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MATHEMATICS

Paper 9709/11
Paper 11

Key messages

Candidates should be aware that when an answer is provided in the question the working out which leads to that answer should be convincing. When an answer is provided it often gives candidates access to a subsequent part of the question, as in **3(i)**, **6(i)** and **8(i)**, which an incorrect answer would have lost. When candidates are unable to reach a given result they should look closely at any subsequent parts of the question to see if they can use the given answer to solve them.

General comments

The presentation of non-exact numerical answers is described on the front cover of the exam and candidates should be reminded to refer to this. Although no GCSE topics are directly examined it should be noted that basic algebraic and trigonometric techniques are required to answer some A Level questions.

Comments on Specific Questions

Question 1

The use of the general term of this type of expansion provided the quickest route to the required coefficients. Complete expansions leading to correct coefficients were also seen regularly. Those who bracketed the $-2x$ term were a lot less likely to make sign errors.

Answer: $\frac{-9}{8}$

Question 2

(i) Setting the scalar product of the two given vectors to zero provided a very quick route to the solution. The use of the full expression of the scalar product with the magnitude of the vectors found prior to using cos 90° = 0 often resulted in very confusing algebra.

(ii) The idea of a unit vector and its calculation was shown to be well understood in attempts at this question. Those able to find $\overrightarrow{BC}$ mostly went on to reach a correct expression for the unit vector.

Answers: **(i)** $p = 6$ **(ii)** $\frac{1}{5\sqrt{5}}(2\mathbf{j} + 11\mathbf{k})$ or equivalent

Question 3

(i) Most correct solutions involved a common denominator on the LHS and use of the trigonometric identity in the numerator. As the final result was given, factorisation of the simplified numerator was required for full marks to be awarded. A small number of candidates found common denominators of $\sin\theta$ or $\sin^2\theta$ by multiplying the second fraction on the LHS by $\frac{(1-\cos\theta)}{(1-\cos\theta)}$. From this form the RHS quickly became apparent. Those who chose to simplify the denominators through the whole expression were able to gain full credit when clear explanation was provided.

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(ii) The expected route of equating the RHS of the identity in part **(i)** to the RHS of the equation in part **(ii)** enabling $\tan\theta$ to be found, was often seen. Those unsuccessful in part **(i)** rarely appreciated that they could use the result from part **(i)** in part **(ii)**. Most candidates who found the first angle realised that a second angle within the range was required.

Answers: **(ii)** 33.7° and 213.7°

Question 4

(a) Finding d from the first and fifth terms then n, then S_n proved to be the most successful method for the majority of correct solutions. The relatively small number of terms encouraged some to list and sum all the terms.

(b) For those who identified the series as a GP and identified r and n the solution was easily reached. As in part **(a)** the small number of terms resulted in a number of listing and addition methods, the most accurate of which gained full credit.

Answers: **(a)** 50 **(b)** £22400

Question 5

(i) Many correct smooth curves through (0,2) were seen and those which showed the 'flattening' at each end gained full marks. Although it was not necessary to label the x-axis, if a single cycle was sketched it was essential if multiple cycles were drawn.

(ii) The correct use of the distance formula was often seen, even when the y-coordinates of P and Q had not been properly calculated. Correct answers were not always rounded to one decimal place as requested.

(iii) Successful candidates generally found the exact gradient of PQ and used the coordinates of P or Q in a straight line equation. Some were equally successful using simultaneous equations with $y = mx + c$ and the coordinates of P and Q. Those who chose to find the gradient of PQ as a decimal could not justify the given exact value of h It was acceptable (but rarely seen) to verify $y = 0$ at $h = \frac{5\pi}{9}$.

Answer: **(ii)** 3.7 **(iii)** 2.5

Question 6

(i) This proved to be an unexpectedly difficult question. Those who sketched the prism and its base generally produced better solutions. Finding the exact area of the base was strongly suggested by the form of the given result. Some candidates who then found a correct expression for h did not consider all five faces of the prism in their final calculation. Those who did had to display good algebraic reasoning to reach the given result.

(ii) The use of the first derivative and the need to set it to zero was appreciated by many. Those who dealt successfully with the x^{-2} term usually found the correct value of x.

(iii) Most completely correct solutions came from the calculation of the second derivative and substitution of the value obtained in part **(ii)**. Those who elected to investigate the change in sign of the first derivative needed to show clear working and reasoning to justify their conclusion.

Answer: **(ii)** 20 **(iii)** minimum

Question 7

(i) The need to integrate $\frac{dy}{dx}$ was appreciated by most and those who correctly used a constant of integration often went on to reach a completely correct solution.

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(ii) Dealing with the negative coefficient of x^2 proved to be an obstacle to obtaining a correct solution. Those who did were able to isolate and deal with the last two terms of the expression. Some chose to reverse the signs in the expression, completed the square and obtained full marks if they then corrected the initial sign reversal.

(iii) Some correct solutions used the completed square form from part **(ii)** and some the original expression. Although many candidates set these to zero and obtained the correct critical values few went on to express the required set correctly.

Answers: **(i)** $y = 7x - \frac{x^3}{3} - 3x^2 + 5$ **(ii)** $16 - (x+3)^2$ **(iii)** $-7 < x < 1$ or equivalent

Question 8

(i) The quickest route to the given result was found by those candidates who realised *OAX* and *OBX* were congruent isosceles triangles and therefore $A\hat{X}B = \pi - O\hat{A}X$. The more popular route was finding *AM* and *XM,* then $A\hat{X}M$ which was then doubled. It was not uncommon to see a lot of the working out in the diagram above the printed question.

(ii) The arc length formula was often used, but only those candidates who saw that the question stated that *XA* was the radius of the shaded segment and attempted to use it gained any credit.

(iii) Candidates who completed part **(ii)** successfully mostly completed this part. The assumption that the shaded area was a semi-circle was often incorrectly made.

Answers: **(ii)** 27.8 cm **(iii)** 38.0 cm^2

Question 9

(i) Calculating the inverse through change of subject was well understood and applied by many candidates. Most realised at some point f(x) (or y) and x must be interchanged. Although the correct result could be reached by considering a flowchart and the associated inverse operations, few candidates who selected this method could reach a correct solution.

(ii) Forming the composite function was well understood by many candidates and most of these produced completely correct solutions. Occasionally candidates confused $\mathrm{f}(-1)$ with $\mathrm{f}^{-1}(1)$.

(iii) Most successful candidates found $\mathrm{g}^{-1}(x)$, equated this to $\mathrm{f}^{-1}(x)$ and set the discriminant of the resulting quadratic equation to zero. Some realised that this quadratic must have the form $(x \pm 2)^2 = 0$, expanded the LHS and equated the middle term to the middle term of their quadratic. A few realised that equating $\mathrm{f}(x)$ to $\mathrm{g}(x)$ would also give the two values of a but only a few completely correct solutions followed this path.

Answers: **(i)** $\mathrm{f}^{-1}(x) = \frac{(3x-2)}{2x}$ or equivalent **(ii)** $\frac{7}{5}$ **(iii)** -10 and -2

Question 10

(i) The need to find the gradient of the curve by differentiation and the differentiation of this type of function appeared to be well understood. Of those who found an expression for $\frac{dy}{dx}$ many went on to use the gradient relationship and a straight line equation to find the normal equation. Errors in dealing with the negative signs in the differentiation and the gradient relationship were seen in solutions which otherwise would have gained full marks.

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(ii) Whilst the differentiation of this type of function appeared to be well understood, the integration resulted in fewer correct solutions. To gain any credit, solutions had to show an attempt to integrate y^2 . The omission of the lower limit was rarely seen as most attempts showed candidates realised the substitution of $x = 0$ would not result in zero. Most answers included the π at some point in the solution. Candidates who produced answers without any evidence of integration gained no credit.

Answers: **(i)** $y = -\frac{1}{3}x + \frac{7}{3}$ **(ii)** $\frac{8\pi}{5}$ or equivalent

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MATHEMATICS

Paper 9709/12
Paper 12

Key messages

When solving Trigonometric equations, it is important that only answers in the appropriate quadrant or range are given.

Candidates would benefit from reading the questions carefully at least twice and then extracting the relevant information from them. Not fully comprehending the information given or making false assumptions often leads to a significant amount of time and effort being wasted and no worthwhile progress being made.

General comments

The paper seemed to be generally well received by the candidates and many good and excellent scripts were seen. There were a number of questions that were reasonably straightforward, particularly near the beginning of the paper, giving all candidates the opportunity to show what they had learned and understood and some which provided more of a challenge, even for candidates of good ability. The vast majority of candidates appeared to have sufficient time to complete the paper and the standard of presentation was generally good with candidates setting their work out in a clear readable fashion. It is important that all working out should be done in the answer paper and not in extra booklets or blank pages and then copied into the answer booklet.

Comments on specific questions

Question 1

Part (i) proved to be an accessible start to the paper with a great many candidates being able to correctly find the required coefficient of x. Some candidates wrote down the full binomial expansion and then selected the necessary term but many more picked out which term would be needed and calculated the coefficient of that term. There were some candidates who forgot to cube the 2 but these were a small minority. **Part (ii)** proved to be more of a challenge for weaker candidates with some failing to realise that the coefficient of $\frac{1}{x}$ in the original expansion was now needed so that it could be multiplied by $3x^2$ and a combined coefficient of x found. Those who only used their answer from **part (i)** received no credit and perhaps need to look at the number of marks for the question as an indication of the amount of work that will be needed.

Answers: **(i)** 80; **(ii)** –40.

Question 2

Part (i) was very well done with the vast majority of candidates obtaining full marks for it. Some candidates mistakenly thought that the gradient of AB was $-\frac{1}{3}$ rather than $-\frac{2}{3}$ but they were generally still able to find an equation and so receive some credit. **Part (ii)** was less well done with many candidates thinking that they had found B when they solved the simultaneous equations. The majority though did realise that they had only found the midpoint of AB and continued on and found B correctly.

Answers: **(i)** $3y + 2x = 14$; **(ii)** $(4,2)$.

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Question 3

Part (i) proved to be a real challenge for many candidates and some very poor algebraic manipulation was seen, such as simply squaring each term rather than fully expanding both brackets. Many candidates though, did make some progress, but the final step of factorising the numerator to $(1-\sin\theta)(1-\sin\theta)$ rather than the more usual $(\sin\theta-1)(\sin\theta-1)$ proved too difficult for many. **Part (ii)** was much more straightforward for the vast majority of candidates. A very small minority missed this part of the question out if they had failed to prove the first part or failed to use the given result and made no progress. Those who did use the given result were almost always successful although sometimes four answers, one in each quadrant, were given rather than simply the two correct answers. Candidates need to realise that this approach leads to marks being lost. There were also a few candidates who truncated 19.47 to 19.4 and lost a mark.

Answer: **(ii)** 19.5°, 160.5°.

Question 4

The fact that the angle in this question was 2θ rather than simply θ caused problems for many candidates, particularly in **part (ii)**. These candidates quoted formulae that they had learnt but failed to replace θ with 2θ and so lost most of the marks available. This happened with the arc length AB in **part (i)** and the areas of the triangle and sector in **part (ii)**. The straight length AB in **part (i)** was usually found by considering a right-angle triangle and hence finding half of the length required although more complicated versions involving using the cosine or sine rule were acceptable alternative approaches. In **part (ii)** some candidates failed to realise that the area of the rectangle, triangle and sector were all required, with some simply subtracting the sector from the rectangle. A good number of fully correct solutions were seen though, with roughly equal numbers using the rectangle minus the segment as using rectangle minus sector plus triangle.

Answer: **(ii)** 22.7.

Question 5

Many fully correct solutions were seen for this question although a minority of candidates misinterpreted what was required in both parts. In **part (i)** candidates were generally able to differentiate the function correctly, although a sizeable minority did forget the (–1), and find the value of x when the curve crossed the x axis. A small number did put x rather than y equal to zero though. Some weaker candidates failed to use the equation of a straight line for the tangent and instead integrated. In **part (ii)** candidates nearly always realised that the chain rule was needed although a number failed to use their $\frac{dy}{dx}$ from **part (i)** or didn't attempt this part.

Answers: **(i)** $4y = 3x - 18$ **(ii)** 0.12.

Question 6

This question required candidates to find the volume generated when the area between 2 curves was rotated. A significant number of candidates found this concept, or what was required to find the volume, difficult to understand, although many fully correct solutions were seen. A small number of candidates missed this question out altogether whilst some others equated the 2 curves and then tried to integrate the new expression which resulted. Others subtracted the equations before squaring them or added the 2 volumes rather than subtracting them. The most common successful approach was to consider the 2 volumes separately and then subtract at the end although a good number of strong candidates were able to subtract the squared expressions first and then consider a single integral. The actual integration that was done was usually done correctly.

Answer: 9π or 28.3.

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Question7

Both parts of this series question were slightly different to normal but many candidates coped very well with the slight changes and often obtained full marks for them. In **part (a)** the question was very standard except that the total that the sum had to exceed was very large. The vast majority of candidates realised the need to use the sum formula and used it correctly to obtain a quadratic equation. This didn't factorise but many of the candidates were able to either use the quadratic formula or the solve function which many calculators now have. Almost all candidates realised that the number of terms couldn't be negative and rejected this solution but a significant number left the final answer as 69.2 or rounded down to 69. **Part (b)** was very well answered and only the weakest candidates did not use the sum to infinity formula or were unable to solve the resulting equation correctly to find *r*. Many candidates went onto square *a* and *r* and correctly found the new sum to infinity although some only squared one of these values. An alternative approach was to square each term in the old series to obtain the new values of *a* and *r*. This was again usually successful.

Answers: **(a)** 70; **(b)** 64.8.

Question 8

Part (i) was a standard vector question and very many candidates obtained the correct answer by using the scaler product as instructed. Very few used the cosine rule but it should be noted that those who did, lost some of the marks available. Some weaker candidates used the wrong vectors or reversed the direction of one of them but this was rare. **Part (ii)** proved to be much more difficult. Candidates were generally able to attempt to find vector $\overrightarrow{AB}$ but sign errors were common and many assumed that the values for $\overrightarrow{OA}$ and $\overrightarrow{OB}$ found in **part (i)** still applied. Many candidates seemed unfamiliar with the implications of $\overrightarrow{AB}$ being parallel to $\overrightarrow{OC}$. Some equated the lengths of the vectors and many more simply equated the **i** and **k** components and ignored the fact that this meant that 2 = 4 if the **j** components were equated. Stronger candidates realised that the values of the **j** components implied that one vector was twice the other and could successfully solve the resulting equations.

Answers: **(i)** 68.2; **(ii)** 2.5, 4.

Question 9

Unfortunately, many weaker candidates misunderstood the square root notation to mean $8\sqrt{(x-2x)}$ rather than the $8\sqrt{x} - 2x$ it was meant to be. A number of these candidates realised their mistake when the resulting non-sensible answers were obtained and restarted but some did not. In **part (i)** the vast majority who understood the notation were able to differentiate correctly and set this to zero but solving the resulting equation was difficult for many and a smaller number, having successfully found the *x* coordinate, forgot the corresponding *y* value. In **part (ii)** again the differentiation was generally done correctly but some candidates seemed unsure of what to do next and a small number described the function as either increasing or decreasing. If it is clear, as in this case, that the second derivative will be negative then simply stating <0, therefore maximum is sufficient to obtain the mark. If it is not clear then the *x* coordinate at the stationary point needs to be substituted into the second derivative, it needs to be evaluated and then the conclusion stated. In **part (iii)** candidates who knew how to proceed employed two equally valid methods of solution. Some realised that when the line intersected the curve then the equation that was obtained was a quadratic in $\sqrt{x}$ .These candidates were usually able to then solve the quadratic and then square the resulting answers to obtain the required *x* values, although a small number square rooted these values instead. The other approach was to isolate $\sqrt{x}$ or $8\sqrt{x}$ and then square both sides. Some candidates simply squared each term in the original equation and received no credit. **Part (iv)** proved to be difficult for all but the best candidates. Only a very small number saw the link with **part (i)** of the question and those who were successful instead considered when the discriminant would be < 0.

Answers: **(i)** (4,8), **(iii)** 1, 9 **(iv)** $k > 8$.

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Question 10

Part (i) of this question proved more challenging than expected. Most candidates were able to rearrange the equation to $\tan\left(\frac{1}{2}x\right) = -\frac{2}{3}$ but many did not then find the final correct answer in radians. Candidates would benefit from further practice on this type of question. In **part (ii)** the vast majority of candidates were able to find the inverse function but only the better candidates were able to find the correct domain. In **part (iii)** only the very best candidates were able to successfully obtain full marks. Most were aware that the inverse function would be a reflection of the original function in the line y = x, it was drawing the original function which proved to be the challenge. Some candidates drew a tangent graph over a longer period than the domain specified and so did not have a one-to-one function. Others attempted to plot values but these often led to a graph that wasn't tangential in shape. Those who were successful, restricted themselves to the given domain and considered how $y = \tan x$ would be transformed to the given function and the consequential resulting changes in the graph.

Answers: **(i)** –1.2; **(ii)** $2\tan^{-1}\left(\frac{x+2}{3}\right)$, $-5 \leqslant x \leqslant 1$.

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MATHEMATICS

Paper 9709/13
Paper 13

Key Messages

1 There were numerous occasions when candidates failed to achieve the required accuracy in their answers. Unless requested otherwise, the requirement is that answers are given correct to 3 significant figures. The implication of this is that candidates need to carry at least 4 significant figure accuracy in their working. The exception to the requirement for 3 significant figures is for answers in degrees, where the requirement is 1 decimal place. It should be noted, however, that angles in radians should be given to 3 significant figures, and confusion over this point caused some candidates to work in Question 7 with 0.93 radians, or even 0.9 radians rather than 0.927(3) radians – resulting in an unfortunate loss of marks.

2 Completing the square can generally be requested (with variation in the signs, of course) in one of two forms: $a(x-b)^2+c$ or $(ax-b)^2+c$. In Question 9, the latter form was requested and significant numbers of candidates failed to read the question carefully and presented their answers in the other form, often causing the loss of up to 2 marks.

General Comments

The paper was generally well received by candidates and many very good scripts were seen. Most candidates seemed to have sufficient time to finish the paper. It is pleasing to be able to report that candidates have taken notice of comments in previous reports and were prepared to show more working in their progress to the answer. It is also pleasing to report that candidates were more aware of the amount of space allowed for their working and in the great majority of cases were able to fit their working into the given space without the need to resort to supplementary sheets.

Comments on Specific Questions

Question 1

This question was answered well. Binomial coefficients were generally correct and evaluated accurately and the vast majority scored full marks.

Answer : 2 / 3.

Question 2

In part **(i)** almost all candidates used the $a/(1-r)$ formula correctly and gained the first mark. Where the answer is given, a full and clear method must be shown, even if a step might appear to be obvious. In this part of the question, sometimes the handling of the minus sign was not fully evident which prevented the second mark being scored. The simplest way is to factorise the numerator in the form $(1-r)(2-r)$ rather than the more obvious $(r-1)(r-2)$; cancelling the brackets $(1-r)$ can then be achieved immediately. By way of contrast, part **(ii)** was not done at all well. Candidates were not able to use the restriction on r and then apply it to $S = 2 - r$.

Answer : **(ii)** $1 < S < 3$.

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Question 3

Most candidates were able to eliminate *y* and simplify to a 3-term equation, scoring the first mark. The most successful approach from this point was to employ a dummy variable *u*, for example , solve the quadratic equation in *u*, transform the solutions back to $x^{1/3}$ and cube the solutions to find *x* and then *y*. However, a variety of errors were seen which meant that a substantial numbers of candidates scored the first mark only. Some candidates chose to use *y* as their dummy variable and, despite the danger of confusion, a greater proportion of these candidates were able to progress to correct solutions. Another group of candidates, from their equation in $x^{1/3}$, cubed individual terms of the equation, or cubed the whole equation. Some candidates spotted one solution ($x = 8$) but not the second solution and yet other candidates tried to use logarithms. None of these approaches scored any of the last 3 marks.

Answer : (8, 3), (–1, 0).

Question 4

This question was generally well done with many candidates scoring full marks (6) for the whole question. In part **(i)**, a few candidates reversed the two vectors, subtracting $\overrightarrow{OB}$ from $\overrightarrow{OA}$. Some did not find $\overrightarrow{OP}$ until part **(ii)**, presumably not realising that $\overrightarrow{OP}$ and 'the position vector of *P* ' were the same thing. Part **(ii)** was answered successfully by almost all candidates. In part **(iii)** the majority of candidates knew that they had to establish whether the scalar product was zero or not, although a significant number of candidates followed a more circuitous route and found that the cosine of the angle between the vectors was zero and hence the angle was 90°.

Answers : **(i)** $\begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$; **(ii)** √30 or 5.48; **(iii)** Perpendicular.

Question 5

Most candidates were successful in part **(i)** in showing that the equation could be expressed in the given form. While some candidates followed a long and complicated route most candidates got there quite quickly by replacing $\tan\theta$ by $\sin\theta / \cos\theta$ and multiplying by the product of the denominators. In part **(ii)** almost all candidates transformed the equation to one of three forms: $\tan^2\theta = 1/2$ or $\cos^2\theta = 2/3$ or $\sin^2\theta = 1/3$. Some candidates using the first two forms forgot ± and hence found only one solution.

Answer : **(ii)** 35.3°, 144.7°.

Question 6

The correct method is to determine the gradient of the tangent and equate this to the derivative of the curve, which leads to $x = 4$. Substitution into the two given equations leads to the required value of *k*. Candidates who used this method almost always scored full marks. However, very many candidates treated the normal as the tangent, eliminated *y* between the two given equations and then applied $b^2 - 4ac = 0$ and were unable to score any marks at all from a completely incorrect method.

Answer : $k = 11$.

Question 7

A high proportion of candidates obtained the correct answer in part **(i)**, the main mistake being to give the answer to an insufficient degree of accuracy, which also impinged on part **(ii)**. A few gave the answer in degrees. In part **(ii)**, most candidates knew the processes required to get the right answer. In addition to premature rounding in part **(i)** and/or part **(ii)**, marks were also lost by forgetting to double angle *ABC* when calculating the areas of triangle *BCD* and/or sector *BCD*.

Answers : **(i)** 0.927; **(ii)** 55.8

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Question 8

Most candidates used the gradient condition successfully in part **(i)**, although a few had the gradient inverted. Almost all candidates were then able to rearrange to make *b* the subject of the expression. In part **(ii)** a significant proportion of candidates first tried approaches involving equations of lines, before many, after making no progress, resorted to a method based on equating the distance between (*a*, *b*) and (–1, 1) with the distance between (–1, 1) and (10, –1) and then substituting for *b* using the relationship found in part **(i)**. While the majority of candidates used a correct method, arithmetic and algebraic errors were not infrequently seen which prevented candidates from reaching the correct solutions. There was also a rarely seen, but ingenious, vector method employed by a few candidates who realised that the vector $\begin{pmatrix} 5 \\ 10 \end{pmatrix}$ possessed the requirements of having gradient 2 and a magnitude of √125, the magnitude of vector **AB**. The solution then followed from $\begin{pmatrix} -1 \\ 1 \end{pmatrix} \pm \begin{pmatrix} 5 \\ 10 \end{pmatrix}$.

Answers : **(i)** $b = 2a + 3$; **(ii)** (4, 11), (–6, –9).

Question 9

In part **(i)** many candidates did not read the question carefully enough, as reported in the **Key Messages** section above, and did not present their answers in the required form. Many of these took 3 or 9 out as a factor before completing the square and, even if they tried, could not successfully put the 3 or 9 back inside the brackets. In part **(ii)** most candidates realised that 1/3 was the value of *p* required but did not always express the answer perfectly. However, examiners treated generously answers which could have been expressed more appropriately. In part **(iii)** most candidates showed they knew how to derive the inverse of a given function. Candidates who started with the wrong form from part **(i)** were still able to arrive at the correct inverse, and even candidates who had made an error in part **(i)**, were able to score 2 marks out of 3 by following a correct sequence of operations. The mark for the domain was not often scored, however, with some confusion being shown between range and domain. Sometimes $f^{-1}(x) \geq 5$ was offered instead of $x \geqslant 5$ and sometimes there was no response at all. In part **(iv)**, candidates should have been alerted to the fact that no working was necessary by the form of the question 'State …' for which only 1 mark was on offer for a correct answer. The set of values of *q* can be deduced immediately by inspection of the answer to part **(i)**. However, although there was still a good proportion of correct answers, the majority of candidates opted for the standard longer approach and set the discriminant of the resulting equation to be negative. This was an unnecessarily time consuming exercise with the potential of producing errors.

Answers : **(i)** $(3x-1)^2 - 5$; **(ii)** Smallest value of *p* is 1 / 3; **(iii)** $f^{-1}(x) = \dfrac{\sqrt{x-5}+1}{3}$ for $x \geqslant 5$; **(iv)** $q < 5$.

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Question 10

Part **(a)(i)** involves the revolution of a region about the <u>*y*-axis</u>, which necessitates integrating x^2 with respect to *y*. Since $x^2 = y + 1$, this leads to integrating $y + 1$ with respect to *y* resulting in $\frac{1}{2}y^2 + 1$. Limits 0 and *h* need to be applied leading to the given answer. However, candidates did not always follow this route and because the answer was given the onus was on candidates to ensure that examiners were left in no doubt that the answer had been obtained in a legitimate way. Some candidates integrated $x + 1$ with respect to *x* with limits 0 to *h* and this, on the face of it, unnecessary interchange of variables, needed to be made clear in order to convince examiners. Because *h* is a particular value of *y*, some candidates, no doubt with an eye on the given answer, integrated $h + 1$ with respect to *y* (or sometimes *x*). This gained no credit. In part **(a)(ii)** it had been hoped that since part **(a)(i)** involved revolution about the *y*-axis, the region in part **(a)(ii)** would be envisaged as an area between the curve and the *y*-axis. This involves a straightforward integration of $\sqrt{y+1}$ with respect to *y* with limits 0 and 3. A greater proportion of candidates, however, opted for the more difficult route which had a number of pitfalls on the way. This route involves integrating $x^2 - 1$ with respect to *x*, with limits 1 and 2 (which need to be calculated) and subtracting this from the area of the rectangle. The area of the rectangle is more easily calculated as 3×2, but some candidates chose to integrate 3 with respect to *x* with limits 0 and 2. Some of these candidates, however, chose to combine the two integrals not appreciating that they have different limits. As a result, this second route was far less successful than the former route. In part **(b)**, although candidates generally realised they needed to find $\mathrm{d}V/\mathrm{d}h$, a significant number could not do so accurately. There was also some confusion in notation used, both in differentiation and in the following implementation of the chain rule. As a result it was not uncommon to see $2 \times \mathrm{d}V/\mathrm{d}h$ instead of $2 \div \mathrm{d}V/\mathrm{d}h$.

Answers : **(a)(ii)** 14 / 3; **(b)** 1 / (2π) or 0.159.

Question 11

Part **(i)** was usually well answered but it was common to see the constant of integration omitted or not evaluated. This might sometimes have been because candidates had not processed the information given in the question. The fact that f had a minimum value at $x = 2$ meant that $f'(x) = 0$ at $x = 2$. Failure to evaluate the constant of integration lost a mark in part **(i)** and had an adverse effect on part **(iii)**. The focus of part **(ii)** was the arithmetic progression. Some candidates failed to appreciate this and tried, unsuccessfully, to proceed with integration. Follow-through marks were available for candidates who had made an error in part **(i),** or had forgotten the constant of integration, but were methodically correct in dealing with the arithmetic progression. Part **(iii)** tested the use of constants of integration. Not all candidates realised they had to use their value for f(0) from part **(ii)** to evaluate the second constant of integration. Then candidates needed to use again the fact that the minimum value of f occurs when $x = 2$ and substitute $x = 2$ to obtain the minimum value.

Answers : **(i)** $\frac{1}{2}(4x+1)^{1/2} - \frac{3}{2}$; **(ii)** f(0) = –3; **(iii)** minimum value = $-\frac{23}{6}$.

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MATHEMATICS

Paper 9709/21
Paper 21

Key messages

Candidates are reminded of the necessity of ensuring that a question has been answered in full. They should also ensure that working is done to the required level of accuracy (at least 4 significant figures) and that final answers should be to 3 significant figures unless otherwise requested. Calculators are a useful tool but must not replace working which should always be shown, particularly when either an exact answer is needed or a question uses the word 'hence' which requires use of work from a previous part of the question.

General comments

Most candidates appeared to have sufficient time to attempt all eight questions with varying degrees of success. In most cases candidates had sufficient room on their examination paper to write their solutions. Those candidates that needed extra room made use of the blank page in the booklet. Having a given space for candidates to write their solutions appeared to make candidates more conscious of the need to set their work out in a clear and logical fashion. It was evident from some of the scripts seen, with a large number of blank spaces where the solutions should have been, that candidates had not prepared enough for the examination.

Comments on specific questions

Question 1

Most candidates were able to take logarithms of each side of the given equation and apply the power rule to both sides. Problems occurred when trying to manipulate the equation to obtain the form $y = \frac{\ln 5}{4\ln 3}$, with some candidates attempting to use the laws of logarithms incorrectly. There were instances of candidates working with less than the appropriate level of accuracy resulting in a final answer of 0.365. This resulted in the loss of the final accuracy mark.

Answer: 0.336

Question 2

Most candidates chose to square both sides of the given equation and then attempt to solve the resulting quadratic equation. Many obtained the correct critical values, but as in previous sessions, seemed to be unable to deal with the inequality correctly. Those candidates that chose to deal with two linear equations, usually had greater success with obtaining the final answer, provided they had dealt with the modulus correctly. Candidates were penalised if, having obtained the correct critical values they then chose to give their answer in the form $-1 \geqslant x \geqslant \frac{7}{3}$ or equivalent, by withholding the final accuracy mark. Answers using alternative notation correctly were also acceptable.

Answer: $x \leqslant -1,\ x \geqslant \frac{7}{3}$

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Question 3

The most common error in this question was to have an incorrect coefficient for $e^{\frac{1}{2}x+3}$. However many candidates did obtain a correct integral and form the correct equation $8e^{\frac{1}{2}a+3} - 8e^3 = 835$. Some chose to evaluate $8e^3$ and attempt a simplification of $8e^{\frac{1}{2}a+3} = 674.32$. These candidates usually had greater success at finding *a* using a correct method using logarithms than those candidates who chose to leave each term on the left-hand side of the equation in terms of e and take logarithms erroneously. The incorrect statement $\ln e^{\frac{1}{2}a+3} - \ln e^3 = \ln 104.375$ or similar involving coefficients of 8 was far too common.

Answer: 3.65

Question 4

(i) Usually done well by most candidates. Marks were lost when candidates did not give their iterations to the correct level of accuracy or their final answer to the correct level of accuracy. It was felt that candidates were helped by knowing what value they were to use at the start of the iteration.

(ii) Very few candidates knew how to go about solving this part of the question. The appreciation of the implication of the word 'exact' was lost on most candidates. Many thought that they had to do a greater number of iterations, but many did not attempt this part.

Answers: **(i)** 2.08 **(ii)** $\sqrt[3]{9}$

Question 5

(i) Most candidates were able to score marks in this part of the question. Some candidates gave a solution for α only and then chose to give $R = 3$ in part **(ii)**. These candidates were not penalised but it is important that the demands of the question are read carefully. In this particular question it was evident that the focus of the candidates' attention was on the final phrase of the demand 'giving the value of α correct to 2 decimal places' and completely forgetting about *R*. In some cases even this was not read carefully with some instances of $\alpha = 48.2^\circ$ being seen. The other most common error was to obtain the inverse ratio for tangent and obtain $\alpha = 41.81^\circ$.

(ii) Provided a correct method had been used in part **(i)** most candidates were able to apply another correct method to obtain at least one solution for the given equation. Some candidates gave extra solutions within the range and were penalised by the loss of the final accuracy mark. There were many instances of completely incorrect methods of solution, usually involving dealing with the cosine of the compound angle incorrectly. Candidates are to be reminded of the meaning of the word 'Hence' in questions of this type and the indication that what they have obtained in part **(i)** should be utilised in part **(ii)**.

Answers: **(i)** $3\cos(\theta - 48.19^\circ)$ **(ii)** 118.7°, 337.7°

Question 6

(i) Correct attempts at the trapezium rule were in the minority. It was evident that many candidates were not familiar with the trapezium rule and those that were invariably got the incorrect *y*-values or used *x*-values in the formula.

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(ii) Candidates not making use of the identity $\tan^2 2x = \sec^2 2x - 1$ were usually unable to gain any credit in this part of the question. Some did make sign errors with the identity and were still able to gain method marks. It was evident that many candidates were completely unable to attempt this question in a meaningful way. For those candidates that did make a correct attempt at the integration there were often sign and coefficient errors and for those candidates that did integrate successfully, all too often the meaning of the word 'exact' was lost and decimal answers were given. As a result, few correct solutions were seen.

Answers: **(i)** 0.378 **(ii)** $\pi\left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right)$

Question 7

(i) Most candidates were able to obtain $\frac{dy}{dx} = \frac{4t^3 - 6t^2 + 8t - 12}{3t^2 + 6}$, with the occasional sign error. Problems with the algebraic long division were very common with many candidates being unable to obtain the value of $\frac{4}{3}$ as the coefficient of t.

(ii) Most candidates realised that the gradient of the straight line was $\frac{1}{2}$ but were unable to proceed much further. It was intended that candidates equate their answer from part **(ii)** to -2 and thus obtain the value of the parameter at the appropriate point. Some candidates did equate their original $\frac{dy}{dx} = \frac{4t^3 - 6t^2 + 8t - 12}{3t^2 + 6}$ to -2 and proceed from there with varying degrees of success.

Answers: **(i)** $\frac{4}{3}t - 2$ **(ii)** $(1, 5)$

Question 8

(i) Provided candidates realised that they needed to make use of the product rule and differentiation then most were able to gain at least one mark. The main problems candidates had was simplifying their differentiation to a correct form. Some also had problems differentiation $\ln\left(\frac{x}{6}\right)$ correctly. Some candidates did not realise that they needed to find the value of x at the point P and some of those that did were unable to formulate and solve the necessary equation.

(ii) Many candidates did not have their first derivative in the required form, due to being unable to simplify their first derivative correctly. For those that did, many resorted to the use of their calculator and gave the coordinates in decimal form, not taking notice of the word 'exact'. Very few completely correct solutions were seen.

Answers: **(i)** 18 **(ii)** $\left(6e^{-\frac{1}{2}}, -54e^{-1}\right)$ or exact equivalent.

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MATHEMATICS

Paper 9709/22
Paper 22

Key messages

Candidates are reminded of the necessity of ensuring that a question has been answered in full. They should also ensure that working is done to the required level of accuracy (at least 4 significant figures) and that final answers should be to 3 significant figures unless otherwise requested. Calculators are a useful tool but must not replace working which should always be shown, particularly when either an exact answer is needed or a question uses the word 'hence' which requires use of work from a previous part of the question.

General comments

Most candidates appeared to have sufficient time to attempt all eight questions with varying degrees of success. In most cases candidates had sufficient room on their examination paper to write their solutions. Those candidates that needed extra room made use of the blank page in the booklet. Having a given space for candidates to write their solutions appeared to make candidates more conscious of the need to set their work out in a clear and logical fashion.

Comments on specific questions

Question 1

Most candidates chose to square both sides of the modulus equation. Problems occurred with the resulting quadratic equation which although in terms of x also contained terms in a. Many candidates appeared to be unable to solve quadratic equations of this type, often making errors with the terms involving a, or using incorrect methods which resulted in solutions in terms of both a and x.

Candidates dealing with two separate linear equations obtained from the modulus equation usually had more success with obtaining the correct solutions.

Answer: $6a$, $\frac{4a}{3}$

Question 2

An application of logarithms to each side of the given equation, together with the application of the power law for logarithms meant that most candidates were able to obtain the first two marks. Some candidates were careless with the use of brackets , writing $x + 4\log 3 = 2x\log 5$, rather than the required $(x+4)\log 3 = 2x\log 5$. This then caused problems with the simplification to obtain a value for x. Use of logarithms to base 10 was just as common as logarithms to base e, with some candidates making use of either base 3 or base 5 logarithms which was equally successful.

Answer: 2.07

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Question 3

(i) Too many candidates were unable to sketch the graph of $y = x^3$ and were thus unable to gain marks for this part of the question. Of those that did sketch the graph of $y = x^3$, successfully, most sketched a straight line in the correct position to show the graph of $y = 11 - 2x$. It was expected that candidates make a comment about there being only one point of intersection, thus implying only one solution of the given equation. Many candidates lost the final mark as no appropriate comment was made.

(ii) Questions involving the iteration process are usually done successfully by the majority of candidates and this was no exception. Most chose a sensible starting point for their iterations and carried out the appropriate number of iterations, usually making use of the 'Answer' function on their calculator. There are still some candidates that do not make use of their calculator in this way, which tends to lead to more errors being made not to mention the extra time needed for the process. Errors tended to be usually when candidates did not give their final answer to 4 significant figures and occasionally when the iterations were not given to the required level of accuracy.

Answer: **(ii)** 1.926

Question 4

Most candidates realised that they needed to use the quotient rule to differentiate the given equation. This was usually done successfully, with the occasional error, the most common being $\frac{d}{dx}\left(e^{4x}\right) = e^{4x}$. With an expression for $\frac{dy}{dx}$, most candidates made the substitution of zero to obtain a gradient which was then used to form an equation of the tangent at the point where $\left(0, \frac{1}{3}\right)$. This question was a prime example of candidates not ensuring that they had completed the full demands of the question, with many not giving their final answer in the required form, thus using the final mark.

Answer: $10x - 9y + 3 = 0$

Question 5

This was the question that was probably one of the most demanding on the paper. Most candidates were able to make the statement $\ln y = \ln K - 2x\ln a$ and also find the gradient of the straight line correctly. Two approaches were possible with most candidates opting to use the gradient in an attempt to find the value of *a*. Unfortunately, many candidates equated the gradient to $\ln a$ rather than to $-2\ln a$. Errors in substituting a value for *a*, in order to find *K*, were often made with logarithms being used inappropriately in the majority of incorrect cases. Some candidates chose to use the coordinates given and the equation $\ln y = \ln K - 2x\ln a$ in order to solve a pair of simultaneous equations. Although not as common as the first approach, this method tended to be more successful, provided the coordinates were used correctly, as the error involving the gradient in the first approach was not a problem. Too many candidates did not work to the required level of accuracy throughout the question and often values for *K* were inaccurate even though a correct approach had been used.

Answer: $a = 1.3$, $K = 8.4$

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Question 6

(i) The great majority of candidates were able to score at least 4 marks for this part of the question. The factor theorem was usually applied in one form or another and most candidates obtained the quotient $6x^2 + x - 35$, usually by algebraic long division, but occasionally by observation. It should be noted that synthetic division is not regarded as a valid method for **showing** $(x+2)$ is a factor of the given cubic equation. Many candidates thought that the final answer was $(x+2)(6x^2+x-35)$, not recognising that the quadratic quotient could be factorised.

It was also obvious that some candidates had made use of the equation solver on their calculator and work 'backwards' to the factors $(x+2)\left(x+\frac{5}{2}\right)\left(x-\frac{7}{3}\right)$ which are not the factors of the given cubic equation. Candidates should be judicious about the use of their calculators.

(ii) Candidates should take note of the word 'Deduce', which implies, together with the mark allocation of 2 marks, that the solutions are obtained by little work. Many candidates chose to start the question again, usually with little success. However there were a significant number of candidates who did recognise the link between the 2 equations and give the correct solutions.

Answers: **(i)** $(x+2)(2x+5)(3x-7)$ **(ii)** $-\frac{1}{2},\ -\frac{2}{5},\ \frac{3}{7}$

Question 7

(a) Most candidates realised that they needed to expand the brackets before making an attempt at integration. Unfortunately, it was evident that some candidates thought that integrating each term in the brackets was the way forward and thus failed to obtain any marks for this part of the question. Fewer candidates realised that the use of the appropriate double angle formula was needed before attempting integration. Of those candidates that did make use of the correct approach, many then 'forgot' that the variable involved was θ and integrated the constant term as $2x$ rather than 2θ.

(b) (i) Most solutions were in terms of logarithms with errors usually being in the multiples of the logarithmic functions.

(ii) Many candidates applied the limits to their integral in terms of logarithms correctly. However it was evident that many were unsure of how to simplify their logarithms to a single logarithm.

Answers: **(a)** $\frac{1}{2}\sin 2\theta - \sin\theta - 2\theta + c$ **(b)(i)** $2\ln(2x+1) + \frac{1}{2}\ln 2x$ or $2\ln(2x+1) + \frac{1}{2}\ln x$ **(ii)** $\ln 18$

Question 8

(i) Most candidates were able to find $\frac{dx}{dt}$, but very few candidates were able to find $\frac{dy}{dt}$ correctly, being unable to differentiate the powers of the trigonometric functions. Most realised that $\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$ and that use of the double angle formula for $\sin 2t$ was needed, but were unable to obtain the given result. Candidates should be aware that when an answer is given, it is inadvisable to try to contrive the given result from clearly incorrect work. By doing this, they are sometimes failing to obtain method marks that they would otherwise have obtained.

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(ii) Usually done well, with most candidates making use of the given result from **part (i)**. The great majority of candidates obtained $\tan t = \frac{3}{2}$ but often made errors in calculating the cartesian coordinates.

(iii) Many were able to identify the correct value of the parameter and go on to attempt the gradient of the normal. Many failed to realise the importance of the word 'exact' and resorted to using their calculator, thus failing to gain the last 2 marks. Some candidates also gave the value of the gradient of the tangent rather than the normal.

Answers: **(ii)** $(2.38, 2.66)$ **(iii)** $\frac{4\sqrt{2}}{3}$

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MATHEMATICS

Paper 9709/23
Paper 23

Key messages

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General comments

Most candidates appeared to have sufficient time to attempt all eight questions with varying degrees of success. In most cases candidates had sufficient room on their examination paper to write their solutions. Those candidates that needed extra room made use of the blank page in the booklet. Having a given space for candidates to write their solutions appeared to make candidates more conscious of the need to set their work out in a clear and logical fashion.

Comments on specific questions

Question 1

Most candidates chose to square both sides of the modulus equation. Problems occurred with the resulting quadratic equation which although in terms of x also contained terms in a. Many candidates appeared to be unable to solve quadratic equations of this type, often making errors with the terms involving a, or using incorrect methods which resulted in solutions in terms of both a and x.

Candidates dealing with two separate linear equations obtained from the modulus equation usually had more success with obtaining the correct solutions.

Answer: $6a$, $\frac{4a}{3}$

Question 2

An application of logarithms to each side of the given equation, together with the application of the power law for logarithms meant that most candidates were able to obtain the first two marks. Some candidates were careless with the use of brackets , writing $x + 4\log 3 = 2x\log 5$, rather than the required $(x+4)\log 3 = 2x\log 5$. This then caused problems with the simplification to obtain a value for x. Use of logarithms to base 10 was just as common as logarithms to base e, with some candidates making use of either base 3 or base 5 logarithms which was equally successful.

Answer: 2.07

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Question 3

(i) Too many candidates were unable to sketch the graph of $y = x^3$ and were thus unable to gain marks for this part of the question. Of those that did sketch the graph of $y = x^3$, successfully, most sketched a straight line in the correct position to show the graph of $y = 11 - 2x$. It was expected that candidates make a comment about there being only one point of intersection, thus implying only one solution of the given equation. Many candidates lost the final mark as no appropriate comment was made.

(ii) Questions involving the iteration process are usually done successfully by the majority of candidates and this was no exception. Most chose a sensible starting point for their iterations and carried out the appropriate number of iterations, usually making use of the 'Answer' function on their calculator. There are still some candidates that do not make use of their calculator in this way, which tends to lead to more errors being made not to mention the extra time needed for the process. Errors tended to be usually when candidates did not give their final answer to 4 significant figures and occasionally when the iterations were not given to the required level of accuracy.

Answer: **(ii)** 1.926

Question 4

Most candidates realised that they needed to use the quotient rule to differentiate the given equation. This was usually done successfully, with the occasional error, the most common being $\frac{d}{dx}\left(e^{4x}\right) = e^{4x}$. With an expression for $\frac{dy}{dx}$, most candidates made the substitution of zero to obtain a gradient which was then used to form an equation of the tangent at the point where $\left(0, \frac{1}{3}\right)$. This question was a prime example of candidates not ensuring that they had completed the full demands of the question, with many not giving their final answer in the required form, thus using the final mark.

Answer: $10x - 9y + 3 = 0$

Question 5

This was the question that was probably one of the most demanding on the paper. Most candidates were able to make the statement $\ln y = \ln K - 2x\ln a$ and also find the gradient of the straight line correctly. Two approaches were possible with most candidates opting to use the gradient in an attempt to find the value of *a*. Unfortunately, many candidates equated the gradient to $\ln a$ rather than to $-2\ln a$. Errors in substituting a value for *a*, in order to find *K*, were often made with logarithms being used inappropriately in the majority of incorrect cases. Some candidates chose to use the coordinates given and the equation $\ln y = \ln K - 2x\ln a$ in order to solve a pair of simultaneous equations. Although not as common as the first approach, this method tended to be more successful, provided the coordinates were used correctly, as the error involving the gradient in the first approach was not a problem. Too many candidates did not work to the required level of accuracy throughout the question and often values for *K* were inaccurate even though a correct approach had been used.

Answer: $a = 1.3$, $K = 8.4$

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Question 6

(i) The great majority of candidates were able to score at least 4 marks for this part of the question. The factor theorem was usually applied in one form or another and most candidates obtained the quotient $6x^2 + x - 35$, usually by algebraic long division, but occasionally by observation. It should be noted that synthetic division is not regarded as a valid method for **showing** $(x+2)$ is a factor of the given cubic equation. Many candidates thought that the final answer was $(x+2)(6x^2 + x - 35)$, not recognising that the quadratic quotient could be factorised.

It was also obvious that some candidates had made use of the equation solver on their calculator and work 'backwards' to the factors $(x+2)\left(x+\frac{5}{2}\right)\left(x-\frac{7}{3}\right)$ which are not the factors of the given cubic equation. Candidates should be judicious about the use of their calculators.

(ii) Candidates should take note of the word 'Deduce', which implies, together with the mark allocation of 2 marks, that the solutions are obtained by little work. Many candidates chose to start the question again, usually with little success. However there were a significant number of candidates who did recognise the link between the 2 equations and give the correct solutions.

Answers: **(i)** $(x+2)(2x+5)(3x-7)$ **(ii)** $-\frac{1}{2},\ -\frac{2}{5},\ \frac{3}{7}$

Question 7

(a) Most candidates realised that they needed to expand the brackets before making an attempt at integration. Unfortunately, it was evident that some candidates thought that integrating each term in the brackets was the way forward and thus failed to obtain any marks for this part of the question. Fewer candidates realised that the use of the appropriate double angle formula was needed before attempting integration. Of those candidates that did make use of the correct approach, many then 'forgot' that the variable involved was θ and integrated the constant term as $2x$ rather than 2θ.

(b) (i) Most solutions were in terms of logarithms with errors usually being in the multiples of the logarithmic functions.

(ii) Many candidates applied the limits to their integral in terms of logarithms correctly. However it was evident that many were unsure of how to simplify their logarithms to a single logarithm.

Answers: **(a)** $\frac{1}{2}\sin 2\theta - \sin\theta - 2\theta + c$ **(b)(i)** $2\ln(2x+1) + \frac{1}{2}\ln 2x$ or $2\ln(2x+1) + \frac{1}{2}\ln x$ **(ii)** $\ln 18$

Question 8

(i) Most candidates were able to find $\frac{dx}{dt}$, but very few candidates were able to find $\frac{dy}{dt}$ correctly, being unable to differentiate the powers of the trigonometric functions. Most realised that $\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$ and that use of the double angle formula for $\sin 2t$ was needed, but were unable to obtain the given result. Candidates should be aware that when an answer is given, it is inadvisable to try to contrive the given result from clearly incorrect work. By doing this, they are sometimes failing to obtain method marks that they would otherwise have obtained.

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(ii) Usually done well, with most candidates making use of the given result from **part (i)**. The great majority of candidates obtained $\tan t = \frac{3}{2}$ but often made errors in calculating the cartesian coordinates.

(iii) Many were able to identify the correct value of the parameter and go on to attempt the gradient of the normal. Many failed to realise the importance of the word 'exact' and resorted to using their calculator, thus failing to gain the last 2 marks. Some candidates also gave the value of the gradient of the tangent rather than the normal.

Answers: **(ii)** $(2.38, 2.66)$ **(iii)** $\frac{4\sqrt{2}}{3}$

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MATHEMATICS

Paper 9709/31
Paper 31

General comments

The standard of work on this paper varied considerably and resulted in a wide spread of scores from zero to almost full marks. Some excellent scripts were seen but there were also a large number of candidates who appeared poorly equipped for the paper and made little progress.

Most candidates made good progress with the following topics/questions: modular inequality and binomial expansion (**Questions 1 & 2**), arithmetic with complex numbers (**Question 7(i)**) and the partial fractions (**Question 9(i)**). Over a third of all candidates offered no attempt at **Question 5** (circular measure and iterative solution of equations). Other topics with a very low response rate were: gradient of a normal (**Question 4(ii)**), equations of parallel planes (**Question 6(ii)**) geometrical relationship between line segments (**Question 7(iii)**), solution of trigonometric equation (**Question 8(ii)**) and use of product rule (**Question 10(ii)**).

Much of the work was clearly set out and easy to follow. Candidates who erase their initial attempt or use a pen to overwrite work originally done in pencil should be aware that the process of scanning the work picks up all versions with equal clarity, producing work that can be very difficult to read – it would be better to put a line through the original work and rewrite it further down the page.

When a question expressly forbids the use of a calculator (as in **Question 7**), or asks for the answer to be given as an exact value (as in **Question 4(ii)** and **Question 10(i)**) then candidates should be aware that calculator methods will not gain credit. Similarly, when a result is given (as in **Question 3(i)**, **Question 3(ii)**, **Question 5(i)** and **Question 7(ii)**) then a full and detailed explanation is expected.

The failure to use brackets correctly was a common problem. While many candidates do recover, with their subsequent work implying that the brackets existed in their minds, this is sometimes not the case and marks are thereby lost unnecessarily.

Where numerical and other answers are given after the **Comments on specific questions** that follow, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on specific questions

Question 1

Many candidates obtained the correct solution. The most concise solutions often started with a sketch of the two functions. However, the most popular approach was to square both sides of the inequality to obtain a non-modular inequality or equation. Some candidates made errors in squaring the brackets, and it was common to see solutions where the factor of 3 on the right hand side had not been squared. Most candidates who found the correct critical values also stated the correct region.

Answer: $x < 1$ and $x > 7$

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Question 2

The majority of candidates stated the given fraction correctly in index form and applied the binomial expansion correctly. The most common error was to omit the 6 in $(6x)^2$ and $(6x)^3$. Some candidates spoiled a solution which was otherwise correct by multiplying their answer by 3 to achieve integer coefficients.

Answer: $1-2x+8x^2-\frac{112}{3}x^3$

Question 3

(i) Many candidates combined the two log terms correctly and reached $e^x=\frac{1-y}{y}$. They changed the subject to obtain $y=\frac{1}{e^x+1}$ and often stopped at that point because they could not see how to get from there to the given answer. Those candidates who found or obtained an expression for e^{-x} early in their working were more likely to succeed.

(ii) Relatively few candidates recognised this integral as $-\ln\left(1+e^{-x}\right)$ although some got there eventually using a substitution. After correct substitution of limits, some candidates struggled to combine the two terms correctly. The question asks candidates to demonstrate an exact answer – showing approximate decimal equivalency is not sufficient.

Question 4

(i) Many candidates used the chain rule correctly to obtain $\frac{dy}{dx}$ expressed in terms of trigonometric functions. Some of those candidates who did not know that $\sec^2\theta=1+\tan^2\theta$ did eventually reach a final answer in terms of $\tan\theta$, albeit via a rather long route.

(ii) Many candidates did not connect the gradient of the normal being 1 with $\frac{dy}{dx}=-1$ and therefore made no progress here. Several candidates with a correct quadratic in $\tan\theta$ tried to solve $\tan^2\theta+\tan\theta=2$ by considering factors of 2. Candidates with a correct solution for $\tan\theta$ often went on to work in decimals, so they could not score the final mark.

Answers: **(i)** $\frac{\tan^2\theta-2}{\tan\theta}$ **(ii)** $\frac{3}{4}\pi-1$

Question 5

(i) Many candidates made a very poor start to this question, showing little or no understanding of how to find the area of a sector of a circle or of a triangle. The candidates who did start with a correct unsimplified equation invariably reached a correct conclusion.

(ii) Candidates who could not demonstrate the result in **part (i)** could go on to complete the rest of the question, however many were clearly put off because over 30% offered no attempt at this part of the question. Those candidates who remembered that they should be working in radians and did substitute the given values into a suitable function or functions usually obtained correct values and reached the correct conclusion.

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(iii) Again, many candidates did not attempt this question, possibly because they simply did not continue after not completing the first part. However, those candidates who have worked through the past papers for this syllabus were expecting a question of this type and often gave fully correct solutions. Candidates should be aware that in this type of question we do expect them to work to the accuracy requested.

Answer: **(iii)** 1.374

Question 6

(i) Many candidates were successful in tackling some aspect of this problem. Several found the point of intersection of the line and the plane. A majority found the direction of the line and compared it with the direction of the normal to the plane. Those candidates who started by finding the coordinates of the midpoint of *AB* were often successful in tying everything together. The majority of solutions were almost correct, but the explanation did not justify the statement that the plane bisects the plane at right angles; the most common problem was that candidates did not treat 'bisects' as a precise term and were satisfied by demonstrating that the line intersected the plane.

(ii) This proved to be one of the most difficult questions on the paper. Many candidates did not attempt to use the fact that the equation **r.n = a.n** gives you the distance of the plane from the origin if **n** is the unit normal vector. A few candidates reached the correct answer, sometimes via quite difficult routes. Many candidates did not appear to recognise that the equation of a parallel plane would be of the form $2x + 2y - z = d$

Answer: **(ii)** $2x + 2y - z = 2$

Question 7

(i) The work on the arithmetic of complex numbers was good, with many correct responses seen. Some marks were lost needlessly through arithmetic errors and poor use of brackets.

(ii) Very few candidates recognised that angle *AOB* is equal to the argument of $\frac{u}{w}$. There were some correct solutions using cosine rule, and some that treated the complex numbers as vectors and used scalar product. Some candidates lost marks because they used the trigonometry of a right angled triangle without demonstrating that they had a right angle triangle. The question is asking for proof of a result, so a complete proof is needed.

(iii) Many candidates had correct expressions for the complex numbers represented by *OB* and *CA* but often reached only one conclusion about the geometrical relationship.

Answers: **(i)** $-7 - \mathrm{i}$, $1 + \mathrm{i}$ **(iii)** parallel, $|CA| = 2|OB|$

Question 8

(i) Many candidates started with a correct expansion of $2\sin(x - 30°)$. In order to obtain an exact value of *R* they needed to substitute exact values for $\sin 30°$ and $\cos 30°$ and remember to subtract the $\cos x$. Most candidates demonstrated a clear understanding of the process for finding *R* and α , even if their final answers were not to the required level of accuracy.

(ii) Many candidates used their results from **part (i)** correctly to solve this equation.

Answers: **(i)** $R = \sqrt{7}$, α=49.11° **(ii)** 71.3°

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Question 9

(i) Finding the partial fractions was a straight forward task for most candidates. Many candidates showed little working, so if their solution was incorrect it was not possible to be sure whether or not they had a correct method.

(ii) Most candidates separated the variables correctly and attempted to use their partial fractions. The majority obtained terms of the correct form when they integrated, but the signs and coefficients on the x terms were sometimes incorrect. Most candidates had a correct method for determining the constant of integration and went on to try to combine terms. Many slips in the algebra and arithmetic meant that there were few correct final answers.

Answers: **(i)** $\frac{1}{3x} - \frac{2}{3(2x+3)}$ **(ii)** 1.29

Question 10

(i) The first mark in the integration by substitution was for stating $\frac{du}{dx} = -\sin x$. Many candidates did not attempt this step. Of those who did, the notation was often very poor and left a lot to the imagination of the examiner. The second key step was to use the double angle formula to express $\cos^2 2x$ in terms of $\cos x$ and complete the substitution. Although many candidates completed the trig. substitution successfully, they did not combine it with a full substitution in the integral. In the substitution of limits, candidates needed to use exact values, not decimal equivalents, to reach an exact answer. Fully correct solutions were unusual.

(ii) Most candidates who offered an attempt to answer this question did apply the product rule and chain rule. Many obtained a correct expression for the derivative, and indicated that they intended to solve $\frac{dy}{dx} = 0$. Rather than take out common factors, many candidates started by expanding their derivative, obtaining a long and complicated expression. With care, this could simplify to a correct value for a trig. ratio and a correct final answer.

Answers: **(i)** $\frac{1}{15}\left(7 - 4\sqrt{2}\right)$ **(ii)** 0.32

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MATHEMATICS

Paper 9709/32
Paper 32

General comments

The standard of work on this paper varied considerably and resulted in a wide spread of scores from zero to full marks. Most candidates made good progress with the early questions, and the majority offered solutions to all questions.

While many candidates did well in questions which were similar to tasks they had seen before, they did not necessarily have the skills to adapt their knowledge to something slightly different. In the longer questions, candidates made good progress with the trigonometric identity and calculus (**Question 7**), the partial fractions and binomial expansion (**Question 8**), and use of the product rule and integration by parts (**Question 10**). Those questions that were less well done were solving the variable separable equation (**Question 5**), complex numbers (**Question 6**) and vector geometry (**Question 9**).

Much of the work was clearly set out and easy to follow. Candidates who erase their initial attempt or use a pen to overwrite work originally done in pencil should be aware that the process of scanning the work picks up all versions with equal clarity, producing work that can be very difficult to read, it would be better to put a line through the original work and rewrite it further down the page.

When a question expressly forbids the use of a calculator (as in **Question 6**), or asks for the answer to be given as an exact value (as in **Question 7(iii)** and **Question 10(iii)**) then candidates should be aware that calculator methods will not gain credit. Similarly, when a result is given (as in **Question 7(i)**, **Question7(ii)** and **Question 10(i)**) then full and clear working is expected.

The failure to use brackets correctly was a common problem. While many candidates do recover, with their subsequent work implying that the brackets existed in their minds, this is sometimes not the case and marks are thereby lost unnecessarily.

Where numerical and other answers are given after the comments on individual questions that follow it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on Individual Questions

Question 1

Many candidates made a strong start, manipulating the logarithms correctly and obtaining a correct expression for x. Several of these candidates lost the final marks through rounding incorrectly or by failing to reject the negative root. Some candidates had little or no idea how to manipulate logarithms and exponentials; some of these earned a mark for the use of $2\ln x = x^2$. The false statement $\ln(x^2 + 1) = \ln x^2 + \ln 1$ was a common error.

Answer: 0.763

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Question 2

The most efficient approach to this question is to start with a sketch, as this gives a clear indication of where the solutions lie. The most popular approach – squaring both sides to obtain a non-modular equation or inequality, and solving the resulting quadratic equation, usually led candidates to the correct critical values. Very few then went on to reject $x < \frac{1}{2}$. Common errors included slips in the arithmetic, errors in squaring brackets, or squaring only the modular side of the inequality.

Answer: $x > \frac{7}{4}$

Question 3

(i) The majority of candidates expressed $\cot\theta$, $\tan\theta$ and $\sin 2\theta$ correctly in terms of $\cos\theta$ and $\sin\theta$ and went on to form a quartic in $\cos\theta$. The most common errors were slips in the arithmetic and algebra, which prevented several candidates from reaching a correct final answer.

(ii) Solving the equation posed many difficulties for candidates. Some correctly identified the equation as a quadratic in $\cos^2\theta$, but did not then go on to find the square root of their solution to find $\cos\theta$. Many candidates considered only the positive square root and consequently did not find any solution in the given range. A significant number of candidates rewrote the equation as $\cos^2\theta\left(2\cos^2\theta + 1\right) = 2$ and proceeded to set each factor equal to 2, demonstrating a significant weakness in basic algebra.

Answers: **(i)** $2\cos^4\theta + \cos^2\theta - 2 = 0$ **(ii)** $152.1°$

Question 4

(i) Most candidates used the chain rule correctly to express $\frac{dy}{dx}$ in terms of *t*. The most common error was to obtain $\frac{d}{dt}\ln(2t-1) = \frac{1}{2t-1}$. It was fortunate for many candidates that full marks were available for a correct unsimplified answer, because there were several slips in the attempts to simplify answers. A small number of candidates tried unsuccessfully to rewrite the parametric equations as a cartesian equation before differentiating.

(ii) While the method for forming the equation of a straight line was well understood, a significant number of candidates found the equation of the tangent, not the normal. Candidates who did not give their final answer in the form requested did not score the final mark.

Answers: **(i)** $\frac{8t-2}{2t(2t-1)}$ **(ii)** $x + 3y - 14 = 0$

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Question 5

(i) Although some candidates answered this question efficiently and accurately in just a few lines, many made things complicated for themselves by not adopting an easy approach. It was common to see factors such as $\frac{-1}{0.2}$ retained without simplification and equations with both sides negative were not tidied up. There was frequent misreading of $1+t^2$ for $(1+t)^2$, with some candidates alternating back and forth between the two versions in the course of their solution. Some candidates integrated $\frac{1}{(1+t)^2}$ incorrectly to obtain a ln function. Some candidates attempted to split the fraction into partial fractions. Several candidates never formed a differential equation in *y* and *t* and made no progress at all.

(ii) Few candidates gained both marks. Of those who did understand what was required, many lost the mark for the limiting value of *B* by giving it as a decimal approximation when the question asks for an "exact value". There were many vague statements about the mass of *A* such as "gets smaller", "decreases", which gained no credit as they do not indicate that the mass of *A* tends to zero for large *t*.

Answers: **(i)** $\ln y = \frac{2}{1+t} - 2 + \ln 100$ **(ii)** $A \to 0$, $B \to 100e^{-2}$

Question 6

(i) The most successful responses to this question started by using the given root together with its conjugate and forming the quadratic factor $x^2 - 4x + 5$. Combined with algebraic division, this approach often reached the correct conclusion. The majority of candidates started by substituting the given root to form a complex equation in *a* and *b*. Only a few went from this stage to considering the real and imaginary parts of the equation. Most also substituted $x = 2 + \mathrm{i}$ to form a second equation in *a* and *b* and attempted to solve their two equations as simultaneous equations. A large number of algebraic and arithmetic slips meant that this approach was rarely successful. The question clearly states that "**the use of a calculator is not permitted**", so solutions with clear evidence of this use (e.g. using the modulus argument form of the root or stating $(2-\mathrm{i})^3 = 2 - 11\mathrm{i}$ with no working shown) did not score marks.

(ii) Over all, the quality of the Argand diagrams seen this time was an improvement on previous sessions, with many fully correct solutions. Most candidates did use the clear space on the question paper for their sketch, although a minority attempted to draw an accurate diagram on graph paper. Most candidates achieved roughly equal scales on both axes and drew a circle of the correct dimensions in the correct position. Many candidates recognised the second locus as a straight line, however some lines were drawn at $z = -\mathrm{i}$ and some did not look as if they were intended to be parallel to the real axis. A small number of candidates with correct diagrams then selected the wrong region.

Answer: **(i)** $a = -2$, $b = 10$

Question 7

(i) This is a task that has been set before and many candidates gave clear, correct demonstrations of the given result. Those candidates who said that $\frac{1}{\cos\theta} = \sec\theta$ and hence the result follows as a known result scored no marks. A small number of candidates started with $\frac{1}{\cos\theta} = \frac{\tan\theta}{\sin\theta}$ and used differentiation of a quotient. This was a more complicated approach than that expected, but was accepted if the candidate completed it successfully.

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(ii) Candidates were equally divided over whether to approach this identity from the left hand side or from the right. Both approaches were equally successful and several correct proofs were seen. Some candidates working from right to left obtained the quadratic numerator $\sin^2\theta + 2\sin\theta + 1$ and did not recognise this as a perfect square. A small number of candidates worked from both sides and their work met in the middle. Here again, many errors were caused by algebraic slips rather than by lack of a correct approach.

(iii) Several candidates saw the links between the three parts of this question and gave concise, correct answers. The question asks for an exact solution, so a decimal approximation was not acceptable. The most common error here was that candidates could not integrate $\sec\theta\tan\theta$ – they often offered a complicated expression involving logarithms.

Answer: **(iii)** $2\sqrt{2} - \frac{1}{4}\pi$

Question 8

(i) Most candidates started by assuming a correct form for the partial fractions, and many of these scored full marks. Those who started by setting $x = -\frac{2}{3}$ and finding $A = 2$ usually had a fully correct solution. Others who set up 3 equations in 3 unknowns were more likely to get lost in complicated algebra and make an error along the way. A minority of candidates started with an inappropriate form for the partial fractions and made little progress.

(ii) Many candidates expanded $(1+\frac{3}{2}x)^{-1}$ and / or $(1+\frac{1}{5}x^2)^{-1}$ correctly. Several candidates were able to provide accurate solutions, and those who had incorrect coefficients from **part (i)** often scored 4 out of the 5 marks available. The most common error was to mishandle the extracted factors of 2 and 5. A small number of candidates continue to spoil their final answer by multiplying through to achieve integer coefficients.

Answer: **(i)** $\frac{2}{3x+2} + \frac{x-3}{x^2+5}$, **(ii)** $\frac{2}{5} - \frac{13}{10}x + \frac{237}{100}x^2$

Question 9

(i) Although very straight forward, this question was not similar to anything that the candidates had seen on recent papers, and many of them struggled to find a correct approach. Those candidates who did form an appropriate scalar product often made a sign error in their solution, and obtained $+\frac{5}{2}$ as the value of the parameter. Some candidates found the vector from *A* to the foot of the perpendicular, but not the position vector of the point. Many candidates made no attempt to find the position vector of the reflection of *A*. Of those who did try to find it, several showed little understanding of where it would be, often giving the negative of the position vector of *A*, or the negative of the position vector of their foot of the perpendicular.

(ii) Only a minority of candidates identified two correct directions on the plane, or two correct points on the plane. Many thought that the point *A* would be on the plane. Very few recognised that if the plane passes through the origin then $d = 0$. The most common approach was to use the vector product, although there were correct attempts to find the ratio of *a*, *b* and *c*.

(iii) The majority of attempts used the formula for distance of a point from a plane, rather than the equivalent approach of using the distance between parallel planes. The most common error in applying the formula for distance of a point from a plane was to use the modulus of $\mathrm{i} + 2\mathrm{j} + 4\mathrm{k}$ in the denominator rather than the modulus of their normal vector.

Answers: **(i)** $2\mathrm{i} + \mathrm{j} + 2\mathrm{k}$, **(ii)** $x + y - z = 0$, **(iii)** $\frac{1}{\sqrt{3}}$

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Question 10

(i) Most candidates started with a correct use of the product rule for differentiation. Apart from a few with errors in their constants, most reached the stage of stating $\tan 2p = \frac{1}{p}$ and went on to reach the given answer.

(ii) Most candidates selected an initial value between 0 and $\frac{1}{4}\pi$ and applied the iterative process correctly. There is clearly some confusion over the level of accuracy required, with several candidates not working to at least the 4 decimal places specified in the question. Some candidates with correct working truncated their final answer to give 0.53, rather than the rounded value 0.54. A small number of candidates were clearly working in degree mode and scored no marks.

(iii) Most candidates recognised the need to use integration by parts. The most common errors were to see 2 in place of $\frac{1}{2}$, or sign errors. A number of candidates integrated $\cos 2x$ to obtain $\sin x$. Several candidates did complete the integration successfully. There was often no evidence that both limits had been used in reaching the final answer, and many answers were given as decimals despite the question asking for an exact answer. A small number of candidates attempted to find the volume, not the area.

Answers: **(ii)** 0.54, **(iii)** $\frac{1}{32}\left(\pi^2 - 8\right)$

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MATHEMATICS

Paper 9709/33
Paper 33

General comments

The standard of work on this paper varied considerably and resulted in a wide spread of scores from zero to full marks. The questions or parts of questions that were generally done well were **Q1** (trigonometry), **Q3** (solving disguised quadratic equation), **Q4** (integration by parts), **Q6(iii)** (iteration), **Q7(i)** (finding minimum point), **Q8(ii)** (solution of differential equation), **Q9(i)** (partial fractions), **9(ii)** (evaluation of definite integral), **Q10(i)** (intersection of two lines) and **10(ii)** (vector equation of plane). Those that were done less well were **Q2** (binomial expansion), **Q5** (chain rule and reformulation of result in terms of tan *x*), **Q6(i)** (position of root), **6(ii)** (convergence of iterative formula), **Q7(ii)** (trapezium rule), **7(iii)** (explanation of errors in numerical integration), **Q8(i)** (explanation of formulation of differential equation), **Q11(a)** (solution of pair of simultaneous equations involving two complex unknowns) and **11(b)** (understanding the meaning of the product of two complex numbers).

In general the presentation of candidates' work was good, although this was not the case in **Q11(a)**. The inability to distinguish 'i's from '1's and 'z's from '2's in their own handwriting led some candidates to make multiple errors. Likewise there was often bad mathematical notation with ln *x* appearing more like ln ^{x}. Since candidates' scripts are now scanned for marking, legible handwriting is even more important. For clarity, it is essential that candidates write with a black/dark blue pen since other colours or pencil markings do not scan so well.

It was pleasing to see that candidates had taken on board many of the points in previous reports, in particular the necessity of showing sufficient working to justify their steps, whether or not they are working towards a given answer. Examples in this paper were solving a quadratic equation and substituting limits into integrals (**Q3**, **Q4** and **Q9(ii)**).

However, there were two points in previous reports which would benefit from repetition. Firstly, if a question clearly requests an exact value, as in **Q4**, a decimal answer is not acceptable. In questions of this type, a method that is only possible with the use of the calculator receives no credit. Likewise if candidates take a suitable approach but then introduce decimals, from that point onwards no further marks are possible. The other point relates to accuracy:– in questions demanding a specific level of accuracy, candidates are required to show this in their answers (see **Q6(iii)** and the final part of **Q7(ii)**).

Q11 of this paper was a non-calculator question, with candidates given clear instructions not to use a calculator. This was because the question was testing their understanding of the methods and ability to demonstrate they could apply them in problem-solving. They were therefore required to show all steps in their working, and this is discussed in more detail under the Specific Questions section of this report.

In this paper, around half the questions were testing whether candidates could apply correctly a basic mathematical technique, for example in **Q4** (integration by parts), **Q7(i)** (determine minimum point), **7(ii)** (trapezium rule), **Q8(ii)** (solution of differential equation using separation of variables), **Q9(i)** and **(ii)** (partial fractions and definite integral) and **Q10(i)** and **(ii)** (intersection of lines and formulation of planes). In the remaining questions, candidates were required to think carefully and choose the appropriate line of attack. **Q1** illustrates this: candidates should have looked at the RHS of the expression, realised that cos2*x* was an expression that they knew in terms of cos *x* and sin *x*, and therefore taken the logical approach of converting the LHS into such terms, with the proof following in a couple of lines. Instead many decided either to square the expression or to multiply the numerator and denominator by the numerator or the denominator. In both cases these led to large amounts of un-necessary algebra, and little chance of avoiding errors, making it difficult to reach the required result. This poor choice of strategy will be discussed in detail in the Specific Questions section.

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Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on specific questions

Question 1

Many candidates proved the required statement by converting to sin x and cos x and using the Pythagoras relationship between $\cos^2 x$ and $\sin^2 x$. Since this is a proof, such a relationship should have been clearly stated and not just left for the examiner to assume. The other sensible approach was to convert all the trigonometric quantities into tan x, and then use the Pythagoras relationship involving $\tan^2 x$ and $\sec^2 x$. Whilst other successful lines of attack were seen, they usually required more time to succeed and often rambled to the correct expression. Starting from the RHS was generally unsuccessful.

Question 2

There were many correct solutions, however some candidates then spoilt their excellent work by multiplying their correct answer to eliminate fractions. Many knew that the easiest approach was to extract 3 from the bracket and expand $(1+(2x/3))^{-3}$. Usually at least the first couple of terms were correct in their unsimplified and simplified forms. However, the extracted term caused problems: a variety of terms of the form 3^n were seen, where $n = 0, \pm1, \pm2, \pm3$. Even if it had been correctly extracted as 3^{-3} this was sometimes then written as 27.

Answer: $\frac{1}{27} - \frac{2}{27}x + \frac{8}{81}x^2$

Question 3

The majority of candidates scored full marks, although a few candidates stopped too early, once when they had $e^x = 2/3$. A very limited number of candidates failed to realise that the equation could be reformulated as a quadratic equation in u. Most candidates followed the instruction to give their final answer to 3 significant figures. It was disappointing to see some candidates going straight from their quadratic equation to the solution, hence risking losing the method mark should any error occur.

Answer: $x = -0.405$

Question 4

Whilst most candidates were familiar with the method of integration by parts, some were evidently confused about integrating and differentiating trigonometric functions. Errors in the use of the chain rule also caused some problems with the coefficients throughout the integrals. Candidates should indicate that they have considered substituting the lower limit, even if, as in this question, the terms are both evaluated as zero. Errors in substitution of the upper limit were also often present. As mentioned in the General Comments section, some candidates interpreted 'exact answer', to mean a decimal rather than the correct version including a surd. Fractions should always be simplified where possible in the final answer.

Answer: $(4 - \pi)/\sqrt{2}$

Question 5

(i) Although many candidates could differentiate the ln function correctly using the chain rule, either by finding the derivative of $1 + 3\cos^2 x$ or by converting the expression into one involving cos $2x$, many found it challenging to progress to an expression in tan x as the question demanded. Converting to a different trigonometric function is not a sensible approach as it often leads to errors even before the differentiation process has commenced. Poor manipulation of ln was seen, with candidates attempting to split the function incorrectly or modify it, introducing errors. Writing the derivative in terms of tan x should have been relatively straight forward, had candidates noted that in the numerator they would need to divide by $\cos^2 x$ and a similar operation to the denominator introduces $\sec^2 x$. The use of Pythagoras involving $\sec^2 x$ and $\tan^2 x$ then completes the process. By looking at what is required candidates can avoid moving further away into tan$2x$ and becoming overwhelmed by the complexity of the expression.

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(ii) Few candidates had a suitable expression from **(i)** to enable them to proceed to solving a 3-term quadratic in this part, although it was quite possible to solve from the initial correct derivative in terms of sin x and cos x.

Answers: **(i)** $dy/dx = -4\tan x/(4 + \tan^2 x)$ **(ii)** $x = 1.11$

Question 6

(i) Normally such questions require candidates to show that a root lies between two stated values, but here the question required candidates to show that the root α exceeded 2.5. This was a question where candidates needed to think what to choose for the other value and the domain of the function provided the clue. In spite of the clear indication that one value should be 2.5 and the other in the domain above 2.5, it was common to see values such as 0, 2, 2.4, 3.5 being substituted. In fact a considerable number of candidates made no attempt at this section or at **(ii)**.

(ii) Most candidates who attempted this realised that a consistent limit was required, whether x or, more correctly, α. The real difficulty in the question was either the removal of π if they started from the iterative formula or the introduction of π if they started from the equation. Few candidates performed this task adequately. Going from the iterative formula, they needed to expand $\tan(x - \pi)$ while starting from the equation they needed to state that tan x has a period of π.

(iii) Candidates usually scored full marks here, although a few used tan x instead of $\tan^{-1}x$ in their iterative formula. There were the usual errors of failing to show convergence, working only to 4 decimal places or giving the final answer to 2 decimal places.

Answer: $\alpha = 2.576$

Question 7

(i) This question was answered extremely well. The few errors seen related to the omission of the denominator in the quotient formula, muddled signs in the product rule or taking ln of both sides and applying the law for $\ln(ab)$ incorrectly.

(ii) Many candidates scored full marks, however some had serious errors, such as 6 ordinates, incorrect step size or incorrect formula. Most candidates correctly rounded their answer to 2 decimal places.

(iii) Few candidates were able to explain whether the estimate with two intervals would be greater than or less than the estimate with four intervals. Many omitted the question, others just made a guess without any reasoning. Candidates should have either drawn the curve and two trapezia or stated that the curve was 'convex' hence E would be an overestimate, then shown the diagram with four trapezia or explained that the next estimate would be more accurate or closer to the curve, resulting in another estimate being less than E.

Answers: **(i)** $x = 2$ **(ii)** 2.93

Question 8

(i) Candidates either omitted this question or rewrote in terms of the masses A and B. The question required the two statements involving the masses to be translated into the variables x and y, i.e. $x + y = 50$ and $dx/dt = ky$ before combining them to produce the given equation.

(ii) Many candidates tackled this question extremely well. Only a few candidates were unable to separate the variables correctly. The errors that did arise were due to misplacing of the constant k, incorrect sign in the $\ln(x - 50)$ term, or poor ln work when trying to express x in terms of t.

Answer: $x = 50(1 - 2^{-t/10})$

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Question 9

(i) The partial fraction work was of a high quality, with many candidates producing a perfect solution. The usual arithmetical errors were seen when determining the constants. Other errors seen were the omission of the term A/x or its inclusion twice, once as A/x and again in $(Bx + C)/x^2$, so producing a non-unique partial fraction unless rectified at the end. A few candidates incorrectly introduced a constant term within their partial fraction, but fortunately most were able to recover by finding that the value of this constant was zero.

(ii) This question was answered well by many candidates. Occasionally the $1/x^2$ was integrated incorrectly; otherwise errors were mainly restricted to incorrect coefficients of terms of the correct form. Again it should be stressed that candidates needed to show all the detailed steps in manipulating the ln terms to justify the given answer**.**

Answer: **(i)** $\frac{3}{x} - \frac{2}{x^2} - \frac{6}{3x+2}$

Question 10

(i) Most candidates found this question very straight forward. However, a few candidates used the vector ***AB*** instead of the equation of the line to find the point of intersection. Occasionally in establishing the equation of the line candidates muddled the position of the parameter, including it with the point *A* or *B,* instead of with ***AB***.

(ii) While many candidates were successful in this section, others made serious mathematical errors, such as evaluating the vector product using two vectors that did not lie in the plane. Sign errors in the vector product were surprisingly common, sometimes in all three components.

Answers: **(i)** $m = 3$ **(ii)** $2x - y + z = 6$

Question 11

(a) The majority of candidates did not perform well on this question. There were many reasons for this, and the need for careful presentation has already been mentioned as one contributing factor. Some candidates set both z and w to $x + \mathrm{i}y$. Some failed to realise, in their comparison of real and imaginary parts, that z and w were complex numbers. Only a minority of students managed to complete this question to gain full marks, although nearly all who attempted it gained some credit, usually the mark for $\mathrm{i}^2 = -1$.

This question has a clear rubric stating that the use of a calculator is not permitted. Hence candidates who obtained an equation for either z or w and then simply produced two correct answers scored just the first method mark while those who reached a correct fractional expression for either of these quantities and then gave two correct answers fared a little better. Since they had failed to show the working required to multiply by the conjugate of the denominator, despite producing correct answers for z and w, the final three marks were unavailable.

Given that the use of the calculator is not permitted, taking real and imaginary parts of two equations in two unknowns, resulting in four equations in four unknowns, is not a very sensible approach. However, taking real and imaginary parts of a single equation in one unknown, is a possible approach, resulting in two equations in two unknowns for w or z. Candidates should have solved the two linear equations for z or w, obtaining an expression for either z or w in the form $(a + \mathrm{i}b)/(c + \mathrm{i}d)$, then undertaken the necessary work to convert this to $x + \mathrm{i}y$. It should be noted that there were a couple of perfectly correct different expressions for z, and four, possibly more, for w, that appeared from different ways of solving the equations. A few candidates who found the 'correct' solutions then lost a mark because they failed to leave their answer in the required $x + \mathrm{i}y$ form.

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(b) The rubric clearly stated that calculators were not permitted **throughout this question**. Hence to apply Pythagoras, together with the equation of the straight line *BC*, or the scalar product of the vectors ***AB*** and ***BC***, with all the arithmetical detail needed for this coordinate geometry approach, was not really appropriate. It was extremely unlikely to lead to a correct solution, or even any solution.

Candidates need to understand the ideas behind the multiplication of two complex numbers, in addition to being able to perform the technique. As mentioned in the General Comments section this was one of the questions requiring candidates to do more than just undertake a simple procedure. Multiplying one complex number by another stretches the initial complex number by the modulus of the second complex number as well rotating it through the argument of the second complex number. Hence multiplying the complex number $v - u$ by 2i doubled its length and rotated it through $\pi/2$, so producing the complex number $z - v$ from which z immediately follows.

Answers: **(a)** $w = -\frac{1}{5} + \frac{2}{5}\text{i}$, $z = \frac{3}{5} + \frac{4}{5}\text{i}$ **(b)** $4\sqrt{3} - 1 + 6\text{i}$

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MATHEMATICS

Paper 9709/41
Paper 41

General comments

There were some excellent candidates who produced very good answers on this paper. Overall a wide range of performance was seen but the paper was generally well answered.

Some students lost marks due to not giving answers to three significant figures as requested and also due to prematurely approximating within their calculations leading to the final answer. This was often seen in **Questions 2**, **3** and **7**. Students should be reminded that if an answer is required to three significant figures then their working should be performed to at least four significant figures. In **Question 4** the sine of an angle was given in the question. In such questions it was not necessary to determine the actual angle to 1dp as this often leads to premature approximation and frequently also to loss of accuracy marks.

One of the rubrics on this paper is to take $g = 10$ and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve a correct given answer unless this value is used.

Comments on specific questions

Question 1

Since it was not stated in the question that air resistance was constant, this problem should be solved using work-energy principles. The initial potential energy can be found and during the motion of the particle this is transformed into kinetic energy and the work needed to overcome air resistance. This can be expressed using the work-energy equation in the form PE loss = KE gain + WD against air resistance. Many candidates did in fact assume that air resistance was constant and found the acceleration of the particle and the magnitude of the air resistance force using Newton's second law of motion before using the definition of work done in the form WD = Force $\times$ distance to evaluate the required work done against air resistance. Full marks were only available to candidates who used work-energy principles.

Answers: Work done against air resistance is 18 J

Question 2

(i) Most candidates started this question correctly using Newton's second law applied to the particle in order to find the required acceleration. It was first necessary to resolve forces perpendicular to the plane in order to find the normal reaction and then apply $F = \mu R$ to establish the frictional force. Resolving forces acting on the particle parallel to the plane involved two forces, the component of the weight and the friction force. Combining these two forces using Newton's second law gives the required acceleration. Some candidates forgot to include the component of the weight and others used $R = 0.8\,g$ when finding the friction force. In fact, as the motion is taking place on an inclined plane the correct form is $R = 0.8\,g \cos 10$. This was a question in which some candidates did not work with enough significant figures and produced answers which were incorrect due to prematurely approximating, particularly with their trigonometric values.

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(ii) Almost all candidates chose the correct method here even if they had found the incorrect acceleration in **2(i)**. Use of the constant acceleration equations with $u = 12$, $v = 0$ and the value of the acceleration found in **2(i)** gave the value of the distance travelled up the plane before coming to rest. Although it is possible to solve this question by using work-energy methods, it is a much longer method and very few candidates attempted to do this.

Answers: **(i)** The acceleration of the particle is $-5.68\ \text{ms}^{-2}$ (to three significant figures) **(ii)** Distance moved up the plane before coming to rest is 12.7 m

Question 3

In this question it was first necessary to realise that as the system is in equilibrium, the tension in the left hand string is A N and the tension in the right hand string is B N in order for the two particles to be in equilibrium. As the tension is the same throughout each string, one method of approach is to consider the vertical and horizontal balance of forces acting on the 25 N particle. These give two simultaneous equations in A and B, and solution of these gives the required values of A and B. An alternative method which many used, was Lami's theorem applied to the three forces acting on the 25 N particle, namely A, B and 25 with the relevant angles between the forces. Candidates who used Lami's theorem generally performed better than those using the method of resolving forces.

Answers: $A = 17.1$, $B = 13.3$ (to three significant figures)

Question 4

(i) Candidates generally performed well on this question. Most knew the correct definitions of gravitational potential energy and kinetic energy and were able to apply these definitions to the given problem. The angle was given in terms of $\sin\theta$ and it was not necessary to actually find θ but many did this and in doing so lost some accuracy in their calculations. Another error that occurred occasionally was that rather than use the correct definition of change in KE as $\frac{1}{2}m\left(v^2 - u^2\right)$ use of the incorrect formula $\frac{1}{2}m(v-u)^2$ was seen. Overall some very good answers were seen here.

(ii) Most candidates realised that it was necessary to use the definition of power as rate of doing work which in this case leads to WD by the engine = 32 000 × 12. Once this had been found it was necessary to apply the work-energy equation in the form WD by the engine = KE gain + PE gain + WD against resistive forces. By using the values found in **4(i)** the required WD against resistive forces can be found. Some candidates applied the correct method but had some sign errors in their work-energy equation.

Answers: **(i)** Change in kinetic energy = 52 800 J Change in gravitational potential energy = 144 000 J
(ii) The total work done against the resistive forces during the 12 seconds is 187 200 J (187 000 to three significant figures)

Question 5

(i) Many candidates produced good solutions to this question. As it was given in the question that acceleration is constant and the velocities and distances were involved, it was necessary to use the constant acceleration equation $v^2 = u^2 + 2as$. Some attempted to use equations involving time but made the error of assuming that the time taken to travel between A and B and between B and C were the same. Application of the correct equation applied to the motion between A and B and for the motion between B and C produced values for the distances AB and BC in terms of the unknown acceleration. By dividing these two expressions the required ratio can be found.

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(ii) Although it was possible to use the results of **5(i)** in order to solve this problem, many candidates started again and most correctly found the acceleration by considering the motion from *A* to *D* using the given distance of 80 m and the given initial and final velocities of 20 and 0 respectively. Once the acceleration had been found, the required distance, *BC*, can be found by application of the constant acceleration formulae between points *B* and *C*.

Answers: **(i)** *AB* : *BC* = 64 : 27 **(ii)** The distance *BC* is 21.6 m

Question 6

(i) Most candidates performed well on this question. Firstly it was necessary to use the given information to determine the values of the constants *q* and *r* as *q* = 6 and *r* = –2 and most candidates who attempted this question found these values correctly. This gave an expression for the velocity as $v = 6t - 2t^2$ and most realised that it was necessary to differentiate this to find the acceleration. Since the answer is given in this question, it is particularly important to show all working leading to the answer in order to gain full marks. Clear substitution of *t* = 0.5 into the expression for acceleration gives the required result. A few candidates wrongly chose to integrate their expression for velocity in order to find acceleration.

(ii) Here it was necessary to use the expression found in **6(i)** for velocity and set this equal to zero in order to find the values of *t* when the particle is at instantaneous rest. Most who had found an expression for *v* attempted this question correctly. One error seen was that some candidates only gave one of the two values, usually forgetting the value of *t* = 0.

(iii) Most candidates who attempted this part knew that they needed to integrate their expression for *v* found in **6(i)** in order to find an expression for the displacement. In this case it is important when integrating to include a constant of integration so that the given condition can be applied. Equating the displacement to zero when *t* = 3 gives the value of the constant of integration. By substituting *t* = 0 into the expression for displacement gives the displacement at *t* = 0 as –9 and hence the required distance can be found.

Answers: **(i)** Acceleration at *t* = 0.5 is 4 ms^{-2} (Answer given) **(ii)** The values of *t* when *P* is at instantaneous rest are *t* = 0 and *t* = 3 **(iii)** The distance of *P* from *O* when *t* = 0 is 9 m

Question 7

(i) This question involves connected particles, each on an inclined plane and as there is no friction involved, the best method is to apply Newton's second law parallel to the plane in each case. For both particles there are two forces acting, the component of the weight and the tension. The tension is the same throughout the string but some wrongly used different tensions acting on each particle. When the two equations of motion are stated, they produce two equations each involving the tension *T* and the acceleration *a*. These equations can now be solved simultaneously for the two required values of *T* and *a*. The main cause of error was candidates using the wrong component of weight parallel to the plane such as 1.2 *g* cos 60 for particle *B* or even not using a component but using 1.2 *g* parallel the plane for *B*, and equivalent errors at *A*.

(ii) In this part it is first necessary to resolve forces perpendicular to each plane in order to determine the normal reaction in each case so that the friction forces can be found using $F = \mu R$. The two normal reactions are different but some wrongly took them to be the same as one another. The forces acting parallel to the plane for both *A* and *B* are in equilibrium and in both cases these forces consist of the weight component, the tension in the string and the friction term. The two equations will involve the unknown tension *T* and μ. Solving these simultaneously and eliminating *T* gives the required value of μ. One error which was often seen was that candidates wrongly assumed that the tension in this case was the same as that found in **7(i)**. Similar errors with components were seen here as were highlighted in **7(i)**.

Answers: **(i)** The acceleration of *A* is 3.20 ms^{-2} (to three significant figures) The tension in the string is 6.56 N (to three significant figures) **(ii)** The value of μ for which the system is in limiting equilibrium is 0.494 (to three significant figures)

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MATHEMATICS

Paper 9709/42
Paper 42

General comments

The paper was generally well done by many candidates although as usual a wide range of marks was seen.

The presentation of the work was good in most cases and as the papers are now scanned, it is important to write answers clearly using black pen.

Some candidates lost marks due to not giving answers to three significant figures as requested and also due to prematurely approximating within their calculations leading to the final answer, particularly in **Questions 4**, **5** and **6**. Candidates should be reminded that if an answer is required to three significant figures then their working should be performed to at least four significant figures.

In **Question 6** the sine of a required angle was given. In this question it was not necessary to calculate the angle itself as the sines and cosines required could be evaluated exactly. However, many candidates often proceeded to find the relevant angle to one decimal place and immediately lost accuracy and in some cases marks.

One of the rubrics on this paper is to take $g = 10$ and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve a correct given answer unless this value is used.

Comments on specific questions

Question 1

Most candidates first found the distance travelled as $12 \times 1.5 = 18$ m and then used the definition of work done as WD = $20 \cos \theta \times 18$ and equated this to 50 and solved for θ. Another method was to use the definition of power as Power = Work Done/time = 50/12 and then use the equation $P = Fv$ with $F = 20 \cos \theta$ and solve for θ. Either method was perfectly acceptable. Most candidates scored well here. One common error was to use $F = 20 \sin \theta$ instead of $20 \cos \theta$

Answers: $\theta = 82.0$

Question 2

(i) Almost all candidates scored this mark. Since the answer was given then it is particularly important to show all working. One method is to use the constant acceleration equation $v^2 = u^2 + 2as$ with $u = 0$, $a = 2.5$ and $s = 5$. Candidates must show this in the form $v^2 = 2(2.5)5 = 25$ in order to explain the given answer. Alternatively it can be shown that the time of travel from A to B is 2 seconds. This can be determined by using the equation $s = ut + \frac{1}{2}at^2$ with $s = 5$, $u = 0$ and $a = 2.5$ and then using $v = u + at$ with $u = 0$, $a = 2.5$ and $t = 2$ to show the given result.

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(ii) **(a)** Many candidates attempted to use the constant acceleration equations in this problem when in fact this is not possible. This is because in any problem where a particle is constrained to move along a curve, the acceleration is not constant. The only method available, within the bounds of the syllabus for this paper, when solving problems of motion along a curve is the use of the work-energy principle. Measuring potential energy from *B*, the PE loss between *B* and *C* is PE loss = 0.2 $\times$ 10 $\times$ 6 sin 30 = 6. The gain in KE as the particle moves from *B* to *C* is KE gain = $\frac{1}{2}(0.2)(v^2 - 5^2)$, where *v* is the speed of the ring at C. As the wire is smooth, the work-energy equation takes the form PE loss = KE gain and this will produce the required value of *v*. If the point from which PE is measured is taken as *A* then it must be remembered that there is a frictional force acting between *A* and *B* and so the work done against friction must also be included. Very few candidates scored full marks on this question.

(ii) **(b)** Here once again it is not possible to use the constant acceleration equations and so once again the work-energy principle must be used. The greatest speed will occur at the lowest point of the semicircle. Using *B* as the reference point, the work-energy equation takes the form of 0.2 $\times$ 10 $\times$ 6 = $\frac{1}{2}(0.2)(v^2 - 5^2)$, where *v* is the speed of the ring at the lowest point. It is also possible to use either *A* or *C* as the reference point for PE provided the relevant modifications are made to the work-energy equation as before.

Answers: **(i)** The speed of the ring at *B* is 5 ms^{-1} (Answer given) **(ii)(a)** The speed of the ring at *C* is 9.22 ms^{-1} **(b)** The greatest speed of the ring is 12.0 ms^{-1} (to three significant figures)

Question 3

(i) There was some confusion in the answers given to this question. The time *t* that was referred to in the question was measured from when particle *B* passes through *O* but many used *t* as the time measured from when particle *A* passes through *O*. This often lead to an incorrect answer of 10*t* for the displacement of particle *A* and an incorrect factor of $t - 2$ in the expression for the displacement of particle *B*. Use of the constant acceleration formulae for each particle gives the required result.

(ii) For this part of the question it was first necessary to find the value of *t* at which particle *B* came to instantaneous rest. Most candidates found this correctly. Constant acceleration equations can be used to give $16 - 2t = 0$ and $t = 8$ as the value when particle *B* comes to rest. This value can now be substituted into the expressions found in **3(i)** and the value of the distance between the particles, $s_A - s_B$ can be found

(iii) Very few candidates managed to successfully complete this part of the question. There are a variety of different approaches which can be taken. The general expression for the distance between the particles at time *t* is $s_A - s_B = 10t + 20 - (16t - t^2) = t^2 - 6t + 20$ and this can be differentiated to find the value of *t* at which the maximum or minimum value occurs. Alternatively the method of completing the square on this expression enables the minimum value to be found very simply. Another approach is to use the fact that the minimum distance between the particles occurs when the speeds are the same, namely 10 ms^{-1} and the time when this occurs can be found. In all cases the critical time is at $t = 3$

Answers: **(i)** Displacement of particle *A* is $s_A = 10t + 20 = 10(t + 2)$. Displacement of particle *B* is $s_B = 16t - t^2$ **(ii)** Distance between the particles when *B* is at instantaneous rest is 100 – 64 = 36 m **(iii)** The minimum distance between the particles is 11 m

Question 4

(i) **(a)** Most candidates performed well on this question. As the car is moving at a constant speed the net force on the car is zero and so the driving force must balance the resistance force of 850 N. Use of the formula $P = Fv$ is then made to determine the required value of the power. Some candidates failed to give their answer in the required units

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(i) **(b)** In this part of the question, it was first necessary to add 6 kW to the value of the power found in **4(i)(a)** and then use this new value of P to determine the new driving force on the car. Newton's second law of motion can now be applied to the car. An error made by many was to forget to include the constant resistance force of 850 N when applying this equation to the car.

(ii) In this part of the question it was first necessary to determine the driving force acting on the car. Almost all candidates correctly found this by using the formula $F = P/v$. As the car is moving up the hill at a constant speed the driving force must balance the combination of the resistance force and the component of the weight of the car down the hill in the form $80\,000/24 = 850 + 1200\,g \sin\theta$. However, many again forgot to include the resistance in their calculations, while some used $\cos\theta$ instead of $\sin\theta$ as their component.

Answers: **(i)(a)** The power developed by the car's engine is 35.7 kW **(b)** The instantaneous acceleration of the car is $5/42 = 0.119\ \text{ms}^{-2}$ (to three significant figures) **(ii)** $\theta = 11.9$ (to three significant figures)

Question 5

In this question it is necessary to consider the two extreme cases when the particle is about to slip up the plane and also when it is about to slip down the plane. Although candidates performed well on this question, many only considered one of these cases. An error that was often seen was that candidates took the normal reaction as $0.12\,g \cos 40$ but when resolving forces perpendicular to the plane, in fact the normal reaction R is given by $R = 0.12\,g \cos 40 - P \sin 30$. This form of R can be used in both cases but it should be noted that the numerical value of R is different in each case since it depends on the value of P. It is then necessary to apply Newton's law for equilibrium parallel to the plane in the two cases and these only differ in the sign of the friction term μR, namely $P \cos 30 = 0.12\,g \sin 40 \pm \mu R$. This equation and the equation for R can now be solved simultaneously for each case giving the two extreme values of P. As the question asked for the range of values it was necessary to give the final answer in the form of an inequality and many lost the final mark due to merely quoting the two extreme values of P.

Answers: The set of possible values of P satisfies $0.676 \leqslant P \leqslant 1.04$ (to three significant figures)

Question 6

(i) The most straightforward method of approach to this question is to apply Newton's second law along the horizontal plane for particle A and along the sloping plane for particle B, using T, the tension in the string and a the magnitude of the acceleration of the particles. For particle A the only force acting along the horizontal plane is the tension, T. However, many candidates wrongly included the weight of the particle which in fact acts at right angles to this horizontal plane. When considering particle B, the forces acting along the plane are the tension T and the component of the weight along the plane, $1.5\,g \sin\theta$. Some candidates wrongly used the weight itself $1.5\,g$ and others chose the wrong component $1.5\,g \cos\theta$. The two equations can then be solved simultaneously for the required values of T and a.

(ii) The first part of this question required a simple application of the constant acceleration formulae using the given information to find the acceleration a, and most candidates found this successfully. Since both planes are now rough, the extra frictional force must be included when considering each particle. Since $F = \mu R$ must be applied in each case it is clear that the two frictional forces are different since the normal reactions are different. In order to find these forces it was necessary to resolve forces perpendicular to each plane in order to determine the normal reaction acting on each particle. When considering the motion of particle A along the horizontal plane, there are now two forces acting, namely the tension T and the frictional force F_A. For particle B the forces acting parallel to the sloping plane are the component of the weight, the tension T and the frictional force F_B. Many candidates wrongly assumed that the tension in the string was the same as that found in **Question 6(i)**. When Newton's second law of motion is applied to each particle using the value of the acceleration a which was found earlier, this produces two simultaneous equations in T and μ. Elimination of T between these two equations enables the required value of μ to be found.

Answers: **(i)** $T = 1.5$ N $a = 5\ \text{ms}^{-2}$ **(ii)** $\mu = 2/5 = 0.4$

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MATHEMATICS

Paper 9709/43
Paper 43

Key messages

Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates are also reminded to maintain sufficient accuracy in their working to achieve this level of accuracy in their final answers (e.g. **Question 2, Question 5(ii) and Question 6(i)(b)**).

When using integration, candidates are reminded of the need to consider the constant of integration as in **Question 4(ii)**.

General comments

The examination provided opportunities for candidates to show what they knew at all levels. Much work of a high standard was seen including many excellent and clearly presented scripts. **Question 2** and **Question 6(i)** were found to be the easiest questions whilst **Question 5(i)** and **Question 7(ii)** provided the most challenge.

Comments on specific questions

Question 1

(i) Although candidates usually knew that the work done by the 35 N force was 35 · 12cos20°, they frequently combined this with the work done due to the resistance. Thus 395 + 180 = 575 J and 395 – 180 = 215 J were both common incorrect answers for the work done by the man. There were also candidates who believed that the wheelbarrow was pushed down an incline rather than along a horizontal road with 420 J of work done.

(ii) This part of the question was answered better. The more popular method was to restart, find the acceleration and then apply one or more constant acceleration formulae (usually $v^2 = u^2 + 2as$) to obtain the speed after 12 m. Alternatively, those who used a work/energy equation sometimes continued with erroneous values from **Part (i)** e.g. ½ · $25v^2 = 575$.

Answers: **(i)** 395 J **(ii)** 4.14 ms^{-1}

Question 2

The majority of candidates found this to be a straightforward question and frequently gained full marks by resolving in two perpendicular directions and solving the resulting equations. When solving $2P\cos\theta = 3P\cos55°$, a surprising number of candidates substituted their approximate value for P into the equation rather than considering the solution of $2\cos\theta = 3\cos55°$. The value for P was sometimes given as 8.1 correct to two significant figures rather than the required three significant figure value.

Answer: 8.14 N 30.6°

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Question 3

(i) Although this was a sketch graph, candidates were expected to show a steeper line for the deceleration than the acceleration and also to indicate the relevant values on each axis. Some graphs looked symmetrical while a few clearly showed the first section steeper than the last.

(ii) This part of the question was more difficult. Some misunderstood 'the length of time' and attempted to find the distance travelled at constant speed. Ideally, candidates could write down the expression $180 - T - 1/2T$ using an understanding that the deceleration took half the time taken for the acceleration. In practice, some lengthy attempts were seen including the use of constant acceleration formulae and a variety of unknowns.

(iii) The shorter solutions formed an equation for the area of the trapezium, solved it to find T and used this to calculate the required area. More often candidates calculated the three areas separately first. Either way, a correct equation depended on a correct expression for time in **Part (ii)**. A number of candidates found the distance travelled when not decelerating (2900 m) rather than the distance whilst decelerating. A few believed that the deceleration took twice as long as the acceleration leading to $\frac{1}{2}(180 + 180 - 3T) \cdot 25 = 3300$ and a distance of 800 m.

Answers: **(ii)** $180 - 1.5T$ **(iii)** 400 m

Question 4

Most candidates recognised that calculus was needed for this question.

(i) **Part (i)** involved the use of $a = \mathrm{d}v/\mathrm{d}t$ and then the solution of $\mathrm{d}v/\mathrm{d}t = 54$. Those who differentiated using the chain rule and then formed the equation $6(2t - 5)^2 = 54$ sometimes found just one value for t from solving $2t - 5 = 9$ instead of solving $2t - 5 = \pm 9$. Those who expanded before differentiation usually obtained a quadratic equation with two solutions as expected. Other errors included differentiation to $k(2t - 5)^2$ with e.g. $k = 3$, or occasionally an attempt to include the use of a constant acceleration formula despite the variable acceleration.

(ii) Whilst candidates knew that $s = \int v\,\mathrm{d}t$, and were frequently able to integrate the given function correctly, those who found $s = 1/8(2t - 5)^4$, often omitted the constant of integration from their expression for the displacement or sometimes assumed incorrectly that C = 0. Some found the constant of integration to be + 625/8 instead of – 625/8. For the fewer candidates who expanded first, the constant of integration did not present a problem since C = 0 for $t = 0$ in this case. A few candidates believed that they needed to use their values of t from **Part (i)** and substituted to find numerical values for the displacement rather than an expression in t.

Answers: **(i)** $t = 1$ or 4 **(ii)** $s = (2t - 5)^4/8 - 625/8$

Question 5

This question was found to be more challenging mainly due to the two second time difference between the projection of the particles.

(i) A majority of candidates attempted to apply $s = ut - \frac{1}{2}gt^2$ to the two particles. Some candidates worked with $t_1 = t - 2$ instead of $t_1 = t + 2$. Others used t as the time taken for the first particle instead of the second, leading to $t = 15/7$ or 2.14. This was sometimes resolved by realising that $15/7 - 2 = 1/7$ was the required answer. Some candidates calculated the time and distance to the greatest height for the first particle or the displacement and velocity after two seconds, but were often unable to use these to solve the problem. A few were successful in starting the first particle at maximum height and considering $s_1 + s_2 = 7.2$, or in starting the particles after two seconds and considering $s_1 + s_2 = 4$. A number of candidates oversimplified, treating the motion as if the particles were travelling at constant speed, solving $12(t + 2) = 20t$ leading to $t = 3$.

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(ii) Whilst the method was straightforward, the displacement of 2.76 m was not common, sometimes because of previous errors in finding the time taken and sometimes due to premature approximation e.g. $s = 20 \cdot 0.14 - 5 \cdot (0.14)^2 = 2.70$. The weaker candidates who used constant speed in **Part (i)** continued with $s = vt$ to obtain e.g. $20 \cdot 3 = 60$ m.

Answers: **(i)** 0.143 **(ii)** 2.76 m

Question 6

This question was straightforward for many candidates and fully correct solutions were common in all parts.

(i) (a) Most candidates applied $P = Fv$ to obtain 400 N. The best solutions made it clear that the driving force and resistance were equal for equilibrium.

(b) Candidates were expected to apply Newton's Second Law to find the acceleration. A few applied $P = mav$ ignoring the resistance. Occasionally 'increased to 22.5 kW' was treated as 'increased by 22.5 kW' so that the question was answered for a power of 38.5 kW. The final answer was quite often seen as 0.083 (correct to two rather than three significant figures), and also seen as 0.833.

(ii) Whilst many candidates formed a correct equation $16000/v - (590 + 2v) = 0$, it was quite common to find slips e.g. $16000/v - 590 + 2v = 0$. Some candidates included an additional term for 'ma', using an acceleration from **Part (i)(b)** e.g. $16000/v - 590 + 2v = 1200 \cdot 1/12$ despite the constant speed. Most attempted to solve their quadratic equation but did not always reject the negative solution.

Answers: **(i)(a)** 400 N **(b)** 0.0833 ms^{-2} **(ii)** 25 ms^{-2}

Question 7

(i) In this question the answer $m = 5.94$ was given, so candidates were expected to show sufficient working for the solution of the equilibrium equation. The main difficulty was for those who applied Newton's Second law without realising that for equilibrium the acceleration is zero.

(ii) Although this was a challenging final question, most candidates attempted either a partial or a complete solution and there were many fully correct solutions seen. Candidates needed to recognise the two stages of motion and needed to consider all forces acting at each stage. The majority set up equations of motion for the two particles rather than work/energy equations. A few ignored the inclined plane and treated the system as a pulley with both particles hanging vertically. Some omitted either the weight component or the frictional force due to the rough slope when forming the equations of motion for particle A. For the second stage of motion the main error was to use an erroneous acceleration e.g. $a = g$ or $a = g\sin 30°$ or the same acceleration as before the string became slack. A few candidates calculated $0.5 + 2s_2$ instead of $0.5 + s_2$.

Answer: 0.710 m

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MATHEMATICS

Paper 9709/51
Paper 51

General comments

The paper was of a very similar standard to the one set last June.

Very few candidates now use g = 9.8 or 9.81.

Candidates should refer to the formula booklet provided if in doubt about a formula.

Some candidates are still losing marks for not giving answers to 3 significant figures and also due to premature approximating within their calculations leading to the final answer. Candidates should be reminded that if an answer is required to 3 significant figures then their working should be performed to at least 4 significant figures.

The easier questions proved to be 4**(ii)**, 4**(iii)**, 7**(i)** and 7**(ii)**

The harder questions proved to be 2**(ii)**, 3**(i)**, 3**(ii)** and 5**(ii)**

Comments on specific questions

Question 1

Too many candidates when using $v = u + at$ did not realise that the v value should have been negative as it is travelling downwards.

Answer: 1.27 s

Question 2

(i) By resolving horizontally and using $T = \lambda x/L$, the required value of λ could be found. Some candidates tried to use an energy method which was not relevant here.

(ii) Many candidates found this part of the question very difficult. A 3 term energy equation was required involving 2 EE terms and 1 PE term.

Answers: **(i)** 5 **(ii)** 0.8

Question 3

This question proved to be rather difficult.

(i) Most candidates knew that it was necessary to take moments about O. A number of candidates used an incorrect formula to find the centre of mass of the hemi-sphere. Sign errors often appeared in the candidates equation.

(ii) Candidates again knew to take moments about O. Very few correct answers were seen. The required equation should have been $24x + Wy = (24 + W)x\ 0.15$, where x was the distance of centre of mass from part **(i)**, y was the centre of mass of the hemi-sphere H and W the weight of H.

Answers: **(i)** 0.225 m **(ii)** 40 N

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Question 4

(i) Some candidates used the trajectory equation quoted in the formula booklet and so did not obey the instructions given in the question. It was necessary to use horizontal and vertical motion to set up the equations $x = 10t$ and $y = 15t - \frac{1}{2}gt^2$. $t = x/10$ can then be substituted into the y expression to find the required result.

(ii) This part of the question simply required $y = 0$ to be substituted into the equation found in part **(i)**.

(iii) This part of the question required $y = -14$ to be substituted into the equation found in part **(i)**. This resulted in a quadratic equation and when solved gave the required answer.

Answers: **(i)** $y = 1.5x - 0.05x^2$ **(ii)** distance < 30 **(iii)** 37.5 m

Question 5

(i) The incorrect formula was used by some candidates for the centre of mass of the lamina. The distance should have been $2 \times 0.7 \sin(\pi/2)/(3\pi/2)$ and not $2 \times 0.7 \sin(\pi/2)/(\pi/2)$. The reaction at P will be perpendicular to the plane. With this in mind the candidate needed to take moments about A.

(ii) Very few correct answers were seen. Here candidates needed to resolve horizontally and vertically for the forces acting at P and then to use $\tan\theta$ = vertical force/horizontal force, where θ is the required angle.

Answers: **(i)** 7.12 N **(ii)** 65.6°

Question 6

(i) This part of the question can be solved by using $T = \lambda x/L$ and Newton's Second Law horizontally with the acceleration equal to $r\omega^2$. This gave the correct value of ω, the angular speed. The final step was to resolve vertically to find θ.

(ii) Candidates now needed to find the required KE. Before they could do this, the velocity v had to be calculated by using $v = r\omega$. The EE can be calculated by using an extension of 0.1. Finally the difference can be found.

Answers: **(i)** 6.32 rad s^{-1}, 60° **(ii)** 0.4125 J

Question 7

(i) Candidates needed initially to work out the coefficient of friction, μ. This was $\mu = F/R = 0.6 \times 0.5^2/(0.5g) = 0.03$. Newton's Second Law can now be applied giving $0.5\,dv/dt = 0.6t^2 - 0.03 \times 0.5g$. This would then give the correct equation.

(ii) The candidate had to now integrate the equation from part **(i)**. By substituting $t = 0.5$ and $v = 0$, the value of the constant of integration was found. A number of candidates assumed this value to be zero.

(iii) Candidates now had to integrate again to find the displacement. By substituting $t = 0.5$ and $v = 0$ the constant of integration was calculated. Finally $t = 1.2$ was used to find the required displacement.

Answers: **(i)** $dv/dt = 1.2t^2 - 0.3$ **(ii)** $v = 0.4t^3 - 0.3t + 0.1$ **(iii)** 0.0926

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MATHEMATICS

Paper 9709/52
Paper 52

General comments

The paper was of a very similar standard to the one set last June.

Very few candidates now use g = 9.8 or 9.81.

Candidates should refer to the formula booklet provided if in doubt about a formula.

Some candidates are still losing marks for not giving answers to 3 significant figures and also due to premature approximating within their calculations leading to the final answer. Candidates should be reminded that if an answer is required to 3 significant figures then their working should be performed to at least 4 significant figures.

The easier questions proved to be 1**(i)**, 2**(i)** and 4**(i)**.
The harder questions proved to be 3**(i)**, 3**(ii)**, 6**(iii)** and 7**(iii)**.

Comments on specific questions

Question 1

(i) Many candidates scored the 1 mark available by using $v = r\omega$. A number of candidates made it more difficult by using $a = v^2/r = r\omega^2$.

(ii) This part of the question was misunderstood by a number of candidates who thought that the string was at an angle to the vertical instead of the horizontal. The tension could be found by using Newton's Second Law. Having found the tension and using $T = \lambda x/L$, λ the modulus of elasticity could be calculated.

Answers: **(i)** 0.8 m **(ii)** 12 N

Question 2

(i) This part of the question was generally well done. Some candidates when resolving vertically used sin 60 instead of cos 60.

(ii) Most candidates knew that they had to use Newton's Second Law horizontally. Again some trigonometric errors occurred.

Answers: **(i)** 0.1 kg **(ii)** 7.25 m s^{-1}

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Question 3

This question proved to be very difficult, particularly part **(ii)**.

(i) When taking moments about the base too many candidates omitted to include the weight of the base.

(ii) Very few candidates were able to find the distance of the centre of mass of the lid from the base. This should have been $(0.2 + 0.1\sin\theta)$, where θ is the required angle.

Answers: **(i)** 0.08 m **(ii)** 41.8°

Question 4

(i) Most candidates were able to set up the required equation using Newton's Second Law.

(ii) Candidates realised that they needed to integrate. Too many of them failed to recognise that the integral resulted in a logarithmic expression. Of those who acquired the correct integration, some failed to deal with the algebraic manipulation.

(iii) Quite a number of candidates just substituted $x = 4$ into their expression for part **(ii)** instead of $x = 4$ and $x = 8$. The difference was then needed.

Answers: **(i)** $v\mathrm{d}v/\mathrm{d}x = 10 - 0.5v^2$ **(ii)** $v = \sqrt{\left(20 - 20\mathrm{e}^{-x}\right)}$) **(iii)** $0.0404\,\mathrm{m\,s^{-1}}$

Question 5

(i) Quite a number of candidates found the required extension and then omitted to include an elastic potential energy term in their energy equation. Each year it is pleasing to see more candidates with a much clearer understanding of this topic.

(ii) This part of the question was well done by many of the candidates. A few candidates failed to include a potential energy term.

Answers: **(i)** $4.9(0)\,\mathrm{m\,s^{-1}}$ **(ii)** 2.18 m

Question 6

(i) This part of the question could be done by taking moments about A. If $\bar{x}$ was the distance of the centre of mass of the rod from A then $3 \times 0.6 = 8\cos 60\,\bar{x}$, so $\bar{x} = 0.45$.

(ii) A new diagram could be drawn here in order help with the solution. By taking moments about A, the value of P can be found.

(iii) By using $\mu = F/R$ in both parts, 2 values of μ, the coefficient of friction, can be found.

The larger value gave the correct result. This needed to be stated.

Answers: **(i)** 0.45 m **(ii)** 6 N **(iii)** 0.472

Question 7

(i) By comparing the general trajectory equation quoted on the formula sheet with that given in the question, the value of $\tan\theta$ could be found.

(ii) By substituting $x = y = a$ in the given equation and also by considering horizontal motion using $v = u + at$, 2 equations are found. These can be solved simultaneously to give the required values of V and a. There are a number of different solutions to this part of the question.

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(iii) The horizontal and vertical velocities at A need to be calculated. The candidate now needs to use $\tan\alpha$ = (vertical velocity)/(horizontal velocity). This gives α, the angle with the horizontal.

Answers: **(i)** $\tan\theta = 2$ **(ii)** $V = 44.7$, $a = 80$ **(iii)** 0° to the horizontal

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MATHEMATICS

Paper 9709/53
Paper 53

General comments

The paper was of a very similar standard to the one set last June.

Very few candidates now use g = 9.8 or 9.81.

Candidates should refer to the formula booklet provided if in doubt about a formula.

Some candidates are still losing marks for not giving answers to 3 significant figures and also due to premature approximating within their calculations leading to the final answer. Candidates should be reminded that if an answer is required to 3 significant figures then their working should be performed to at least 4 significant figures.

The easier questions proved to be 4**(ii)**, 4**(iii)**, 7**(i)** and 7**(ii)**

The harder questions proved to be 2**(ii)**, 3**(i)**, 3**(ii)** and 5**(ii)**

Comments on specific questions

Question 1

Too many candidates when using $v = u + at$ did not realise that the v value should have been negative as it is travelling downwards.

Answer: 1.27 s

Question 2

(i) By resolving horizontally and using $T = \lambda x/L$, the required value of λ could be found. Some candidates tried to use an energy method which was not relevant here.

(ii) Many candidates found this part of the question very difficult. A 3 term energy equation was required involving 2 EE terms and 1 PE term.

Answers: **(i)** 5 **(ii)** 0.8

Question 3

This question proved to be rather difficult.

(i) Most candidates knew that it was necessary to take moments about O. A number of candidates used an incorrect formula to find the centre of mass of the hemi-sphere. Sign errors often appeared in the candidates equation.

(ii) Candidates again knew to take moments about O. Very few correct answers were seen. The required equation should have been $24x + Wy = (24 + W)x\ 0.15$, where x was the distance of centre of mass from part **(i)**, y was the centre of mass of the hemi-sphere H and W the weight of H.

Answers: **(i)** 0.225 m **(ii)** 40 N

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Question 4

(i) Some candidates used the trajectory equation quoted in the formula booklet and so did not obey the instructions given in the question. It was necessary to use horizontal and vertical motion to set up the equations $x = 10t$ and $y = 15t - \frac{1}{2}gt^2$. $t = x/10$ can then be substituted into the y expression to find the required result.

(ii) This part of the question simply required $y = 0$ to be substituted into the equation found in part **(i)**.

(iii) This part of the question required $y = -14$ to be substituted into the equation found in part **(i)**. This resulted in a quadratic equation and when solved gave the required answer.

Answers: **(i)** $y = 1.5x - 0.05x^2$ **(ii)** distance < 30 **(iii)** 37.5 m

Question 5

(i) The incorrect formula was used by some candidates for the centre of mass of the lamina. The distance should have been $2 \times 0.7 \sin(\pi/2)/(3\pi/2)$ and not $2 \times 0.7 \sin(\pi/2)/(\pi/2)$. The reaction at P will be perpendicular to the plane. With this in mind the candidate needed to take moments about A.

(ii) Very few correct answers were seen. Here candidates needed to resolve horizontally and vertically for the forces acting at P and then to use $\tan\theta$ = vertical force/horizontal force, where θ is the required angle.

Answers: **(i)** 7.12 N **(ii)** 65.6°

Question 6

(i) This part of the question can be solved by using $T = \lambda x/L$ and Newton's Second Law horizontally with the acceleration equal to $r\omega^2$. This gave the correct value of ω, the angular speed. The final step was to resolve vertically to find θ.

(ii) Candidates now needed to find the required KE. Before they could do this, the velocity v had to be calculated by using $v = r\omega$. The EE can be calculated by using an extension of 0.1. Finally the difference can be found.

Answers: **(i)** 6.32 rad s^{-1}, 60° **(ii)** 0.4125 J

Question 7

(i) Candidates needed initially to work out the coefficient of friction, μ. This was $\mu = F/R = 0.6 \times 0.5^2/(0.5g) = 0.03$. Newton's Second Law can now be applied giving $0.5\,dv/dt = 0.6t^2 - 0.03 \times 0.5g$. This would then give the correct equation.

(ii) The candidate had to now integrate the equation from part **(i)**. By substituting $t = 0.5$ and $v = 0$, the value of the constant of integration was found. A number of candidates assumed this value to be zero.

(iii) Candidates now had to integrate again to find the displacement. By substituting $t = 0.5$ and $v = 0$ the constant of integration was calculated. Finally $t = 1.2$ was used to find the required displacement.

Answers: **(i)** $dv/dt = 1.2t^2 - 0.3$ **(ii)** $v = 0.4t^3 - 0.3t + 0.1$ **(iii)** 0.0926

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MATHEMATICS

Paper 9709/61
Paper 61

Key Messages

To do well in this paper candidates must work with 4 significant figures or more in order to achieve the accuracy required. Candidates are advised to ensure that workings are submitted within their solutions to communicate their thinking, as there are occasions when a correct answer value may not be sufficient to justify full credit. The workings also enable marks to be awarded where there are errors in the process. Candidates should label graphs and axes including units, and choose sensible scales.

General comments

It was pleasing to see that some of the candidates who took this paper had a good knowledge of the syllabus. However there were still many candidates from centres who did not appear to have prepared sufficiently for the exam. They did not appear to have had any practice with past papers and consequently a large number of candidates were unable to recognise the statistical terms used, let alone attempt the paper.

Comments on specific questions

Question 1

This question was one of the worst attempted on the paper. Many candidates did not appreciate that k is just the difference between the mean and the coded mean i.e. 50.5 – 315 / 30. The variance can be found directly using the mean of the coded squares – (the coded mean)^2 i.e. 4022 / 30 – (315 / 30)^2.The mistake most candidates made was to have one part of the formula in coded form and one part in uncoded form.

Answer: 40, 4.88

Question 2

This question was attempted by almost everyone apart from candidates at the centres who had not covered the syllabus. It involved finding three probabilities, namely P(R), P(T) and P(R∩T). Candidates then had to see whether multiplying P(R) by P(T) gave the answer of P(R∩T) .If the answer is Yes, then events R and T are independent. Some good candidates obtained full marks but many assumed P(R∩T) was equal to P(R) multiplied by P(T) and then stated since P(R∩T) equals P(R) multiplied by P(T) then the events R and T are independent. This circular argument only obtained 2 marks, for P(R) and P(T). It is important that candidates realise that they must say what P(R∩T) is first, before multiplying out and checking.

Answer: Yes, events R and T are independent.

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Question 3

(i) The question asked for a fully labelled tree diagram. One mark was given for the correct shape of 3 branches for match 1 and 3 by 3 branches for match 2. One omission of a line or a probability or a label was condoned for the initial method mark. The second mark was for all correct probabilities with the correct shape. Probabilities in the form of mixed decimals and fractions of the form 6.5/10 were not allowed.

(ii) This was an easy conditional probability question and well done by those who knew what they were doing. There are many examples of such exercises in text books.

Answer: **(ii)** 9 / 82

Question 4

(i) It was pleasing that almost half the candidates knew they had to find the frequency density before drawing the histogram. Not all of these managed it correctly but for those who did it was an easy method mark. The Accuracy mark was for plotting the heights correctly on the graph paper with their scale. It was most disappointing to see how many candidates used thick pencils and drew freehand wavy lines for the histogram. This meant that many candidates lost a mark unnecessarily. Another point of loss was that candidates failed to label axes. All they had to label was frequency density (fd was acceptable) and time in seconds. All in all, this question should have been an easy 4 marks but it was one of the more poorly done.

(ii) A nice question with many candidates scoring the full 2 marks. Candidates who didn't, either used the upper class or lower class boundaries instead of the mid points, or just added values of t and divided by 5.

Answer: **(ii)** 45.8

Question 5

(i) Many candidates recognised that this was a binomial-type question but had difficulty in finding p, the probability of a cracked egg. Credit was given for an attempt to find the probability of 2 cracked eggs using any value of p, even one they made up.

(ii) Candidates who attempted to find 1 – P(0) cracked eggs or 1 – P(0, 1) cracked eggs were awarded a method mark for knowing what to do, but only if they had a probability p in the first place. Many candidates attempted to use a normal distribution but this gained no marks.

(iii) Many candidates used their value of p^n instead of their value for **(ii)**^{n}. Those who did use the correct probability appeared to be able to solve the resulting inequality successfully, either by logs or by trial and error.

Answer: **(i)** 0.252 **(ii)** 0.766 **(iii)** 18

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Question 6

(a) (i) This is a normal distribution question. Candidates are awarded marks for standardising and choosing the correct probability (Φ or $1 - \Phi$). Then a mark for the correct answer. Candidates who just type the numbers into a calculator and write the answer down will only get 1 mark out of 3. It is important that teachers realise this so that candidates write down the standardising equations. See the mark schemes to find out how this works.

(ii) Another situation where just writing the answer down will not get full marks. This question was generally well done by those who knew what to do. Candidates who rounded the standard deviation to 3 significant figures had an answer which was not correct to 3 significant figures. It is important that candidates work with 4 or more significant figures so they do not lose the final accuracy mark.

(b) It was hoped that candidates would use symmetry to enable them to find the appropriate probabilities but many didn't so gave themselves some extra work. A diagram would have helped candidates to see that the centre part under the normal curve was required, and this was clearly greater than 0.5.

Answer: **(a)(i)** 5.82 **(ii)** 0.221 **(b)** 788 or 789

Question 7

(a) This question proved to be one of the best ones attempted by the candidates. Candidates chose either to insert the youngest children between the remaining 5 children, one at a time, or to find the total number of ways of arranging all 8 children and then subtracting ways with 3 youngest together, together with 2 youngest together.

(b) Many questions have been asked on this theme and it was pleasing that those candidates who had been well taught gained full marks. In fact, this question probably had the best mark on the whole paper.

(c) This question was also well done. The method marks were given even for candidates who used replacement, despite being told that no character can be repeated. Not everybody realised that each option should be multiplied by 4! because the characters had to be in a particular order. Even so, many people gained 2 marks on this part question.

Answer: **(a)** 14400 **(b)** 194 **(c)** 519480

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MATHEMATICS

Paper 9709/62
Paper 62

Key Messages

Candidates are reminded that to achieve non-exact answers correct to 3 significant figures, all calculations should be carried out using at least 4 significant figures.

Candidates are advised to ensure that workings are submitted within their solutions to communicate their thinking, as there are occasions when a correct answer value may not be sufficient to justify full credit. The workings also enable marks to be awarded where there are errors in the process.

The move to a structured paper does not appear to have had a detrimental impact on solutions. Occasionally candidates, when answering question **5(ii)**, did not use the additional space provided at the top of the facing page, which did result in some solutions being difficult to understand because candidates tried to fit answers into the smaller area.

General Comments

Many candidates seemed well prepared for the examination, and there were many praiseworthy solutions seen to each of the syllabus areas.

Candidates who used diagrams to interpret the information stated in the questions were often more successful, especially when considering the normal distribution. Many candidates were able to gain credit in question 6 because their diagram indicated the approach that they were seeking to achieve, even if there was confusion over the details of the data provided.

Candidates who had prepared well appeared to have sufficient time to attempt all questions. However, a significant number of candidates failed to follow the instructions stated within questions, or to appreciate that the skills developed in Pure Mathematics 1 may be used in this component.

Comments on Specific Questions

Question 1

This question was focusing on interpreting the mean and standard deviation. There were a surprising number of candidates who did not use the data provided fully.

(i) This was attempted well by almost all candidates. Good solutions created an algebraic equation which was then solved to identify the cost of a single jumper. A surprising number of candidates stated the total cost of 3 jumpers as their final answer. Weaker solutions used a logical approach to calculate separate costs and to reach the price of the single jumper without any algebraic statements.

(ii) There were many attempts at this question which gained little credit as the candidate did not explain their reasoning as to how they knew that Diksha paid $104 for the 4 shirts. Good solutions clearly stated that because the standard deviation is $0, then all the items must cost the same amount. Weaker solutions assumed all the items had the same cost, and then provided calculations to justify this assumption.

Answer: **(i)** $40

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Question 2

(i) The majority of candidates correctly identified the median number of caterpillars and read the value accurately from the graph. There was more confusion for the interquartile range. Weaker solutions often found the difference of the upper and lower quartile number of caterpillars and so having the same value as the median. Many solutions also showed a lack of accuracy in using the graph, either misinterpreting the horizontal scale or not drawing their reading lines horizontally or vertically with care.

(ii) Good solutions stated the separate values for the boundary lengths and then found the difference, whereas weaker solutions simply stated the calculation or final answer. There was inaccuracy in reading the vertical scale in some solutions. A number of candidates failed to answer the actual question and provided the value for 3.5 cm

(iii) Good solutions calculated 94% of the caterpillar population before reading from the graph. However, the majority of solutions failed to answer the question and calculated the length which 6% of the caterpillars were smaller than. A few candidates then assumed that the cumulative frequency graph was symmetrical and subtracted their value from 5 cm.

Answers: **(i)** median=3.2, IQR = 1.1 **(ii)** 110 **(iii)** 4.5

Question 3

A significant number of candidates answered this question well

(i) Candidates were required to show that the 2 probabilities were equal, and therefore a full algebraic approach and simplified final expression was needed for each term. Weaker solutions often replaced the k with a consistent value, or simply did not include it in the expression, which presumably was meant to indicate that the constant had been cancelled.

(ii) It was encouraging that most solutions included an appropriate probability distribution table, even if the values for the probabilities were inaccurate. Weaker students often appear to have calculated the value of k initially and then placed their numerical value in the table. It was unexpected that a number of negative probabilities were calculated, either as the stated value for k or when substituted back into the probability distribution table.

(iii) Many good solutions were seen. The majority of candidates substituted their value of k obtained in part **(ii)** accurately. Good candidates often worked in their probability distribution table in part **(ii)**. These candidates would be well advised to indicate to the examiner that there working is elsewhere in the answer space of this part.

Answers: **(ii)** 1 / 25 **(ii)** 63 / 25

Question 4

Many candidates found the unstructured nature of this question challenging. Candidates who presented their work in a structured manner were most successful in conveying their approach to the question, and evidence was seen that corrections were made where errors had been identified within the solution.

The best solutions often included a tree diagram. This clearly identified the required outcomes, and presented the information in a more familiar style. A clear algebraic approach was then used throughout, often with additional description to explain the stages being followed.

However, many solutions used an outcome space approach. The linking of P(6) with the single outcome was frequently seen, although there was less consistency finding the product of the probability of (3,3) as many solutions found the sum of the terms. Solutions then tended to follow the same technique for the remaining outcomes. Some ‘product’ solutions ignored that there were 2 possible ways of scoring 5 which led to inaccuracy. It was good to see that almost all candidates realised that the sum of the three probabilities was 1 and used this approach to calculate the final value.

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The number of solutions with a probability of q greater than 1, which then calculated a negative probability of p was unexpected at this level.

Answers: $p = ½$, $q = ⅓$, $r = ⅙$

Question 5

Although this was a fairly standard normal distribution question, many solutions became inaccurate because of premature approximation where calculated values are used to only 3 significant figures in further parts of the question.

(i) Good solutions often included a sketch of the normal distribution curve to assist in determining the appropriate z-value to obtain. Most solutions substituted the values correctly into the standardisation formula. Weaker solutions often failed to solve accurately the equation that was stated. A number of 2 significant figure answers were noted, despite the instructions on the front of the question paper.

(ii) Good solutions recognised the symmetry within the normal distribution to minimise work needed, and a sketch of the normal distribution again helped the understanding of the question requirements. Most solutions used the standardisation formula twice, and then attempted to find the required probability area. A common error was to use convert 'half a minute' to '0.3 minutes', which was not acceptable within the standardisation formula. A sizable proportion of solutions only calculated the probability using 4.4 minutes. This was often the final expression at the bottom of the page and it was unclear if candidates had not identified that additional working space was available at the top of the next page.

(iii) Many candidates failed to follow the instruction 'without further calculation' in this question. Good solutions explained how increasing the standard deviation would increase the spread of the lengths of the video and therefore decrease the probability of being within the same time range. The alternative approach was to indicate that increasing the standard deviation would decrease the z-value as this was the determinant, which would lead to a reduced probability, but this was often less successful as calculations were often present.

Answers: **(i)** 0.328 **(ii)** 0.873 **(iii)** less than in **(ii)**

Question 6

Many solutions displayed a lack of understanding of the context of this question. Evidence was frequently seen that candidates assumed that 'identical books' were clearly individually identifiable, with additional arrangements calculated, while at the same time removing the repeats for other groups of books.
The best solutions included diagrams to illustrate the set conditions.

(i) The majority of solutions recognised that there were 2 different options to consider (A … A and B … B), and that these were to be summed. Most solutions identified that there were only 9 books which could be rearranged, but there was less consistency in removing the repeated outcomes. When not fully correct, the errors tended to occur in the A … A term as the 2 remaining As were not considered within the middle books for repeats. Weaker solutions considered that there were 11 books on the shelf to arrange and then unsuccessfully attempted to subtract the large number arrangements that were not required.

(ii) Solutions which used the 'intersperse' approach were generally more successful. Good solutions often included a narrative for the process, explaining each stage of the method. Common errors in this approach were to introduce · 4! for the different ways of ordering the identical As or failing to divide by 2! to remove the repeated Bs. Solutions which attempted the 'subtraction' approach often became very confused. Good solutions considered the number of ways that could be made if just the As were together and then when both the Bs and the As were in separate blocks. Again the narrative often ensured clarity of thinking and the difference was then found. Common errors were failing to remove the repeated outcomes or summing the values found.

Answers: **(i)** 882 **(ii)** 126

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Question 7

Candidates should be reminded that probability can be stated as a fraction, and where stated this is an exact answer which does not require any rounding.

(i) All but the weakest candidates successfully answered this question. Good solutions frequently included a tree diagram and clear identification of the required outcomes. A few candidates used 0.5 rather than 0.05 for the probability that Khalid hurt himself while riding his bicycle. As the answer was exact, there was no requirement to state a 3 significant figure final answer.

(ii) Many good solutions were seen for this question. Most candidates identified that the value calculated in part **(i)** was required for the denominator. It was unfortunate that a large number of candidates seemed to assume that a 2 significant figure answer was appropriate. However, as the decimal answer was non-exact, 3 significant figures is the minimum requirement. A few candidates appeared to have ignored their work in part **(i)**, although in mathematics it is often expected that question parts support each other.

(iii) As the question required the variances to be shown to be the same, candidates must remember that their solutions need to have sufficient detail. Although some good solutions were seen, many solutions failed to state which variance was being calculated which could not lead to full credit. Weaker solutions found the variance of one activity and stated that they were identical. Candidates should be reminded that the final conclusion needs to be clearly stated.

(iv) Successful solutions recognised that this was a binomial distribution, as the 2 activities were mutually exclusive. The best solutions identified that 'at least 2 days' was the same as 'not 0 or 1 day', calculated each probability separately before subtracting from 1. Weaker solutions either just calculated the probability for 2 days or subtracted 2 days with the other probabilities. A number of solutions were seen where candidates appeared to have exchanged the probabilities.

Answers: **(i)** 0.33 **(ii)** 0.909 **(iv)** 0.954

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MATHEMATICS

Paper 9709/63
Paper 63

Key messages

The move to a structured paper does not appear to have had a detrimental impact on solutions, although candidates need to be aware that reaching the bottom of the page does not necessarily mean that they have completed their solution. This was a particular issue with **Question 6(b)(ii)** where a number of candidates used a long method to find the total number of ways with fewer than 3 chocolate biscuits and did not proceed to find the required probability. Candidates are recommended to request extra paper if there is not enough room available for a particular question rather than continuing their solution in the previous answer space. Candidates are expected to use the grid printed within the question rather than a separate sheet of graph paper.

Candidates are advised to ensure that workings are submitted within their solutions to communicate their thinking, as there are occasions when a correct answer value may not be sufficient to justify full credit. The workings also enable marks to be awarded where there are errors in the process.

General comments

This paper proved to be accessible to most students and there was a pleasing improvement in the labelling of graphs. **Question 6** on permutations and combinations proved to be the most challenging and, in **Question 6(b)**, many candidates over-complicated their responses by considering all three types of biscuit separately each time. However, the question that was least well answered was **Question 5(i)** which asked for the required conditions for X to have a binomial distribution. Only the strongest candidates appreciated what was being asked and only a few managed to score both available marks.

Some candidates did not take notice of the instruction to give answers correct to three significant figures and lost marks through giving answers to one or two significant figures or premature approximation. To achieve non-exact answers correct to 3 significant figures, all calculations should be carried out using at least 4 significant figures

Comments on specific questions

Question 1

Most candidates found this question accessible and many produced good solutions, clearly identifying that there were four ways to sum to 9, correctly obtaining the probability of the die landing on 6 and knowing to multiply two probabilities together to find any one of the four possible outcomes. Most of those who lost marks either incorrectly found the value of P(6) to be 0.7 or only identified two ways of summing to 9. Only a few ignored the fact that the die was biased.

Answer: 0.1

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Question 2

This question was well answered by most candidates. Only a few ignored the instruction to use an approximation. Most candidates appeared to be familiar with this type of question and standardised correctly. Although a pleasing number remembered the continuity correction, many of those used 100.5 instead of 99.5. Another less common error was to give the wrong area and produce an answer greater than 0.5. Those who approximated prematurely to give a *z*-value of 1.23 lost the last mark as they did not obtain the correct final answer.

Answer: 0.110

Question 3

(i) Most Candidates recognised the need to multiply two pairs of decimals and obtained full marks for this question. Almost all of those who drew a tree diagram produced the correct answer.

(ii) Those candidates who recognised the need for a Conditional Probability usually obtained the correct answer. However, many did not realise that the denominator would be the answer to **Question 3(i)** subtracted from 1 and recalculated by summing two 2–factor probabilities. The most common error was to find the probability of a large Café Standard and not to divide by the probability of the jar being large.

Answers: **(i)** 0.653 **(ii)** 0.252

Question 4

(a) This question was well answered by most candidates. Those who knew to standardise with 0 and did not use *X* generally obtained the correct *z*-value. Only a few squared or took the square root of sigma. However, a significant number subtracted 0.933 from 1 and lost the final mark.

(b) Most candidates used the tables correctly to find a z-value rather than a probability and went on to equate their z-value to a correct standardisation and obtain the correct final answer.

Answers: **(a)** 0.933 **(b)** 43.4 or 43.5

Question 5

(i) This question seemed to catch the candidates by surprise and very few produced both conditions. Many attempted to give the conditions for a normal approximation. A significant number of those who were familiar with the conditions for a Binomial distribution concentrated on the facts that there should be a fixed number of trials and only two possible outcomes. They did not appreciate that those conditions were already satisfied by the context. A pleasing number wrote that the probability of success had to be constant but it was clear that many candidates did not understand the meaning of the word independent. The successful candidates wrote about independent trials or events and did not refer to there being just two ‘events’ i.e. success and failure.

(ii) A few candidates ignored the instruction to use a binomial distribution and a significant number who followed the instruction forgot that Hebe attempts one crossword puzzle a day and used n=10 instead of n=7. Most of those who used a binomial distribution with n=7 and p=0.7 obtained the correct answer.

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(iii) A surprising number did not realise that they could subtract their answer to **Question 5(i)** from 1 to find the probability of Hebe completing 4 or fewer puzzles in a week. Those who went back to scratch often omitted 'none' from their calculation and others tried to use a normal distribution. Most correctly applied the binomial distribution with n=10 to their probability of 4 or fewer puzzles but a significant number multiplied by 3.

Answers: **(i)** 1 Constant probability (of completing) 2 Independent trials/events **(ii)** 0.647 **(iii)** 0.251

Question 6

(a) (i) Many candidates obtained the correct answer although a significant number did not understand that numbers 'between 3000 and 5000' can only start with 3 and 4 and not 5. Those who tried to list the possible numbers were rarely successful.

(ii) The strongest candidates identified that there are two possible choices for the first digit, 5· 5 ways of choosing the middle two digits and three choices for the last digit and they correctly multiplied 25 by 6. A number of candidates forgot about repeats in the middle two digits and thought there were 5P2 choices for the middle two digits. Some separated the number of ways of choosing the middle two digits with and without repeats (5P2+5) and then multiplied by 6. As in the previous part, many candidates incorrectly considered numbers starting with 5 as well as 3 and 4 and there were some who confused odd with even numbers.

(b) (i) The majority of candidates realised that there were 18 non-oatmeal biscuits of which 4 were to be chosen and correctly evaluated 18C4. Others thought that they needed to consider chocolate and ginger biscuits separately and most of them successfully added five two-factor products with n=14 and 4 although some made slips along the way.

(ii) This proved to be the most challenging question on the paper. A significant number assumed constant probability and tried to answer the question using a binomial distribution.

There were many ways to tackle the question correctly, some of which were much longer and more complicated than necessary. The stronger candidates realised that they only needed to consider chocolate and non-chocolate biscuits. If they chose to add the number of ways with 0, 1 or 2 chocolate biscuits, only the best multiplied 16C5 by 4C1 and 16C4 by 4C2. Similarly, if they chose to subtract the number of ways with 3 or 4 chocolate biscuits from 20C6, many did not multiply 16C3 by 4C3. Of those who chose the harder route by separating out the ginger and oatmeal biscuits, only the strongest correctly found the nine different ways with 0, 1 or 2 chocolate biscuits or the six different ways with 3 or 4 chocolate biscuits. Many took up the whole page to reach their total of 36400 and then forgot to find the required probability and divide by 20C6.

There was another group who chose the probability method and, corresponding with previous methods, many omitted to multiply the probability of 1 chocolate by 6C1 or the probability of 2 chocolates by 6C2 or, if they used the method subtracting from 1, they omitted to multiply the probability of 3 chocolates by 6C3 and the probability of 4 chocolates by 6C4.

Answers: **(a)(i)** 48 **(ii)** 150 **(b)(i)** 3060 **(ii)** 0.939

Question 7

(i) Most candidates answered this question well. A few considered the class widths to be 6, 6, 11, 6 rather than 5, 5, 10, 5. Others knew to multiply the frequency densities by class widths but summed the frequencies instead of listing them. A small minority divided the frequency densities by the class widths which produced some very small numbers to work on for the rest of the question.

(ii) Most candidates produced well labelled cumulative frequency graphs with sensible scales and correct heights which were carefully plotted. A number of candidates plotted at the mid-points and a significant number plotted 0.5 away from the correct upper end points of 5, 10, 20 and 25. A small minority did not understand what a cumulative frequency graph would look like.

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(iii) The candidates who drew lines on their graph to find the quartiles and median generally produced accurate results. A number of candidates did not notice that the lower quartile of 10 was exact and a surprising number did not use their quartiles to find the requested interquartile range.

(iv) Although a few candidates used class widths or upper points instead of mid-points, most candidates produced accurate answers.

Answers: **(i)** 10, 40, 120, 30 **(iii)** median = 14.2; IQ range = 8.5 **(iv)** 14

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MATHEMATICS

Paper 9709/71
Paper 71

General comments

In general, candidates scored well on **Questions 2**, **3(a)**, **4(ii)** and **(iii)**, whilst **Questions 3(b)** and **6** proved more demanding. There was a complete range of scripts from good ones to poor ones. There were some candidates who seemed unprepared for the demands of this paper.

Most candidates kept to the required level of accuracy, though, as is often the case, there were situations where candidates lost marks for giving final answers to less than three significant figure accuracy.

Timing did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

Comments on specific questions

Question 1

The approximating distribution required here was a Poisson distribution. There were some candidates who correctly used this distribution with λ=0.2, however, there were also candidates who used a binomial distribution, or who did not know how to approach the question. Candidates who used the required Poisson distribution often reached the correct answer and some were able to justify their approximating distribution. Others misinterpreted 'more than 2' and omitted or included extra terms in their calculation.

Answers: **(i)** 0.00115 **(ii)** n large, $np<5$

Question 2

This was a reasonably well answered question. Many candidates set up correct null and alternative hypotheses (though some candidates were not precise enough, incorrectly writing, for example, H_0=64.0). Some candidates found a correct z value (errors included omission of $\sqrt{100}$ in the calculation) and it was important that a comparison of their z value with 1.96 (or equivalent area comparison) was then clearly seen. The conclusion should then be written as a non-definite statement (as below).
Candidates rarely stated a necessary assumption for the test to be carried out.

Answers: Assume standard deviation unchanged, No evidence that heights are shorter

Question 3

Whilst many candidates knew how to calculate a confidence interval, there were some common mistakes made. An incorrect z value was often seen, and the variance (which should have been 2.6, was often taken as 2.6^2, due to misinterpretation of the information given in the question). The final answer should be given as an interval, not as two separate values.
Very few candidates correctly answered **part (b)**, whilst in **part (c)** many candidates gave acceptable answers.

Answers: **(a)** 6.77 to 7.43 **(b)** 0.000064 **(c)** e.g. Particular day or time of day was used

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Question 4

This question was reasonably well attempted, particularly **parts (ii)** and **(iii)**.
In **part (i)** not all candidates fully appreciated the properties of a probability density function, then relating this to the context of the question was also not always found to be straightforward.
Showing that k was 10 in **part (ii)** was done well by a large number of candidates, though as this was a 'show that' question it was important that full working was shown; there were occasions where the final mark was withheld for lack of essential working.
Finding E(X) in **part (iii)** was also well attempted, with many candidates correctly integrating $xf(x)$.
An incomplete formula for Var(X) caused a loss of marks for some candidates in **(iv)**.

Answers: **(i)** Greater area where $x < 7.5$ than $x > 7.5$ **(iii)** 6.93 **(iv)** 1.95

Question 5

Well prepared candidates made a reasonable attempt here, though a variety of errors were made which mainly involved finding the variance in both parts of the question. In **part (i)** there were candidates who did not include the weight of the carton, and also those who confused standard deviation and variance. In **part (ii)** the most common error was to calculate the variance of C−2B as '171.88' + 2 × 12.1 rather than '171.88' + 2^2 × 12.1 (or equivalent) and some candidates subtracted the terms rather than added.

Answers: **(i)** 0.223 **(ii)** 0.965

Question 6

In general, this question was not well attempted. In order to find the critical region (CR) for the test in **part (i)**, it was necessary to calculate both P($X \leqslant 1$) and P($X \leqslant 2$) in order to fully justify that the CR was $X \leqslant 1$. Many candidates merely calculated individual probabilities, showing no summation, whilst others just calculated P($X \leqslant 1$) hence not fully justifying their answer. In **part (ii)** a text book definition of a Type I error was not acceptable; a statement in the context of the question was required. In **part (iii)**, having found the CR in **part (i)**, the expected method was to use this CR stating that 2 was not in the CR and therefore conclude that there was no evidence that the mean number of injuries had decreased. Many candidates did not realise what was required here and did not use their answer to **(i)** often doing further (sometimes incorrect) calculations to carry out the test. Although credit was available for correct further calculations, this was not often gained. The conclusion to the test should be written as a non-definite statement (as below). **Part (iv)** was the best attempted part of the question, the main error noted was a lack of, or an incorrect, continuity correction.

Answers: **(i)** $X \leqslant 1 = 0.0103$ **(ii)** Wrongly conclude that the mean number of sports injuries has decreased **(iii)** No evidence that the mean number of sports injuries has decreased **(iv)** 0.0543

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MATHEMATICS

Paper 9709/72
Paper 72

General comments

In general, candidates scored well on **Questions 3(i), 6** and **7** whilst **Questions 2** and **5** proved more demanding. Candidates were largely able to demonstrate and apply their knowledge in the situations presented, though there were occasions when candidates did not read the question carefully and whilst they were able to apply their knowledge they did not answer the question as set (see comments below). There was a complete range of scripts from good ones to poor ones.

Most candidates kept to the required level of accuracy, though, as is often the case, there were situations where candidates lost marks for giving final answers to less than three significant figure accuracy. Where the question specifically requests a particular level of accuracy (as in **Question 2(i)**) it is important that this request is adhered to. Many candidates lost marks here even though they knew how to answer the question.

Timing did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

Comments on specific questions

Question 1

Whilst many candidates were able to correctly calculate the required 95% confidence interval, there were other candidates who did not seem to realise that the question required a confidence interval for a proportion, and were incorrectly constructing their interval around 1602. A correct z value of 1.96 was usually used, though errors were made. It was important that the final answer was given as an interval.

Answers: 0.784 to 0.818

Question 2

It is important that candidates read the question carefully; marks were lost here by not adhering to the instructions given regarding the accuracy of the answer, and by not relating their answer for **part (ii)** to their answers in **part (i)** as requested. In **part (i)** many candidates did the correct calculation, but used the usual three significant figure accuracy rule rather than the 3dp requested in the question; unless previous figures were seen this caused a loss of marks. **Part (ii)** was also not well attempted, as a large number of candidates did not relate their answer to **part (i)** as requested in the question. The answer required was that E(X) was approximately the same as Var(X), as found in **(i)**. Answers quoting general requirements for a Poisson distribution or approximation were often seen which was not what the question required. **Part (iii)** was reasonably well attempted.

Answers: **(i)** E(X)= 4.197, Var(X)=4.196 **(ii)** E(X)≈ Var(X) **(iii)** 0.590

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Question 3

Part (i) required the calculation of unbiased estimates of the mean and variance. This was generally well attempted, with relatively few candidates confusing the two alternative formulae for the unbiased estimate for the variance. In **part (ii)** many candidates were able to follow the steps required for a significance test, errors seen were either not showing a comparison clearly, or making an invalid comparison. The final conclusion on a significance test should be a non-definite statement (as below).

Answers: **(i)** 2.31, 2.61 **(ii)** No evidence that incomes in the region are greater

Question 4

Calculating the probability of a Type I error was required in **part (i)**. Questions on Type I and Type II errors are often not well attempted, but this was not the case here with many candidates scoring well. The significance test in **part (ii)**, however, was not quite so well attempted; some candidates used a normal distribution (despite the question stating that a binomial distribution should be used) and others calculated P(1) rather than the tail probability P(0,1). Again, the final conclusion on a significance test should be a non-definite statement (as below), and should not be ambiguous in any way.

Answers: **(i)** 0.0913 **(ii)** There is evidence that the proportion is less than 1 in 4

Question 5

This question, which tested an understanding of probability density functions, proved to be challenging for many candidates. There were various approaches that could be used to answer the questions in **(i)**; some candidates correctly deduced that f(x) must be of the form kx, whilst others used the area of a triangle to find its height and hence the gradient and some used the idea of similar triangles. There were many candidates who tried the question repeatedly from different start points, often making little progress or making false assumptions; it is important that solutions given can be followed so that the method used is clear. There were some candidates who gave answers to the probability (X<2) as a value greater than 1; candidates must look at how reasonable their answer is.
Candidates generally knew how to calculate the median in **part (ii)** using the f(x) they found in **(i)**.

Answers: **(i)** P(X<2) =1, a=2, f(x)= 0.5x **(ii)** $\sqrt{2}$

Question 6

Part (i) was particularly well attempted with a large number of candidates scoring full marks. Equally, in **part (ii)** there were many fully correct solutions, the errors seen included a lack of or an incorrect continuity correction. **Part (iii)** was quite well attempted, errors included an incorrect value for λ; the value required was 3.6 + 12 ÷ 7, many candidates used a value of just 3.6 or alternatively just 12 ÷ 7.

Answers: **(i)** 0.261 **(ii)** 0.761 **(iii)** 0.224

Question 7

This question was also well attempted, though mistakes were made when calculating the variance on both parts of the question. Standardising, with their values of E(X) and Var(X) was usually correctly performed but errors were made, on both **part (a)** and **(b)**, when finding the final probability from the z value obtained.

Answers: **(a)** 0.152 **(b)** 0.0246

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MATHEMATICS

Paper 9709/73
Paper 73

Key messages

Candidates should consider how plausible an answer is. Probabilities must lie between 0 and 1, so answers outside this range must be incorrect.

When interpreting Type I and Type II errors in context, candidates should consider both hypotheses rather than giving general answers.

Candidates should understand that when they are asked to 'write down' an answer, little or no calculation is required.

General comments

Many fully correct solutions were given in all questions, with only **Question 7(iii)** being answered incorrectly by many candidates. **Questions 3**, **5 and 6** were consistently answered well, with clear solutions covering all the key aspects required. Many very high marks were seen.

Comments on specific questions

Question 1

The majority of candidates were able to correctly identify the method being used to generate members of the sample using the list of random numbers given. There were two common errors, the inclusion of 109, when this member was already in the sample, or the inclusion of 765 when there were only 654 members to choose from. Examiners awarded marks to alternative answers, provided the methodology could be understood. A number of candidates were unable to answer the question, or were unable to use the list of numbers in a way which could be followed. Some candidates reused numbers already used for the first two numbers.

Answer: 573, (0)43 289

Question 2

(i) The correct confidence interval for p was found by most candidates. Some candidates used an incorrect value for z (most often 1.406) while some candidates made errors finding the variance, often using $\sqrt{200}$ twice in this calculation. Candidates almost always expressed their final answers as an interval.

(ii) Most candidates correctly identified the probability that a 92% interval would not include p as 8%. Those who gave alternative answers, often wrote 0.92 or 0, as well as alternatives using the binomial distribution.

Answers: **(i)** 0.453 to 0.577 **(ii)** 8%

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Question 3

The requirement to work with the sum of 10 days production of iron ore was correctly interpreted by most candidates as a normal variable with parameters 10×7 and 10×0.46^2. Many of these candidates then correctly used the normal distribution to calculate the probability that the total production in 10 days would exceed 71 tonnes. There were two errors seen frequently in candidates work. The first was to treat the sum as the product of 10 individual days leading to an incorrect variance of $10^2 \times 0.46^2$. The second was to apply division by 10 to the variance, as if the calculation comparing 71 and 70 was comparing two means.
A small minority of candidates applied a continuity correction, which is inappropriate in this situation.

Answer: 0.246

Question 4

(i) Nearly all candidates were able to correctly calculate unbiased estimates for the population mean and variance of the measure of pollutant. Very few candidates calculated the biased variance, while some other candidates made errors in substitution in the variance formula, the most frequent error was using the figure for the mean as Σx.

(ii) The majority of candidates performed the significance test correctly, although the hypotheses were not always stated. Those who did state the hypotheses, correctly stated H_1 as indicating a drop in the level of pollutant. Most candidates carried out the test correctly, with all valid methods being accepted. The most frequent was comparing the test value of z, with the figure of –1.645, assuming the test statistic was negative. Almost all candidates then correctly stated that there was insufficient evidence of a drop in pollution.

Answers: **(i)** $\bar{x} = 0.0335$, $s^2 = 0.0000339$ **(ii)** Accept H_0, no evidence of a decline in pollutant.

Question 5

(i) (a) The approximating distribution was correctly identified as the normal by the majority of candidates, with nearly all of these candidates then identifying the correct parameters. A significant number of candidates did not apply the required continuity correction, or applied an incorrect one, and therefore lost at least one mark.

(i) (b) The majority of candidates who applied the correct approximation were then able to justify it, most often by stating that the mean was large. A minority of candidates stated that n was large, although there was no n in the question.

(ii) (a) Examiners were expecting that candidates would apply the Poisson distribution as $np < 5$, and $p < 0.1$. Those candidates who identified the distribution as Poisson nearly always correctly found $P(X > 2)$ as $1 - P(X \leqslant 2)$. *Due to an issue with this question, careful consideration was given to its treatment in marking in order to ensure that no candidates were disadvantaged.*

(ii) (b) Examiners expected candidates to identify that np was less than 5 or that n was large and p small as a justification for the application of the Poisson distribution. *The comment in part **(ii)(a)** also applied.*

Answers: **(i)(a)** 0.65(0) **(b)** $\lambda > 15$ **(ii)(a)** 0.121 **(b)** $np < 5$ or large n, small p

Question 6

(i) Almost all candidates were able to provide the working required to show that $k = 6$. A small number of candidates did not recognise that integration was needed together with the result being equated to 1.

(ii) The majority of candidates obtained the correct values for $E[X]$ and $Var[X]$. Many candidates spent unnecessary time finding $E[X]$, rather than using the symmetry of the function, in order to deduce that $E[X]$ was 0.5. A minority of candidates did not calculate $Var[X]$, but calculated only the value of $E[X^2]$.

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(iii) The majority of candidates did calculate the required probability in this part, but the limits of the integration were often incorrectly stated as 0.4 to 2, leading to a negative answer. These candidates did not take sufficient notice that the function was valid only in the range 0 to 1. A smaller number of candidates applied the inappropriate use of the normal distribution, using the values obtained in part **(ii)**.

Answers: **(i)** $k = 6$ **(ii)** $E[X] = 0.5$, $Var[X] = 0.05$ **(iii)** 0.648

Question 7

(i) This question addressed the topic of significance testing in the context of a discrete distribution. This question was answered well by many candidates, who recognised the requirement to calculate the test statistic as $P(X \leqslant 2 \mid \lambda = 5.64)$. The most common error was to calculate only $P(X = 2)$ meaning that the test was invalid. Some candidates attempted to use the normal distribution, which was an inappropriate method. The majority of candidates who calculated the correct statistic then performed the test correctly by comparing the test statistic with 0.05, correctly concluded that there was insufficient evidence of a drop in the number of accidents. Nearly all candidates remembered to state hypotheses, and most of these used a correct parameter, examiners accepting both μ and λ.

(ii) The correct interpretation of a Type II error here was that the number of accidents would be concluded to be unchanged, when actually it had reduced, was stated by many candidates. Some stated that the number of accidents had changed rather than reduced, while some candidates did not put their answers in context. Some candidates defined a Type I error.

(iii) This part of the question was the most challenging on the paper. Candidates needed to identify the critical region of the test in part **(i)**, and having identified this to then calculate, with the new mean, the probability that the number of accidents in this new situation would not fall within the critical region. A small number of candidates recognised the need to find the critical region. Many candidates solutions were incorrect, with the most common being to calculate $P(X > 2)$ as this was the alternative region to part **(i)**. Not all candidates used the new value of the mean

Answers: **(i)** Accept H_0 no evidence that the number of accidents had fallen
(ii) Conclude that the number of accidents was unchanged, when it had decreased **(iii)** 0.122

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