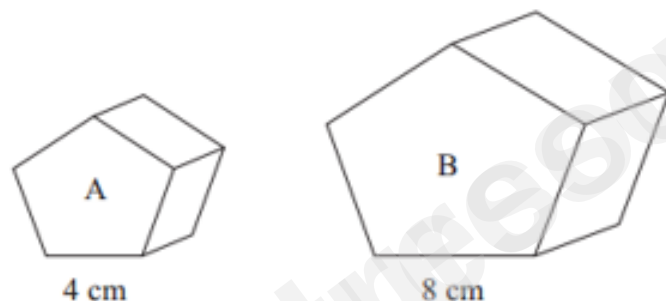
Solid A has a volume of 80cm^3 .

(a) Work out the volume of solid B .

?

(b) If solid B has a total surface area of 160cm^2 , work out the total surface area of A .

?

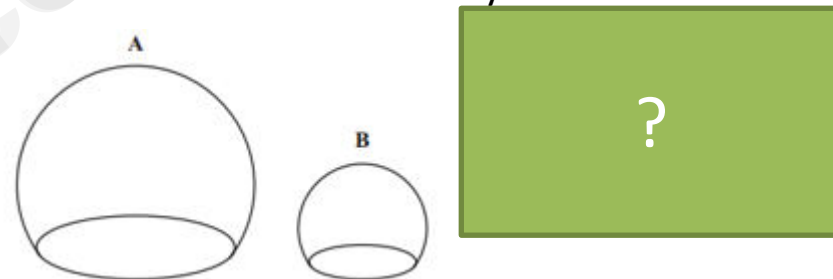


- 9 [June 2010 Q23] A and B are two solid shapes which are mathematically similar. The shapes are made from the same material.

The surface area of A is 50cm^2 , and of B is 18cm^2 . The mass of A is 500 grams.

Calculate the mass of B .

Note that given a fixed density, mass scales in the same way as volume.



The surface area of shapes A and B are x and y respectively. Given that the length of shape B is z , write an expression (in terms of x , y and z) for the length of shape A .

?

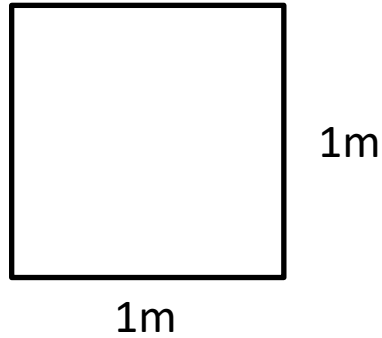
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Units of Area and Volume

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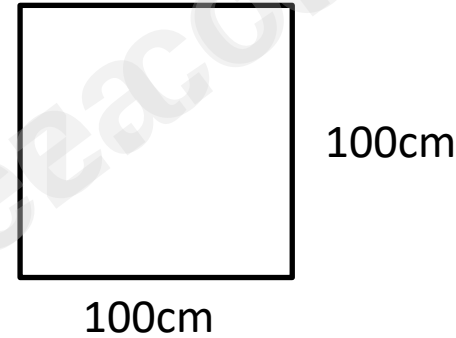
We can use the same principle to find how to convert between units of volume and area.

A unit (metre) square:



$$\text{Area} = \boxed{?} \text{ m}^2$$

The same square with cm:



$$\text{Area} = \boxed{?} \text{ cm}^2$$

Example:

What is 8.3m^2 in cm^2 ?

$$8\text{m}^2 \times \boxed{?} = \boxed{?} \text{ cm}^2$$

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Further Examples

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28 cm² in mm²?

?

3.2 m³ in cm³?

?

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Exercise 3

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1 What is 42cm^2 in mm^2 ?

$$42\text{cm}^2 \times \boxed{?} \rightarrow \boxed{?} \text{mm}^2$$

5 What is 5.1cm^2 in mm^2 ?

$$5.1\text{m}^2 \times \boxed{?} \rightarrow \boxed{?} \text{mm}^2$$

2 What is 2m^2 in mm^2 ?

$$2\text{m}^2 \times \boxed{?} \rightarrow \boxed{?} \text{mm}^2$$

6 What is 2km^3 in m^3 ?

$$2\text{km}^3 \times \boxed{?} \rightarrow \boxed{?} \text{m}^3$$

3 What is 3m^3 in cm^3 ?

$$3\text{m}^3 \times \boxed{?} \rightarrow \boxed{?} \text{cm}^3$$

7 What is 4.25m^3 in mm^3 ?

$$4.25\text{m}^3 \times \boxed{?} \rightarrow \boxed{?} \text{mm}^3$$

4 What is 13cm^3 in mm^3 ?

$$13\text{m}^3 \times \boxed{?} \rightarrow \boxed{?} \text{mm}^3$$

8 What is 10.01km^2 in mm^2 ?

$$10.01\text{km}^2 \times \boxed{?} \rightarrow \boxed{?} \text{mm}^2$$

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